Urban Transportation Systems Planning
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Lecture-39
Logit Choice Models-III

Welcome to module E, lecture 9. In this lecture also we shall continue our discussion about logit choice models.

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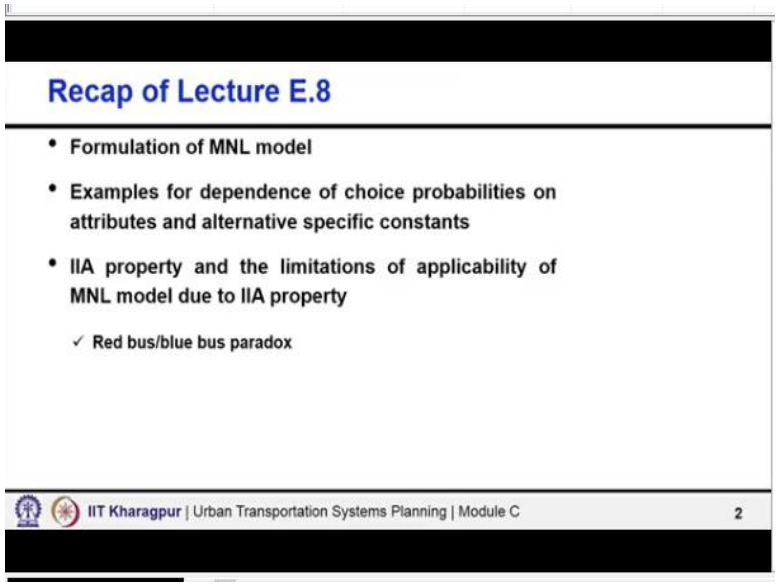

In lecture 8, we mentioned and discussed about the formulation of MNL model multinomial logit model, took examples for to explain the dependence of choice probabilities on attributes and attributes or alternate specific constants, how the choice probability is depend on the attribute levels and also how the probabilities can get influenced by the consideration of the alternate specific constant or AC.

Then, the IIA property was explained clearly independence of irrelevance alternative that was explained and the limitation of the applicability of MNL model due to IIA property was also discussed. Taking example of the famous red bus, blue bus paradox with this background, we shall continue our discussion about logit model today and continue our discussion more specifically on this IIA property.

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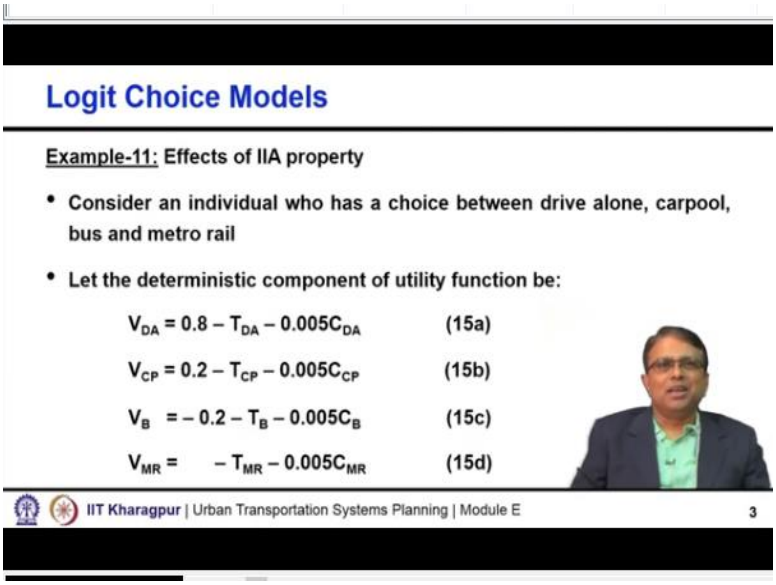

Now, let us take an example first to understand that whatever we said that the ratio of probabilities of 2 alternatives does not get influenced by the utility of any third alternative. So, always this ratio will remain constant, the probabilities will change if any other alternative third or fourth alternative, the utility increases or decreases, accordingly the probabilities will also change, no problem.

But the probabilities will change keeping this ratio unchanged. So, in that ratio the increase or decrease will happen. And the red bus, blue bus paradox clearly helped us to understand how sometimes the results could be illogical. Let us take this example to understand something more in this regard. Consider an individual who has a choice among drive alone, carpool bus and metro.

So, bus or metro are both a public transport, carpool and drive are both private vehicle oriented but operates differently in drive alone person will travel alone and in carpool multiple or known people 2, 3 people 4 people may travel together. Let the deterministic component of the utility functions be as shown here in equation 15a, 15b, 15c and 15 d. So, obviously, the metro is taken as a base alternative. So, compared to metro the bus is found to have a negative alternate specific constant, how we can explain that?

Yes, we can explain probably, let us say somebody people generally may find that the emission will be much more when you travel by road transport bus, you will be more exposed to emission or air pollution in general. Whereas, the metro is underground, railways mode

and completely segregated from the normal road traffic. So, the emission may not be there. So, in that perspective bus having disutility as compared to metro.

