Urban Transportation Systems Planning
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Lecture-40
Logit Choice Models-IV

Welcome to module E lecture 10. This is the last lecture of this module. And in this lecture also we shall continue our discussion about logit choice models.

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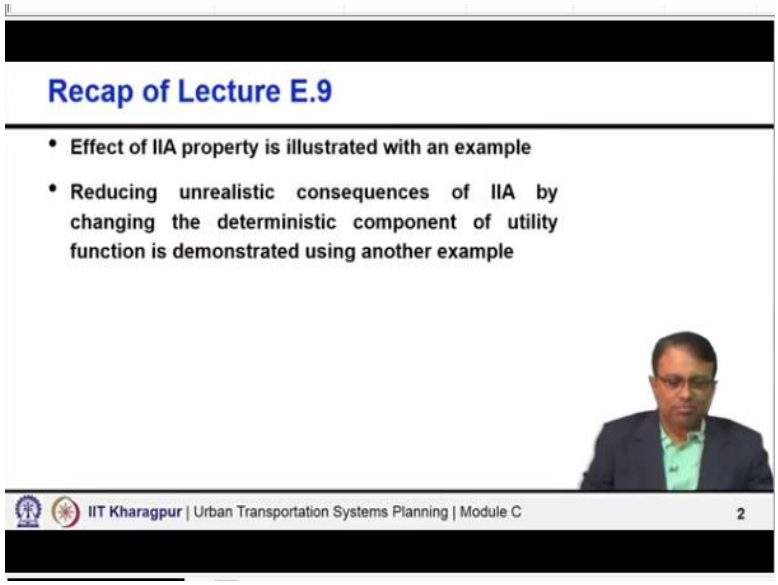

In lecture 9 we discussed about the effect of IIA property particularly we illustrated and explained to clearly how things that can happen with an example. And then we discussed also how by improving the model, specifically the utility equation by incorporating more logical variable, how the unrealistic consequence of IIA can be reduced, it can only be reduced, it cannot be eliminated just by changing the utility function. So, an example was taken and explained to you that how by improving the utility equation; we can reduce the unrealistic consequence of IIA.

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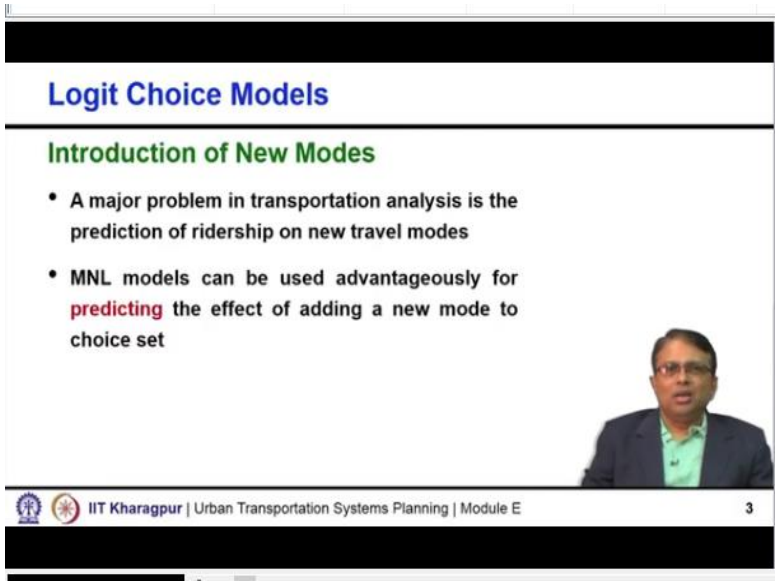

Today, let us first discuss how we can take care of this introduction of new modes, or new alternative in a choice context, because this is a measure this could be quite interesting and quite relevant as a major problem in transportation analysis is the prediction of ridership on new travel mode. So, many of the Indian cities are now trying to develop metro or BRTS, any such project the most fundamental requirement is to estimate for a given layout route or service characteristics, how much demand the system is going to serve?

Because your economic feasibility, financial viability, all these are directly influenced or impacted by the volume of traffic or the demand rather, instead of traffic we should call it demand, how many people are going to use and of course, because of this shift, the road congestion will decrease, emission will come down and so on. So, prediction of the demand for any new system is a basic necessity.

And very relevant in the context of improvement of transportation systems, especially urban transportation system. Now, in all such cases, MNL models can be used advantageously for predicting the effect of adding a new mode to the choice set, that maybe existing modes are there maybe only bus, carpool or taxi or private vehicle and you want to bring BRTS or you want to bring maybe the metro rail as an alternative. So, this introduction of new mode can always be handled very nicely in the MNL model.

