Urban Transportation Systems Planning
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Lecture - 26
Singly Constrained Gravity Model

Welcome to module D lecture 6. In this lecture we shall discuss in details about singly constrained gravity model. In our last lecture that is D 5.

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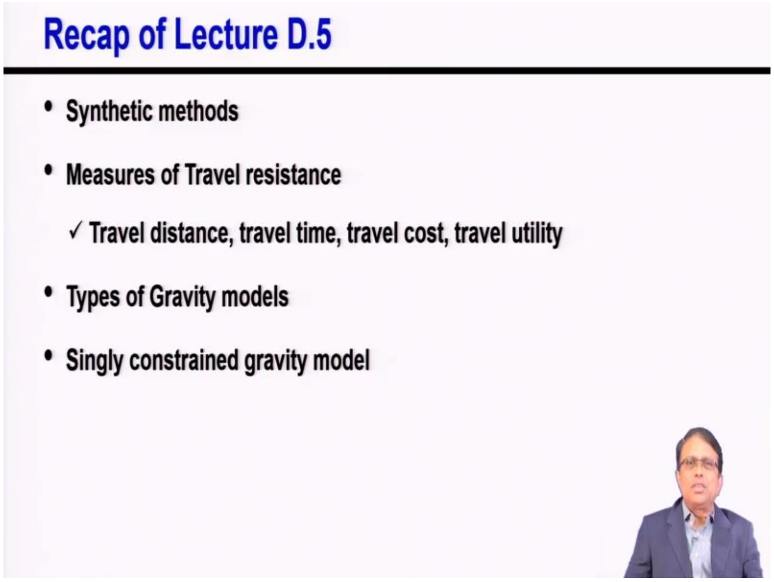

We discussed about synthetic methods, what are uniqueness or how the synthetic methods are different from the growth factor base method? Then we talked about various measures which may be used to express the travel resistance or travel impedance. For example, travel distance, travel time, travel cost, travel disutility or utility etc. Then we talked about two types of gravity model, mainly the singly constrained model and the doubly constrained model.

And we said that why we call them a singly constraint, why we call them as doubly constrained and then we give you an example of a singly constrained gravity model. So, we shall continue today from that example. So, let us get back to the slide.

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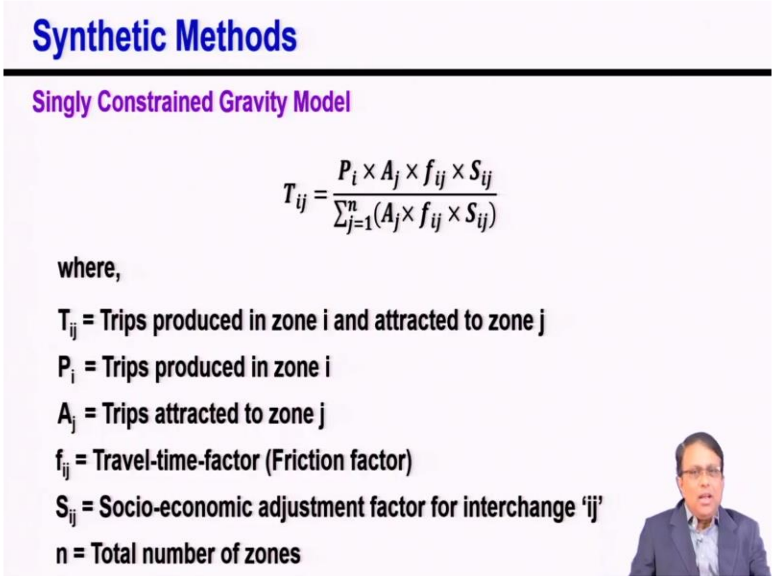

We said here the T ij equal to P i, A j, f ij divided by sum over A j, f ij. Let us for the moment ignore this S ij which is socioeconomic adjustment factor for interchange ij. Let us ignore that part and let us take that values, all S ij value as 1 or unity. So, we have you know, let us start with a simpler formulation, P i, A j, f ij by sum over A j, f ij. How does it relate to the gravity model? I said that basic formulation is what?

Trip distribution is proportional to P i production A j attraction and inversely proportional to some special separation. That means more the distance less will be the trip more, the cost less will be the trip, more the time less will be the trip and so on. So, it is inversely proportional and normally it should be in the denominator. But let us call that we are taking the value of f ij in such a manner, that higher the distance or higher the cost, lesser is the value of f ij.

