Urban Transportation Systems Planning
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Lecture - 45
Approaches to Traffic Assignment

Welcome to Module F, lecture 5. Now we shall really start talking about the approaches to traffic assignment and different techniques that are available.

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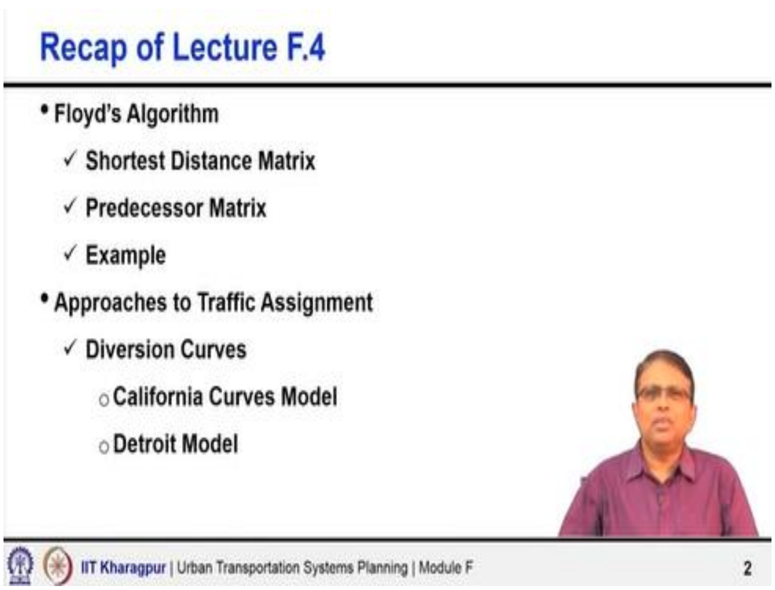

In the previous lecture we discussed about the Floyd's algorithm, how to find out the shortest path using Floyd's algorithm. The shortest distance matrix, the predecessor matrix and also took an example to tell you how step wise you can proceed to get the shortest path from all home node to all home node in n iteration assuming that there are n nodes in the network. That all steps we explained.

We also discussed with you about another approach to traffic assignment particularly the diversion curves, what diversion curves try to do, what are the factors they consider to get the proportion of traffic that are likely to be diverted to a new or proposed road. And two specific models namely the California Curves Model and the Detroit Model those were mentioned briefly.

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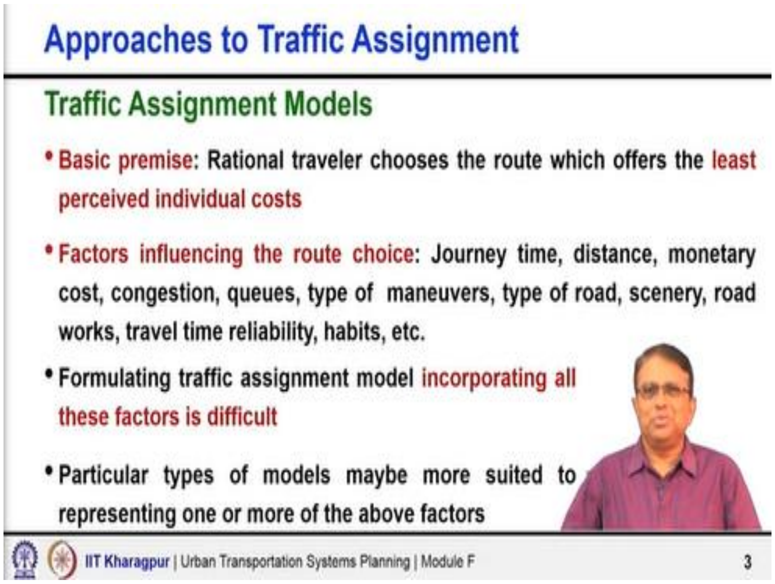

Now, this lecture onwards we shall really talk about the traffic assignment techniques or traffic assignment approaches. What we are trying to say that rational travelers choose the route which offers the least perceived individual cost. That is a very common and basic remark of traffic assignment. Each of us we are trying to choose the route where we think the perceived cost is least. There are several factors which influence the route choice.

For example, people consider what is the journey time they compare along different roads some people may consider distance similarly some may consider monetary cost, congestions, queues, type of maneuvers what one has to do, type of road and also you know any temporary work. What is the travel time reliability? Because travel time is not every day, we may not experience the same thing.

So, some people may consider the average travel time, some may consider even more weightage on the travel time reliability. So, all such kind of things will influence the route choice decision. So, it is very very complex, not at all a simple task. And it is really difficult because human being, people are most complicated elements their thought process what they consider you know you have seen in our choice model also when I was talking in the context of mode choice I said that the considerations itself may be very different.

