Urban Transportation Systems Planning
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Lecture - 23
Detroit Method and Fratar Model

Welcome to module D lecture 3. In this lecture we shall discuss about 2 methods, which are related to growth factors of these methods for trip distribution, and these methods are called as Detroit methods and Fratar method.

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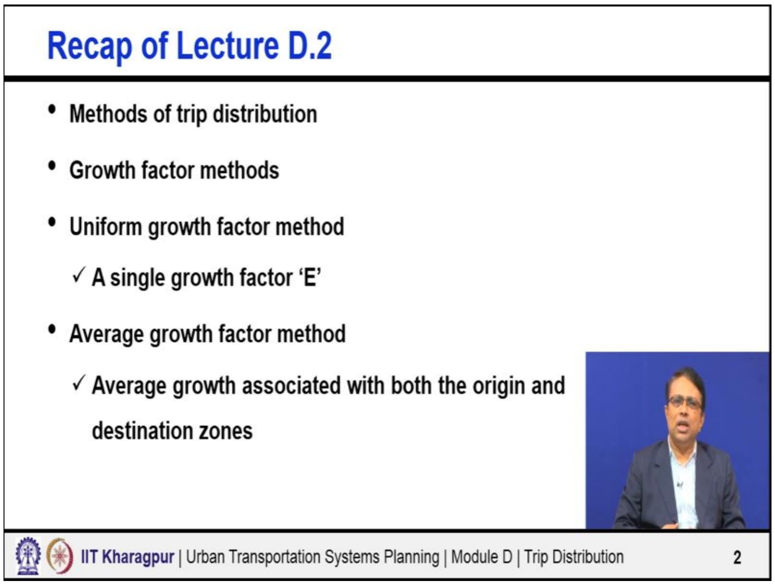

What we discussed in lecture 2, we talked about various methods for trip distributions, growth factor based methods and synthetic methods or synthetic models. Then discussed, about the approach what is being followed in the growth factor based method, we take the present t ij as a value or number of trips from zone i to zone j and then we apply a group factor on that to get our future value of trouble.

How many trips will happen from i to j in the horizon year or design year or in the future year, whatever you say. Then we said that there are different methods all of them can be called as growth factor based methods, because principally all methods do the same thing, but the way the growth factor is calculated and applied that is not same for all the methods, that are different in different methods.

So, then we started discussion about the growth factor based methods, we talked about uniform factor method also discussed about average factor methods and explained to you how really they work, with example problems.

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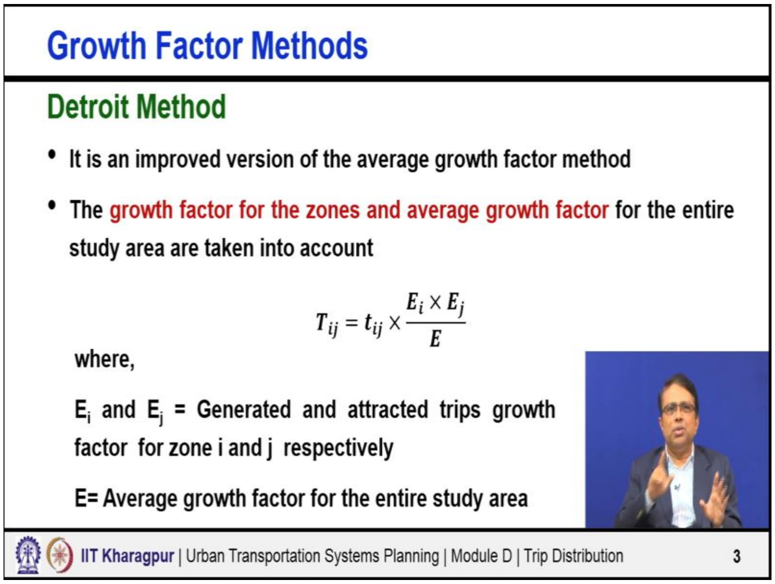

Now with this background today, we shall talk about first a new method, which is called Detroit method. Now, you have seen that, the trip distribution when we are doing for the future year and particularly using growth factor based method. We said that it is logical to assume that growth of a particular travel segment say T ij that means travel from i to j should be logically related to the growth of production shown i and the attraction zone j.

So, how much zone i is growing or how much growth is expected for zone i and how much growth is expected for zone j. So, the average factor method we did something we calculated the growth of zone i, that is E i we said and growth of zone j, E j and we took the average of that, that means E i plus E j this value divided by 2 and that is the growth we applied to the cell t ij. In Detroit method, it is slightly different.

Why how we are calculating the growth factor, look at this part it is not E i + E j by 2, rather it is E i into E j. But then divided by E, what is E? E is, the overall growth of the entire area, it is like the uniform growth. So, it is in a way a combination of uniform the growth factor and average growth factor. So, from average growth factor, we are taking E i and E j values and for uniform growth from uniform growth factor, we are taking the E value.

So, how the growth factor for a particular cell is calculated, that is what will be the growth factor associated with the cell t ij it will be E i into E j divided by E. The overall growth, so growth of zone i, growth of zone j divided by total growth of the study area. That is the way the growth factor for each cell is calculated and as we did in the average factor method, once you go for one iteration.

The row totals and column totals of the matrix what you have estimated and what you want to target may not match exactly. So, what you need to do in that case is to go for another iteration just as you started with an initial matrix and the growth factors at the origin end at the destination ends exactly the same you will go for one more iteration or even more number of iteration and as you make more number of iterations you know match will be you know closer.