That means f ij equal to let say in a very simple manner, it is almost like f ij equal to 1 by T ij or 1 by T ij to the power whatever you say n or cost to the power n whatever you say. So, that means f ij is just use as a multiplier, but it values changes in such a manner that higher the cost, higher the distance or higher the time, then lesser is the value for the f ij. So, simply as I said that if I am dividing it by say a value anything x, I can always multiply it with y, y equal to 1 by x.

So, it is just used as a multiplier. So, up to that part then P i, A j into f ij that is exactly like our gravity model formulation what we have discussed so far proportional to P i proportional to A j and inversely proportional to some special separation say distance cost or type. But then what I said? I said one more thing that in this case zone i is not only connected to one particular zone j, but it is connected to may be n number of zones which are all destination zone.

And for each of those zones there is an attraction and there is in corresponding f ij value. So, it has to be something like in relative sense. So, how much of the trips which are produced in zone i? So, P i number of trips are getting produced to from zone i how much will go to zone j, a particular zone will depend on what is the value of attraction and what is the friction factor for that particular zone, relative to all attraction zones which are connected to i.

Because every attracting zone is competing with each other every attraction zone has got an attraction value of A j every attraction zone has got a value of f ij. So, one particular zone what is that attraction and what is that friction factor relative to all zones destination zones which are connected to production zone i, they are corresponding attraction and f ij value. So,, that is what is done here P i, A j, f ij divided by sum over A j, f ij.

What is sum over all destination zone j and if I have a number of zones, then j equal to 1 to n. So, now this formulation is clear why we called it as a singly constrained model, because simply if you add T ij sum over all j, then what this equation will give this equation itself will give you P i. So, the model form ensures that T ij sum over j is equal to P i. That is why it is singly constrained, but the other end does not come out automatically.

If I add it over some over i does it give me A j no it does not give me A j automatically. So, that means one constraint is satisfied that the production constrained is satisfied, attraction constrained is not satisfied in this case. So, we call it as a singly constrained gravity model, what kind of singly constrained, you can call it as production constrained gravity model. Now one more interesting thing before I proceed further. And one more thing as I said that what is this S ij?

The role of S ij, I shall explain you very clearly when I talk about the calibration of this friction factor, how to get this f ij values when I discuss that in my next lecture that time I shall explain you clearly where and what is the role of this S ij. For the moment let us assume that S ij as one, unity for all the zones. The interesting points what I was trying to mention, I can also write this model as an attraction constrained model.

It will be again a singly constrained model gravity model, but attraction constrained, attraction ends will be constrained. How I can write it, just it is not written here, but follow me in that case I will say T ij equal to A j. Attraction how much total attraction is coming to zone j, A j multiplied by P i, f ij divided by sum over P i, f ij sum over all i, i equal to 1 to n that means total attraction is coming A j.

How much is coming from a particular zone i will depend on what is the production of that zone and what is the f ij value? But note that, that is the only production zone which is connected to this attraction zone j, there are many other production zones which are connected. So, relative to all the productions of each zone which are connected to zone j and the corresponding f ij value? In that equation if you sum it over j it will not give you P i.

But if you sum it over i it will give you A j, because it is A j multiplied by P i, f ij by sum over P i, f ij sum over all i. So, if I sum it over all i, numerator and denominator will cancel and you will find only A j. So, that attraction constrained will be satisfied. But in that case sum over i it will not give you P i. So, the production ends will not get satisfied automatically by the model. So, both ways one can write.

Singly constrained gravity model, we can write it like a production constrained model, I can also write it as attraction constrained model.

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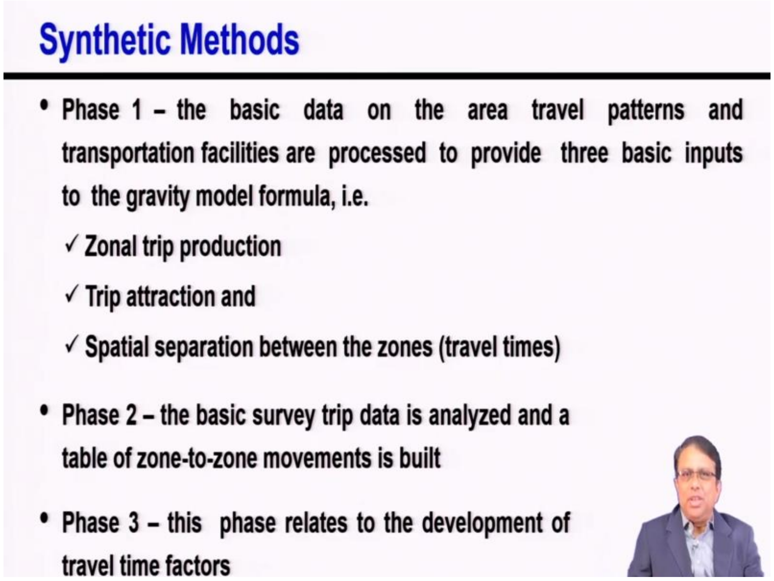

