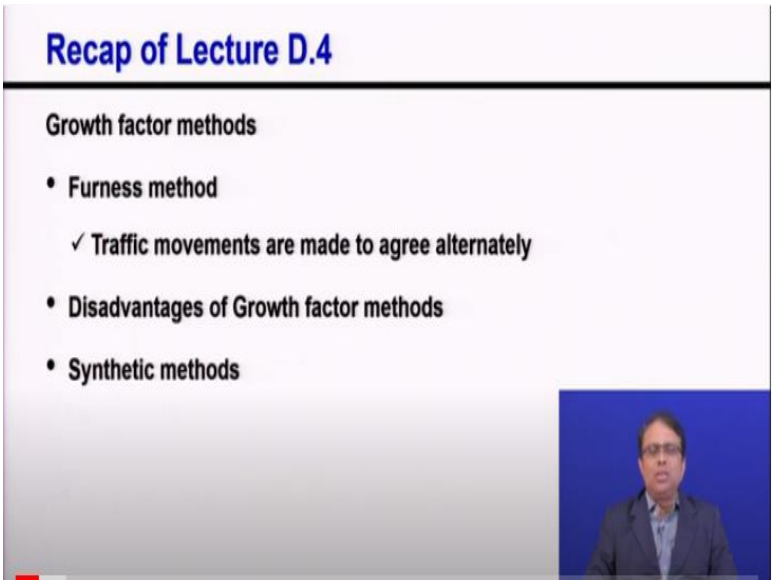
Urban Transportation Systems Planning
Prof. Bhargab Maitra
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture - 25
Synthetic Methods, Measures of Travel Resistance and Gravity Model

Welcome to module D lecture 5. In this lecture we shall briefly introduce to you synthetic methods for trip distribution, and then in particular we shall talk about various measures which may be used for capturing or quantifying the travel resistance or impedance and then we shall discuss about the gravity model.

(Refer Slide Time: 00:43)



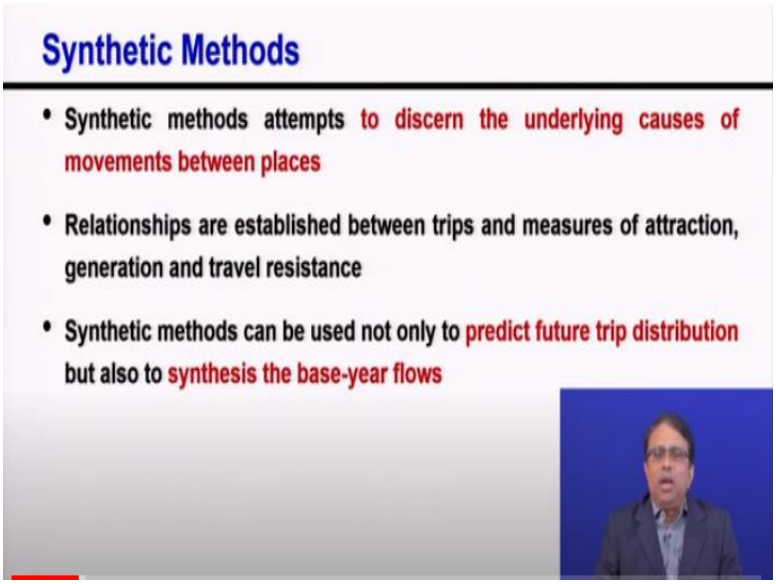
In the previous lecture, in fact previous few lectures, we discussed about various growth factor methods. In lecture 4, we talked about Furness method, how we balance the trips first matching the columns totals and then again matching the row total, again matching the column total, again matching the row total and then how we converge to a target year or design year t ij values or trip distribution matrix which will satisfy the row totals.

And the column totals in the future or the target row totals and column totals. We also discussed the general disadvantages associated with the growth factor methods mainly highlighting that it is only a mathematical technique to upgrade the given matrix to match the future row totals and column totals. We are not actually considering the reasons which influence the travel.

May be the productions, attractions and also the special separation between zones, particularly the influence of the transport network, any changes any improvement in the transport network. All those are not really taken into consideration. But we said that where you can apply it advantageously particularly maybe for short update of the trip matrix, when you know that during that period there is no significant change in the transport network.

So, we talked about the disadvantages but also that clearly indicated that where we can actually overcome these disadvantages. And use growth factor methods advantageously to upgrade our trip matrix. Then before we closed lecture 4, we briefly introduced to you the synthetic methods.

(Refer Slide Time: 03:13)



Basically the synthetic methods try to overcome the disadvantages associated with the growth factor based method. Synthetic methods attempt to discern the underlying causes of movement between places. Why to some destination more trips are attracted? Why to some other destination less trips are attracted? What are the real reasons?