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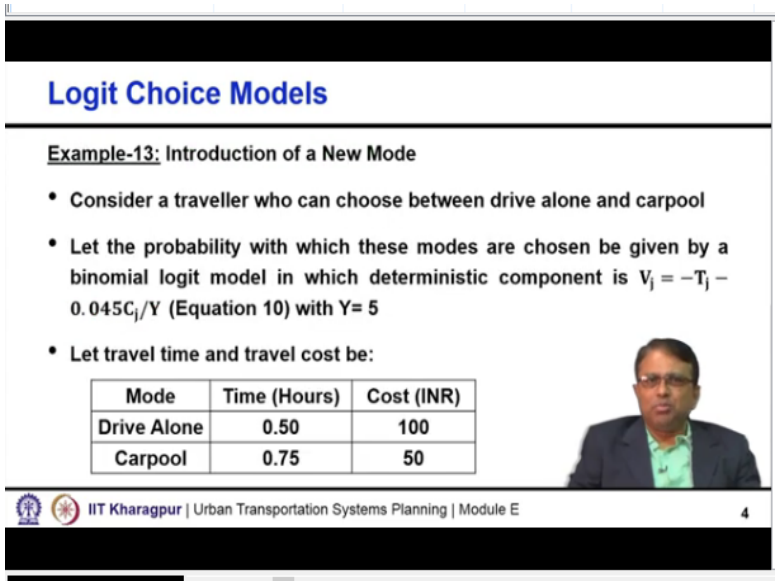

Let us take an example problem to show you how one can calculate it. Consider a traveller who can choose between drive alone and carpool. So, only 2 options are available either drive alone or carpool, public transport is not existing. Let the probability with which these modes can be chosen be given by a binomial logit model in which deterministic component of the utility is – T j -0.045 C j by Y.

I guess the same equation we used earlier also. And we want to find out how the choice probabilities will happen for drive alone and carpool, when the income is 5 lakhs or 500,000 Indian rupees. So, taking Y as 5 and the travel time and travel cost as shown in the slide for drive and carpool.

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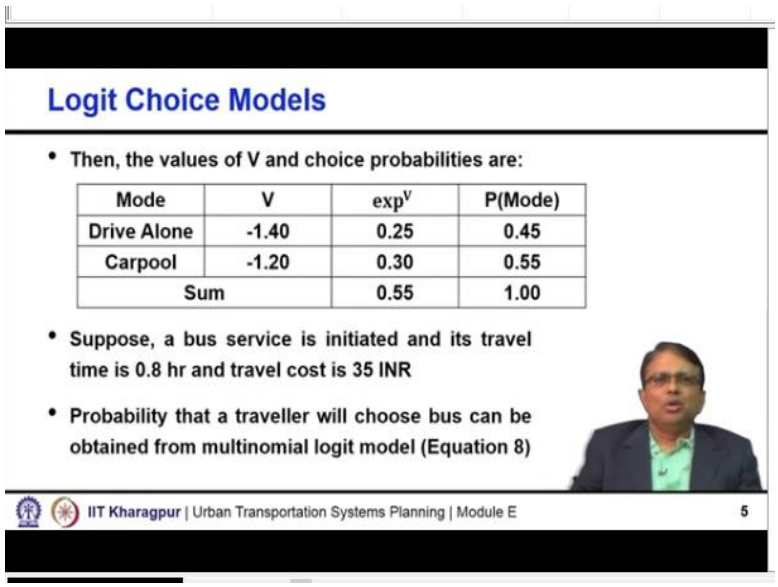

We can calculate the utility deterministic component of the utility obviously. So, that drive alone it becomes -1.4, carpool -1.2, calculate the exponent or exponent of these utilities which are 0.25, 0.3 total 0.55 and we can easily get that probability abusing drive alone is 0.45 and probability of using carpool is 0.55. Nothing new, but we just created the base for the next step.

Now, suppose a bus service is initiated and its travel time is 0.8 hour and travel cost is 35 Indian rupees and with these 2 values suppose we want you to calculate, then how much or what will be the probability of using bus in this context.

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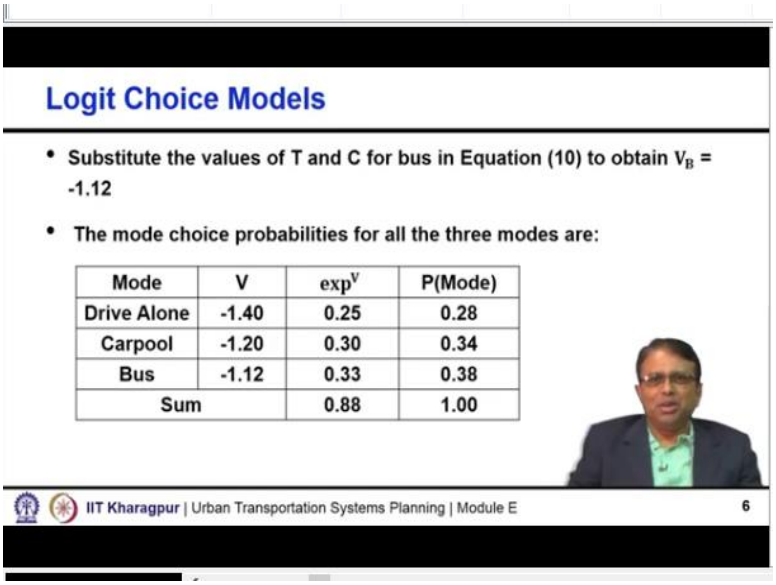

We can use a multinomial logit instead of binomial logic now, there are 3 modes. So, that simply we know the utility of drive alone, utility of carpool was also known, we used it earlier remember that -1.40 -1.20 and now, we know this equation, know the value of travel time as 0.8 and travel cost of 35 rupees. So, we can also find out the utility of bus that works out to be -1.12 in this case.

So, e to the power -1.12, 0.33. So, the sum of e to the power V i will change and using multinomial logit very easily we can calculate that what is going to be the probability of using bus, only one thing this IIA property will remain inside, but you can get the probability of using the new one. So, bus you are getting 0.38. So, where from this 0.38 will come? It will come from drive alone and carpool maintaining the IIA property or as per the IIA property, that is the only point.