Now how when we develop this kind of model, how we go, we go phase wise. First phase we need to have all the basic data, like as we have seen every process to generation process to distribution process, every process starts with some data, some input. Here what are my inputs? My inputs are zonal trip production, zonal trip attraction, that means I must know what is the value of P i for all the production zones i equal to 1 to n.

What are my attractions for all these zones j equal to 1 to n and I need to know the spatial separation between zones. If I want to express it in terms of travel time, then the travel time, if I also want to express it as distance then as distance, if I want to express it in terms of cost then the cost. So, for each pair of zones, what is the travel time or the travel distance or the travel cost that is my basic row input in phase one.

Then phase 2 what I want, I need to have the basic survey trip data. And a table of zone to zone movements, because you require this for the calibration of friction factors f ij we have the T ij travel time value or the cost value or the distance value, but that is to be then translated into friction factors f ij value. So, if I want to translate it to f ij, then that f ij curve or f ij function needs to be calibrating.

Now to calibrate the f ij we also need to know that how much zone to zone movement is happening today, that means giving today's production, giving today’s attractions, giving today's travel times if we travel time is used. Then how much travel or what are the T ij values. Then I shall calibrate my friction factor f ij value with respect to the travel times to reproduce the present distribution.

That is what is phase 3. So, in this phase relates to the development of the travel time factors or friction factors, those f ij values as I have shown in the previous slide or in this slide exactly. So, that is the calibration of f ij that is phase 3.

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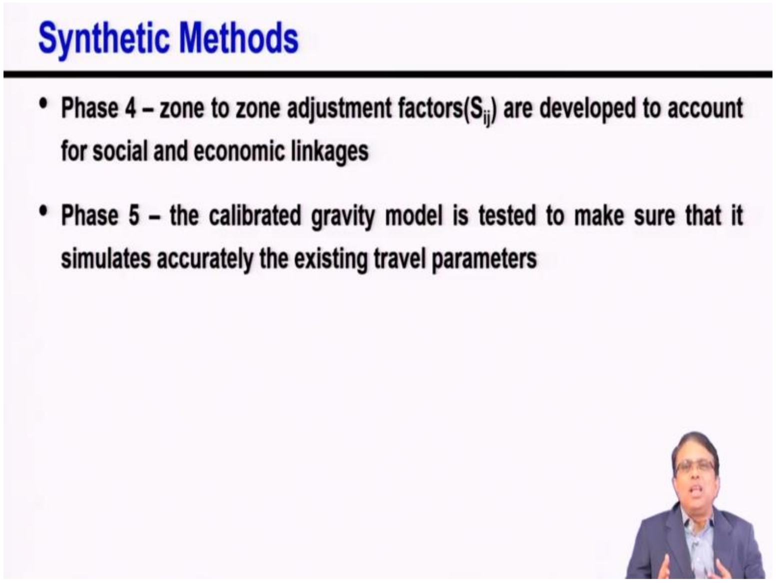

Then phase 4, I also now need to develop this zone to zone adjustment factor. This is the state where you need to develop zone to zone adjustment factors to account for social and economic linkage. Again further details, I have not told or not explained to you so far, why we need this S ij, I will come back. Next lecture when I talk about the calibration of this friction factor then I will explain you that.

Then phase 5 the calibration that gravity model is tested to make sure that it simulates accurately the travelling parameters. That means basically you are now ready for application. Now, let us take an example to tell you how one can apply it. Of course I am not checking that whether I have accurately predicted the trip, I have assumed that all those have done everything is available.

Now, I want to do the synthesis of the current travel pattern. I want to get the cell values how to get it. Let us take the example.

