Advanced Foundation Engineering
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Lecture – 8
Shallow Foundation: Bearing Capacity II

So, last class I have discussed about different shear failures and then Terzaghi's bearing capacity equation and then how we can use Terzaghi's bearing capacity equation for local shear failure because originally it is developed for general shear failure. Now, today I will discuss the other bearing capacity equations. So, first I will discuss about Meyerhof's bearing capacity equation, then Hansen, Vesic and then the IS code recommendations.

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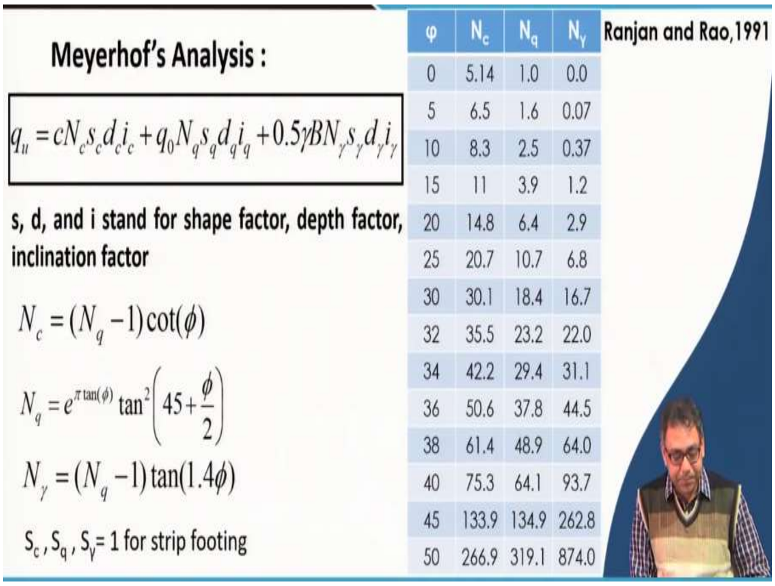

So, the second bearing capacity equation and before that I have also discussed Skempton's bearing capacity equation but that is applicable only for clay soil, and Terzaghi's bearing capacity equation is applicable for clay as well as sandy soil and c-ϕ soil. But in Terzaghi's bearing capacity equation the load is considered perfectly vertical and there is no eccentricity that means load is acting at the center of the footing.

But sometimes load can be eccentric as well as can be inclined, so there is no provision to include those effects into Terzaghi's bearing capacity equation. And then Meyerhof has developed a bearing capacity equation where we can include these options. That means we can include the effect of inclined load, we can include the effect of eccentricity into the bearing capacity equation.

In addition to that, in Meyerhof's bearing capacity equation, we can include the shape effect of the footing, we can include the depth effect of the footing also. So, this is Meyerhof's bearing capacity equation where this is a general type of equation because in next bearing capacity theories like Hansen, then Vesic and IS code the equation is more or less similar type.

So, in this equation cN_c is similar and c is the cohesion of the soil, N_c is the bearing capacity factor, then q_0 is the γD_f and N_q is also bearing capacity factor, N_γ is also bearing capacity factor, B is the width of the foundation, but here s_c, d_c, i_c or s, d, i these are different factors. So s is the shape factor, d is the depth factor and i is the inclination factor. So that means originally these equations are developed for the surface footing.

So, if you have a footing with a depth, then you can incorporate that depth effect into this bearing capacity equation. So as I mentioned if the loading is inclined and different shapes of footing like your rectangular footing with different L/B ratio, then the square footing and also you can include the eccentricity of the loading that I will discuss in later part of this course that how we can include the eccentricity of the loading in detail.

So, these are the factors, but here the bearing capacity factors N_c, N_q, N_γ values are given corresponding to different ϕ values from 0 to 50°. This is similar to Terzaghi's bearing capacity factors, but values are different, these bearing capacity factor are coming from this equation. Those are given here in N_c, N_q and N_γ.

And obviously these equations are originally developed for the strip footing. So for strip footing, all the shape factors are 0. So, this is the bearing capacity equation for Meyerhof.