I may consider travel time, you may consider travel cost. I may my idea about the travel time and your idea of the travel time again could be very different. So, formulating traffic assignment model incorporating all such factors is really difficult. So, what typically is done? Particular types of model generally may be more suited to representing one or more of the above factors.

We tell the principle now that principle is fine, we say cost but cost may be just a common expression of deterrence. Somebody is to somebody the cost may be actually the time cost or the time itself to some people may be the including the other factors, maybe the distance is also maybe a consideration. Or altogether it could be generalized cost also. So, how you define the cost is up to modeller or whatever we would like to consider we can consider. But approaches or the consideration of the deterrence is very important.

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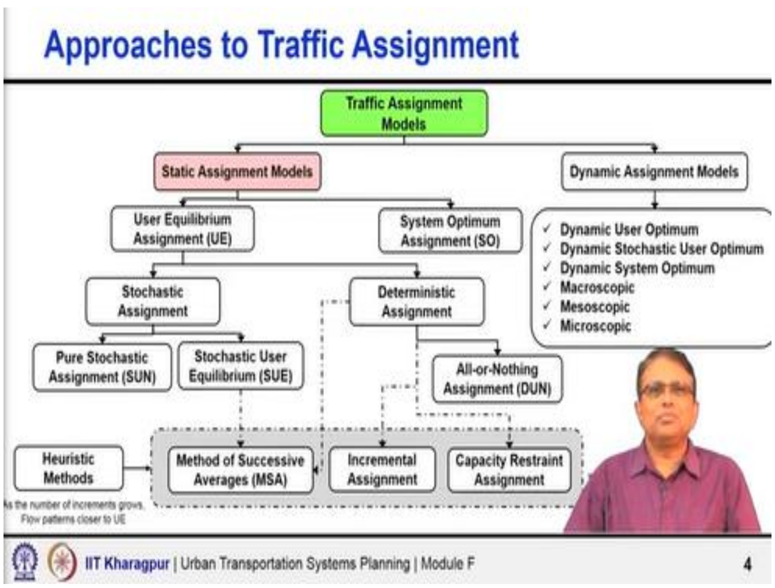

Now, this traffic assignment models, there is extensive development in this area. Like the mode choice and discrete choice models, disaggregate model whatever you say deterministic probabilistic then once you go under probabilistic model there are so many classifications. Whatever even I could discuss is only a part of it, the complete thing it takes much longer time to learn and even to teach also.

So, traffic assignment models are also like that. There are so many under you know, assumptions starting from very simplest thing to we can then keep on building things make it more and more realistic. And you want to make more and more realistic you want to also you have to make it more and more complex as well. So, what happens that? There are different approaches to traffic assignment.

So, now we try to tell you one by one so I am showing the complete kind of a flowchart with all the different techniques. But I am not trying to explain you everything using this flowchart at this stage. What I will do? I will take one box at a time discuss, explain it then go to another box, another approach or within one broad approach there may be further multiple types of assignment those then one by one I want to cover.

So, end of the day you will find you got an understanding of all the approaches. But let me be very clear it is literally impossible to discuss all approaches with equal depth that is impossible. So, we shall mainly focus on some of the approaches where we discuss in little bit more details. And some other approaches we may just give you some basic understanding and some thought process and leave it there.

Because we; will not get really time within this stipulated hour to discuss each and every approach in great detail. And I have no hesitation to you know confess that I shall not be able even to go into much detail and maybe I shall miss many other things. But the basic purpose of this course is to give you some basic understanding that we will do.

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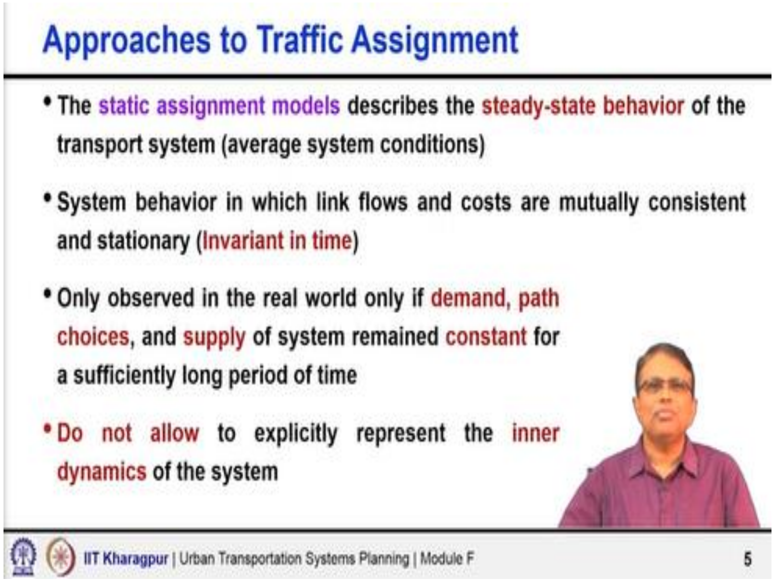

Now first, if you look at this flowchart is the static assignment model. One is static another is dynamic. What is dynamic? We will come later; first let us look at this box static assignment model. Now, static assignment models basically tries to describes the steady- state behaviour of the transport system on an average during a period of time substantial hours the traffic is like this, then how traffic is likely to get distributed to different alternative paths or routes.