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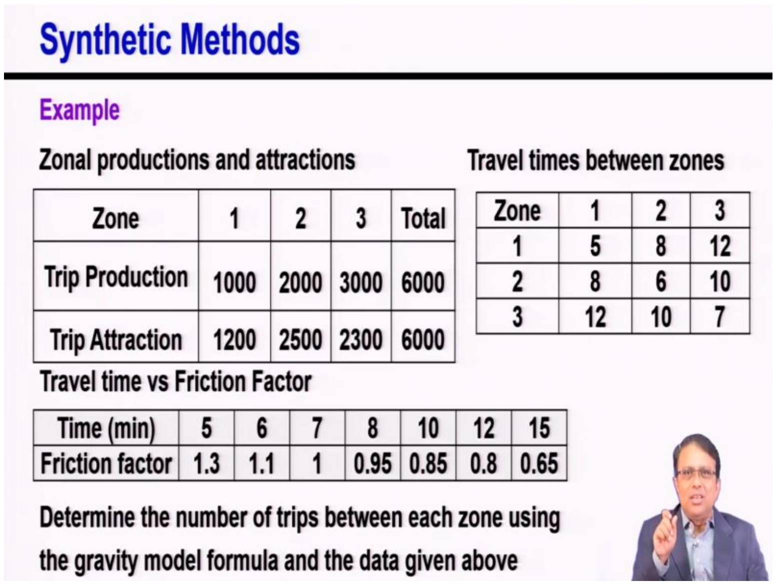

So, let us take that this is only an example. So, I have taken a small example. Let us take that we have only three zones. Zone 1 production is 1000, 2 - 2000, 3 - 3000. So, total production is 1000 + 2000 + 3000. So, 6000, similarly the respective attractions are 1200, 2500 and 2300. So, total is again 6000. So, this checking is important because total productions from all the zones in the study area must match with the total attractions from all the zones in the study area.

This has to match, if there is any imbalance at this stage what we will do? We will remember all this I have done it, this discussion in the previous module, but what we will do? We shall then assume that the total productions are correct accordingly at just these attractions. So, that the; total attractions match with the production. So, this is one input given, then the travel time between zones we are taking travel time here instead of distance or cost.

So, the travel times are also given 1 to 1, 1 to 2, 1 to 3, 2 to 1, 2 to 2, 2 to 3, 3 to 1, 3 to 2, 3 to 3, all these travel times are given and someone has already developed a friction factor car or friction factor values for us with respect to this travel time. So, for 5 minute corresponding friction factor values 1.3, 6 minute 1.1, 7 minute 1, 8 minute 0.95 like that up to 15 minutes.

Somebody has developed this and giving it to you.

How to develop? That will be discussed in lecture 7. Now consider that, look at this interesting thing travel time is more and actual in the original where I introduced gravity model what I said that divided by D ij to the power sum n or whatever it is. So, the distance increases, time increases, the less number of trips are likely to go. But here you see f ij is used as a multiplier. So, higher the value, lesser value of travel time, lesser is the value for friction factor.

Since it is used as a f ij use as a multiplier we are using it in the numerator. So, multiply by f ij. So, that is why the function gets calibrated in a way, so that lesser the travel time, higher the f ij value, higher the travel time lesser the f ij value. So, now with these we want to show you how you can calculate each individual cell of T ij using a production constrained gravity model. Let us go to the next slide.

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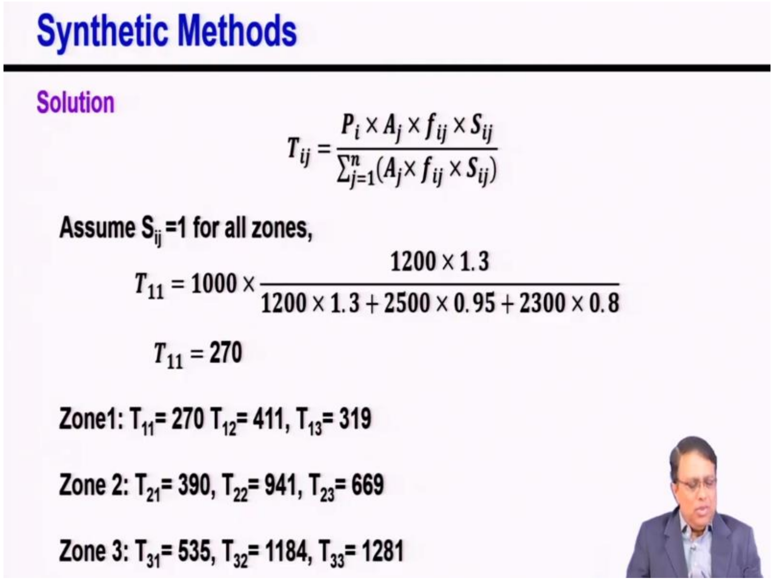

Here that is what is our equation T ij equal to P i, A j, f ij, S ij divided by sum over A j, f ij, S ij. Now, as I said that I have not yet explained you what is S ij. So, let us take S ij as one for all zones. Then what is T i, T 11, P i is what 1000 look at this production from zone 1 is 1000. So, 1000 multiplied by A j. What is A j in this case? A 1 because we; are calculating for the cell T 11 attraction of zone 1, 1200 into f 1 1.