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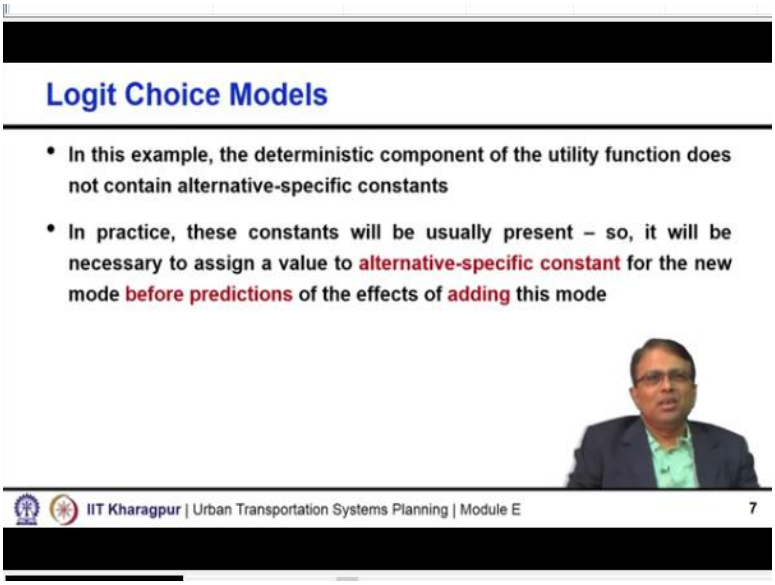

Now, remember that in this example, the deterministic component of the utility function does not contain alternate specific constant. In practice such alternative specific constant will remain present always there will be more biasness, without modes biasness should not really take the equation, in such context where you are considering 3 different modes.

And obviously, here we take maybe some cases 2 variable, 3 variable, 4 variable, but you cannot define there will be so many other factors; this biasness will be there in any choice experiment. The biasness will be there or while developing the model or in real life choice this kind of biasness will be there. So, we need to actually also calculate the mode specific biasness or alternate specific constant for the new mode.

That is one point of course, that also can be done in this course, we are not telling you how to develop such models which are normally a covering a different course. And so that part is beyond the scope of our discussion in this course. But that is one point you can from the perspective of planning you know that how we want to operate the bus and you can estimate the travel time, travel cost.

But also you have to calculate what will be the alternate specific constant for the new mode. That is required.

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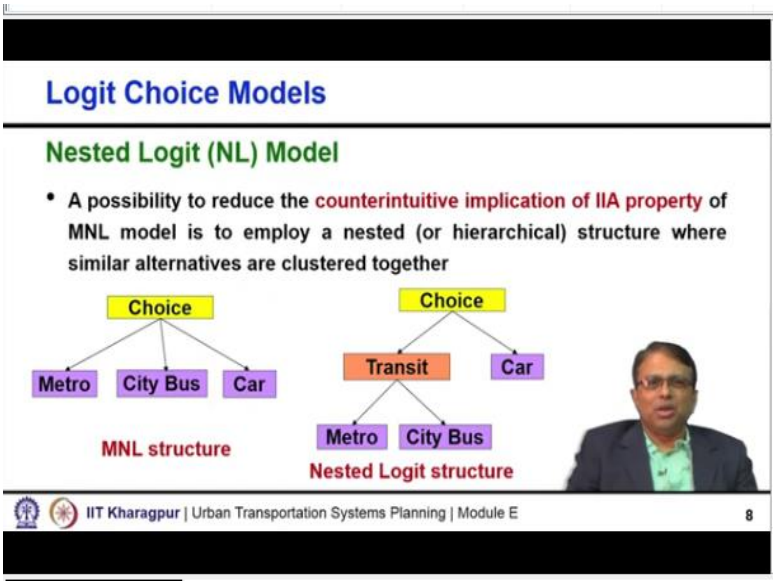

But once you know it, then you can easily actually calculate and take it, the calculation parties again the same only I am trying to say that generally, such utility equation will come with alternate specific constant and for the new mode also we have to take care of this more specific biasness or alternate specific constant. Now, as you have seen in lecture 9 towards the end of lecture 9 that by bringing more relevant variable or redefine the utility equation, we can to some extent, reduce the undesirable consequences of IIA.

Now, I will discuss before I close my next 15, 20 minutes time, we will go on this that how to reduce the impact of IIA? Again remember that we cannot buy this nested logit model you can see the topic is a nested logit model. By using nested logit model you cannot completely avoid or eliminate the effect of IIA, but you can reduce that effect very significantly. So, you cannot make it 0 but you can definitely reduce it very significantly.

So, a possibility to reduce the counter intuitive implication of IIA property of MNL model is to employ in nested or hierarchical structure, where similar alternatives are clustered together or put together, what it is? But what do we mean by that? Let us consider in MNL structure we consider metro, city bus, car, all 3 alternatives at the same level. That means, a person when the decision making is happening, which mode to take among these 3 modes or 3 alternatives, metro or city bus or car.

Now, what we are saying look at this metro and city bus, both are actually common carrier public transport mode. By nature, they are very different**.** They are very similar, not very different. By nature, they are different mode, but they are very similar, because both are

public transport just different form of public transport. And these 2 modes are much different from car.

Car is a private mode personalized vehicle. So, a car is significantly different as a mode from metro and bus. But, metro and bus are very similar as both are public transport mode. Now, because metro and bus are very similar, one can be used probably as a substitute of others in general or generally speaking. So, we expect also if the metro fare is increased, we expect people to come to bus, metro users come to bus, the metro share will reduce, bus share will increase.