Carpool of course is utility because you are shifting from public mode to some extent not completely, but towards private bond, known people travelling together, it is like a group transport and then drive alone the utility is even much higher because as I said the flexibility the privacy, everything is much higher, the security all these are higher.

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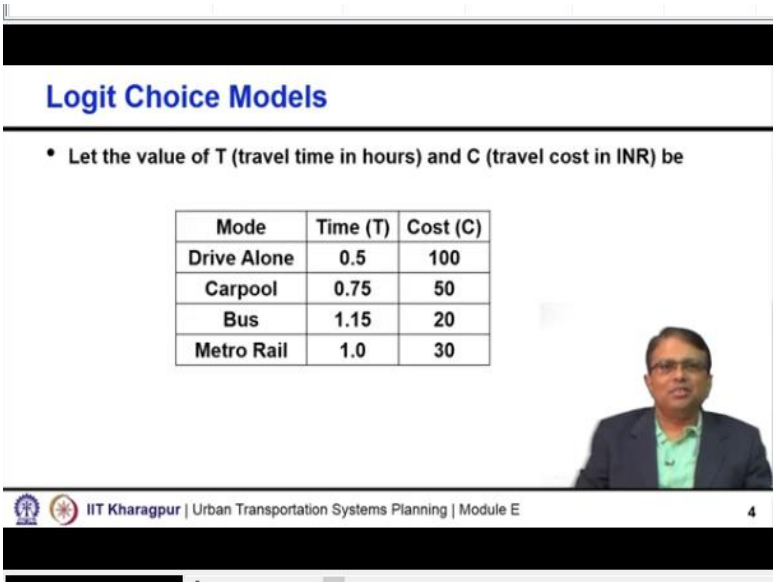

Now with this and considering this travel time and travel costs values, as mentioned here in this slide for drive alone, carpool, bus and metro.

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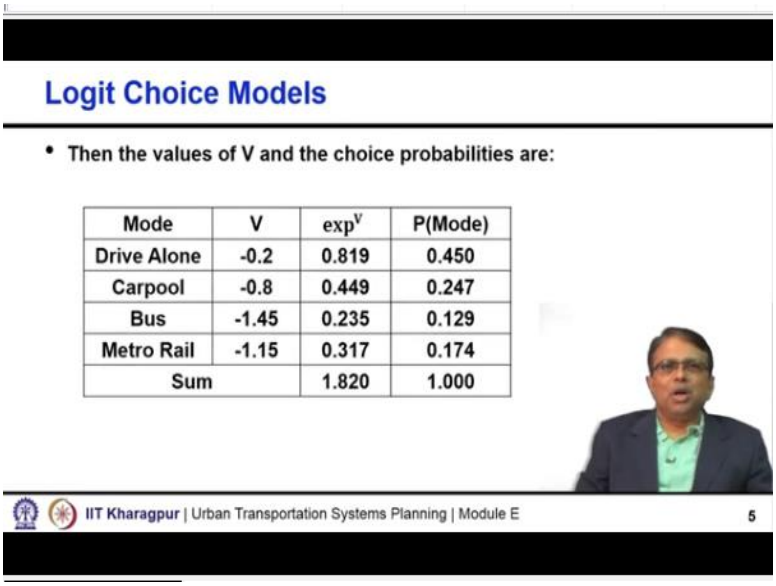

We want now to calculate the utility and the probability using multinomial logit model, this task is simple, you know this values. So, only here in this example, personal characteristics is not considered a socio economic characteristics is not considered. So, only the mode attributes we have taken, just in this example purpose, for this example purpose. So, we can calculate the utility or the disutility values, take the exponent of that, sum it over and then using multinomial logit model, we can calculate the probability of using each of these alternative modes.

So, you get driver alone 0.45, carpool 0.247, bus 0.129, metro 0.174, that is what the probabilities which were calculated using multinomial logit model.

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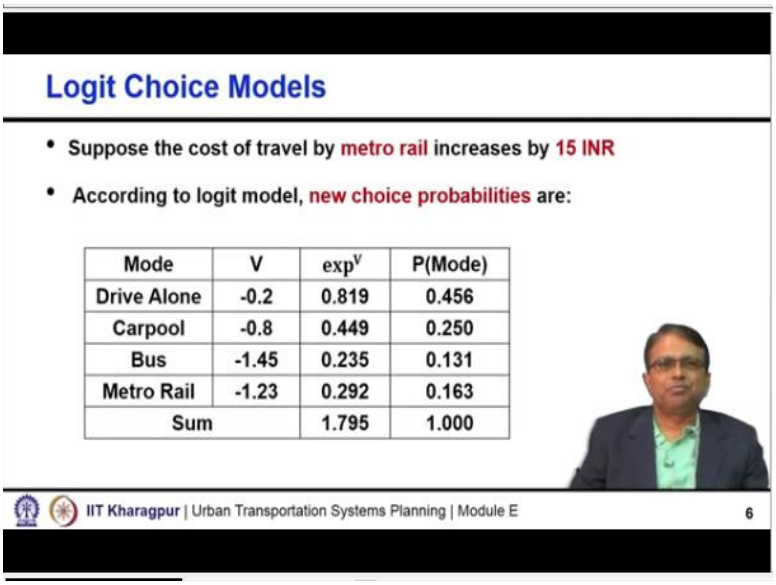

Now suppose the cost of travel by metro rail is increased by 15 rupees. So, what will happen? The utility or the disutility associated with the metro rail the deterministic component that will change. So, you can say the driver 0.2, car 0.8 of course, with negative sign on and bus -1.45, they remain unchanged, because utility of these modes will not get influenced by the utility of changing the characteristics or attributes of any other mode.