What is the value of f 1 1? Look at the previous slide 1 1 means travel time is 5. So, 5 corresponding section factor value 1.3. So, 1200 into 1.3 divided by; obviously all zones which are connected. So, all 3 zones 1, 2, 3. So, corresponding to 1, it will be again this 1200 into 1.3, then A 2, what is A 2 attraction of zone 2, 2500 then 1 to 2 how much is the travel time? 8 minute, 8 minute corresponding friction factor value is 0.95.

So, you go there 2500 into 0.95 + zone 3, what is that attraction 2300 multiplied by let us see what is the value of 1 3, 1 3 is 12 minute. 12 minute corresponding f ij value is 0.8. So, come back 2300 into 0.8. So, like that you calculate, so what is the value of T 11, it will be 270 if you calculate it and hopefully my calculation is correct. Same way you can calculate what is then T 12, T 12 what will be there. It will be same like T 11, 1000 into instead of 1200 into 1.3. It will be 2500 into 0.95 divided by the same as it is there in T 11

So, you will get 411 and for T 13 what it would be, again 1000 into 2300 into 0.8 divided by again same, sum over A j, f ij same value. So, you will get 319. So, like that same following the similar procedure, you can calculate T 21, T 22, T 23. Since it is a singly constrained model sum over all j will give you P i production. So, let us check that let us put these values, you add T 11 270, T 12 411, T 13 319.

You add, you get 1000, 1000 is exactly the production from zone 1. Let us see 1000 is the production from zone 1.

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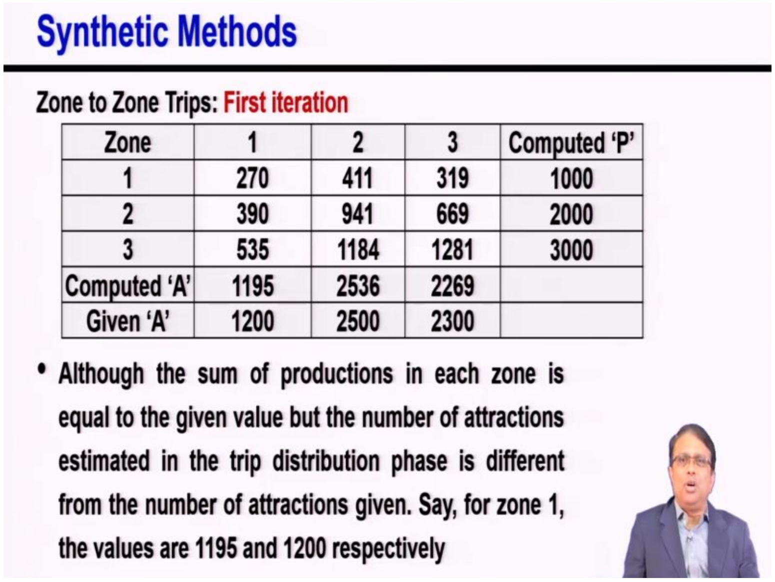

So, it matches, exactly it matches why it matches because you have used a production constrained gravity model. So, automatically by equation it will match you do not have to do anything, by equation, it will match. Similarly for zone 2 also, you see production as matched exactly 2000 zone 3 production exactly 3000. That is fine, that is the good news we expected all these values to match.

But since it is a production constrained model, there is no assurance that attractions will also match automatically, it will not match. Look at this attractions, what we expect, we expect the attraction of zone 1 to be 1200, that is what the values we want and we have used that 1200 values there. But it did not give me 1200, somehow it gave me somewhat closer 1195, still it is better.

But for zone 2 we wanted 2500, we got 2536 some deviation. We use 2500 is the attraction for zone 2. Zone 3 what is the attraction 2300, you should get 2300, how much we get, we got 2269. So, you can see the productions for zone 1, 2, 3 in this example matched exactly with our target productions. But the attractions did not match, because we use a singly constrained model. But we want trip matrix.

