Urban Transportation Systems Planning
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Lecture-38
Logit Choice Models-II

Welcome to module E, lecture 8. We shall continue our discussion in this lecture about logit choice models.

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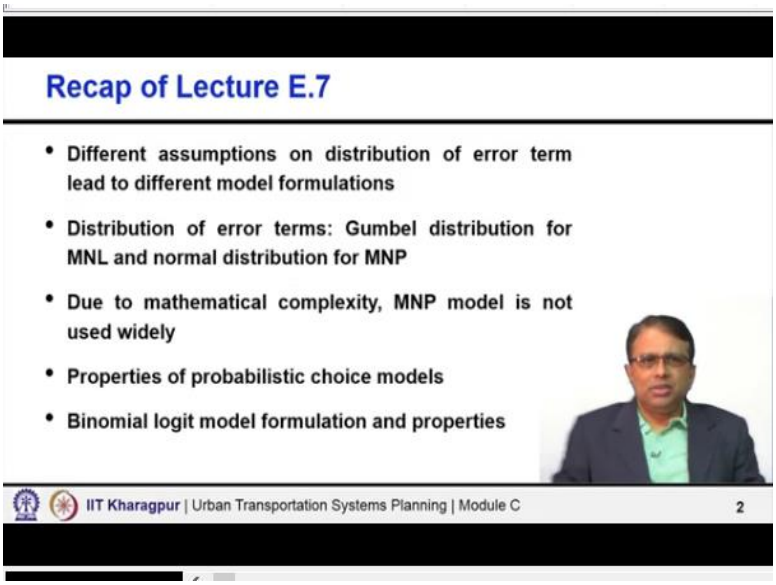

In lecture 7 we mentioned that there are different assumptions on distribution of error term, which lead to different model formulation. For example, if we assume that the error term follows Gumbel distribution, then it brings MNL model multinomial logit model or logit model and if we assume the error terms follow normal distribution, then it results into multinomial probit.

Also, it was mentioned that due to mathematical complexity in model calibration, MNP or multinomial probit is not used widely are not so popular, although it is not that they are not used, they are used definitely, but more wide application and use this for logit models. We also mentioned to you about the properties of probabilistic choice model and then discussed the case of binomial logit model, how the model is formulated and what are the properties of binomial logit model?

And we said that binomial logit model models satisfied 3 out of the 4 desirable properties. The only limitation is it cannot handle more than 2 alternatives and of course, by name we are calling it binomial logit. So, obviously, it handles 2 and it cannot handle more than 2.

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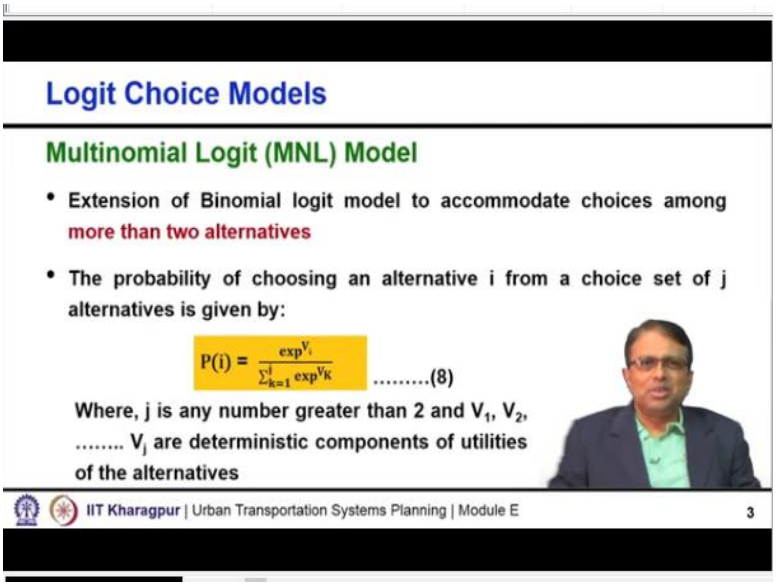

So, in continuation to this today, we shall discuss about multinomial logit model. So, the only limitation of the binomial logit model that it can handle only 2 alternatives not more than 2 that limitation is overcome by this multinomial logit. So, this is an extension of binomial logit model to accommodate the choices among more than 2 alternatives and how it looks like?

So, if we are trying to find out what is the probability of choosing i from a set of j alternatives that means not 2, but we have more than 2, so, the value of j is more than 2. So, you have more than 2 alternatives j number and what is the probability of choosing alternative i, then probability of choosing alternative i equal to e to the power i e Vi, utility of alternative i e to the power Vi divided by sum of e to the power V k where k is equal to 1 to j because we have j number of alternatives.

So, we can say that denominator will be sum of e to the power V k where k is 1 to j. So, j is greater than 2 as I mentioned and V 1, V 2, V j are the deterministic component of the utilities of the respective alternatives.