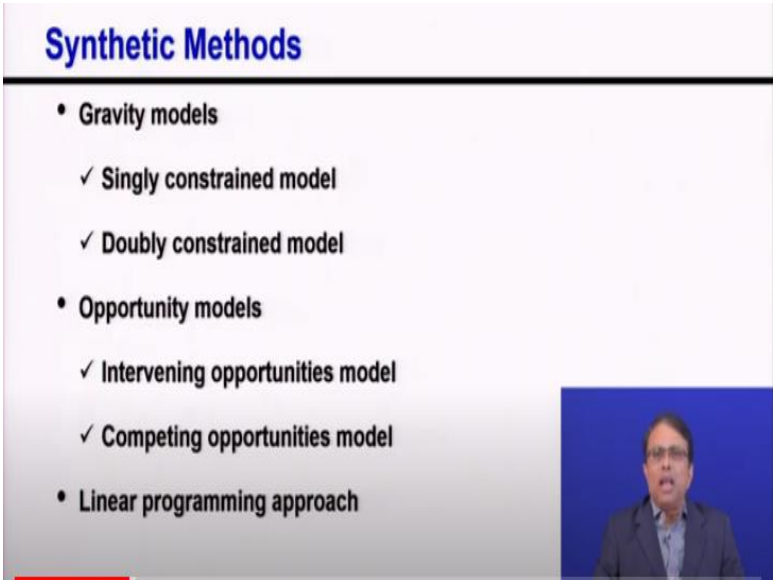
So, those things are actually we try to capture using synthetic methods. So, in synthetic methods, we can say that relationships are established between trips that means trip interchange. How many or what will be the values of t ij relating t ij to the

productions of the origin zone or the production zone to the attraction of the destination zone and also considering the travel resistance between zones.

So, the synthetic method one great advantage is that we can not only predict the future trip but we can also do the synthesis of base-year flows. So, if all inputs are given as we will see we shall discuss even in further details that we can actually do the synthesis of the base-year flow. So, we can actually model the value starting from you the scratch whereas the growth factor based method only once you have given an input matrix.

Then only; based on that applying a growth factor you can get the future t ij values to match the target future productions and attractions.

(Refer Slide Time: 05:22)



Now, there are several synthetic methods which are widely used. Among all those methods the most popular models are known as gravity models. Now gravity models may be singly constrained model, may be doubly constrained model. We shall again explain you very clearly what we mean by singly constrained model what we mean by doubly constrained models.

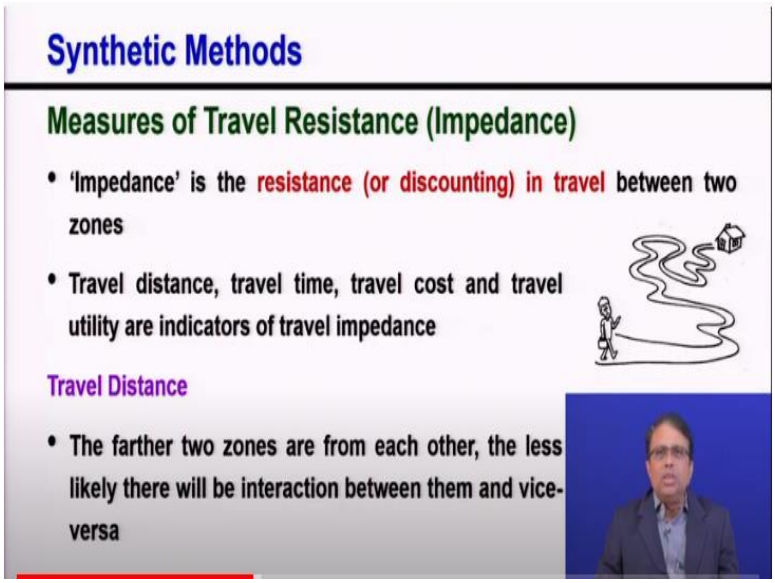
But also it is important to mention that there is not that there is only one singly constrained model or only one doubly constrained model. There are varieties of models. If you open various textbooks you can find out different forms of models which are all called gravity model. The reason is they have certain common

characteristics. So, the common characteristics make all those models as gravity models.

So, gravity model is one category of model rather than one or two models. Then there are also opportunity models which we can again say that they use synthetic methods, the common the synthetic methods. And opportunity models also could be intervening opportunity model, competing opportunity models. And finally what we will discuss in this lecture or in this course not in this lecture in this course is also the linear programming approach which can be used advantageously to formulate trip distribution problem as a linear programming problem.

So, these are the different methods that we are going to discuss in today and in the next, maybe two or three lectures or the remaining lectures in this module.

(Refer Slide Time: 07:34)



So, to start with let us first talk about the gravity model. But before we talk about the gravity model or any of the other models, we need to discuss one very important thing what we call as measure of travel resistance or impedance. Because we said the speciality or the unique characteristics of the synthetic methods is the consideration of the transportation network.