That is the kind of understanding we are trying to give the time dynamics, we are not considering. That means only at one time interval. In reality what will happen morning 8 to 9, 9 to 10, not even 1 hour every 15 minute every 5 minutes the things may be very different. So, there is a time dynamic attached to that. That we are not considering in static assignment model. We are taking a sufficiently longer period maybe over 3 hours, 4 hours, 5 hours or the peak period, what typically happens?

And then what is then the likely average behaviour depending on the average condition or characteristics of the network. So, system behaviour in which link flows and costs are mutually consistent and stationary. So, here I have mentioned in the bracket invariant on in time, that is a very important thing. So, only you can observe such thing in real world if demand, path choices and supply of system remained constant for a sufficiently longer period because the time dynamics is not coming.

In reality whatever happens at 10 o' clock that will also influence the network state at 10:15. The way people have taken route at 10 o'clock will influence in the immediate say if I say t time interval what is happening t + delta t time state that will depend on what happened at t. Time interval t, that kind of thing we are not considering here. Such kind of approaches static assignment models do not allow to explicitly represent the inner dynamics of the system.

But then it is simple grossly if I am doing a planning, I want to know generally you know the total traffic demand is this much and maybe the peak hour will be 8 % or 10 % of the daily demand. And that will be my design our traffic typically for the planning purpose and how then if that is the traffic then how the peak hour, how the distribution will generally happen or how it will look like. That it can tell you.

You want to use for real time traffic control? No, further steps are there. Then the dynamics is very important.

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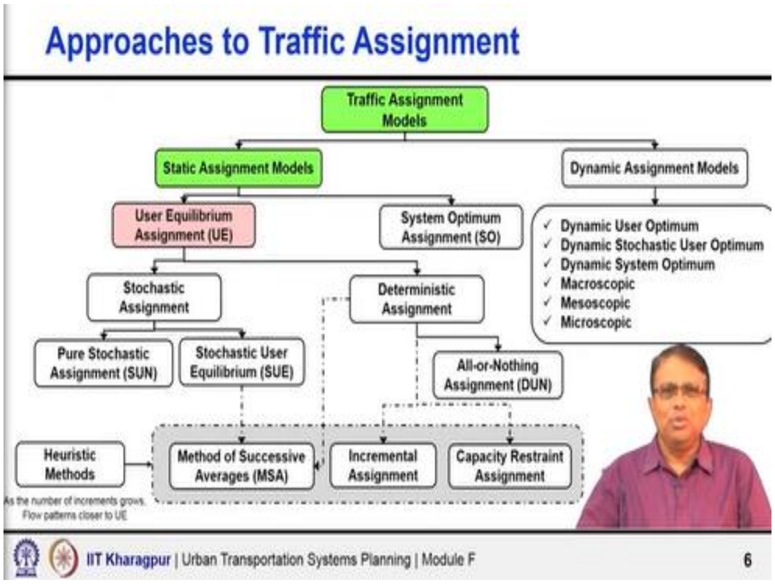

So, that is a brief discussion so whatever I covered I am making them green. My next point is you can see under static assignment models the two broad categories are shown as User Equilibrium Assignment UE. People also call it User Optimal Assignment. The other is which is not highlighted here is the system optimum assignment. You can also call it user optimal, system optimal, user equilibrium, system optimal whatever way.

Now what is this and why this comes into picture under static assignment? So, that static assignment model can crossly be divided under two categories based on what is the basic fundamental principle followed for assignment. One may be user equilibrium another is system equilibrium or system optimal.

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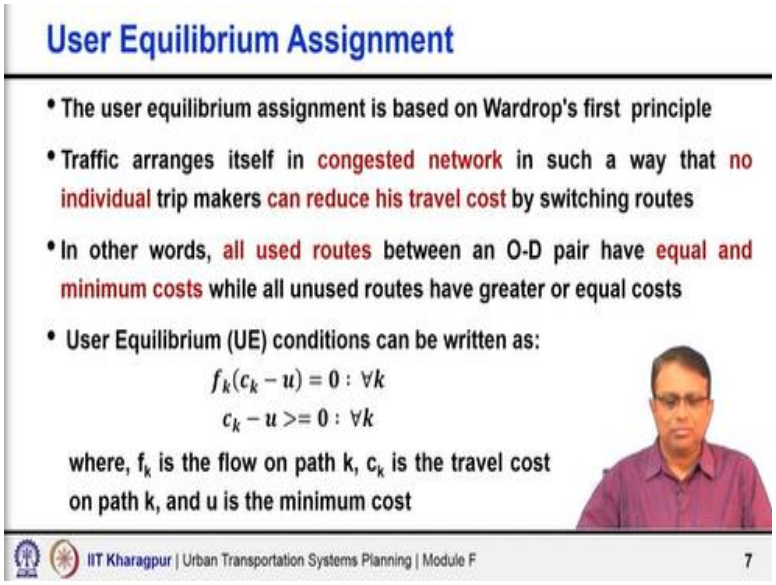