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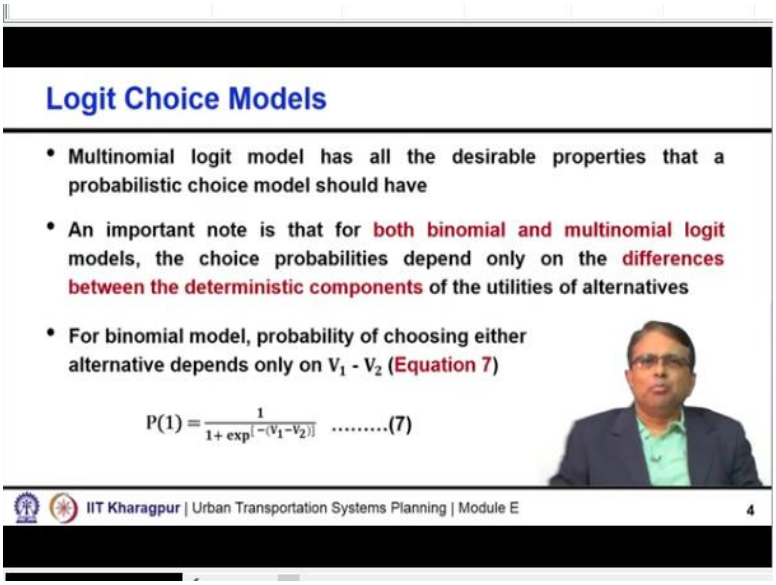

Now, multinomial logit model has therefore, all the desirable properties that a probabilistic choice model should have, we said already that binomial logic has got all the 3 of the 4 properties, the only limitation was that it could handle only 2 alternatives. So, that limitation is also overcome by the multinomial logit model. And important notice that both for binomial and multinomial logic model, the choice probabilities depend on the differences between the deterministic component of that utilities of alternatives.

How the probability is getting controlled by the 2 utilities? It is basically difference not the ratio. And what is said that in binomial logit model probability of choosing alternative 1 is e to the power e 1 divided by e to the power e 1 + e to the power u 2. So, if we divide denominator and numerator both by e to the power e 1, then we get that it is 1 by 1 + e to the power minus within bracket, then V 1 – V 2, remember that it is V 1 – V 2. So it is the difference in utility that is controlling, how it happens when there are more than 2 alternative? More than 2 alternative case also, it is basically the difference.

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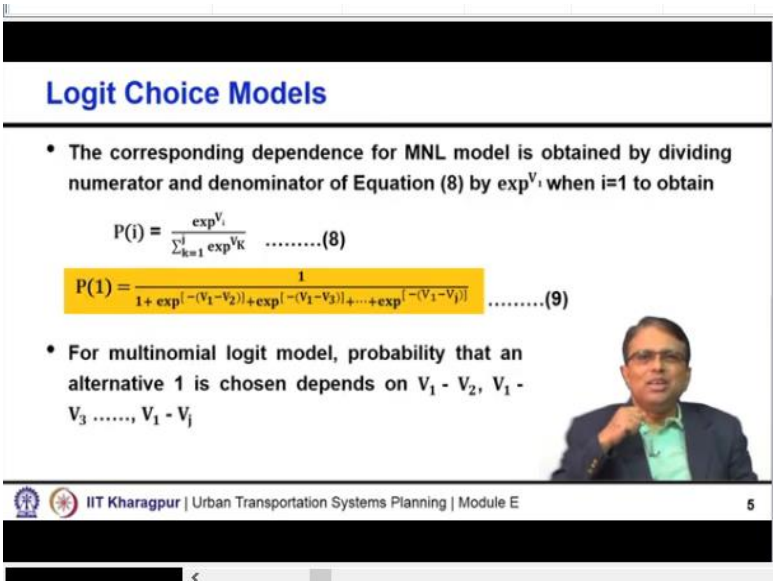

So, if we again divide this general expression e to the power V i divided by e to the power V k, some of our k = 1 to j, then in this case also, what do we do if we suppose divided by e to the power V 1, so what we will get? We will get 1 by 1 + e to the power – V 1 – V i. Similarly, V 2 – V i V3 – V i. So, that is in each case, what is the difference in the utility value? That is what is indicated.

So, in this case also, if we say what is the probability of using car and for the car, bus and metro or the alternatives? Say for example, then probability of using car will depend on the difference of the deterministic components of the utility of car minus car and bus car and metro, where car, bus and metro are the alternatives. So, similar thing is same thing is shown here in equation 9.

So, it is basically the difference that is really controlling. Binomial logit case, it is only the difference between 2 because there are only 2 alternatives. So, difference between deterministic component of utilities of 1 and 2, here it is, 1 and i, 2 and i, 3 and i, 4 and i so on. So, it is basically the difference.

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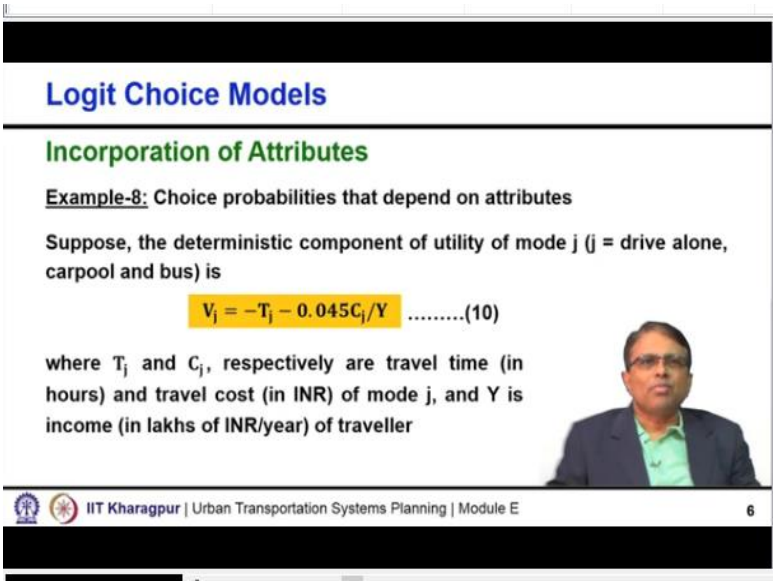

Let us take an example to show you how incorporation of attributes can be done and how we can do the predictions depending on the attributes. So, one example we take for choice probabilities depend on the attributes, that means attributes of alternatives and socio economic attributes. So, we take the same equation, what very similar type of equation if not same, that utility of alternative j V j = - T j - 0.045 C j by Y.