The real reason; why more trips are going to one destination as compared to other destinations and how the properties of transportation network influence the trip distribution or the t ij values. So, this special separation between zones is very

important. And how we represent the properties or the characteristics of transportation system or transport network in the trip distribution context using the measure of travel resistance or impedance.

So, impedance in general is the resistance in travel between two zones. Now what is the resistance? We can generally express like if the travel distance is more between 2 zones, the more the travel distance the more we can consider as a resistance. Because the more the travel distance less number of trips are likely to happen to that destination.

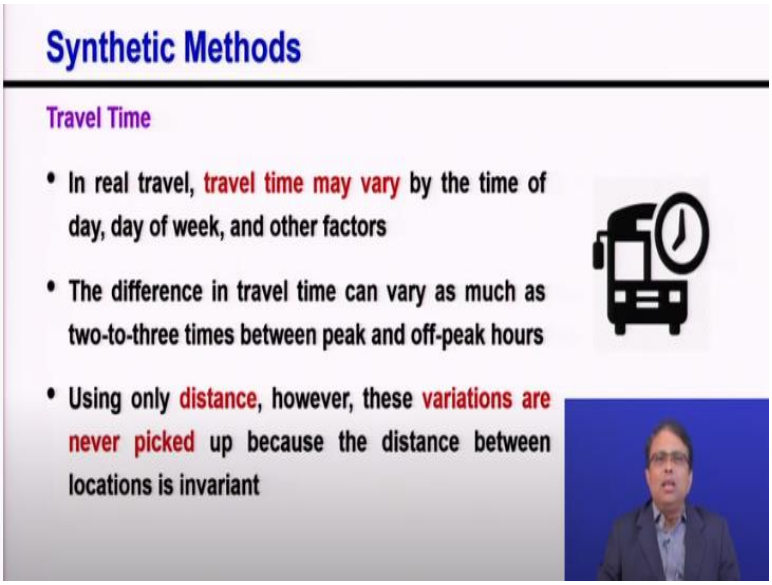
Obviously you think if one zone is connected to two zones and this two zones attraction zones are really identical in terms of their attraction potentials. But one place is maybe 2 kilometers and another place is 5 kilometers away from the origin zone. And let us consider both cases the roads are uncongested. So, it is not a congested network.

So, therefore naturally more trips will happen to the destination which is nearer. That means we can consider or we can say that more the distance then the resistance is more. So, as if the resistance is more so less number of trips are finally reaching to the destination. So, you see that only we can use travel distance. No, one can use travel distance, one can use travel time, travel cost, even travel utility all these are measures of travel impedance.

So, let us discuss little bit more about each of these possible measures of travel resistance or impedance. To start with first let us take travel distance. As I said, the farther to zones are from each other the less likely there will be interaction between them and vice versa. That means if you are considering two zones, zone A and zone B and if these two zones the distance is more, naturally less number of trips will go to that particular distribution.

Because the distance is more, less likely that trip will go there, distance is less more likely the trip will go there. So, if the distance increases lesser number of trips will go, if the distance reduces higher number of trips will terminate.

(Refer Slide Time: 11:35)



So, this is no doubt a measure which may be used to express the travel resistance or the travel impedance. But, you know that the networks often remain congested. So, two places connected by two different distances, but the actual travel time will really matter and actual travel time will depend on how the roads are loaded. That means whether the roads are very narrow or what capacity you have for those roads and how much traffic loading is there.

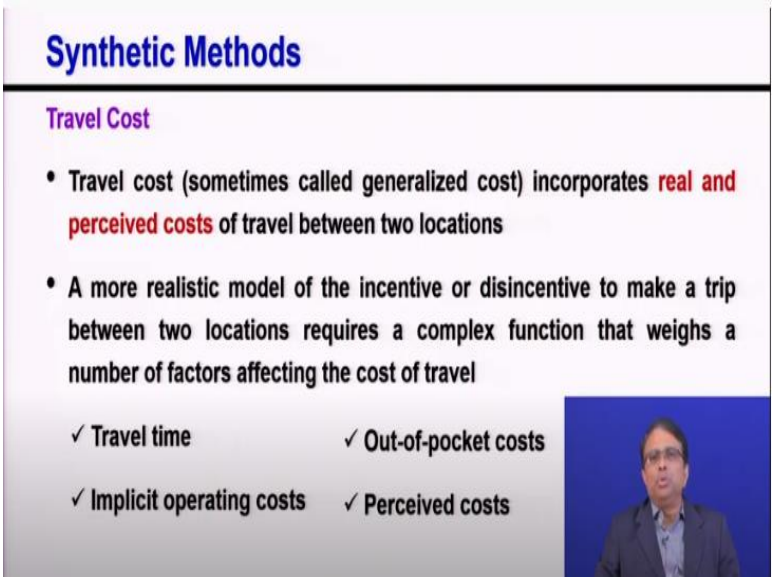
So, depending on the capacity and depending on the traffic load, maybe somewhere a small distance even may take longer time and sometimes even a relatively longer distance you can access in shorter time. So, if we have to logically think then it is not really truly the travel distance. Distance no doubt can be used as a measure of deterrence or impedance.