Now, the user equilibrium assignment is basically based on the first principle of Wardrop's. It is by the name of the researcher Wardrop. So, we call it Wardrop's first principle. And the other box what I have shown I am not discussing it right now the system optimal that is based on Wardrop second principle. So, what this order of first principle say very interesting follow carefully. We say the traffic arranges itself in a congested network.

In such a way that no individual trip makers can reduce his or her travel cost by switching route.

There are alternative routes the travel time is a function of flow. I would like to remind you about the link cost function you know that the link is loaded with more and more traffic the travel time will change. So, it is a kind of equilibrium where people have got distributed to different routes in such a manner.

And that is the equilibrium point, when no one can benefit further in terms of reducing his or her travel time by switching route by switching to another route. And if nobody can benefit further individually by switching route then we can call it, it is an equilibrium no further route switching is happening. So, it is something like people start taking different routes on different days and then they try all the things and now everybody got settled.

They know that this is the best route because by switching to another route I cannot save my travel time. So, in other words when it will be so? When all routes used between O-D pair consider one origin one destination there are multiple paths alternative paths. When all routes which are used there may be unused route, so 1 to 2 I want to go from node 1 to node 2 there may be five different paths.

But traffic may not use all five different paths some cost may be much higher. But whatever routes they will use have equal and minimum cost. And any unused route have either greater or equal cost. If not so, if this condition is not satisfied that will mean still there is another route by switching to that route, I can still save my time further then that equilibrium has not reached. When the equilibrium we can say, when I cannot find a better route that means routes which are being used, for one particular origin to one particular destination.

They are all having equal travel time then only nobody can shift to other routes. And any unused route why it is unused? If unused give you a lesser cost lesser travel time then people would have shifted there. Then it is not equilibrium but if I say it is equilibrium that means any other unused route will have higher cost or maybe at least equal cost. So, I know the travelling by this road travelling by another route it gives me the same time.

But in all probabilities, it will be higher cost. So, mathematically I can write it like this f k into c k - u = 0 for all k and c k - u is greater than or equal to 0 for all k. And equilibrium user optimal equilibrium means at the network level is what? For every O-D pair this is true for no O-D pair nobody can change to get a better one to reduce travel time. That is what is the whole network will be considered under equilibrium.

Now what is the f k? f k is the flow on path k. What is c k? c k is the travel cost on path k and what is the u? u is the minimum cost. Look at this thing equations very carefully c k - u, if c k is the minimum path, then u is the minimum cost, so c k will be equal to u. If people are using a path particular path k then under equilibrium that is also a minimum path. If that is not a minimum path then people would have shifted to the minimum path when no more shifting is happening that means that is also the travel time is the minimum.

So, then c k is equal to u, so c k - u = 0. And if any route, where the c k is not u higher cost then the minimum cost what will happen. Anybody will use that path? No. Ideally it should not; nobody should use that path why people will use a higher cost path then again that is not equilibrium. At equilibrium means they cannot shift to a route where they can further benefit. So, in this case it will contradict.

So, only thing possibility is what then when c k - u is greater than 0, then the flow on that path will be 0. So, f k into c k - u will always be 0 it could be because c k is equal to u and flow is non-zero but then the product will still be 0. Or c k is greater than u but in that case f k will be 0 so again the product is 0 so that is the formulation. I have actually shown that in the next slide.

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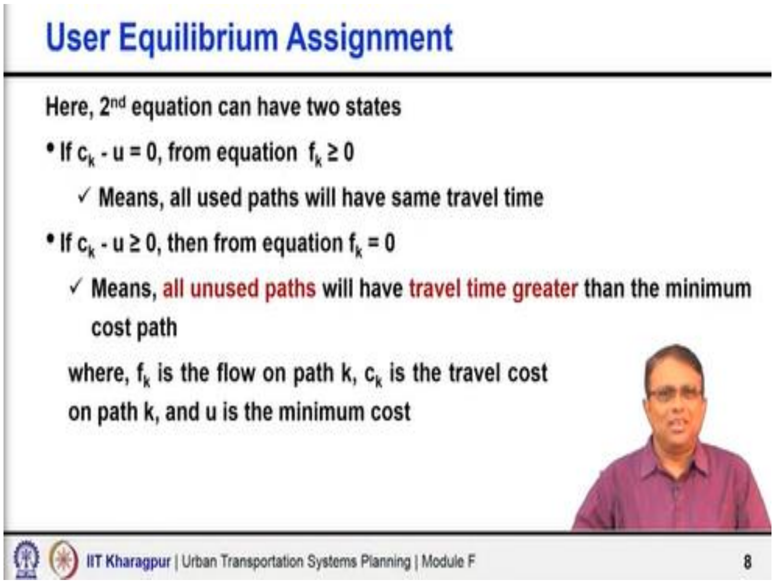

What I said is explained here, so you know why this formulation is valid.