C j is the cost of alternative j, T j is the travel time by alternative j and Y is the income of households in lakhs, lakhs means in 100000s Indian rupees.

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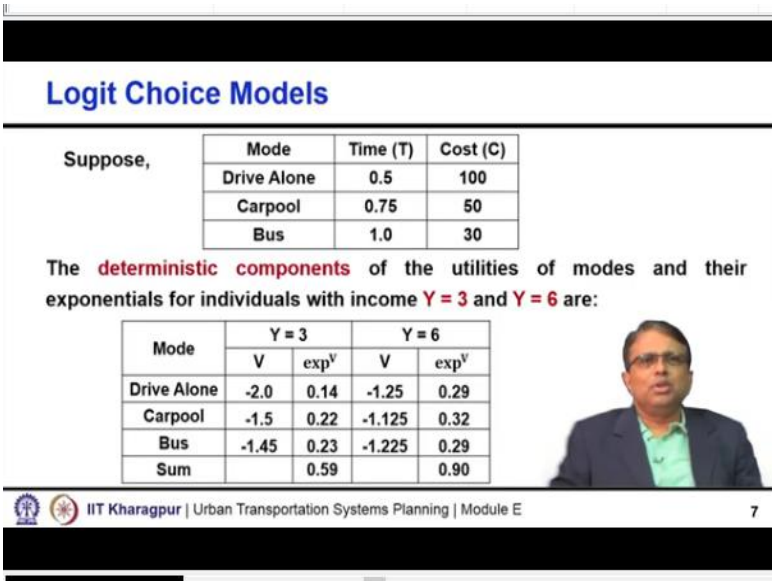

So, if we know that travel time and travel cost by 3 alternative mode, say drive alone, carpool and bus are as given as shown here. Then, if the income levels are 3 lakhs and 6 lakhs Indian rupees per annum respectively for 2 income groups. Then value of Y is 3 and 6 respectively.

So, for value of Y = 3 and 6 as shown here using this drive alone travel time and travel cost and for Y = 3 we can calculate what is then the value of V deterministic component of utility for drive alone.

So, it is – 2. Similarly, for income of 3 lakh per annum Y = 3, we can calculate the utility of carpool and bus of course, the deterministic component of utility. Similarly, we can calculate also when the value of Y is 6, then what is the deterministic component of utility for drive alone, again similar thing for carpool and for bus. Then we can calculate what is then exponential V e to the power V value, then some of e to the power V value.

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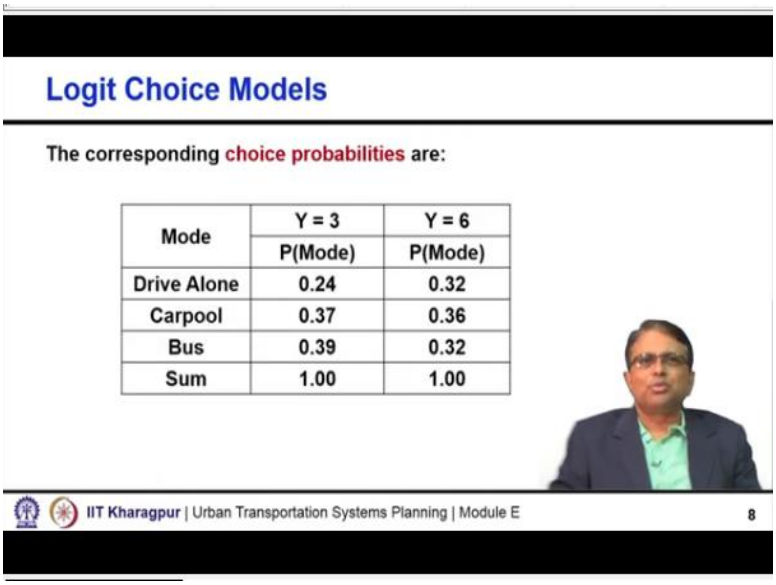

And then accordingly once we know all these we can calculate what is going to be the probability of using different modes for 2 income groups what is shown here and you can see here the results are quite logical, in the sense that when the income increases obviously probability of using bus reduces. So, 0.38 or 39 it goes to 0.32. Similarly the probability of using drive alone increases with the increase in income.

So, as the income is increasing more probability that people will use higher probability of using drive alone and also you observe lower probability of using the bus which is both observations are quite logical, as income goes there will be less probability of using bus and more probability of using car. That is reflected.

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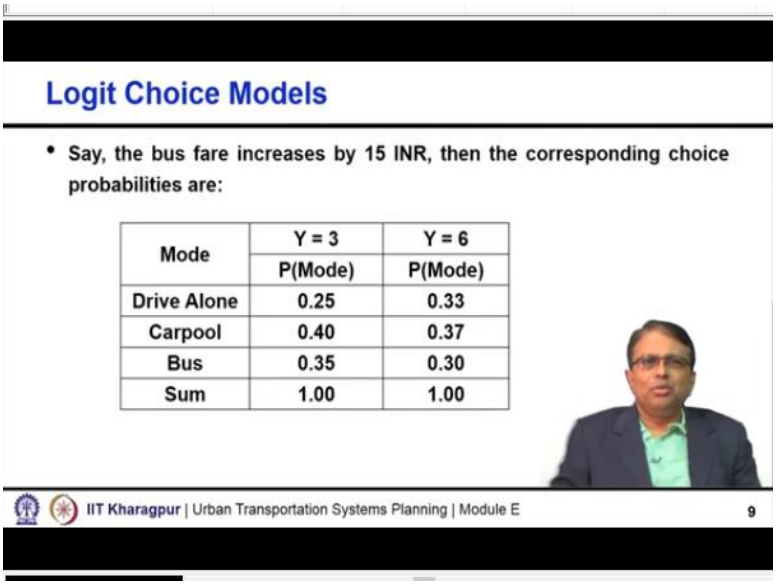

So, that way we can calculate. Now let us see how the attribute changes, say for the bus now let us consider that the bus fare is increased by 15 rupees, what will happen? We know that if the bus fare increases then the deterministic component of utility for the bus will increase for both income groups of course the change may not be exactly the same, but it will change actually and because that bus utility will change, so some of the e to the power V i or e to the power V drive alone, V carpool and V bus.