So, with an acceptable error limit up after some iteration you will probably stop and take that value that ok. I have now got a trip matrix distribution, where the row total and the column totals are reasonably matching with my target values. That is what is basically; the Detroit method.

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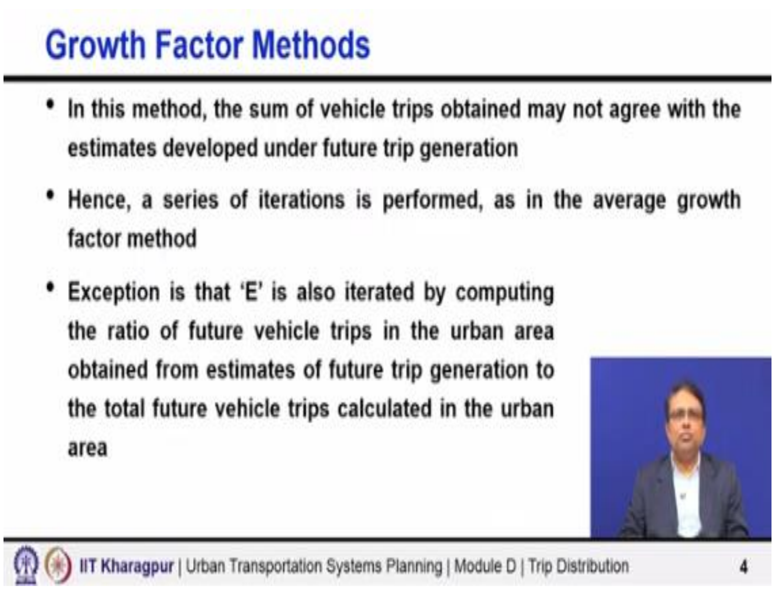

So, that is what is explained here.

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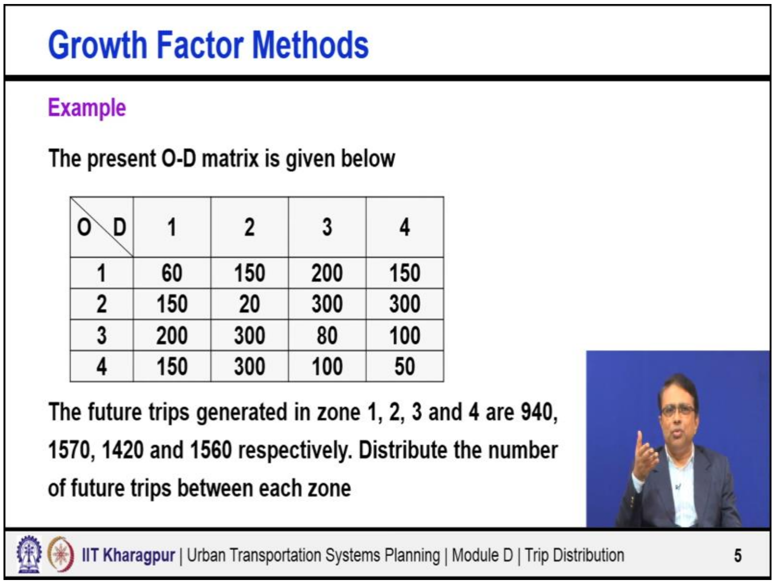

Let us take an example to understand this clearly. Say the present OD matrix is given below. So, 1 to 1 60. 1 to 2 150. 1 to 3 200 and 1 to 4 150. So, we can easily get the present row total 16 plus 150 plus 200 plus 150, like that for 1, 2, 3, 4 all zones. So, we can get the row totals and 4 each zone, whatever is the row total, the column total will again be same, because in all these examples, we are actually taking T ij equal to T ji.

So, now what we do we want to get the future OD matrix, when the future row totals and column totals or row totals or column totals, whatever you will say for zone 1, 2, 3, 4 are expected to be 940, 1570, 1420 and 1560.

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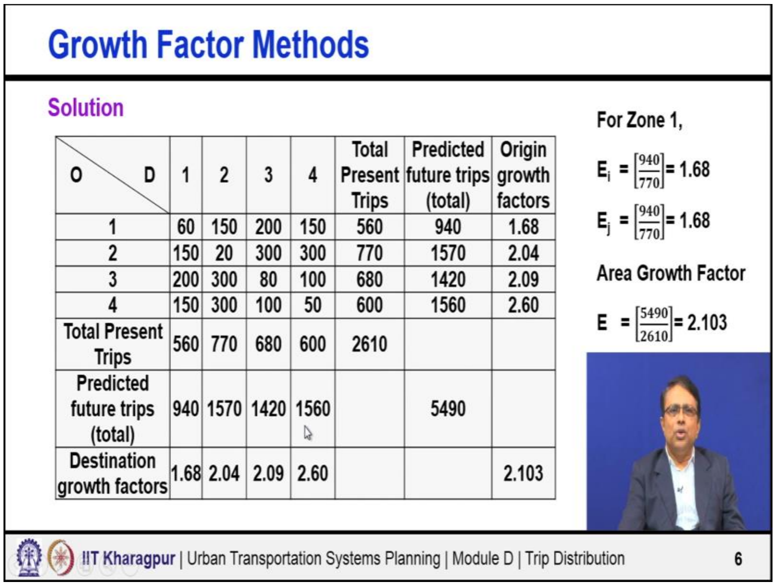