There is nothing wrong people do use it. But obviously travel time is better than the distance because as I said the difference in travel time can vary as much as 2 to 3 times between peak and off-peak hours. And if you use only distance then these variations in the travel time that are never picked up because the distance between two locations is fixed.

Once you say the two locations are 5 kilometers apart then the distance between two zones is simply 5 kilometers. It does not change over time. But the travel time can change. May be during peak hour that 5 kilometer may take 2 to 3 times the travel time as compared to the travel time in the off-peak hour or the lean hour. So, it may

be good to use travel time also to represent the resistance or impedance between two zones.

(Refer Slide Time: 13:53)



Similarly we can also use travel cost. Now travel costs most of you generally tend to think as the actual fare you pay for the public transport or the actual fuel cost may be what you are incurring if you are using a private vehicle. No doubt that is the travel cost. But that is more like the direct cost of travel or out of pocket cost. If you consider the cost does not include only this direct cost or out of pocket cost, but also cost due to other disutilities.

For example, if let say you are traveling in a public transport, then you will obviously think how much time you have to spend if you want to travel by public transport. So, the travel time is one important consideration. And we can also convert travel time into travel cost. If we know the value of time, say for example, if it takes 20 minutes and the value of time is rupees two per minute then we can think 20 minutes is equivalent to 40 rupees.

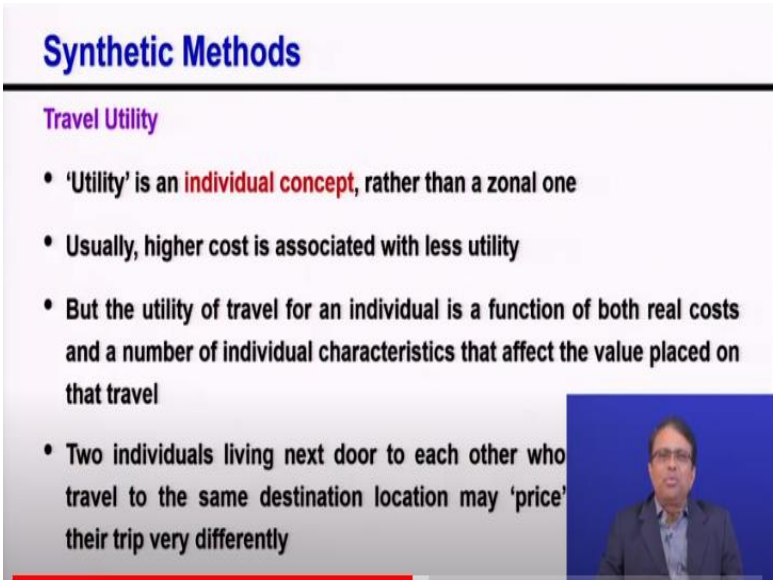
So, there is a time value that can be associated to convert that. Similarly if you are traveling in public transport then what is the level of crowding inside public transport? The crowding is a qualitative aspect but there is a disutility associated with this crowding. If you are traveling comfortably getting a seat in the bus and the bus is not that crowded versus you are traveling as a standee in the peak congested condition both conditions of travel are not the same.

So, there is a cost associated with the disutility or due to crowding. So, that also can be taken. So, if you consider there are several such components which may be some may be direct cost or out of pocket cost which is in rupees, some may be in time so expressed in minute, some may be qualitative, some may be expressed in distance, say for example, how much walking you have to do from your home to the nearest bus stop, this may be expressed in distance or kilometer or meter.

So, if all these units, they are different units, you cannot simply add them together. But if each of these you can convert into equivalent cost in, say Indian rupees and then your fare is also in Indian rupees then you can add all the components together. Because they are all now expressed in rupees and the total cost what you will get you can call it as generalized cost. Most often we call it as generalized cost.

So, generalized cost is a comprehensive measure for cost of travel considering out of pocket cost and all other disutilities of travel converted into equivalent cost. So, that is really a more rational measure than the travel time alone or the distance alone or only out of pocket cost. So, this also can be used successfully to express the deterrence of travel or the friction of travel between two zones.

(Refer Slide Time: 18:05)



Similarly as I said this travel utility in general is also another measure which can be used as an expression of deterrence. I told disutility why? Because actually; it is travel utility but in the context of transport everything is disutility. Because any money you

spend is disutility, anytime you spend is disutility, any waiting you do at the bus stop is disutility, any walking you have to do to reach to the bus stop is disutility.