That sum also will change, the probabilities also will change and you can see here now what will be the probability? The probability of using drive alone for 3 lakh per annum income group will be 0.25, carpool 0.4, bus 0.35 and similarly we can calculate also for the income group of 6 lakhs per annum. Now these are 2 independent calculations very similar calculations only this rupees 15 is added for the bus fare. And naturally the utilities are changed and the probabilities are also very different.

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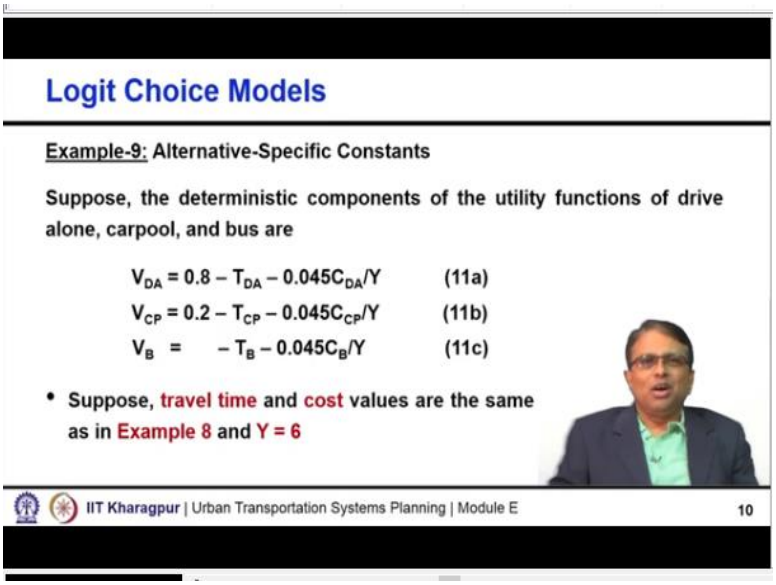

Now how different the probabilities are? You can see that with increase in fare what has happened? Earlier case the bus for 3 lakh income people the bus probability of using bus was 0.39, that has become 0.35. So, bus fare increases, so probability of using bus will reduce because the bus disutility will increase or utility will decrease further. Same thing for 6 lakhs income group it was 0.32, now it becomes 0.30.

So, again it is the results appear to be logical, as the bus fare increases the bus disutility also increases. So, therefore probability of using bus is reduced and that is reduced for both income groups, but still even in other way that it will still remain logical because you see that low income if I consider 3 lakh is low income and 6 lakhs is high income, because there are only 2 income groups, one is lower as compared to the other.

So, I can consider them as low income, high income group you can still say low income group probability of using buses higher and for the high income group probability of using versus lower, for high income group on the contrary probability of using drive alone is higher 0.33 and probability of using drive alone for the low income group is lower 0.25. So, altogether results appear to be logical.

Now in most of the cases you expect also the alternative specific constants to be there because whenever we are considering specific mode and we are developing demand model so the mode specific busness will be there, we explained what is alternative specific constant, the moment it is drive alone many other factor people will probably think that okay I will

have my privacy much bigger, my flexibility much better at any time I mean even if I am 10 minutes late I just simply I take out my vehicle and travel freedom is much better.

So, maybe there will be many other considerations, which will not be reflected through consideration of travel time and travel cost, there will be other attributes which will influence. So, those influences will get reflected. So, suppose, you see that 11a, 11b and 11c are the utility equations, where AC is considered and of course, the bus is the base mode, so, per bus it is 0.

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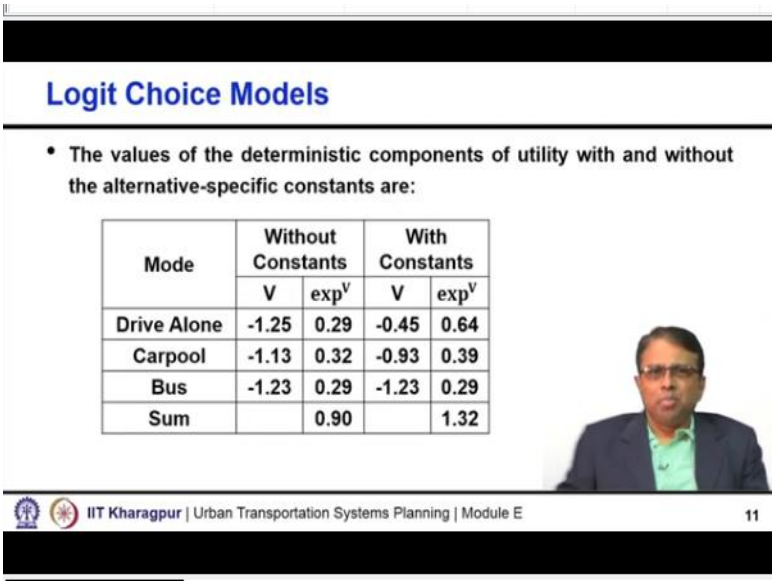

Now, the same example with the similar travel time, travel costs values we can also calculate the utilities exactly in the same manner, calculate e to the power utilities some of the w to the power utilities for all the alternatives considering drive alone, carpool and bus. Then with, with constant and without constant that means, same example, you can see 11a, 11b, 11c, they are in a similar in terms of the travel time and travel cost and income consideration even the coefficient.