That is what is the normal properties of the utility function which was told right in the beginning of the smart choice model lectures or module. But, now, the metro rail value utility was -1.15. Now, it becomes -1.23. So, utility reduce this further or disutility increases further because we have changed the fare. Fare is increased obviously, the e to the power V you can calculate and now sum over that.

And then using logic model you calculate the probabilities. The probabilities are calculated and the revised probabilities are shown here with an increase of metro fare by 15 rupees. Now, obviously, metro rail earlier the probability was 0.174. Now, it will further reduce it becomes 0.163 quite logical because the metro fare has increased. So, utility has come down, disutility has gone down or disutility has become more. And therefore, the probability of using that metro rail has reduced up to this all fine, no issue, no problem.

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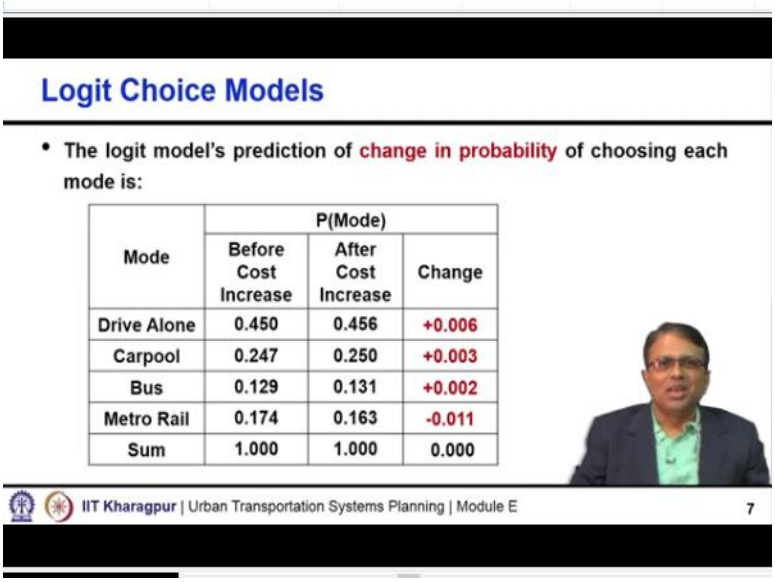

But then look at this change. Now, I compare before cost increase what were the probability values for different modes and now, after cost increase that 15 rupees increase in the fare by metro, what are the probability values? Look at this metro rail there is a reduction, reduction is by 0.11, 0.011 and where the shift has happened. Metro people have using less, where they are going?

They are going 0.002 to bus, 0.003 to carpool and 0.006 to drive alone. No problem people are using metro, metro fare is increased. So, less people will use metro. But why are they are shifting? They are shifting to drive alone, carpool and bus all the 3 modes. And as per our calculation using the multinomial logit model the change in the bus probability of using bus 3 times higher than that change is happening to drive alone.

Does it sound logical, acceptable? I am sure all of you will say that no, but why it is happening? Why more shift happen towards drive alone? Because of this IIA property, before cost increase and after constants crease, these ratios of probabilities are to be maintained, they are independent of what is happening to metro. So, for example, ratio probability of using

drive alone and carpool, before and after metro will remain same because utility of drive alone and carpool there is no change.

So, the ratio remains same. So, in that ratio, people will get attracted, in such a manner that the ratio of the probabilities the probability will change in such a manner that the ratio remains unchanged, same thing drive alone, bus, again the ratios are to be maintained as per IIA. Ratio will remain unchanged, nothing has happened to, no changes happen to the utility of drive alone and no change has happened for the utility of bus.

So, the ratio will remain unchanged. So, with that ratio when the whole thing is getting adjusted eventually from metro people are shifting more to drive alone followed by carpool, followed by bus? Whereas, in a logical manner, I am sure all of you will agree that we expect people largely to go to bus system because metro and bus are alternative public transport mode. So, if metro fare increases, people will go to bus why maximum number of people will go to drive alone.