O \ D	1	2	3	4	Total Present Trips	Predicted future trips (total)	Origin growth factors
1	60	150	200	150	560	940	1.68
2	150	20	300	300	770	1570	2.04
3	200	300	80	100	680	1420	2.09
4	150	300	100	50	600	1560	2.60
Total Present Trips	560	770	680	600	2610		
Predicted future trips (total)	940	1570	1420	1560		5490	
Destination growth factors	1.68	2.04	2.09	2.60			2.103

So, let us go to the next step that is the solution. So, what we do we actually get this present row total and column total and we know that what are our targets. Then accordingly calculate the growth factor for each zone at the origin end at the destination end, they are obviously same, but keep both those factors. Then we also what we do, the present how many trips are there 2610. The future how many trips are targeted, in each zone wise the row total and the column total.

So, we get total number of trips that will be from this study area or that means in all the four zones, in terms of at the origin end or at the destination end. So, then we get the overall value of the growth factor. What is the growth of trips, total number of trips in the study area? That growth factor E, what I said in the previous slide that E value also we calculate, which is 5490 divided by 2610 that is 2.103. So, I can get overall E value calculated and I got E 1, E 2, E 3, E 4, production end and attraction end as well.

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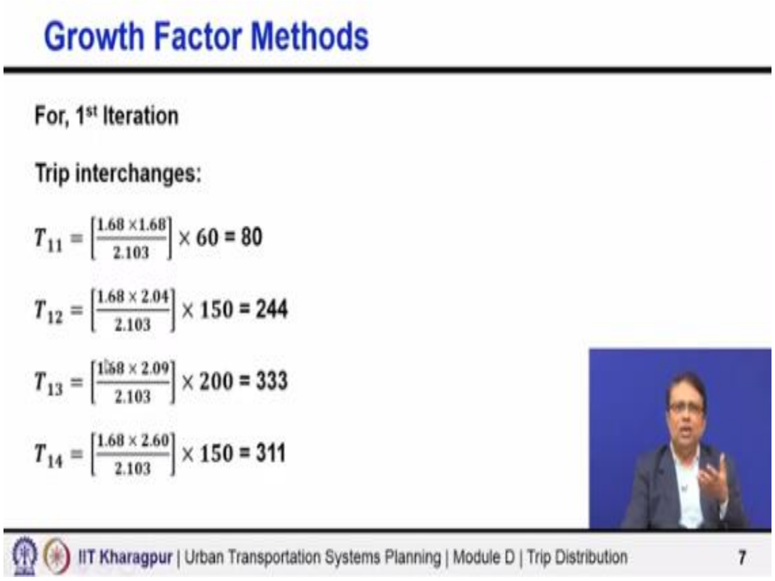

So, now, next is as per the Detroit model, what we will do T 11 growth factor at the origin of one, that is 1.68, destination end also it is 1.68. So, 1.68 multiplied by 1.68 divided by E overall growth of trips in the study areas complete study area, that is 2.103 multiplied by the present value of T 11, that is 60, so you get it as 80. Similarly you calculate the value for 1 2, what we do?

Growth at 1, zone 1 1.68, growth of zone 2 2.04 divided by again overall growth that is E multiplied by the respective cell value now and you get that updated or the future value.

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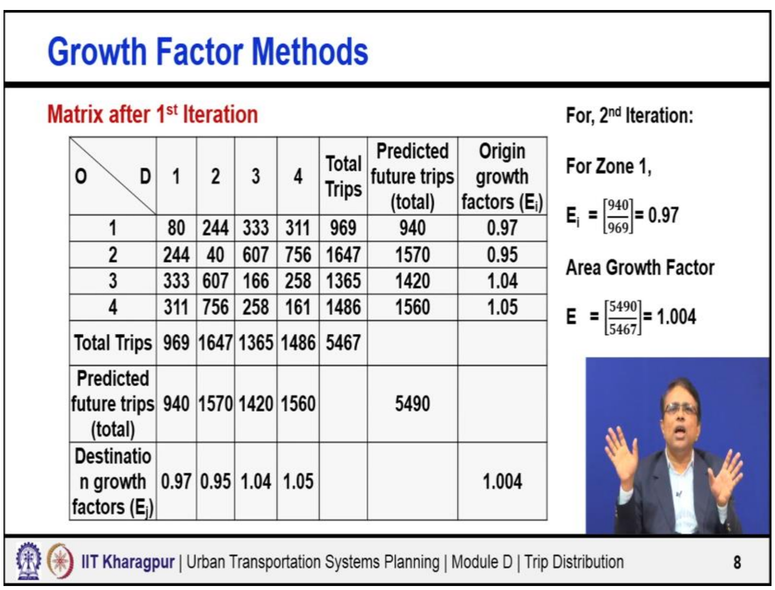

Like that you do so now we have put the updated cell you can see the total trips are nearly matching except for some small difference, they will match and you can see, but the row total and column total are not matching exactly. But does not matter, you know already the procedure, how you started. So, when you started, you just now imagine this given matrix as your starting point and assume that your target remains same in terms of the number of trips, but the factor group factor values will change now.

Because your present matrix is different, target value remains same in terms of production, in terms of attraction. But the growth factors will be different because your present value is different. So, you just make one more iteration, if required one more iteration, so like that a few iterations and as I said, once you have an acceptable error limit, you know that where you can stop accordingly take that matrix.