The only difference is the alternate specific constant. So, if you consider alternate specific constant then with and without alternate specific constant how the values are there? Now, of course, without we have already worked out earlier, but I am again showing it so, that you can just see it side by side and you can compare the values.

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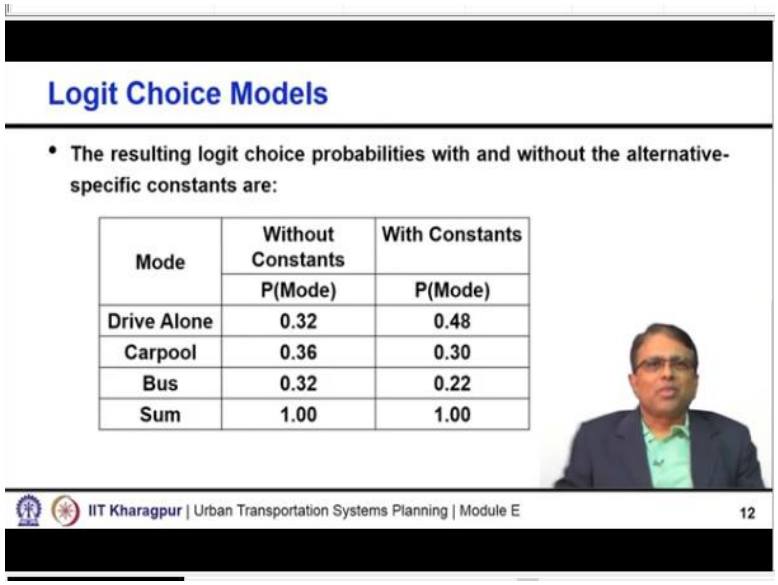

And you see because of this drive alone the high positive value of the AC the e to the power utility value is much higher, see without constant it was 0.29 whereas, when we consider the AC the value goes 0.64. So, utility enhances. So, utility enhances what is expect them? The probability of using drive alone will be higher when you consider the AC. That is what is shown here in this next slide.

Here I am showing then what is the probability of using different mode when you do not consider the constants (()) (17:20) when you consider the constant values AC values are in a more specific busness, then what are the probabilities? You can see here very interestingly drive alone you have now considered the alternate specific busness. So, preference will be where maximum utility will be for drive alone followed by logically carpool and then the bus, bus is based mode 0 compared to bus the positive value for carpool.

And even a higher positive value for drive alone. So, utility of drive alone and carpool increases. So, naturally overall probability distribution of different modes or the probability values probability of choosing the respective modes will change. Now, you see instead of 0.32, the probability of using drive alone becomes 0.48, simply because there is a high mode specific or alternate specific constant.

Similarly, now, because of that the busness will be more towards drive alone and carpool. So, the bus is reduced probability of using bus reduces 0.32 to 0.2. So, you can see very logically when you consider the attributes and their attribute levels, when you consider the change in

the utility and when you also when you bring into consideration the mod specific busness all cases very logical results it will give. So, that way the MNL models are really excellent.

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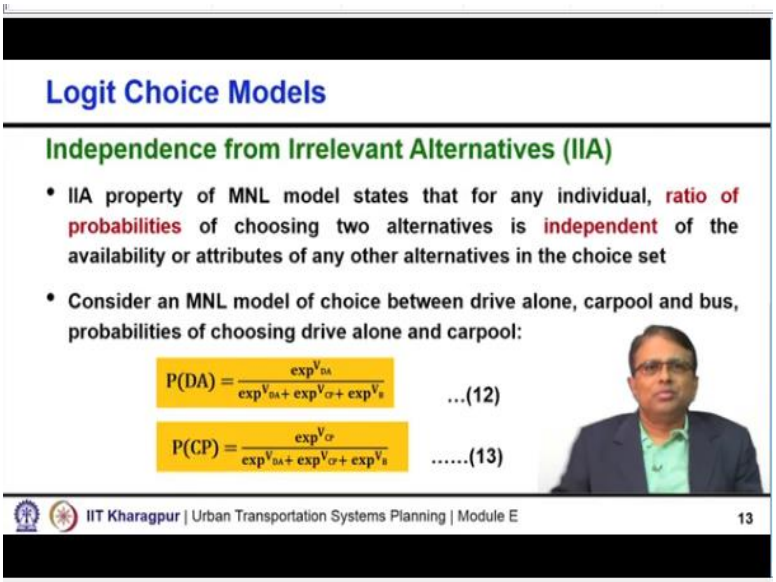

So, now with this now we will go to another property which is a difficult thing with the multinomial logit model. That is actually called independence of irrelevant alternatives, IIA independence from irrelevant alternatives. What is that? IIA property of MNL model states that for any individual think of an individual, the ratio of probabilities of choosing 2 alternatives, ratio of probabilities, it may be probability of using drive alone by probability of using carpool.

Probability of using drive alone by probability of using bus, the ratio of probability of choosing 2 alternatives is independent of the availability or attributes of any other alternatives in the choice. How to explain that? Let us say probability of drive alone will be what e to the power V DA divided by e to the power V DA + e to the power V CP carpool + e to the power V B, when I have drive alone, carpool and bus 3 alternatives.