Although used singly constraint model end of the day, we want to have a T ij matrix, trip matrix which should match both, which should give good match to the row totals and the column totals that means the productions and attractions of each zone should match. So, what we do? We have to do something externally, because internally model, it is a singly constrained model. So, it was to ensure that the productions match.

But there was no assurance that the attraction will not match automatically, so it does not match. But we cannot go ahead with this we have to do something to match this column totals also or the attractions, so how we go ahead?

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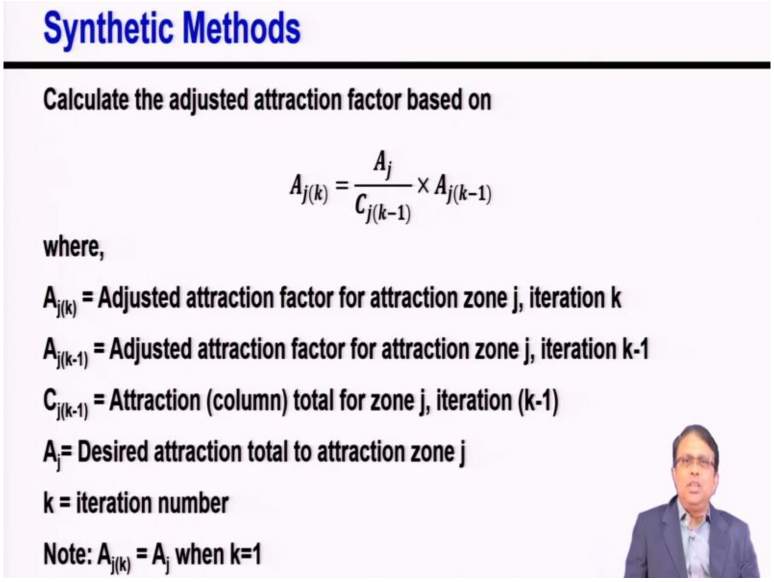

We can use by using this formula, first I will show you as formula, then I will explain you how logically it works. What is the logic behind it? Look at this, what I say A j k that will say to go for an iterative procedure. So, in kth iteration what will be my A j, that means adjusted attraction factor for attraction zone j in iteration k that will depend on what is my A j what is my desired attraction for zone j divided by C j k - 1.

What is this C j k - 1 what attraction total I got for zone j in the previous iteration, because we are thinking of now what will be iteration k - 1, kth iteration. So, we want to know what we got from the previous iteration and A j k - 1 is adjusted attraction factors for attraction zone, you know what we used in iteration k - 1. This formulation is correct, but you can understand it in a very simple manner.

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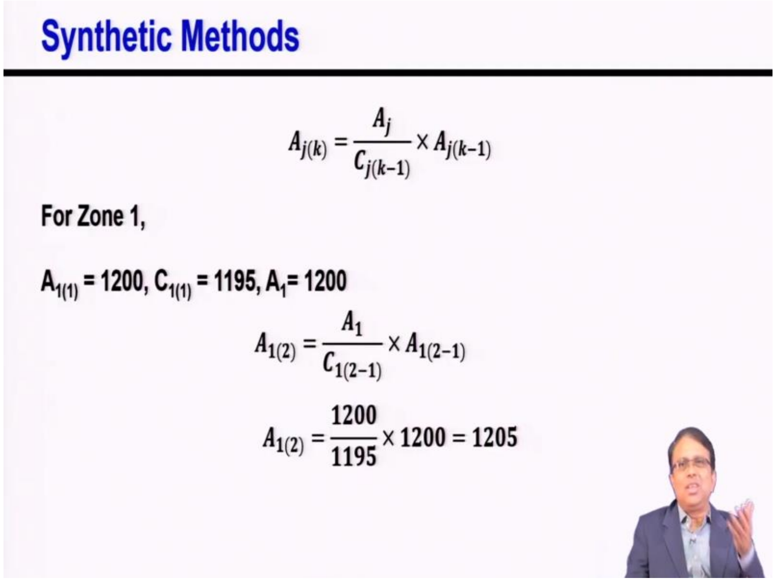

Let me put this question to you suppose in a subject when you are appearing for an exam. You just follow me carefully. In a subject, in exam, you want to get 80, that is what you want end of the day. When you target to get 80, you got 70. Does it tell you something? I repeat you want to get 80, when you target to get 80 you get 70. How you can get 80, the answer is very simple that when I want to get 80 and if I target to get 80, I got 70.