And with that matrix you will have reasonable level of match for the row total and column total. So, that means whatever distributions you will get, if you add the row total. Whatever value you get at the production origin and the destination ends, they will match with your target values with an acceptable error. That is fine, that is what is being done now and you can see here, whatever was the earlier values.

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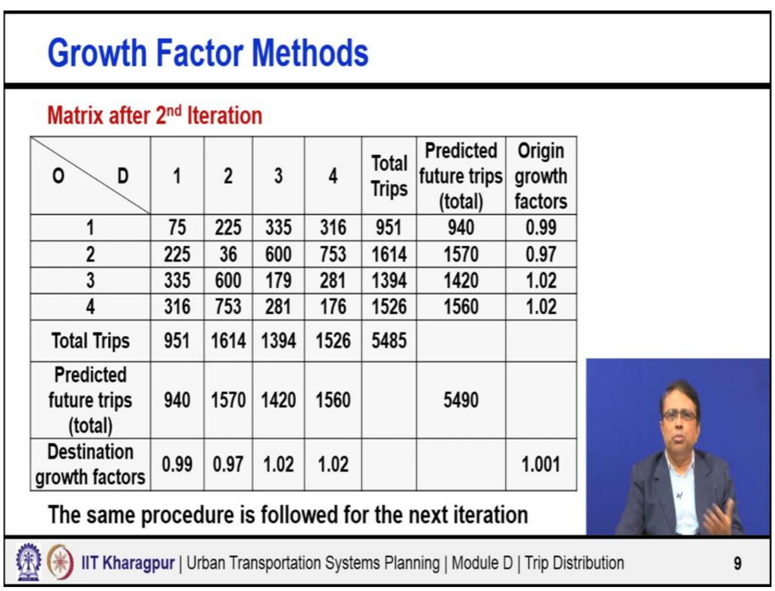

O \ D	1	2	3	4	Total Trips	Predicted future trips (total)	Origin growth factors
1	75	225	335	316	951	940	0.99
2	225	36	600	753	1614	1570	0.97
3	335	600	179	281	1394	1420	1.02
4	316	753	281	176	1526	1560	1.02
Total Trips	951	1614	1394	1526	5485		
Predicted future trips (total)	940	1570	1420	1560		5490	
Destination growth factors	0.99	0.97	1.02	1.02			1.001

Now even you know, you get bit closer, so same procedure you can follow for even for the next iteration you can further go for more and more iteration as a experiment.

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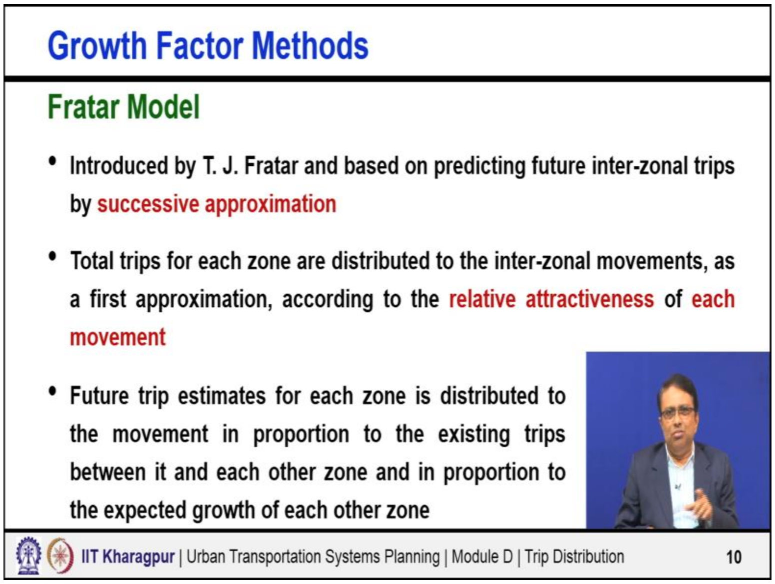

Now coming to the Fratar model, it is another model where again it is an iterative procedure, that is what we follow and what we do the total trips for each zone are distributed to the interzonal movements as a fast approximation, according to relative attractiveness of each movement. Here a new concept came we are saying not just the growth factor, but we say relative attractiveness still no consideration of the transportation network.

No consideration of the transportation network or travel time or travel cost, but still somewhere we are considering the relative attractiveness. So, future trip estimates for each zone is distributed to the movement in proportion to the existing trips between it and each other zone and in proportion to the expected growth of each other actually. I think better instead of explaining using words let me go to that equation.

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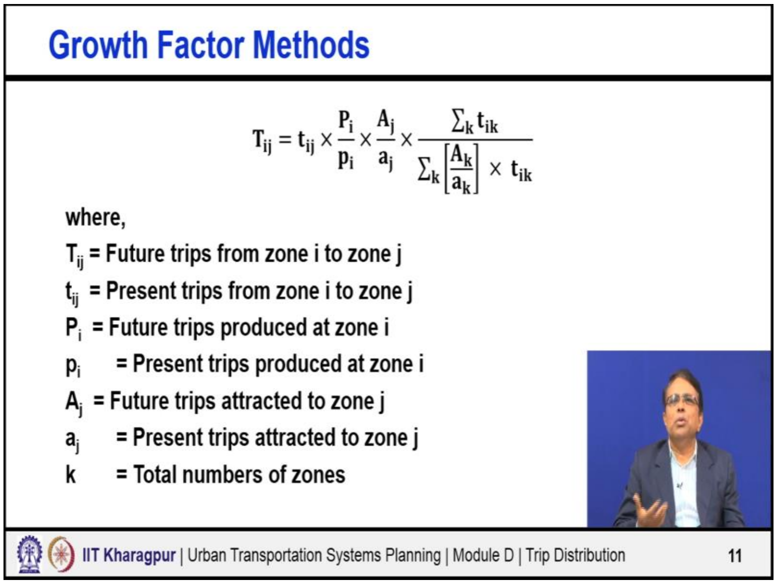