Probability of using carpool is how much? Only the numerator will change, numerator will become instead of e to the power V DA it will become e to the power V CP denominator will remain same as before.

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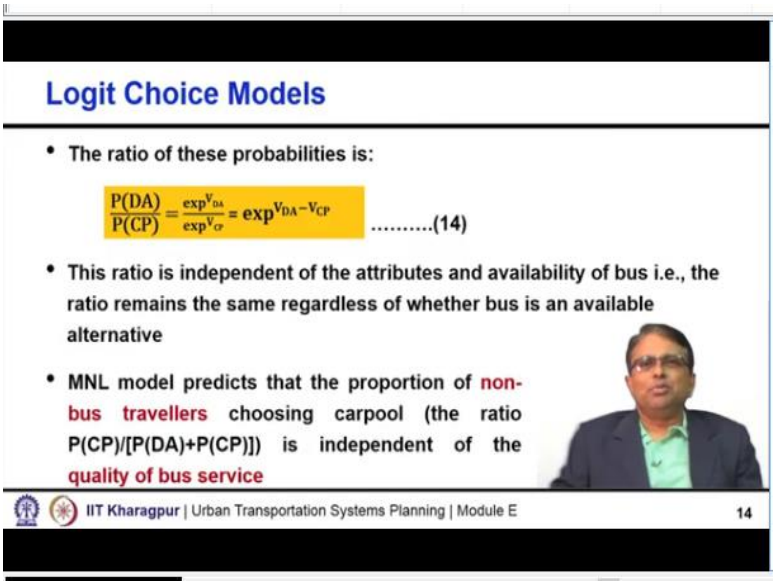

So, now if I take the ratio what it says? It says probability of using drive alone divided by probability of using carpool the ratio of probability of taking 2 alternatives equal to e to the power V DA – V CP, so it is controlled by what? Control by deterministic component of utility of drive alone and deterministic component of utility of carpool, irrespective of what happens to the bus?

Bus utility may go up, may go down, the probability may change, probability of using different modes, but this ratio remains unchanged because this ratio is not a function of the utility of bus. Similarly, if you take the ratio of P DA by P bus ration that only will depends on deterministic component of utility of driver alone and deterministic component of utility of bus, carpool is not going to appear just because of the model form.

So, that mean the ratios depend on only those alternatives, deterministic component of only those alternatives any third, fourth, fifth alternative even if they are present irrespective of their values, this ratio remains unchanged, as long as the probability of suppose in this case the DA and CP do not change, whatever you change for the bus and even if there would have been a fourth, fifth, sixth alternative, any change happens anywhere. These 2 ratios are not going to change. Now, this is the IIA property.

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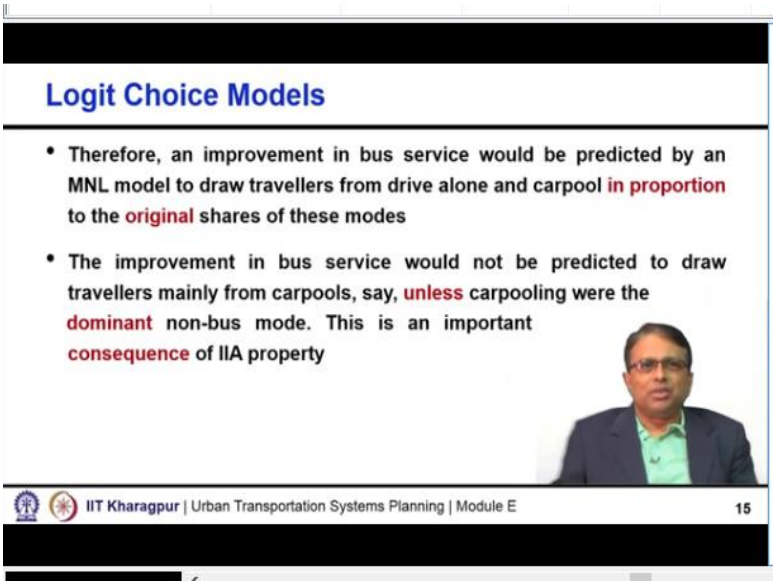

So, even if you are changing now the bus utility changes suppose increase or decrease the probabilities will change, but probabilities will change without changing this ratio, that is in the same proportion whatever has been your original proportion in the same proportion the probabilities are going to be. So, the ratio will remain, if higher more people are going to use bus because of the reduction in disutility of the bus or increase in the utility of the bus whatever you say more people are going to use bus.

Naturally less people will probably use drive alone and carpool, but how people will shift from driving around and carpool, they will shift in the ratio keeping that ratio unchanged. So, in that proportion from these 2 modes the shift will happen. Is it very logical?

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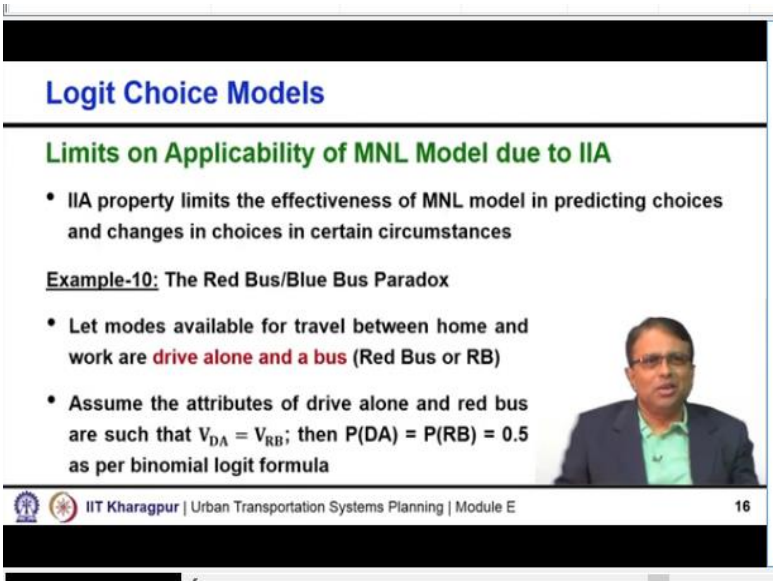