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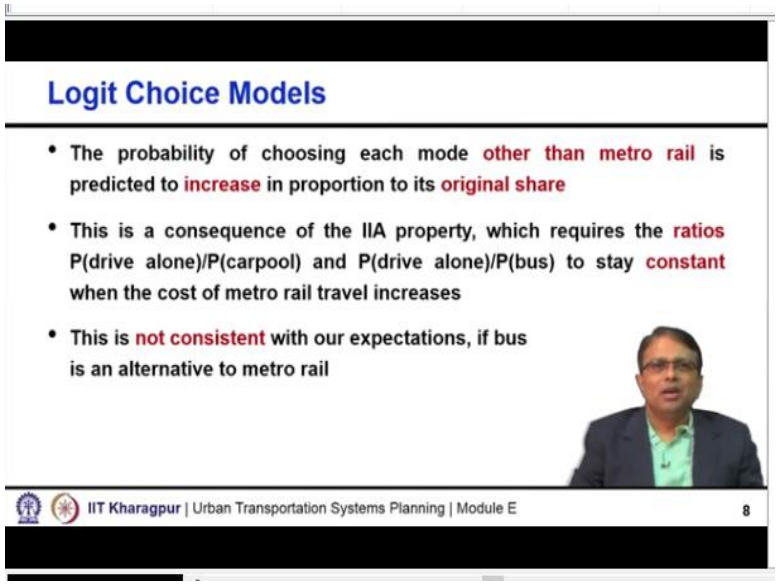

The probability of choosing each mode other than the metro rail is predicted to increase in proportion to its original share. That is where is the catch as I explained earlier also. I have written it here and I have written it also what I said, this is a consequence of IIA property which requires the ratio of probability of drive alone by probability of carpool and probability of drive alone by probability of bus to stay constant when the cost of metro rail travels increases.

So, bring more people in all other transport systems alternatives, but maintaining the original ratios and certainly this is not consistent with our expectation, if bus is an alternative to metro, because obviously most cases it is so.

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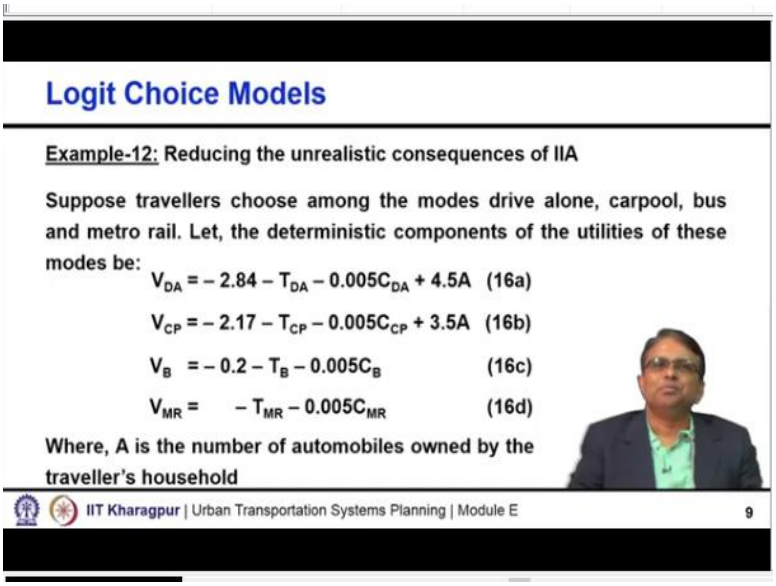

That is what this example shows the typical unrealistic consequences of the IIA property of multinomial logit model, results may not be logical, it gives you result, yes, metro fare is increased, so less probability of using metro that will be there. So, increase and decrease of fare or travel time or travel cost, that component of the result, the probability for that mode using that mode that will be logical to you.

As we have seen here that the fare is increased the probability of using that mode is reduced, decrease. But the IIA property because of that the shift where people are shifting or where from people are shifting, that may not be very, very logical, sometimes rather it is a doubtful result, you logically would not be able to accept it, but we know why it is happening? So, the next question a very relevant question is, is there any way that we can avoid this and reasonable or unwanted consequence which is happening because of the IIA property?

First we shall tell you that maybe you can reduce these unrealistic consequence of IIA, you cannot really eliminate it completely, but you can do some extent reduce by bringing more relevant variable file expressing your deterministic component of the utility This is an example for that, what we have done here additionally we have brought here the car ownership as a variable in this utility equations.

Now, so, it includes travel time, travel cost and also A, A is the number of automobiles own, somebody may have one, somebody may not have automobile ownership for captive riders, no car ownership, somebody also may have 2 automobiles. So, all those can be considered now. So, I have included more relevant variables now as compared to the previous example, although the utility equations given are different, because new sets of variables are there.

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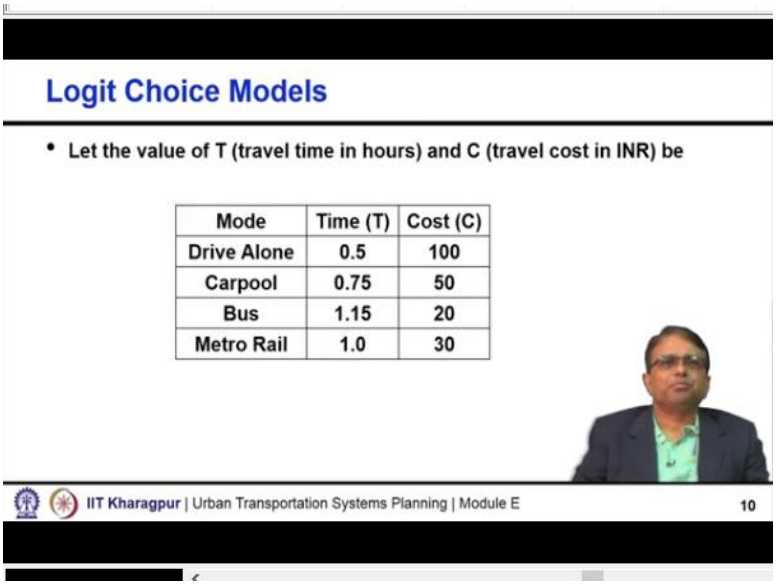

So, now with that we know the travel time and travel cost by all these 3 modes.