Look at this equation, what you are seeing is future T ij equal to present t ij, so t ij into P i by p i, what does it mean the growth of zone i into A j by a j that what does it mean this is growth of zone j into t ik, sum over k divided by sum over k into A k by a k. That means growth of zone k into t ik. Sound too difficult, no. I shall show you a slightly different form of this equation. You will find in most books this same equation is given, but I shall help you to remember this equation very logically.

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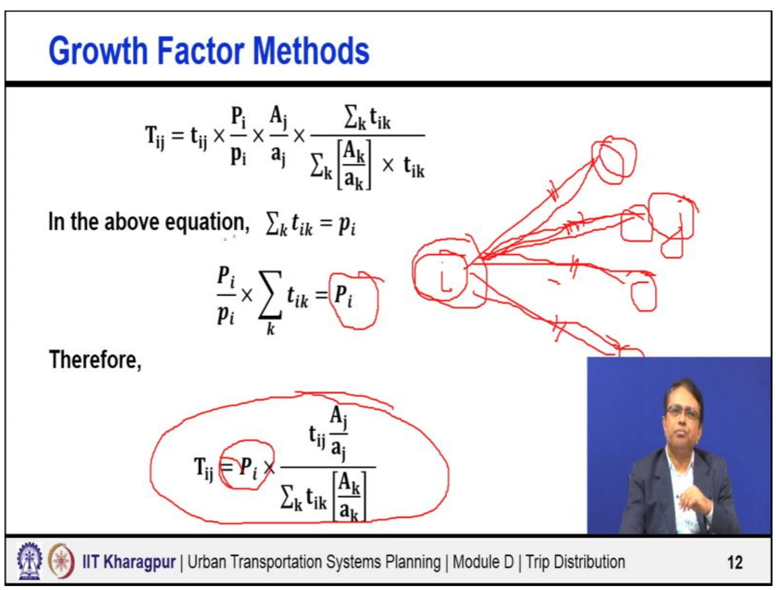

Let us see, this was the equation given T ij equal to t ij into P i by p i into A j by a j into t ik sum over k divided by a A k by a k into t ik sum over k. Now let us see in this above equation take this component. T ik sum over k, what is this, this is nothing but p i. 1 to 1, 1 to 2, 1 to 3. I to 1, i to 2, i to 3, i to 4 like that all k zones. What does it mean production p i? So, if I say P i and divided by small p i into sum over t ik, sum over k. What does it mean?

The whole thing means only p i. So, i can now write this equation like this T ij, future how many trips will happen from zone i to zone j equal to P i, future total production from zone i multiplied by something in numerator something in denominator. I can take it as a factor. What I am trying to indicate future T ij will be from i, how many will go to zone j, will depend on how much is the future trip production. A fraction of that will go to a particular zone j.

Now, what fraction will go, it is T ij, how much will go to a particular zone g j, will depend on the relative attraction of that zone. How we calculate the relative attraction T ij into A j by a j which is nothing but growth of zone j. So, what is the present T ij multiplied by growth of zone j divided by all t ik multiplied by growth of k, sum over all k. That means my i is connected to 1, 2, 3, 4 up to maybe k number of destination zones.

How much will go to a particular zone j, that will depend on what is the present value of T ij, multiplied by the attraction of zone j, divided by t ik multiplied by growth of zone k sum over all k. All the destination zones which are connected to zone i, sum overall k, that means all destination zones which are connected to i. So, that is the fraction, that much or that fraction of P i, future trip production will go to a particular destination zone j.

Now does it sound logical? So, if I have to little bit still help you, I would say this is my zone i, suppose i is connected to many zones different zones it is connected, so many zones are connected we are thinking how much we will go to a particular zone. Now each case there is a cell value here, some trips are happening here, some trips are happening here, some trips are happening here. So, all the zones which are connected some number of trips are happening.

So, what I am saying, how much from this i will go to a particular zone j will depend on how much is going to be produced from zone i in the future, total that is P i. How much is going to be produced or here also P i, multiplied by a fraction of that. Any value between 0 to 1, a fraction of that 2 strips will get terminated or will go to a particular zone j. How we get that fraction? I am saying what is the T ij, how many trips are happening here?

What is this value particular zone j, how many trips are really happening? Now divided by growth of this zone j divided by all such values T ik for different zones multiplied by the corresponding growth of zone k and then sum it over all zone k which are connected to zone i, as distinctions zones. So, if you see this if model what I have shown you in the previous slide and the way I have explained you here in this through this equation.

Both equations are more models are same only written in different forms and if you remember it in this way. So, only we are not going to forget it.