We normally do not expect people to jump to car, but people may jump to car as per MNL model because of this IIA property. So, what we are doing here we are in one level. The first decision making is what we are trying to model is whether to use car or use transit common carrier public transport mode, it is also called transit. So, the first decision is whether to use car or transit because they are different distinctly different, that is the first level decision.

Then we are saying once you know that, transit is the option then within transit next level, whether to use metro or use city bus. So, metro and bus to similar modes are put together and the group is created called transit and the representation is made in an hierarchical or in a nested manner. So, a person making choice, first choice whether to use transit or to use car, if the choice is transit then the next level is whether to use metro or the city bus.

This representation which is shown here is the nested logic structure. The first one is the MNL structure; all 3 are in the same level. So, which one we are going to take among these 3 alternatives, number of alternatives maybe even more, there could be 4, 5, 6. So, as we have created a structure like transit and car the car also suppose you consider car, carpool, hired personnel hired vehicle like we take from a travel agency we hire a vehicle and travel.

Now, again those are more like private use personalized use. So, maybe we could put similar kind of modes together, they are also probably okay personal transit or personal vehicle, under transit metro or city bus, under private again maybe private car, hired private vehicle or hired vehicle, hired private vehicle is not true, hired vehicle like the full vehicle you were hiring or on hire or maybe carpool can also go somewhere there. So, such kind of structure is called nested logit or nested structure.

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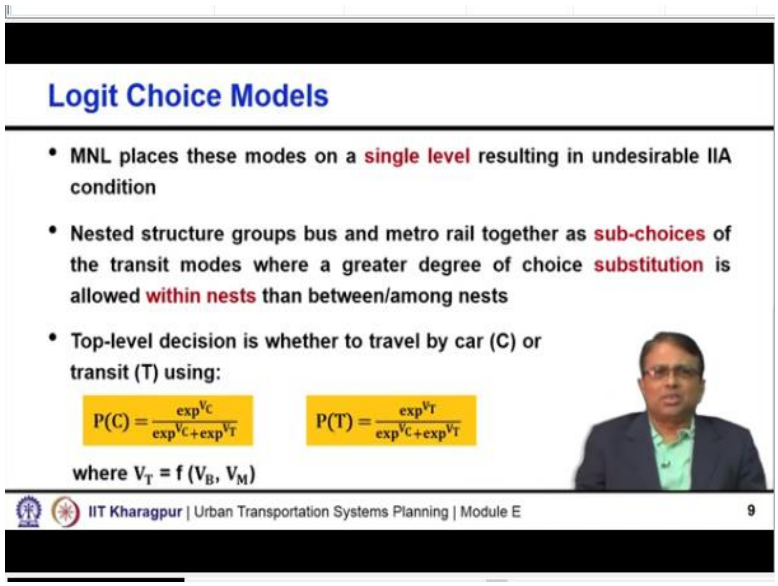

Now, as I said MNL places these modes on a single level resulting in undesirable IIA condition. Nested structure groups bus and metro rail together as a sub choice of the transit mode. First choice is transit or not private, if it is transit then the sub choice is either metro or bus, where a greater degree of choice substitution is allowed within nests than between or among nests this is oneness, this is another nest**.**

And we have been nested again there are further decisions. So, carefully observe a greater degree of choice substitution is allowed within nests, then between or among nests. So, the top level decision is as I mentioned whether to travel by car or by transit. So, if I can define an utility equation for our utility of transit deterministic component obviously, of transit and deterministic component of utility of car then I can calculate what is the probability of using car, it is e to the power V car or V C divided by e to the power V C + e to the power T, T represents transit and what is the probability of using transit?

It is simply e to the power V T, T for transit divided by e to the power V C, C for car + e to the power V T, T for transit fine, simple. So, if we have an equation like transit one equation if we can develop and if we know this equation of car then we can apply this binomial logic model and we can get the probability of using car and probability of using transit. Now, go to the next slide.

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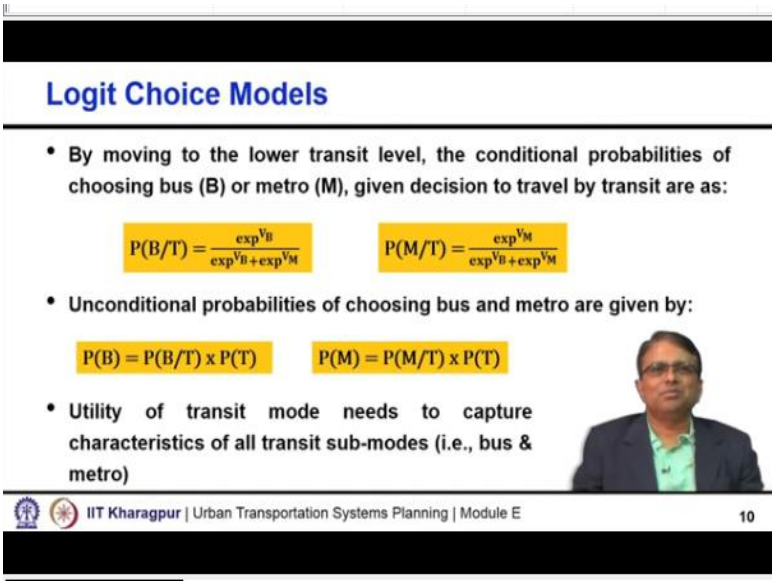

By moving to the lower transit level suppose now somebody is using transit, the conditional probability of choosing bus or metro can be estimated and this I say conditional probability that means given decision to travel by transit. If once the choice has been between transit and car the choice is already transit. So, the second level is given the choice is transit now, what is then the sub choice or what is the conditional probability of choosing bus or metro giving decision to travel by transit.