So, always in all these cases higher the value higher is the disutility. So, in transport case there is nothing utility rather these are all disutility. And in fact when we compare two systems and try to see how much is the benefit we try to only see how the disutility is reduced. A reduction in disutility is actually the benefit. So, as you use generalized cost you can also use travel utility or disutility as a measure of the resistance or the deterrence of travel between two zones.

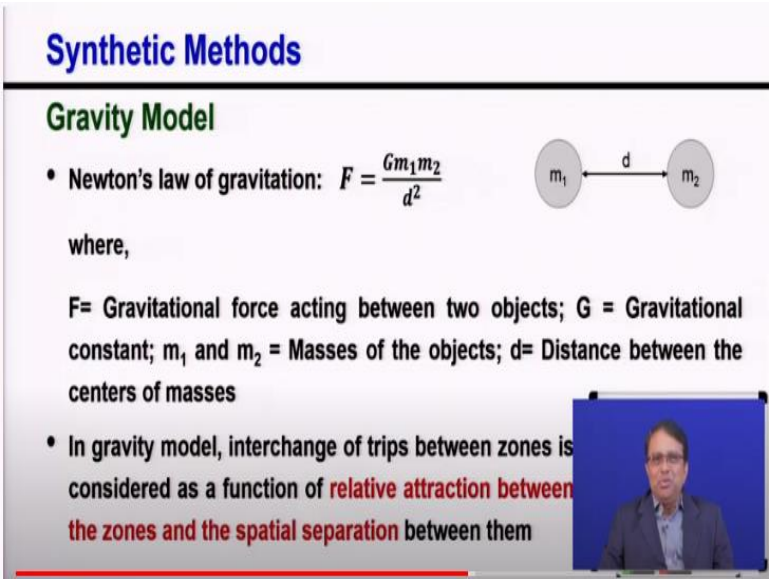
Now remember that utility is individual concept rather than a zone level concept. That means it is individual. So, every individual has his or her own disutility based on the perceived weightage to travel time, travel cost, all other factors which influence the travel decision. But, of course, once you know the individual decision making, the collective decision making or the zonal level what is happening can also be estimated.

But it starts with the individual level or disaggregates level when we are talking about travel disutility. And remember that travel disutility as it depends on the characteristics of the transport system. That means how much will be your waiting time, how much will be your travel time, how many transfers you have to do, what is the direct cost of travel or out of pocket expense or fare, what is the level of crowding you have to face, all these characteristics will influence the travel utility.

Similarly the utility is also influenced by the socioeconomic and trip characteristics of individual. So, the individual characteristics are also important. What is the age, what is the income, what is the gender, what is the purpose of making the trip? All these also will influence the disutility. So, the coefficients; what you are expected to get likely to vary based on these characteristics.

So, two individuals staying next to two adjacent flats may not have the same utility or disutility even though the same options are available to them. It depends on the individual characteristics.

(Refer Slide Time: 21:36)



Now with this background, let us go to the discussion about the gravity model. Look at this sketch. What I have shown? One object with m 1 another object with m 2. They are the masses of the objects, d is the distance between the centers of masses and then what is the F? The gravitational force; acting between two objects. Do you remember this equation?

I am sure all of you do remember this equation. You have studied it in your schools. That is Newton's law of gravitation. Now why we call this trip distribution models as gravity models? Because these trip distribution models are somewhat similar to this Newton's law of gravitation. So, what we say? Gravitational force acting between two objects equal to G into m 1 m 2 by d square if d is the distance between the center of masses.

The same thing here F gets replaced by the number of trips that is expected to happen between two zones. Let us consider instead of two objects let us consider them as two zones. So, i to j so the number of trips that are going to happen are likely to happen between these two zones that is t ij. So, F gets replaced by t ij, m 1 gets replaced by production of zone i, m 2 gets replaced by attraction of zone j.

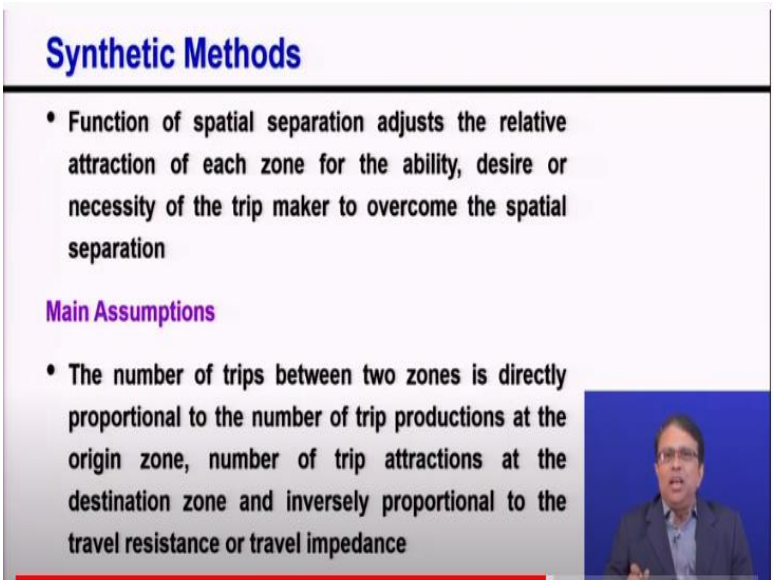
And d is again the distance or time or any measure of the deterrence or special separation between the two zones. And obviously there will also be a constant to make this T ij equal to something like p i aj divided by maybe distance square or some other function. So, this basic form is used in the trip distribution model and

therefore we call all such models anything where fundamentally this formulation is there in some form or other.