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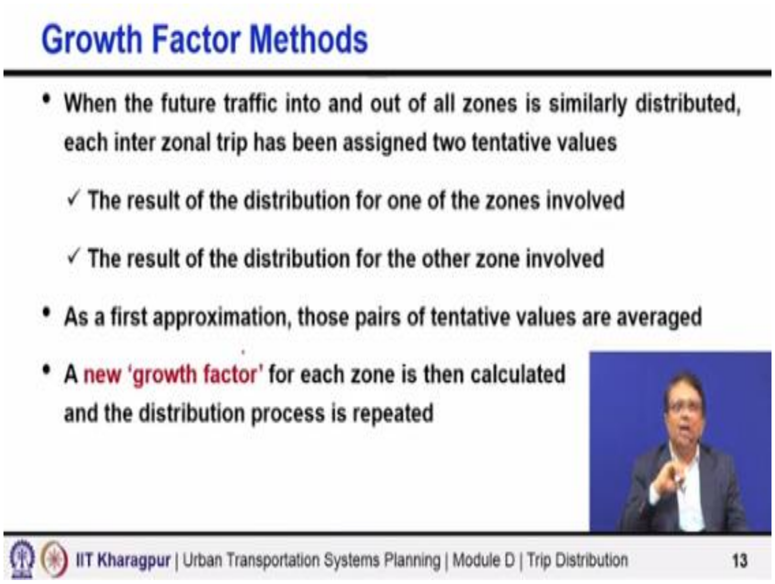

Then as I said that here also we have to go for iterative procedures, you are unlikely to;

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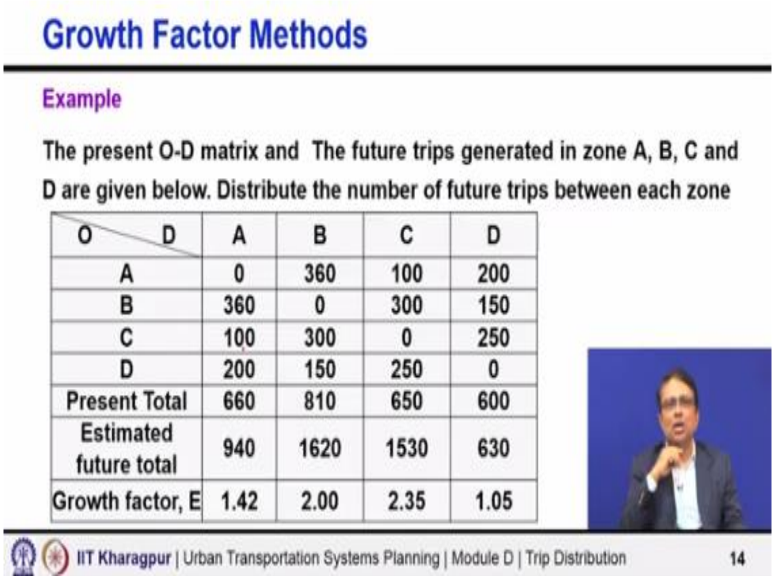

Get it everything all right in one go. So, let us take this example again. Here we have considered all the diagonal elements as 0, because we are interested in this example, we wanted to show how the interzonal trips are getting forecasted. How we can update? So, here again, you can see T ij equal to T ji. So, again, you know production attractions for a zone are going to be sent. We got the growth factor values as given here future estimated total.

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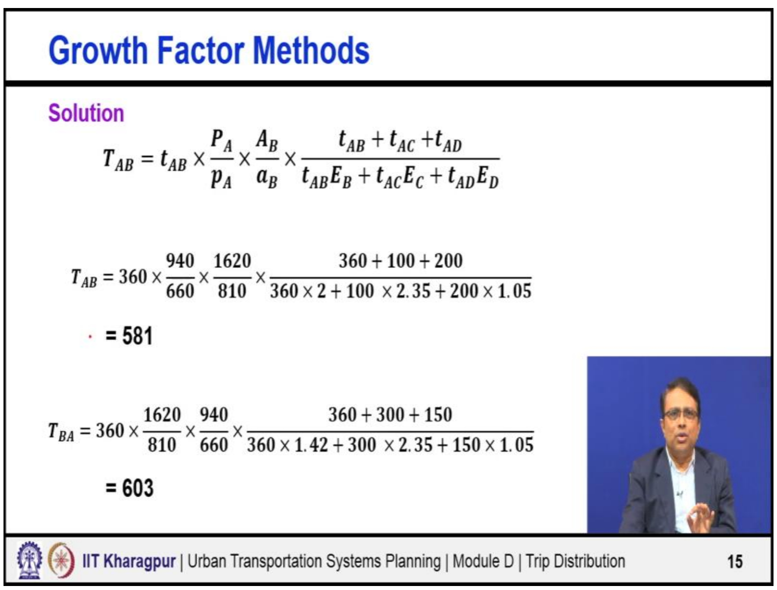

So, we know the growth factor value. You then apply that model whether this given models as it is shown here, that is what I have shown, you can also apply it the way I have shown it here. Both are fine, give you exactly the same result because I have shown these two models are not different. Whatever way I wrote it earlier and whatever we have wrote it, you know at a later stage, same model in a slightly different form, I feel that probably it is easy for you to remember once I show you like this.

The total production and the fraction of that it is guessing terminated to a particular zone. How is the fraction, what is the t ij, what is the growth of the j, divided by t ik into growth of k for all the zones k which are connected to i. So, you get those values, but here as I said you are going to get different value of T AB and T BA, because of this calculation but you know your matrix indicated as it was given initially it shows clearly that T ij and T ji are same.