There we can write it like this probability of using bus given that transit will be used, simply again e to the power bus divided by e to the power bus by + e to the power utility of bus divided by e to the power utility of bus V T + e to the power utility of metro. Similarly, probability of tilting metro is e to the power utility of metro divided by e to the power utility of bus + e to the power utility of metro.

And this utility obviously is the deterministic component of the utility which we can measure. Then, what will be the unconditional probabilities of choosing bus and metro that can be given by these equations as shown here probability of using bus will be probability of using transit multiplied by probability of using bus given that the choice is transit. So, P B given T into P T, similarly, probability of using Metro is probability of using transit multiplied by probability of using metro given that the transit is the option.

That way we can calculate the conditional probabilities. So, what is remaining is utility of transit mode needs to capture characteristics of all transit somewhat, nobody has given me that equation directly, you have given me the equation of utility of bus, utility of metro,

utility of car. So, now, I need if I want to apply this successfully, I need utility of transit modes to capture characteristics of all transit some mode. That is in this case bus and metro.

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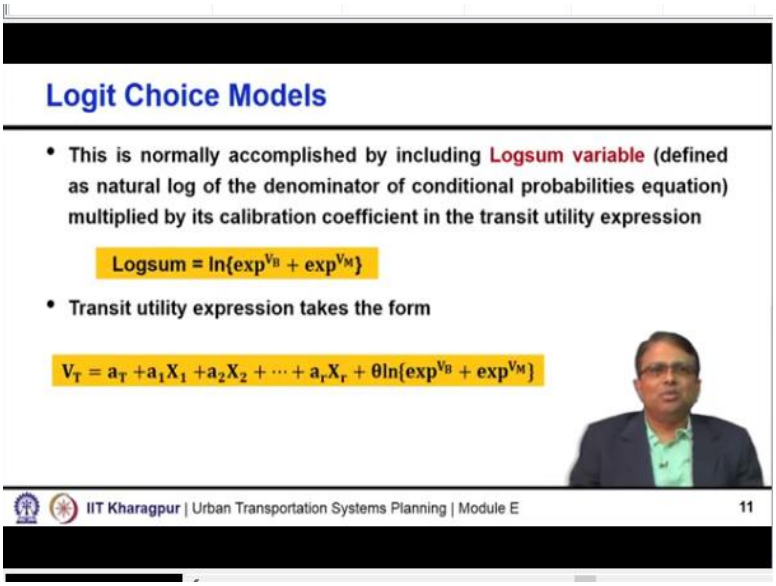

Now, this is normally accomplished by including a new term to you something called logsum variable. Now, what is the logsum variable? It is defined as the natural log of the denominator of the conditional probabilities equation, look at the conditional probabilities equation, what is the denominator of conditional probabilities equation? E to the power V B + e to the power V T that is the denominator of conditional probabilities equation.

So, it is basically taking e to the power V B + e to the power V T. That is what we say that defined as the natural log of this, denominator of conditional probabilities. So, what is the logsum? Logsum equals to ln natural log of e to the power V B + e to the power V M, M stands for metro, B stands for bus. So, thus the utility expression now takes the form V T for transit equal to AT, AT is the alternate specific constant plus, you have variables like as you consider x 1, x 2, x 3, all this plus theta into logsum.

So, theta into ln into within bracket e to the power V B + e to the power V M. So, now, theta comes mean to you. So, I have to explain you what is theta?

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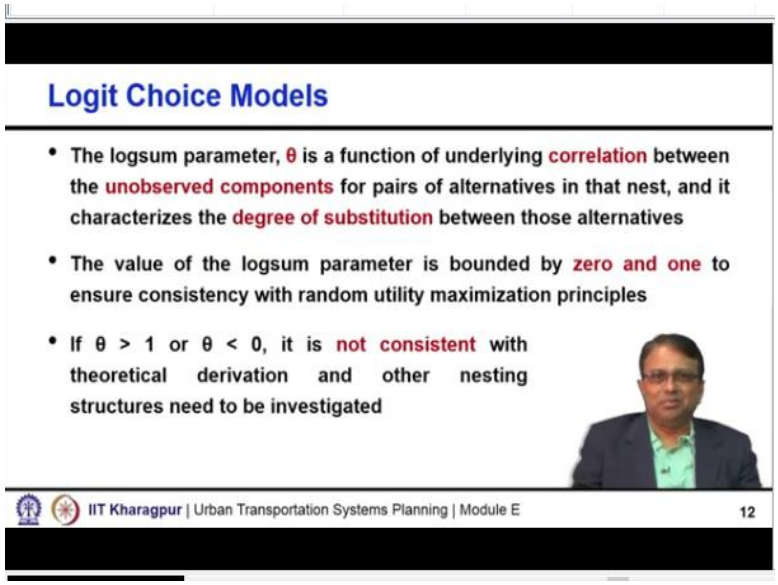

Now the logsum parameters theta is a function of underlying correlation between the unobserved components appears, what is unobserved errors? That is unobserved. So, theta is a function of correlation between the unobserved components of pairs of alternatives in that particular nests, in this case it is transit and it characterizes the degree of substitution between these 2 alternatives, what level?