That is why all those models will be called as gravity models. So, in gravity model interchange of trips between zones is considered as a function of the relative attraction between the zones. What is the relative attraction between the zones and the special separation between them? How much distance apart or what is the travel time between the two zones or what is the cost between the two zones.

And what is the associated utility or disutility of travel between two zones, anything you can consider. And that is why still it is actually looks very much similar to the formulation of this Newton's law of gravitation. So, they are all called gravity models.

(Refer Slide Time: 25:04)



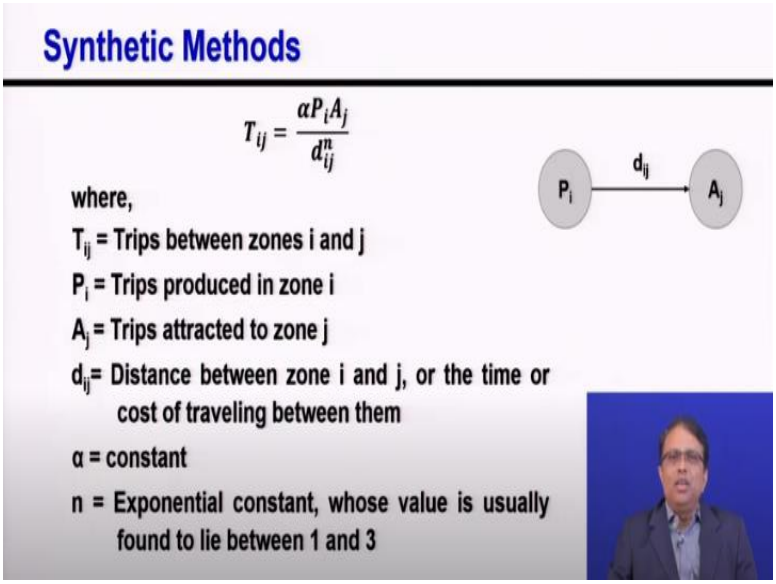
How they look like? We will come back to that but before that a few other things. Now this function of spatial separation; why we bring it? Because we; actually try to adjust the relative attractions of each zone. As you know, that trips are produced in zone i so obviously how much will get attracted to zone j will depend on what is the attraction to zone j.

But then the distance or the cost or the time, the deterrence or the spatial separation it also matters. So, what we are doing that this spatial separation actually is adjusting the relative attraction. It is not only the attraction but attraction multiplied by some function of spatial separation. So, that is what I said. The function of spatial

separation adjusts the relative attraction of each zone for the ability desire and necessity of trip makers to overcome the special separation.

So, it is not attraction, attraction multiplied by some function of spatial separation. So, what are the main assumptions? The number of trips that is what the basic formulation of this gravity model for trip distribution, the number of trips between two zones is directly proportional to the number of trip production at the origin zone, number of trip attraction at the destination zone and inversely proportional to the travel resistance or travel impedance. Exactly looks like gravity formulation.

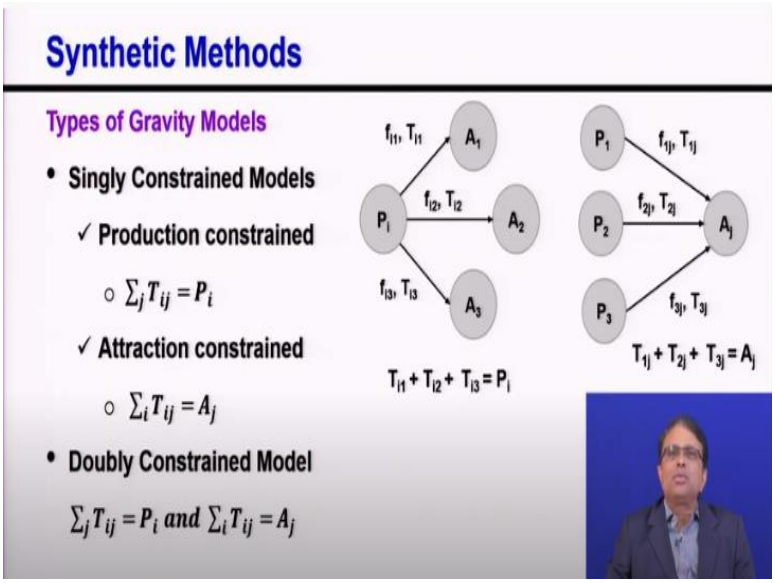
(Refer Slide Time: 27:03)



So, how it looks like? We are now saying T ij the number of trips between zone i and zone j is proportional to P i number of trips produced in zone i proportional to A j number of trips attracted to zone j and inversely proportional let us call distance let us take it as distance. So, we are saying it is inversely proportional to d ij but d ij to the power n.