No, this does not appear logical. Why? Let me give you a very famous example, another world famous example of the red bus, blue bus paradox, what we are saying let modes available for travel between home and work are only drive alone and the bus, nothing else. So, for the moment we do not consider carpool or any other train or metro we do not consider only drive alone and bus.

And just consider that all buses which are available are run by a particular company and all buses are of red color, just an assumption, that to say that this is one group of bus. So, all bases are pain red or painted with red. Now then what will be the and also assume that the utility of drive alone and red bars are same, maybe you have with fare adjustment, with all these things one has done in such a manner that for both the modes available driver alone and a bus the utilities are same.

So, then what do you expect? You expect them probability of using DA equal to probability of using a red bus RB is equal to 0.5 because utility values are same. So, simply I would apply a binomial logit and I will get value in each case the probability of using that alternative is 0.5 because deterministic component of utilities are same, clear up to this.

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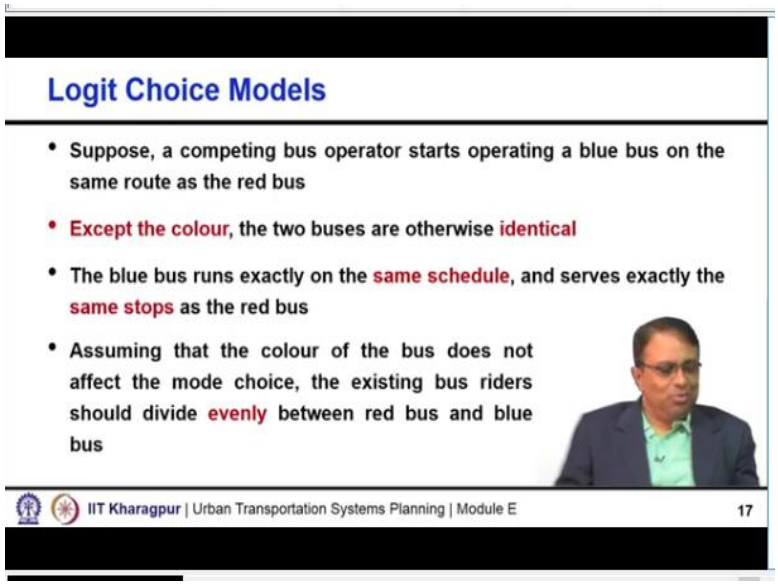

Now, suppose, now, another competing bus operator starts operating bus also in that route. Exactly same number, exactly the same manner, no difference, except now this new bus company starts operating buses which are blue color bus, the painting color is different, painted with blue instead of red. And let us call them as blue bus. Since the utility depends only on time and cost and exactly all are same.

So, naturally the utility then remain unchanged, utility of red bars and utility of blue bus they are not going to be different, they are going to be exactly same and because the utility of red bus and the utility of drive alone were same earlier. So, all 3 alternatives red bus, blue bus and the car all will have same alternatives utility. So, what do we expect logically? What do we expect 50% people were using bus earlier those 50% will get distributed to this red bus and blue bus?

And because both services are exactly the same in every sense, except that one is painted with blue color and other is painted with red color, naturally the share will be equal. So, what do you expect probability of using drive alone will be 0.5, probability of using a red bus which was earlier 0.5 will now become 0.25 and probability of using this blue bus will be 0.25?

So, basically the bus demand will get distributed to these 2 verses 0.25 and 0.25 drive alone will remain 0.5. But what the logic model is going to give you multinomial logit model because of this IIA property, the ratio of probability of using drive alone and probability of using red bus will remain unchanged same. So, what you are going to get?

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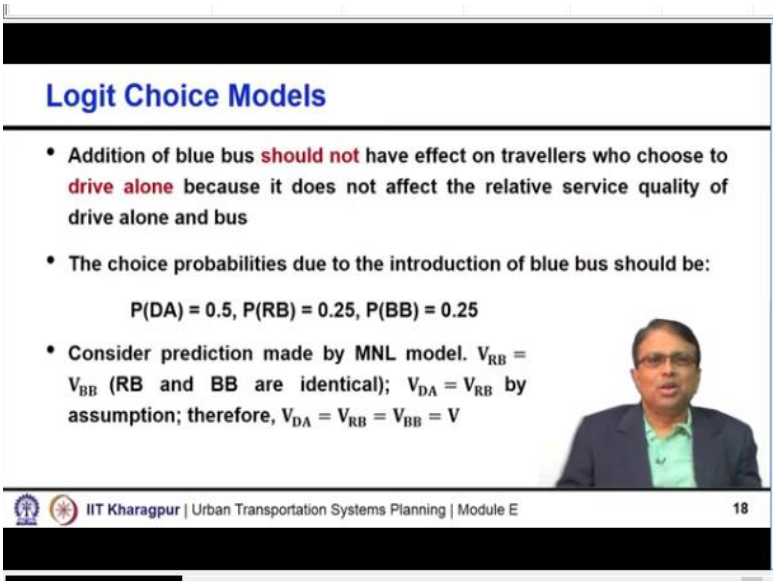

You have 3 modes because of this IIA property remember that what you are going to get? You are actually going to get a distribution like this.