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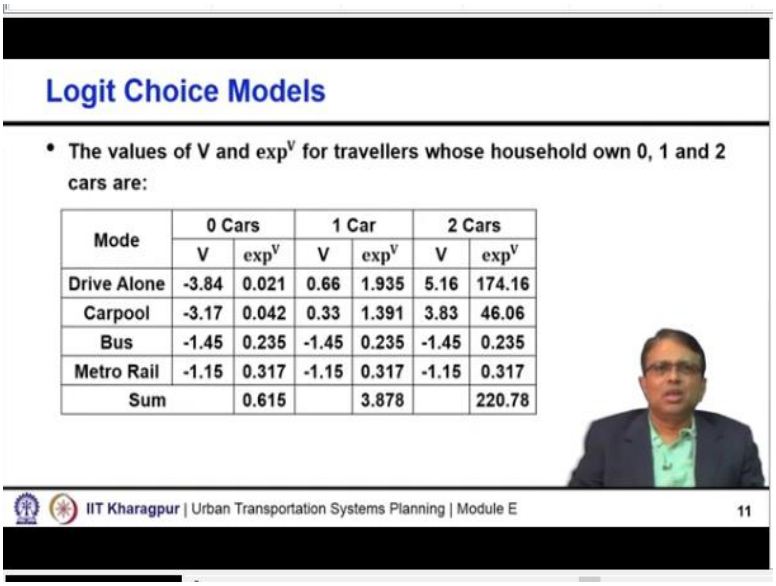

And we can therefore, calculate the value of deterministic component of utility for each of these 4 alternatives drive alone, carpool, bus and metro. Separately for 0 car, 1 car and 2 car users because A is there that automobile ownership or vehicle ownership. So, if it is 0, there

will be some value, if it is 1 there will be some other value there, if it is 2 then again the values are going to change.

So, utility values are going to be different for 0 car, 1 car and 2 car. So, accordingly using this set of equation as given in 16 a, b, c, d, we can calculate using appropriate travel time and travel cost value for different modes and considering the 0 car, 1 car and 2 car and then some of the e to the power utility and some of e to the power utility.

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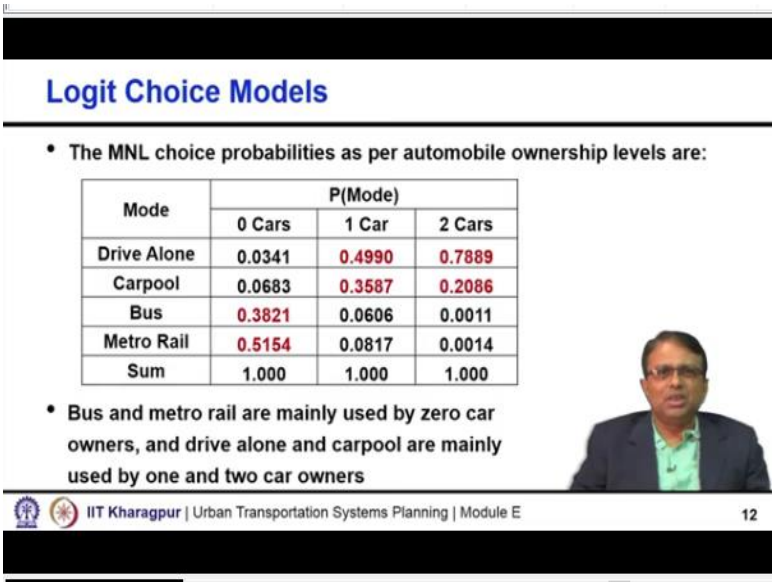

And then accordingly the probability. So, the probability values are calculated. Naturally what you find here? This 1 car, 2 car, when it is 2 car this very high probability that people are going to use drive alone almost 0.788 times, almost near 0.8 80% of the people will use car. Whereas for 0 car ownership more than 80, 85% nearly 80 89% rather, 89% more than that. So, nearly 90% are using either bus or metro. That is what it is.