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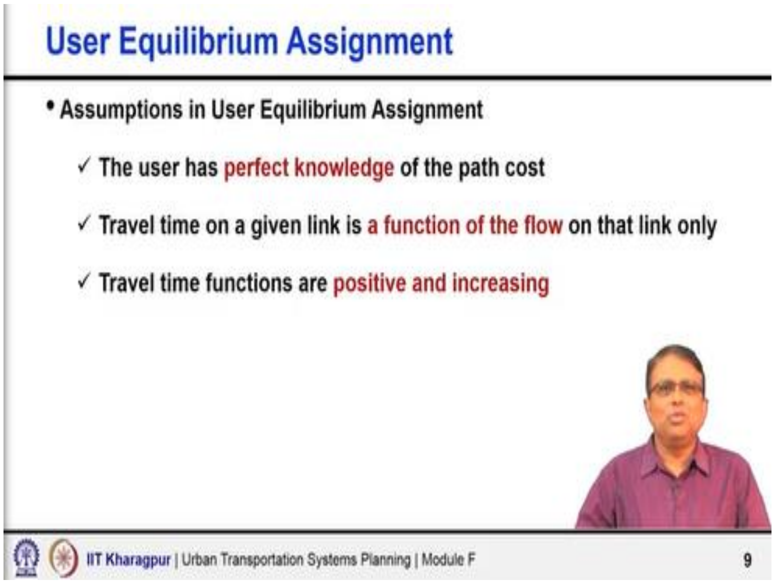

Now, there are certain basic assumptions in user equilibrium assignment. First the user has perfect knowledge on the path cost otherwise how the decision would be taken. So, this is one fundamental assumption, which later on when we discuss other traffic assignment techniques more realistic techniques then we will see how there are also other techniques which can overcome such kind of you know assumptions.

Travel time on a given link is function of the flow of that link only nothing else. So, simply we are saying flow getting distributed because the cost depends on flow only. So, the no shift is happening means the cost will not change if there is no shift then there is no change in the cost. And travel time functions are positive and increasing. Travel time cannot be negative so and any increase in flow will only increase in the value of the travel time.

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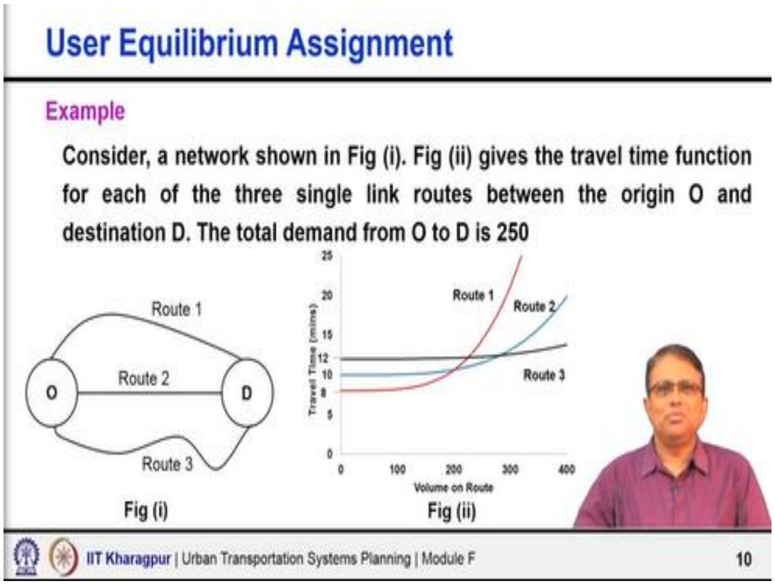

Now, let us take an example consider a network as it is shown there are three parts route 1, route 2, route 3 and I have shown here volume time curves on these three routes. Route 1 is shown by red line, route 2 shown by blue line and route 3 is the other one. Why the curves are like this you know it but still I would like to mention one, why they are different? Because different routes; may be having different capacity.

One road may be a very narrow road and another may be a more wide road another may be a further high capacity road. So, the effect of change of volume with certain volume one you know road may be becoming congested. So, the travel time start increasing very fast as you can see here in all possibility the route one is a low capacity route, also may be the shortest one. Whereas route two or route three may be having high capacity. So, the same volume does not impact the travel time so much.