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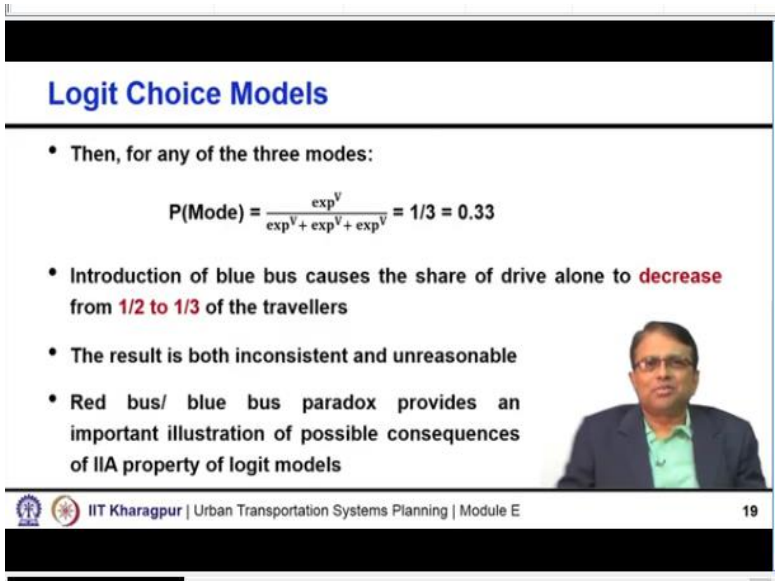

E to the power probability of drive alone divided by or any one more divided by e to the power D drive alone + e to the power V blue bus + e to the power V red bus, this V red bus, B blue bus and the V of the red plus, blue bus and the drive alone are same. So, what is going to happen? You will find that the probability of using each mode will be 0.33. Does it sound alright? No, it does not sound very logical. But then why it happened? It happened simply because of the IIA property of the multinomial logit model. That ratio remains fixed.

So, that is where the problem comes and this red bus, blue bus paradox provide an important illustration of possible consequences of IIA property of logit models, whenever you are going to bring another alternative your original ratio of the probabilities of 2 alternatives any 2 alternatives will remain unchanged. So, what happened in this case exactly is controlled by IIA. So, equal amount of shift equal proportion of shift will happen from the 2 mode to the third mode.

Equal proportion means as per the original ratio and in this case they were equal because the drive alone and carpool both the probabilities were 0.5. So, in this case it is equal that is why I said equal also, equal is also correct. So, same proportion the shift will happen, it has to happen because why because of this IIA property the ratio will remain unchanged. If the ratio to remain unchanged then it can only be multiplied or divided by the same factor.

So, it tells you the result multinomial logit will tell you one third people will now use drive alone, one third will use red bus, one third will use blue bus, but what you actually expect you do not expect drive alone people to shift because you have only considered travel time

and travel cost nothing else and both buses are same in terms of their travel time, travel costs and everything except that one is painted with red and other is printed with blue.

So, you expect the probabilities to be drive alone to remain as 0.5 and only the probability of using bus will now get equally distributed, so 0.25 and 0.25 that is what you expect, but whereas multinomial logit model will say it will be one third, one third, one third for each mode, why this is happening? It is happening because of this IIA property. So, this is a limitation of the multinomial logit model.

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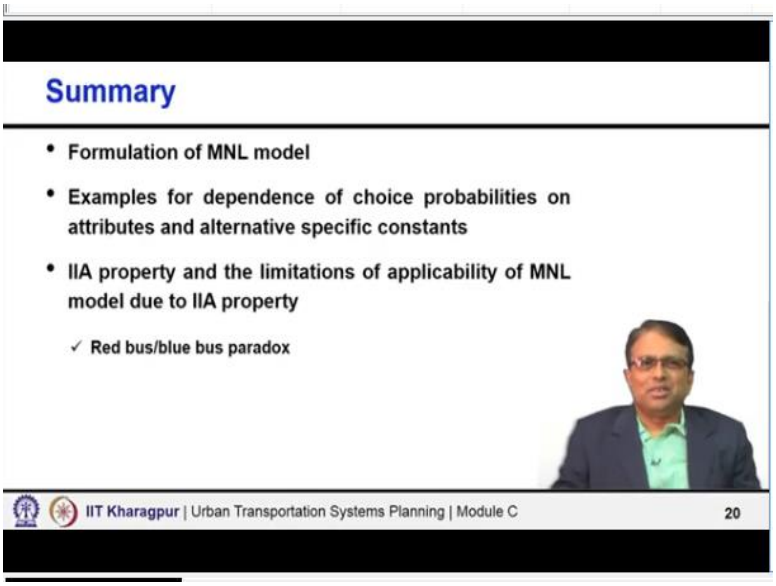

So, in summary we discussed in this class about the formulation of MNL model, we gave you 2 examples of dependence of the choice probabilities, one taking only the attributes and then also taking the alternate specific constants and comparison of results also where was shown to you. Then we explained to you what is the IIA property of multinomial logit model which is a problematic aspect of the multinomial logic model, what IIA we say that what is the independence of irrelevant alternatives, the ratio of probabilities of using 2 alternatives remain unchanged and does not depend in any way on the utilities of any other third alternative.

It only depends on the utilities deterministic component of the utilities have 2 alternatives which are under consideration. The ratio remains same, so whatever happens the probability may increase, may decrease but the ratio will remain unchanged and that will be explained to you very nicely with the help of the very famous red bus, blue box paradox, with this thank you so much and I close this lecture.