So, now you get more logical distribution. Once you think the car ownership at the back end. Earlier we were not knowing the automobile ownership or the car ownership. So, generally the result was appeared to be fine, but when you now consider the car ownership the result and consider across this 3 verticals 0 car, 1 car, 2 car the results look much better**.**

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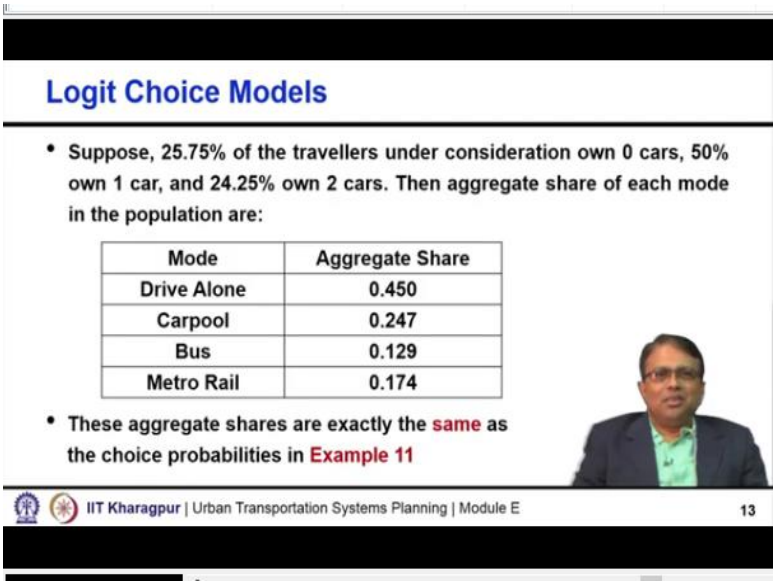

And suppose again because finally, I want to summarize the overall distribution that would be more interesting for you to see. So, I assume that 25.75% of the traveller under considerations own 0 cars 50% own 1 car and 24.2% population own 2 cars. So, now, you know what are the probabilities of using drive alone, carpool, bus and metro by 0 cars user, 1 car owner and 2 car owners.

So, distributions are also known. So, altogether you will get something what is going to be the aggregate share considering 0 car, 1 car, 2 car, all 3 segments together overall aggregate distribution how they look like, they look like this drive alone 0.45, carpool 0.247, bus 0.129, metro 0.174. Now, these aggregate shares are exactly the same as the choice probabilities in example 11. 11 is just the previous example.

We wanted to change the values and share in such a manner somehow just wanted to bring those distributions, what you saw in example 11, it was not a must but just we did it by choice we somehow adjusted the numbers to so, that you get it. So, as the overall distribution is same, but inside stories are different, because now we know that 0 car, 1 car, 2 car ownership, our model include those, include that relevant variable.

Utility equation includes automobile ownership and also in we know the shares,0 percentage of population with 0 car, with 1 car and with 2 cars. So, the inside stories also you know.

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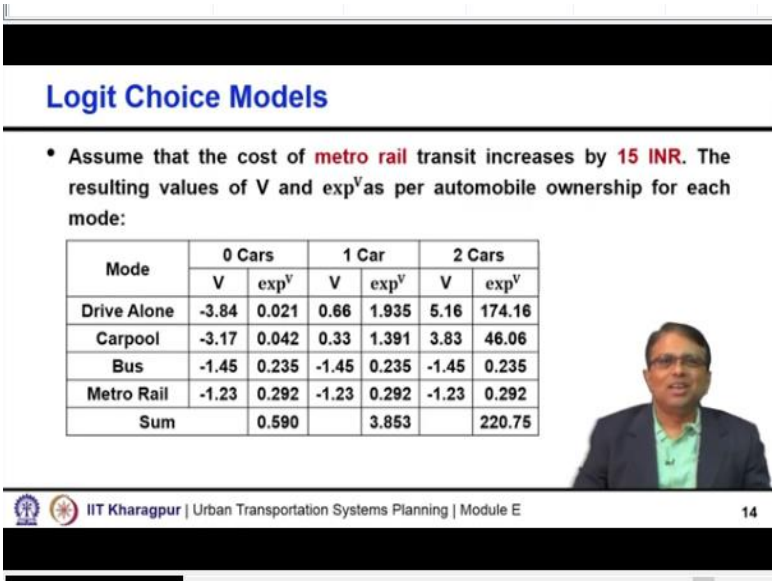

Mode	0 Cars		1 Car		2 Cars	
	V	exp^V	V	exp^V	V	exp^V
Drive Alone	-3.84	0.021	0.66	1.935	5.16	174.16
Carpool	-3.17	0.042	0.33	1.391	3.83	46.06
Bus	-1.45	0.235	-1.45	0.235	-1.45	0.235
Metro Rail	-1.23	0.292	-1.23	0.292	-1.23	0.292
Sum		0.590		3.853		220.75

Now, I bring back the same effect now, what was the thing that was taken there, that the metro fare is increased by 15 rupees Indian rupees. Let us increase that 15 rupees fare. But now, we do not go for the aggregate directly because we know our utility equations are different, 0 car, 1 car, 2 car population is distributed in 3 categories. So, I again calculate the V of course, the V will change only for the metro rail.

And accordingly we calculate e to the power V values all the cases and for each 0 car, 1 car, 2 car in each case each vertical we calculate the sum of the e to the power utility, then the next step is again very simple computationally, just to calculate the probabilities.