Are they really modes which I considered with high level of substitution, that if people are not using A then they will definitely go to B and we are putting A and B together or not that always yes, but surely people will go from A to B, but they can go to other modes as well which are not within this nests. So, the theta indicates that. So, what could be the value? The value of logsum parameter is bounded by 0 and 1 acceptable value 0 to 1 and why they need to be only 0 to 1.

Then only it can ensure consistency with the random utility maximization principle, the whole logit model whole utility, it has to be compatible with that random utility maximization principle. So, then only it will be consistent. So, if theta greater than 1 it may happen for a given condition you calibrate of course, you are not going to discuss the calibration part here.

But suppose you calibrate and once you calibrate, you find theta is greater than 1 or theta will be less than 0, then it is not consistent with theoretical derivation and what is the alternative? The alternative is then think of an alternative nested logit structure. Say for example, what is

alternative nested logit structure? Think of a different, this is one way, I consider common carrier mode private, within common carrier mode bus.

Suppose, I have something I can consider it shared mode or not shared mode, not in this example, because here it is difficult to think immediately about an alternative structure because we have only 3 alternatives, but if you have more alternative, suppose you say 2 types of feeder vehicle and 2 forms of operation, 2 types of feeder vehicle, small feeder vehicle, feeder to the bus system or the metro system, 2 types of feeder vehicle each with 2 different forms of operation.

So, maybe demand responsive or fixed schedule operation, then I can do it, structure it like this, feeder mode type 1, type 2 and under each mode I can think flex you know demand responsive or fixed schedule, I can structure this whole nested logit differently first based on fixed schedule or demand responsive. Under each case, I can consider 2 available modes, mode 1, mode 2.

There also mode 1, mode 2, this is an alternative structure. So, if you find that theta is not coming within this range of 0 to 1, then we have to think of alternative structure.

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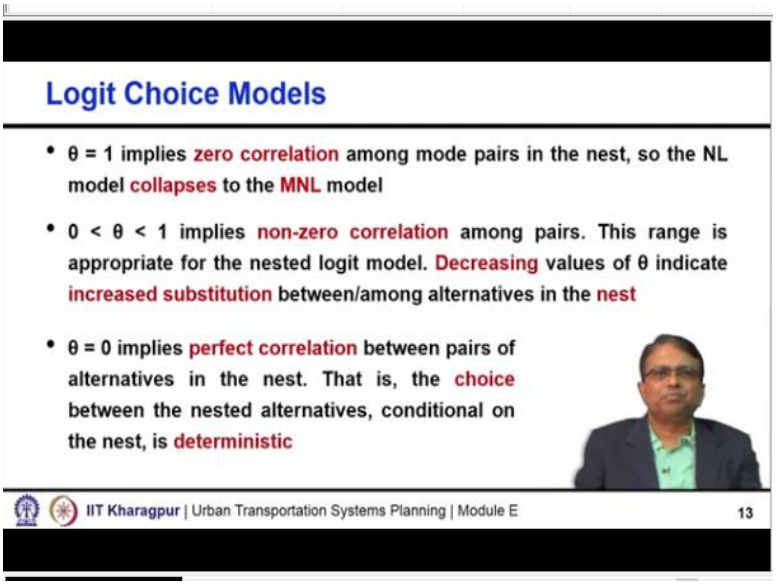

Theta equal to 1 implies there is 0 correlation. That means I am thinking 2 modes or 2 alternatives together thinking that they will probably kind of substitute if people are not using A they will go to B, that is the only heavy significant correlation is there. But if theta is 1

after calibration of the model for the given data that implies that there is 0 correlations among the pairs in the nests.

You have put it together, but there is 0 correlations. So, the nested model, it is no more a nested model, although you have considered them in a nest, but they simply like an MNL model, a multinomial logit model. So, in a way that nest collapse or you can say the nested logit assumption does not work actually, it is not required to put them under that kind of structure, putting them under that kind of structure or as you consider in this case, considering all the 3 model it is A, B and C, simply like a multinomial logit model is all the same.

If theta between 0 greater than 0 or less than 1, it indicates non zero correlation among among pairs the range is appropriate for the nested logit model and of course, the decreasing value of theta indicate increased substitution between an among alternatives in the nest, depends on how many alternatives, in this case only 2 alternatives. So, it is between the 2 alternatives in nest. That means non zero correlation interdependencies higher.

So, more likely people will go to those modes, if something happens with this one. And what is if the theta is 0, it implies perfect correlation between pairs of alternative in the nest. That means that if it simply they are kind of perfect substitutes. So, if theta is 0, then there are 2 alternatives. So, whatever is the increase here, decrease here whole thing will go to the other alternative only.

Nothing will go to any other place, it just within these 2, completely within these 2. That is why it is useful.