Earlier you have seen only two, d square. Here we are saying d ij to the power n, n is an exponent or a constant whose value can be anything between 1 and 3 normally. So, and since I said it is proportional to P i proportional to A j and inversely proportional to the distance or d ij to the power n, so the alpha is the constant to make it equal. So, T ij equal to alpha into P i A j divided by d ij to the power n.

(Refer Slide Time: 28:19)



Now while this is the basic formulation of trip distribution model, basic fundamental trip distribution model, there are two types of gravity models that are used. One is called singly constrained model another is called doubly constrained model. So, now I will tell you from the basic formulation what you have seen here in this slide. There are little bit modifications. What are those modifications?

The first modification is that when I talked about the Newton's law of gravitation, we talked about two masses. Then the basic formulation we say between two zones. Now little bit addition to that. What is that addition? Addition is P i is not only connected to one particular zone A j but it is actually connected to multiple such destination zones, say you want A 1, A 2, A 3.

So, similarly if you consider an attraction zone this attraction zone trip does not come only from one zone P i but it comes from many such origin zones or production zones. Say P 1, P 2, P 3 from different zones trips are getting attracted to one zone. Similarly one zone is produced trip is produced but then getting attracted not to one zone but to, many other zones which are actually connected.

That is what I have shown here. So, some types of relativity of the; relative attractive place that we have to bring. How much of the trips produced in zone i will go to a particular zone j that will depend on attraction of that zone j. Also the friction factor or the impedance between i and j but not only that one but relative to all zones which

are connected to production zone P i and their relative attractions and the corresponding value of the friction factor because somewhere it is to be normalized.

So, it is all like all destination zones are competing with each other, competing in terms of what? Competing in terms of attraction and modified attraction modified by friction factor. So, it is we can say generally attraction into a function of the spatial separation or the friction factor or impedance. So, all destination zones are competing. How much a particular zone j will get?

That will depend on what is the attraction of that and relative to attraction and the friction factor or impedance compared to attraction and friction factors of all zones which are connected to i. Same way you can interpret an attraction zone. That attraction zone is attracting trips from many production zones. So, how much trips will get attracted from a particular zone i that will depend on what is the production from that zone i and what is this friction factor or impedance to zone i?

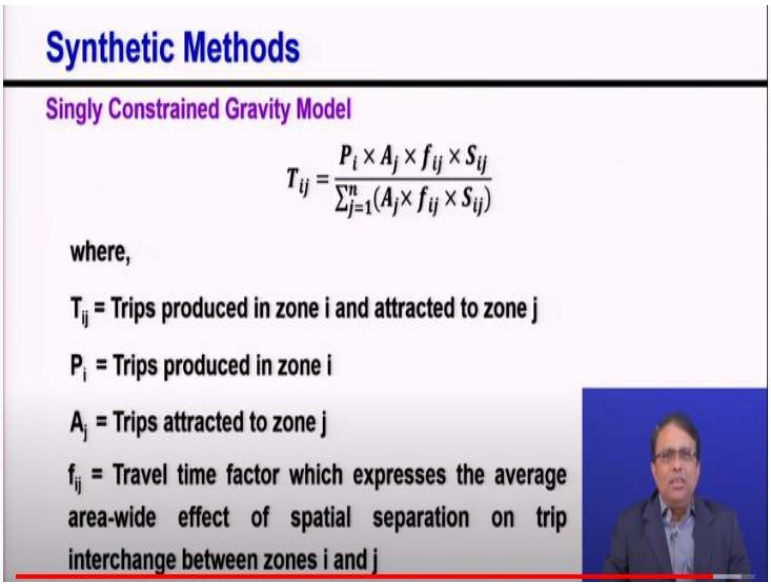
Relative to all zones which are producing trips and are connected to this destination zone j. So, that relative thing will come. The second two types as I say singly constrained and double constrained. What is singly constrained? We are actually taking T ij matrix. So, the if you take a row at all the totals it will give you production. So, T ij sum over all j sum of all destination zone must give you P i.

Similarly T ij if I take one column sum it over all i, 1 to 1, 2 to 1, 3 to 1, 4 to 1, 5 to 1. What should I get? Attraction of zone one. Similarly T ij sum over all i should be A j. Now singly constrained model both these constraints are not satisfied automatically by the model formation. Any one of these two will be satisfied. Either T ij sum over j equal to P i that will be satisfied by the model form or T ij sum over i equal to A j that will be satisfied by the model form.