And it starts increasing at a much higher volume. But then why the zero flow or the free flow travel time is even higher? Maybe the route lengths are different in all possibilities you see a road passing through this busy city area that is the shortest route but having the least capacity for various reasons whereas a bypass route may have a longer capacity higher capacity so it happens. Now if I have to distribute 250 flows, how they are likely to get distributed.

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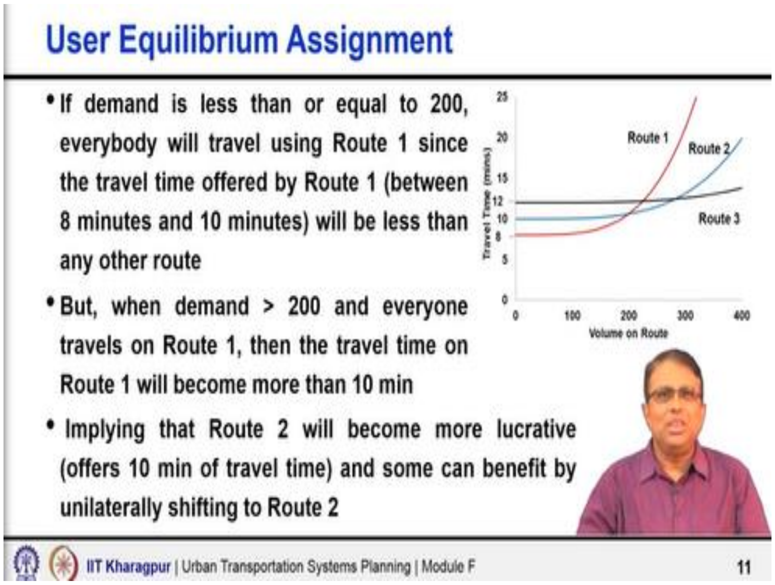

Now it is obvious that you can see that the red line touches the blue line at around 200 demand. So, fast 200 there is absolutely no reason why they will not use route 1 they will use route 1. Because for all of them till that time route 1 travel cost or the travel time will be the least and much lower than the route 2 travel time and route 3 travel time. So, everybody will use route 1, now beyond route 200 flows what happens? If people try to use route 1 then route 1 cost will be higher.

So, what will happen? They will shift to route 3 no route 3 class will also be higher they will start using then route 2 because route 2 travel time will remain about 10 minutes. So, route 1 already got so much of traffic to get a travel time of 10 minutes. Now if 50 flow 50 more vehicles are added you can see clearly from this blue line on which is representing route 2 the travel time on route 2 will still be 10 minutes.

So, 200 people will using route 1 with 10 minute travel time, 50 people will use route 2 again with the same 10 minute travel time. When further distribution will happen till the time route 2 travel time becomes 10, nobody is going to use other routes. Beyond that they will start using route 1 and route 2. And when the travel time on both these routes go beyond 12 minutes then nobody is going to use further route 2 and route 1, they will start using probably route 3. That is the way it is likely to happen because logically we can see that.

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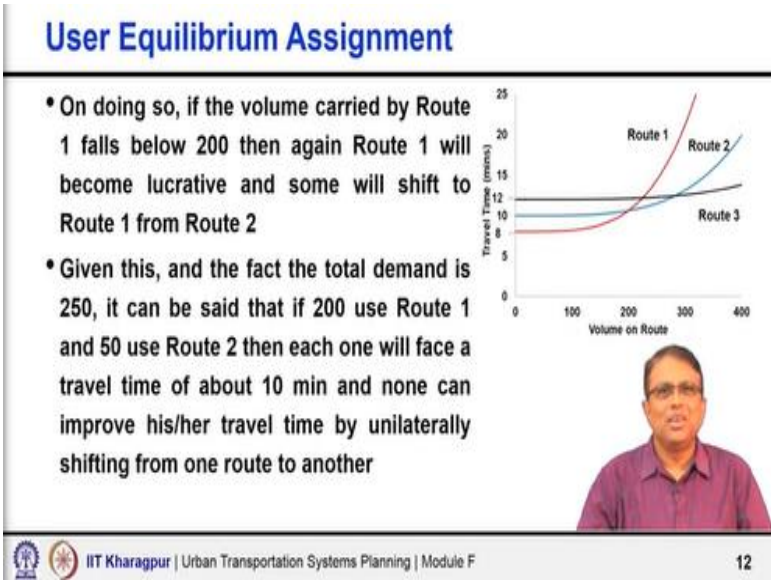

I have explained this here exactly in the; what I said. So, we expect here the best solution 200 will use route 1 and 50 will use route 2. Now anything other than this anything other than this and all will enjoy a travel time of 10 minutes anything other than this you will find there will be an alternative route which will be cheaper. Only under this condition it will you say more people will go to why 50 why not 100 people go there. Now if 100 people go there then Route 1 travel time will reduce it will be less than 10.