So, probably I should target something higher. I should target to get 90. I should prepare or do everything with my target of getting 90, then probably I will get somewhere closer to 80 exactly the same thing we have doing here. That means you want to get 1200, when you target to get 1200 you got actually 1195. So, what I have to do maybe I have to target little higher. So, now my modified attraction exactly should be like this, so 1200 into 1200 by 1195.

So, I should then target little higher, because I am little short of what I want, so I should target little higher probably then I would be very close to 1200. That is what is the adjustment here. Exactly the same way you can calculate the attractions for other zones.

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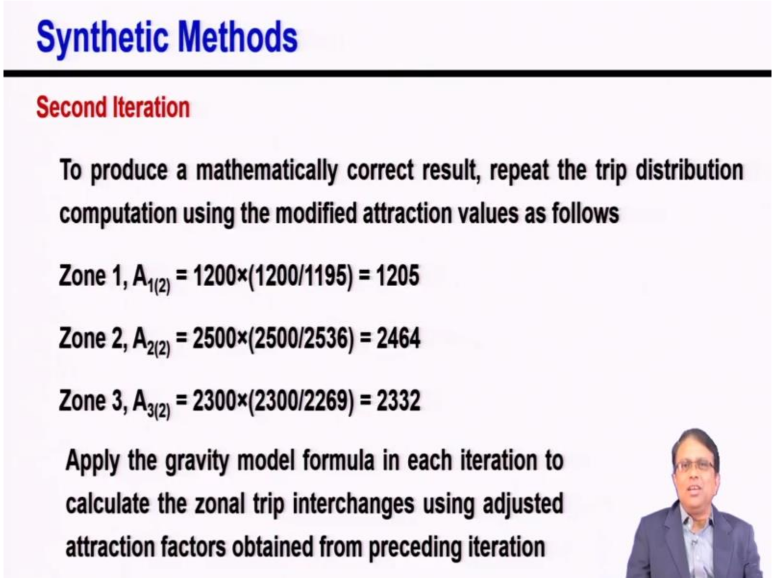

So, I have done this see you know, you wanted to get 1200 you got 1195, so now you are targeting something 1205. You wanted to get 2500, when you target 2500 you get 2536, you got more, so now you target little lesser 2464. When you want 2300 for zone 3 and you target 2300, you got only 2269, so what you have to do, you have to target little higher. That is what we are now targeting not 2300, but we are targeting 2332.

So, that is all new targets so with these new targets, I will not use the attraction as 1200, 2500 and 2300 for zone 1, 2 and 3. Rather, I will use now attractions as 1205, 2464 and 2332 respectively for zone 1, 2 and 3.

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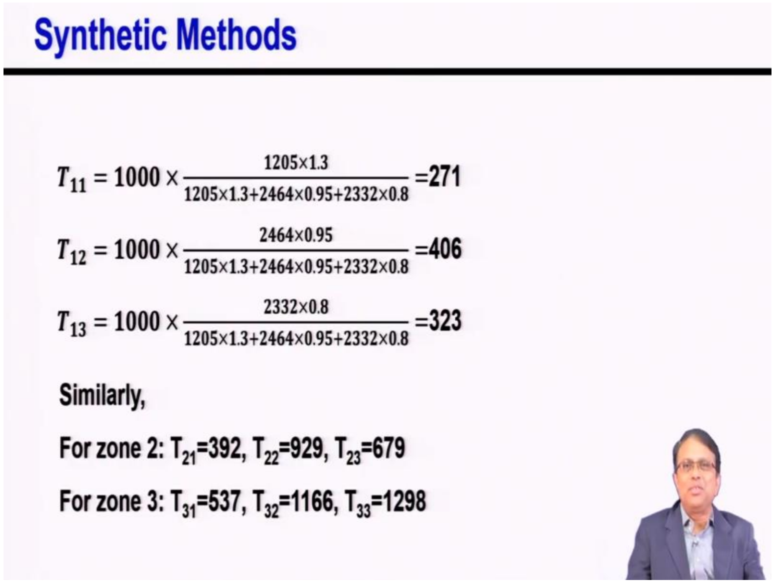

And then I shall go back to the singly constrained gravity model again, everything is same, except now my A 1, A 2, A 3 values are different, my targets are different. I shall again get T 11, T 12, T 13 again if you add all this row totals. It will exactly match your productions it has to match, because of the model form itself the model mathematically ensures sum over j equal to P i, so automatically it will match, and it matched.