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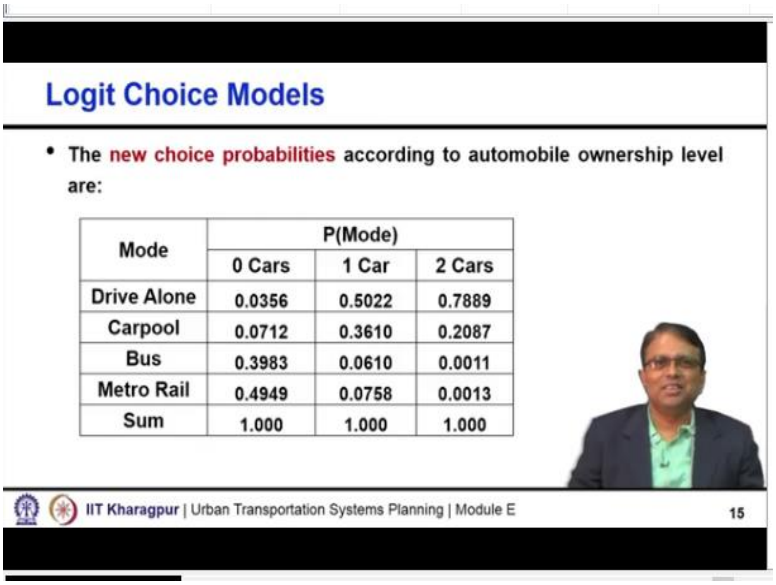

Mode	P(Mode)		
	0 Cars	1 Car	2 Cars
Drive Alone	0.0356	0.5022	0.7889
Carpool	0.0712	0.3610	0.2087
Bus	0.3983	0.0610	0.0011
Metro Rail	0.4949	0.0758	0.0013
Sum	1.000	1.000	1.000

So, the new probabilities as shown here. This is probably interesting, but calculation wise nothing new. We know that let us take each particle metro rail fare has increased, so let us

shift with 0 car. Now it is 0.4949. How much it was earlier? Earlier it was 0.5154. So, obviously metro fare is increased people shift at 0 car group. One car earlier it was 0.0817. Now it becomes 0.0758.

So, it has also reduced. So, you can see that yes, metro fare has increased. So, whatever was the probability value earlier for 0 car segment, for 1 car segment and for 2 car segment in each segment, the probability of using metro has reduced. That is what is expected. And that is what we know that even it is fantastic for that purpose, it can reflect that it is proved once again.

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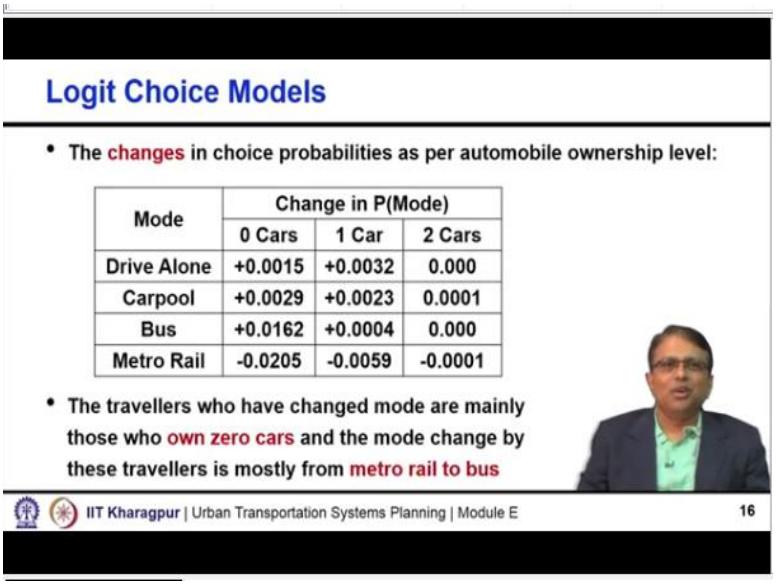

Now let us look at this the change in choice probabilities as per automobile ownership level, what has happened? Let us see the metro rail all cases has reduced. So, there is a minus sign with everything and where that minus is going? Minus is obviously going to all other alternatives, bus, carpool, drive alone and this is for each segment I am saying 1 car and the change in the probability.

One car, 0 car and 2 car segment, now, I know again overall population how much percentage is with 0 car, how much percentage is with 1 car and how much percentage is with 2 car? Those proportions are known.

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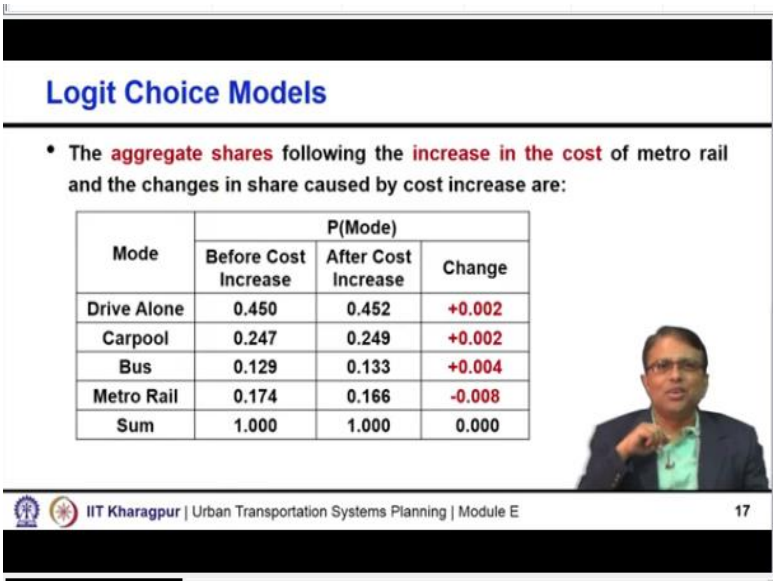