Not both. Of course finally the other end also has to be satisfied but the model automatically will not ensure that. Whereas in doubly constrained model; both these constraints will be satisfied automatically by the model itself. So, the model function is something like that T ij equal to the way we will write. If you sum it over j it will

give you by equation P i. If you sum it over i it will give you A j from the model itself. That is the different between singly constrained and doubly constrained model.

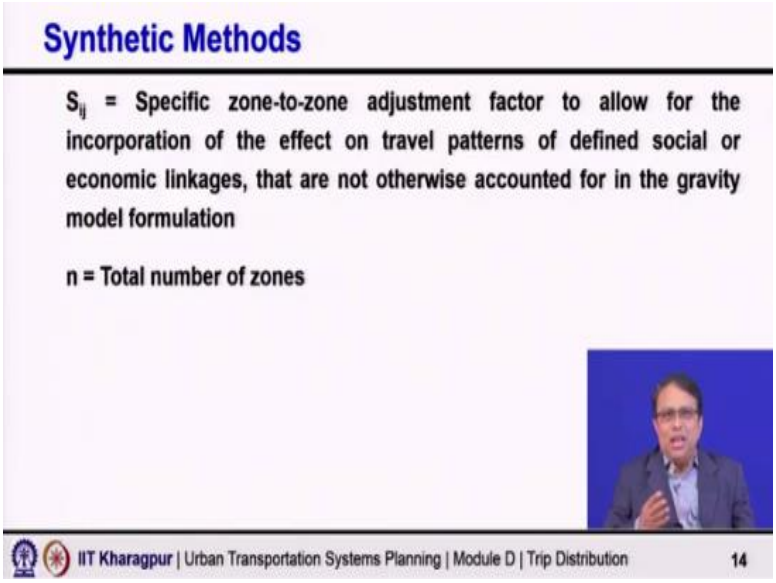
(Refer Slide Time: 34:09)



Let us quickly look at a singly constrained model. T ij equal to as I said P i into A j f ij by sum over A j f ij sum over what sum over all zones j which are connected to i. That is the relative thing. Only thing new here is a new term I have introduced here or mentioned here is S ij. S ij is called zone to zone adjustment factors. What is this S ij? Why we use S ij?

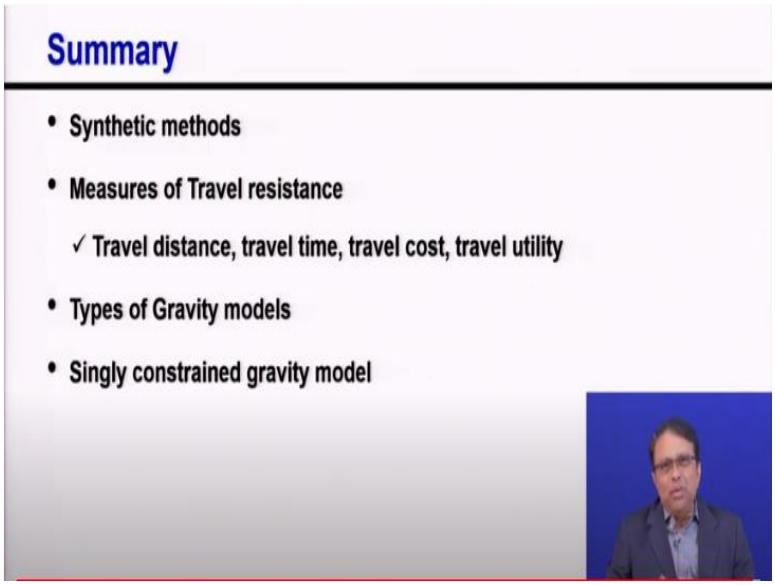
That I shall explain you when I will discuss about the calibration of this simply constrained gravity model in that context. So, you know, this is a singly constrained model and if you add it sum over j by the model itself, you will get you can see look at this thing it will be giving you P i. But if you sum it over i this equation is not automatically going to give you A j. So, it is a production constrained, singly constrained but production constrained gravity model.

(Refer Slide Time: 35:21)



That is what I have indicated here.

(Refer Slide Time: 35:22)



So, what we discussed here, we discussed briefly about the synthetic method. How the synthetic method is different from the growth factor based method. Then we talked about the various methods of travel resistance or impedance in terms of using as a function of travel distance, travel time, travel cost, travel utility and others. Then we talked about what is gravity model. Why we call these models as gravity models and then there are two types of models as I said.

Singly constrained model and doubly constrained model. And then we introduce to you one particular form of singly constrained gravity model. So, with this I close this lecture. We shall continue in the next lecture. Thank you so much.