So, what we will do, we will simply calculate separately T AB equal and value T BA value and then take average of this as a value of both T AB and T BA. Similarly T AC and T CA will be calculated separately and then the average of these two cells will be taken and we will put that value as T AC and also T CA. Similarly T AD, T DA will be calculated separately, average of these two values will be taken and the same value will put in T AD as well as in T DA, that is what I have shown here.

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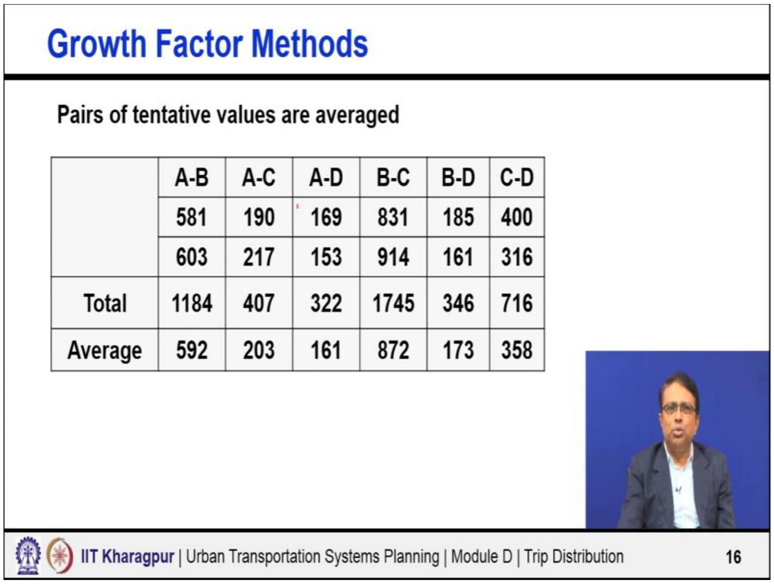

	A-B	A-C	A-D	B-C	B-D	C-D
	581	190	169	831	185	400
	603	217	153	914	161	316
Total	1184	407	322	1745	346	716
Average	592	203	161	872	173	358

In the next slide D AB and T BA values are calculated separately, T AC and T CA values are calculated separately. A D and D are calculated separately, then each cases like that for all the cells and then we take the total, take the average and put this average in the next matrix.

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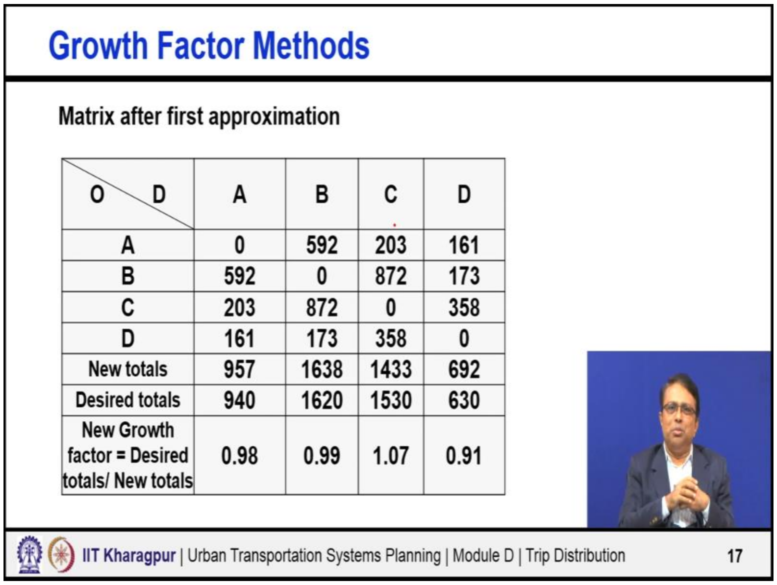

O \ D	A	B	C	D
A	0	592	203	161
B	592	0	872	173
C	203	872	0	358
D	161	173	358	0
New totals	957	1638	1433	692
Desired totals	940	1620	1530	630
New Growth factor = Desired totals/ New totals	0.98	0.99	1.07	0.91

And accordingly you get an update. Now, once you get an update. It will still not match with your desired total exactly we can see there is a difference. Some cases you will get you know lesser than your target total. In some cases, you will get a higher value than your target total. So, indicating that still somewhere it is more somewhere it is less. So, the overall productions and attractions are not matching exactly.

What to do? By this time, you know what you have to do. So, you know that you can go for one more iteration. So, just go for one more iteration. So, take your input matrix as the present matrix what you have got after the previous iteration and keep your target values number of trips same unchanged. Target number of trips will be same in every iteration but your growth factors will change.

Why the growth factors will change because your updated matrix is different in every iteration. Initially, it was the starting matrix for the base year then some updates have been happening in every iteration. So, end of each iteration you will get a value or you will get a matrix which is different from the previous model. So, your, you know, new matrix the row totals and column totals will change.

But you target row totals and column totals will not be changed that is as per the given. In the beginning itself, we have been told that, what should be the target row total and column total.

That is given in the beginning. So, that will those values will remain same. But this factor should be 0. So, growth factors will change and interestingly you will find always in all the subsequent iterations.

Some values will have cells or some zones will have cell, will have growth factor more than one and some cells will have growth factor less than one, because somewhere it has gone more somewhere it has gone less as well. So, as you progress as you make more iteration your factors, new growth factors will be very close to one or unity. And then as I have told you in all other methods here also you decide and acceptable errors because these are all planning estimates.

So, you decide an error how much error is acceptable to you, you decide the limit any limit you decide accordingly the number of iterations will vary, but things will eventually converge and most cases all of us we use computer programs, so no problem once you have built that logic inside and you have written a code simply give the input indicate the error level and it will keep on iterating till the time the acceptable result is object.