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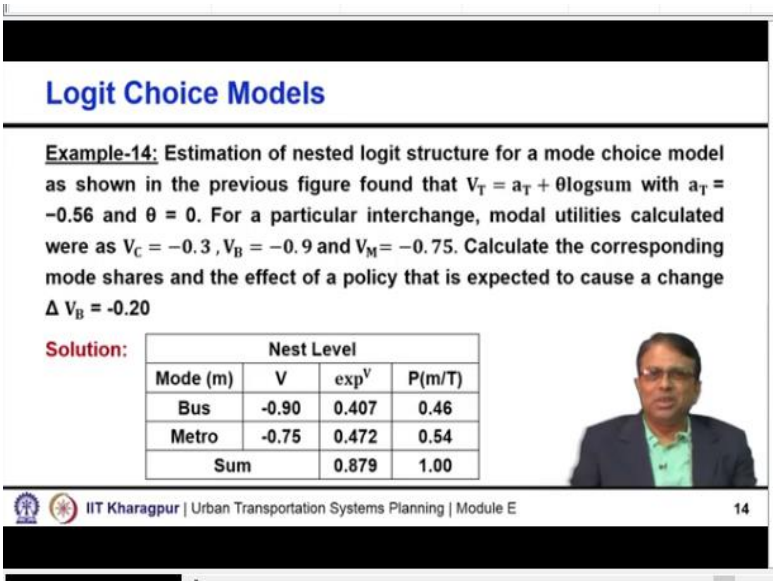

Let us take a small example only for 2 more minutes, estimation of a nested logit model, mode choice model as shown in the previous figure take the same figure found that V T transit is a T plus theta logsum where a T is estimated to be maybe -0.56 and theta in this example is taken as 0. Now for a particular interchange model utilities calculated were V C as -0.3, V B -0.9, V B -0.75.

Calculate the corresponding mode shares and the effect of a policy that is expected to cause a change in V B utility of bus by -0.20. So, the first part is very known we consider bus and metro. Calculate the utility of bus given as already given by -0.9, bus -0.75 you get the probability 0.46 and 0.54 respectively straightforward.

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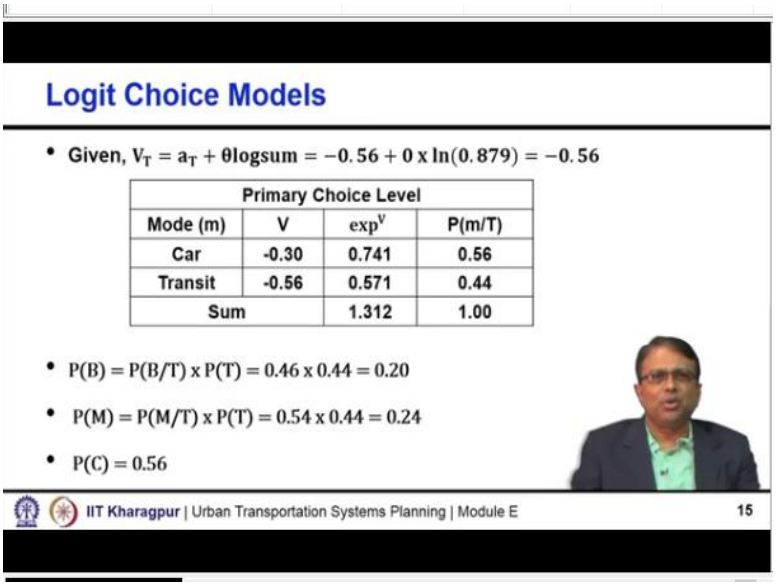

Now, given equation is what transit V T = to a T plus theta logsum, in this case logsum you have calculated and it is 0.879. Look at this, e to the power B bus + e to the power D metro total is 0.879. But ultimately, since the theta is 0, so that component does not come into picture, it is a case as I said theta a perfect correlation. So, V T becomes -0.56. Now once you know V T as -056, car is also known as -0.3, then you can calculate what is the probability of using car and probability of using transit?

So, what is then the conditional probability? Probability of using bus then what 0.46 probability of using transit like 0.44, probability of using transit multiplied by 0.46, that means within bus and metro, the probability of using bus is 0.46, metro is 0.54. So, overall transit probability is 0.44 into 0.46, so 0.20 and what is the probability of using metro? The same way 0.44 into 0.54. So, 0.24. So, what is the distribution? 0.56 car, 0.24 metro 0.2 bus.

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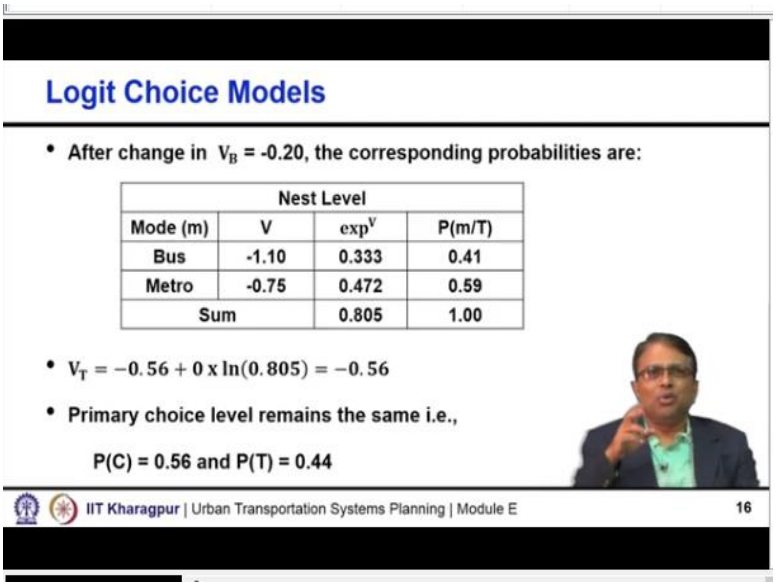

Now what happens if the bus utility changes further by -0.02. So, bus utility becomes now then -1.10, earlier it was –0.9 –0.2 is the change, so -1.10 accordingly bus and metro that distribution changes to 0.41 and 0.59 but then V T, V T still remains 0.56, although the logsum changes, but since the theta is 0, so, V T remains unchanged, perfectly correlation, as it was explained with theta equal to 0 perfect correlation.