So, some people again will shift back to route 1. Or any other distribution you will see still you will be left with alternative route where the travel time can be reduced further. So, at equilibrium or in equilibrium 200 will use route 1 and 50 will use route 2.

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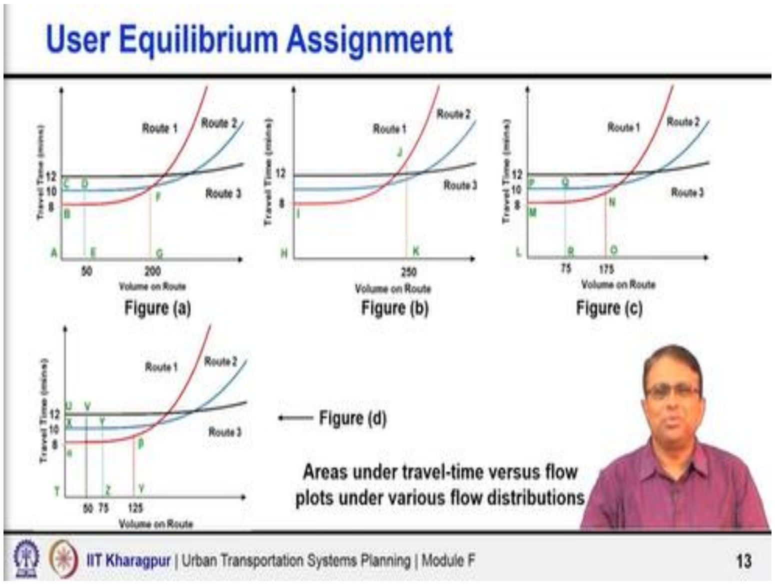

Here I have shown you different possibilities. Let us take this is the ideal solution which is shown in the first figure a, 200 is using route 1 and 50 is using route 2. And then there are various other solutions which are shown in figure b, c and d. Why I shown this? Please look at this examine these graphs carefully and see what is the total area under these curves total area and you will realize that the total area under this curve is minimum for this optimal solution.

That means any other distribution you take other than 200 using route 1 and 50 using route 2 except this solution whatever any other possibility you take the total area under the curves is going to be more. That is very interesting, what it shows? It clearly tells us that in user optimal equilibrium the total area under the curves will be minimum. That is what is the lesson learnt.

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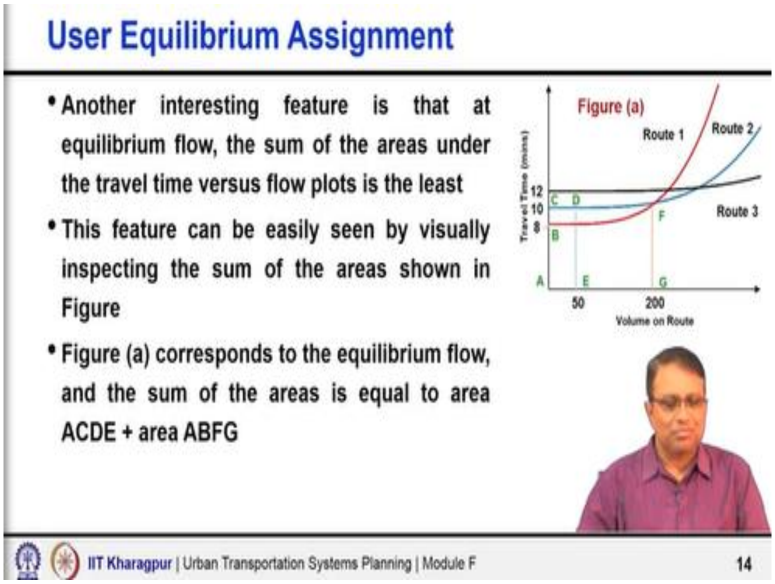

So, it is shown here any possibility you take you will find you will end up with a higher area. That is what is shown with each of these figures.

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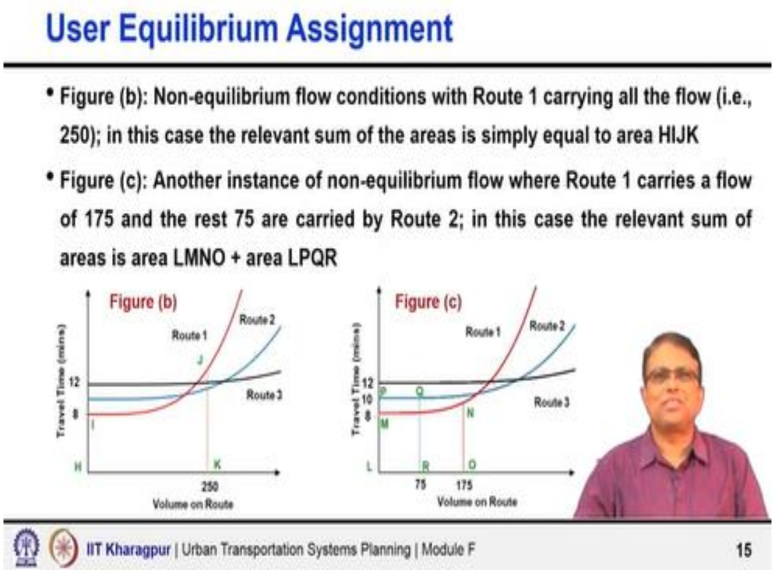