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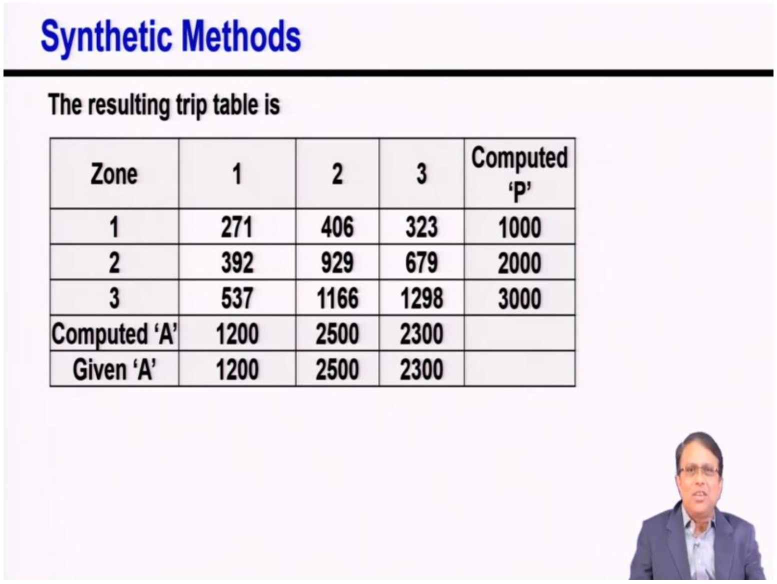

But now what you see here, it is a small example, so maybe in second iteration itself would give very good result. You will find your attraction totals are also matching. Obviously if you take a larger study area multiple number of zones, it may not exactly match in second iteration. It is just because it is a simple example in second iteration itself it gets matched, does not matter go for more iteration.

Again you get, you see what you wanted and what you got accordingly again target the attractions accordingly. Either if you need to target little higher you target little higher or you target little lesser to some adjustment one or two or three or five more iterations does not matter, the computer will do most cases you are not going to do hand calculation. Because any reasonable, you know zone study area will have multiple, you know zones and you will all use computer.

So, set what is your error, acceptable error limit accordingly, iterate and come out and give you the trip distribution when the error is within the acceptable limit. So, that is the way you can actually apply it.

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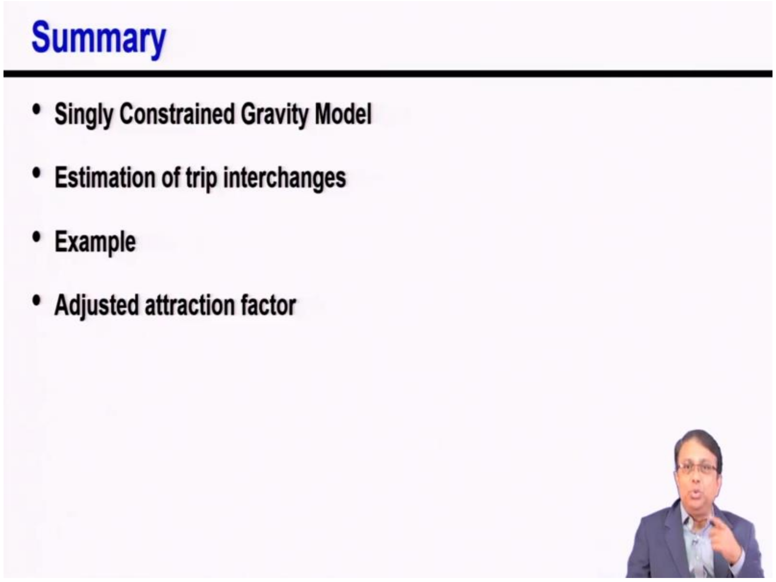

So, what we discussed in this lecture is about the singly constrained gravity model, why we call them singly constrained gravity model, then also mention to you the singly constrained, may be production constrained, may be attraction constrained, why we call them singly constrained. Then how we estimate the trip enter changes, what are the steps we follow, stage wise how we develop, how we go and then we took an example and explained you how you apply it.

Particularly the most interesting point in a singly constrained gravity model one end, this was the production constraint model, so production limit satisfied but the attraction ends the constrained will not satisfied automatically. So, we explained to you what you can do externally, externally what kind of balancing or iteration you can do to match the other end. So, with this I close this lecture.