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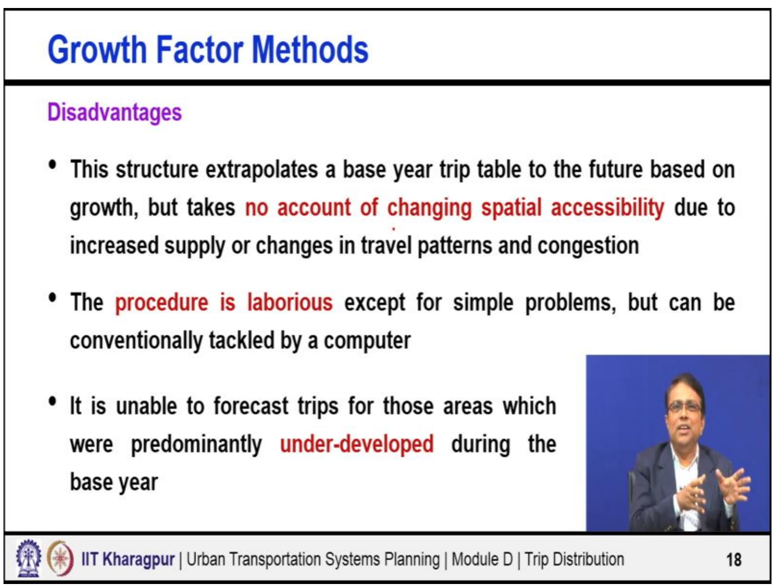

So, the disadvantage is again this structure explored a base year trip table, to the future based on the growth but takes no account of changing special accessibility, due to the increase supply or changes in the travel pattern or congestion. Fundamentally that is the weakness for every method, which are under growth factor based methods, we are not considering the reason for travel.

More travel or less travel because just imagine whatever is in the base year, if I have really developed some of the links and some of the new bus corridor I have introduced and you know the connectivity or to some destinations really improve significantly naturally in the future when more travel happens people will actually travel to those destinations more, because the travel has improved, the transport systems have been proved.

May be the new roads you have created, new transport system you have created, you have you know widened the road you have developed your maybe a few flyovers along certain corridors, such kind of changes are not getting considered it just a number game trying to balance the numbers, somehow to match the row total and the column total. So, that is the fundamental weakness of all these methods, all these growth factor based methods.

The procedure is laborious except for simple problems, but as I said, this is I cannot consider it as a disadvantage, because most of us will use a simple computer program and put the values inside. It is unable to forecast trip for those areas which were predominantly under developed during the base year, again this is not a problem with only frater model but almost or with all the models which are growth factor based models, or growth factor methods.

Because you are using a multiplier that is the basically, logically what happens and area which is already fully developed the scope for further development will be limited in that area. Whereas, it an area where you have more of an open area and presently load in city development or low level of activities, those are the areas which have, you know, more potential for developments. So, logically, I would probably expect.

That the number will grow much higher in area which is presently not so developed and where there are more development opportunities. Rather than areas which are completely saturated, but here because we are getting multiplier, so obviously the higher the number in the base year those numbers are further getting projected to a higher level and which remains lower will remain lower.

And if it is one, any case if it is 0, then anything gets multiplied by 0. So, which is not undeveloped at present may be zero development that will remain as zero development, which is very unrealistic because those are the areas which will develop much faster.

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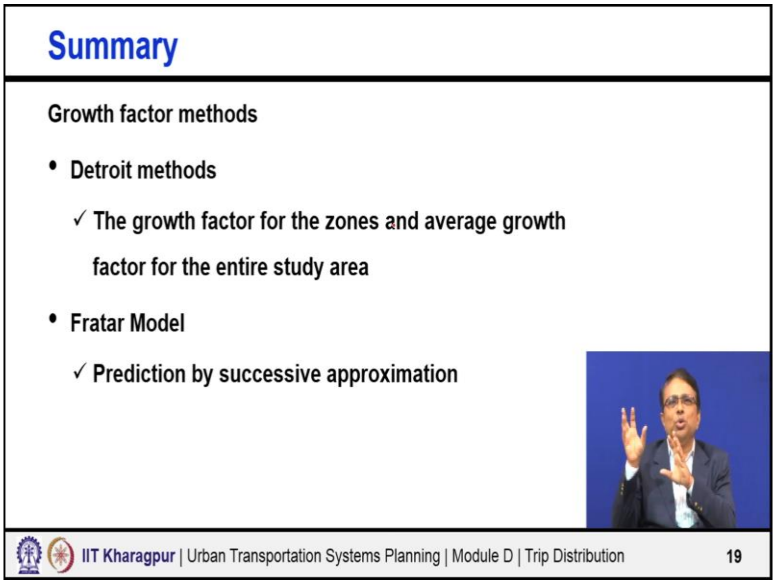

So, in summary what we discussed in today's class, we discussed two methods which are under growth factor methods, they are Detroit methods and Fratar method. You have seen the way the growth factor is applied to each cell is very different. Very different that is why two methods are also different and we took example problems to explain you, how you can apply and how you can produce metrics future t ij or in this context maybe we can say T ij value.

The future values of number of trips between i and j, knowing the present t ij, that present number of trips and knowing the future target totals; row totals and column totals and using this methods the way the growth factor values are calculated. That is all thank you so much.