So, except the first one optimal one any other possible distribution you take route 1, route 2 even something on route 3 you will find the total area under the curve will be higher than what you are supposed to get for the figure a. So, you know that figure is optimal because any other solution is not equilibrium, because you will be left with an alternative where you can reduce your travel time further. So, under the optimal user optimal equilibrium the total area under this curve will be minimum.

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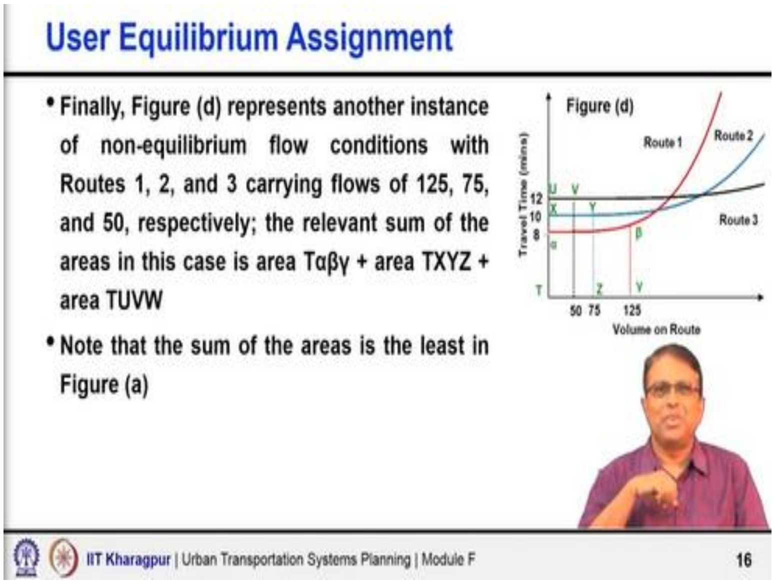

That is what is shown in all the slides with each of these figures now you can explore it further when you get the notes.

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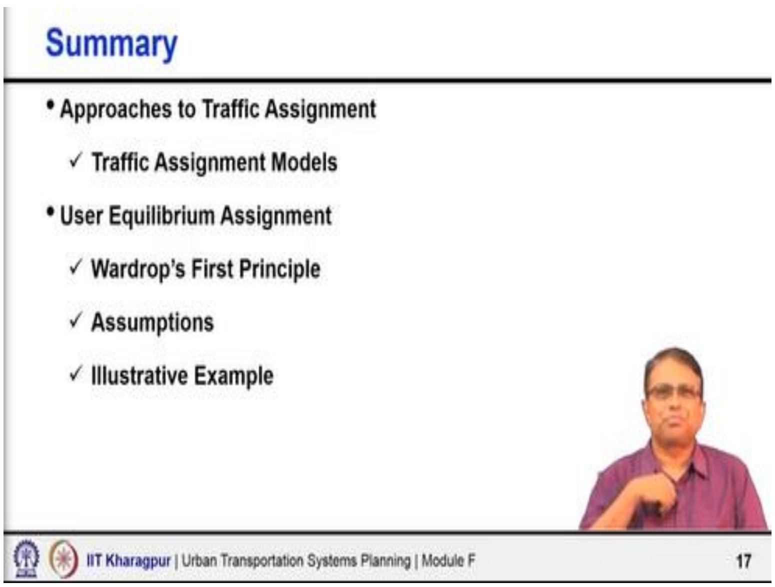

So, what we discussed in this lecture in summary. Broad approaches to traffic assignment we just introduced we did not tell you all the approaches we talk about first the static approach and then indicated that when we say static, so there must be something dynamic. What is the basic difference? One that time is taken as you know the time demand is time variant and static assignment, we are taking over a longer period of time overall demand what is the average thing that is expected.

When you go to dynamic assignment you will see that we consider every time interval how the things are getting linked. And then we tell under static assignment there is equilibrium assignment user equilibrium assignment talk to you about Wardrop's first principle assumptions. And took an example to tell that how you can get or how simple case what the solution is likely to be.

And to tell that the area total area under all the curves and you know time into the flow or the demand that is going to be minimum for the best solution or optimal solution. For any other solution the area is going to be higher. So, with this we close we shall continue in the next lecture. Thank you so much.