So, what happens now, the probability of choice level remains the same as it was there at the base level, that whether to use transit or whether to use car. So, that remains 0.56 and 0.44 as it was earlier because the theta is 0 in this case. So, for the specific example they do not change.

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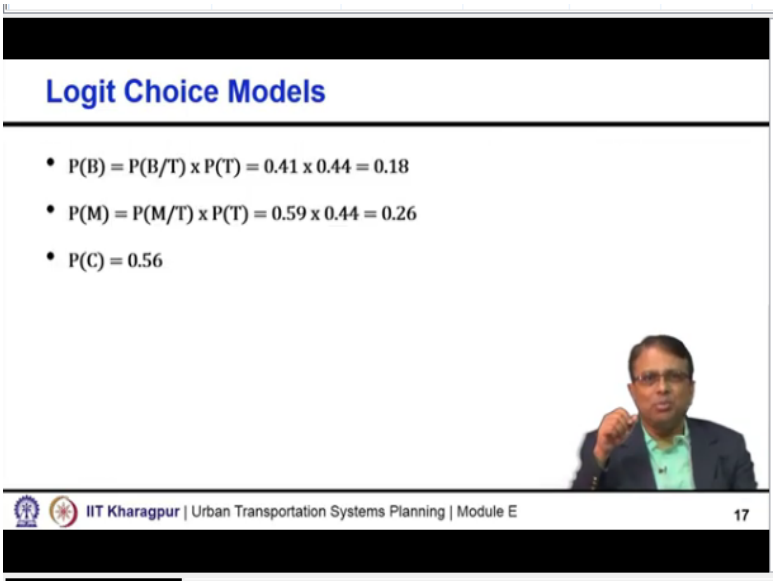

But then what changed is given the choice it is transit the probability of using bus and probability of using metro will change now, because of the change in the bus utility, bus utility reduces further. So, now, how you find out again like this? So, 0.44 remains unchanged as it was earlier also, in earlier case also it was 0.44, but then car was 0.5, this bus was 0.41 and metro was 0.59.

Now, the bus becomes 0.41 and 0.59 now, earlier it was 0.46 and 0.54. So, there is a change within bus and metro. So, that is getting reflected. So, what do you find here? We find here that P B = 0.44 into 0.41 and P M = 0.44 into 0.59, 0.41 and 0.59 comes from this slide. So, whatever we see, we see here because theta is 0. In this case remember, so, the probability of using car does not change, it remains 0.56.

Earlier also it was 0.56, after changing the utility or disutility of bus also it is 0.56, where the changes happening? Changes happening within a nest between bus and metro because both are transit mode and you logically expect that, that something happens with the disutility of bus increase, bus probability will reduce and people will shift to metro, not to car. In this case, that actually gives you the result.

But remember this example is a case very much specific example, where that theta is 0, if the theta is not 0, then the results will be different depending on what is the value of theta. And remember most interesting and important the interpretation of this theta what it means when

it is 1, it is comes down to MNL, when it is 0 perfect correlation, everything will happen within that nest, nothing will come out.

And then theta between greater than 0, less than 1, nonzero correlation some impact will be there depending on what is the value of theta.

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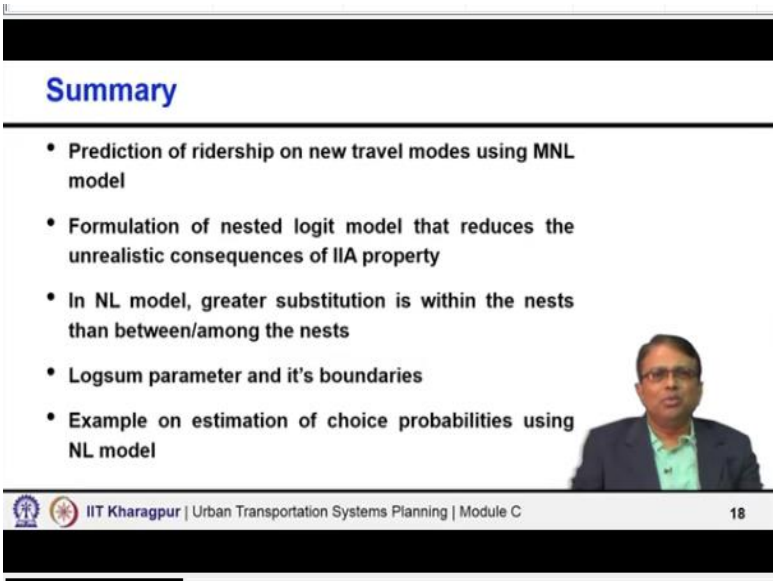

So, altogether to close this and with this we are closing this module as well I would say prediction of ridership on new travel mode using MNL model that we demonstrated with an example explained you the formulation of nested logic model not the theoretical formulation but basic how it looks like and the concept of this hierarchical part or the nested part and that reduces the unrealistic consequence of IIA property.

Remember that in the NL model greatest substitution is within the nest then between or among the nest, told you what is logsum parameters, what are its boundaries, what is the value of theta? How or what is the interpretation of this value of theta. And then finally took a very simple example, on estimation of choice probabilities using nested logit model. So, with this I close this section, thank you so much.