So, with those, I again calculate now the overall aggregate level what is really happening the distribution. So, I tell you and show you before costs increase the probabilities values for drive alone, car, bus, metro rail and now after cost increase with this new sets of equations, which include automobile ownership, how the probability values look like for drive alone, carpool, bus and metro.

And show you that overall aggregate level of what is the change? Look at this metro overall probability is reduced, but where it has gone now? Majority of that has gone now to bus, not to carpool and drive alone. Yes, still something has gone to carpool and drive alone. But what do you expect that if people are shifting from metro majority of them should go to bus, earlier what was there? Whatever shift happened to bus 3 times higher shift happened to car.

You remember just go to that equation bus shift was change was 0.002, drive alone 0.006 3 times higher, whereas here what has happened? Largely the shift has happened to bus, so metro fare is increased fine. So, people majority of them they shift to bus. So, much more logical result, much better result. So, that shows one possible way of reducing the undesirable effect of IIA is to make the utility equation more realistic, bring more pertinent variable into utility equation and make the utility equation or the model even more realistic here. So, by bringing more pertinent variable, logical variable one can reduce the undesirable effect of the IIA.

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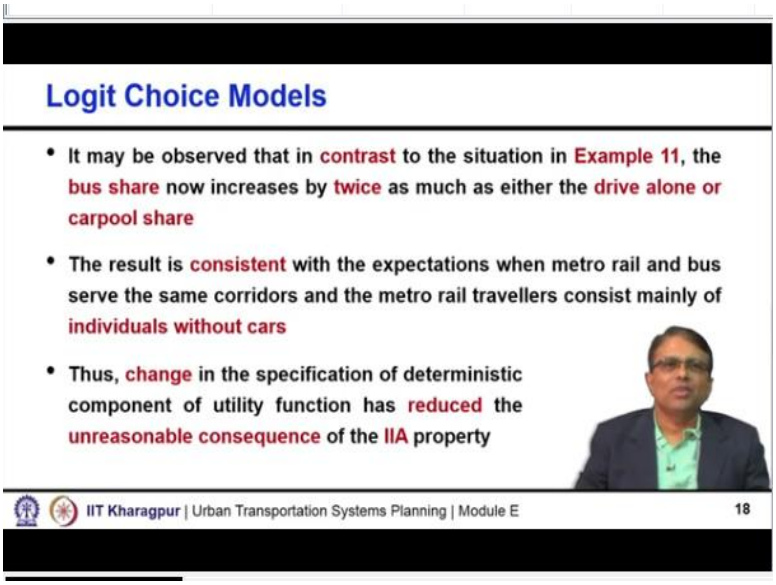

So, whatever I said are mentioned here, that in contrast to previous example, how these results improve and finally, the same conclusion that does change in specification of deterministic component of utility function has reduced the undesirable consequence of the IIA property.

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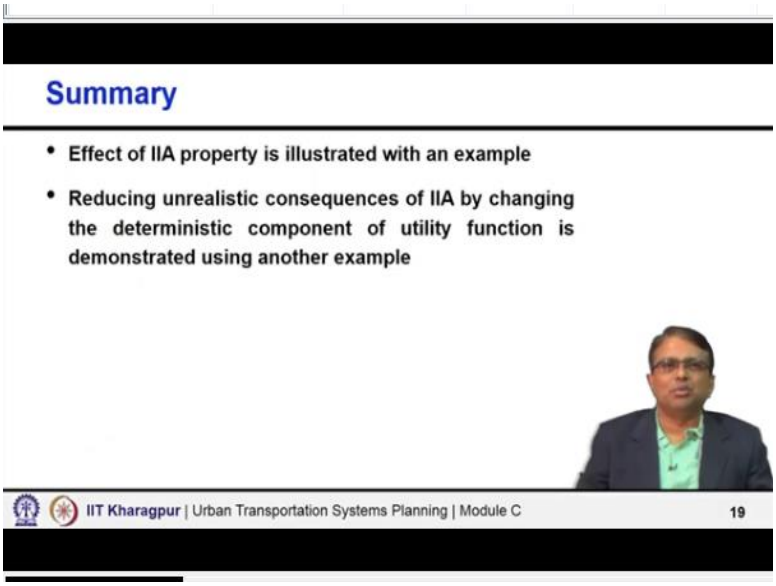

So, to summarize, in this lecture, we discussed about the effect of IIA property further with an example and then we said how we can reduce the unrealistic consequence of IIA by changing the deterministic component of the utility function and that we demonstrate it with an example. So, with this, I close this lecture. Thank you so much.