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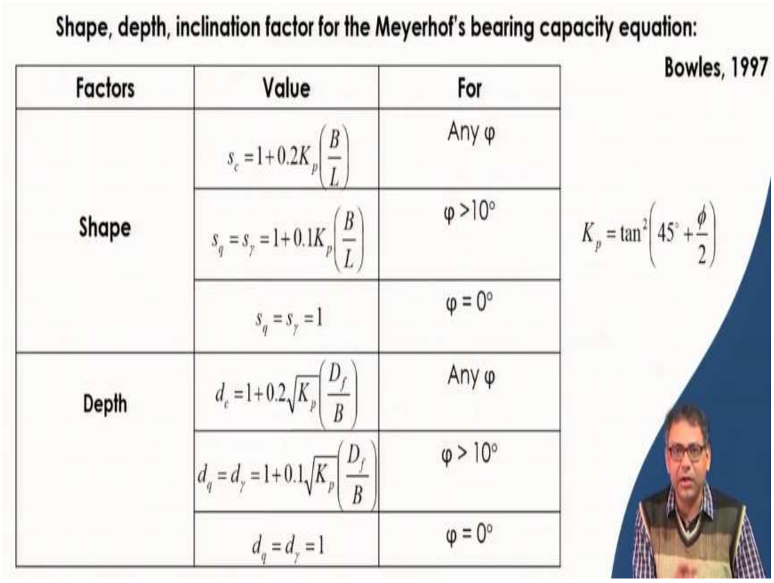
Shape, depth, inclination factor for the Meyerhof's bearing capacity equation:

Bowles, 1997

Factors	Value	For
Shape	$s_c = 1 + 0.2K_p\left(\frac{B}{L}\right)$	Any φ
	$s_q = s_\gamma = 1 + 0.1K_p\left(\frac{B}{L}\right)$	φ >10°
	$s_q = s_\gamma = 1$	φ = 0°
Depth	$d_c = 1 + 0.2\sqrt{K_p}\left(\frac{D_f}{B}\right)$	Any φ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\left(\frac{D_f}{B}\right)$	φ > 10°
	$d_q = d_\gamma = 1$	φ = 0°

$K_p = \tan^2\left(45^\circ + \frac{\phi}{2}\right)$

And so these factors can be calculated from these tables. So, shape factors for any ϕ value, s_c can be calculated by using these equations. So, you can use these tables to determine the shape factor, depth factor. I am just explaining that if it is N_q and N_γ then we can use this expression where K_p is $\tan^2\left(45° + \frac{\phi}{2}\right)$, ϕ is the friction angle of the soil. Similarly, s_q, $s_\gamma =$ 1, if ϕ = 0°.

Similarly, we can calculate the depth factor also, D_f is the depth of the foundation, B is the width of the foundation and again for ϕ = 0°, d_q, $d_\gamma = 1$ and d_c is $1 + 0.2\sqrt{K_p}\left(\frac{D_f}{B}\right)$ for any ϕ, but $d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\left(\frac{D_f}{B}\right)$ if ϕ > 10°. So, either you will get ϕ = 0° or ϕ > 10° in most of the cases either it is a cohesive soil that means your ϕ = 0° or if it is a c-ϕ soil then if ϕ > 10° then we can use these expressions.

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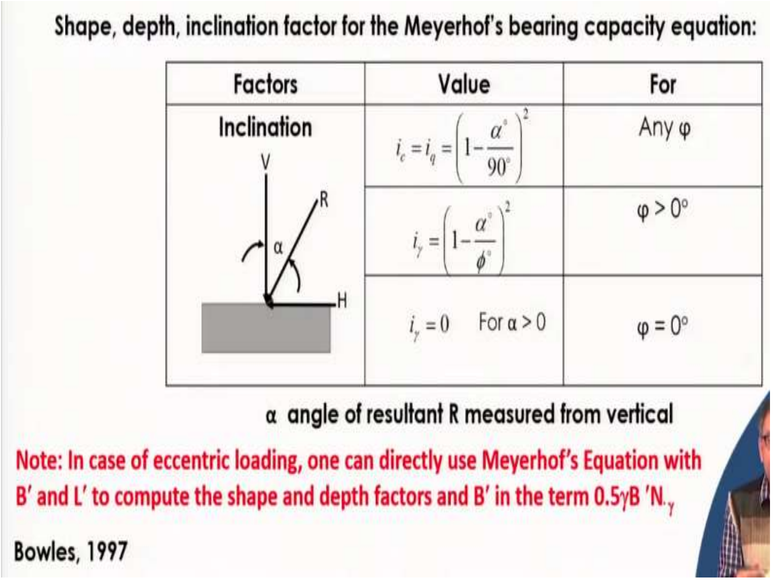

And then for inclination factor this R is the load which is inclined and this R has two components, obviously one is vertical component V, another is horizontal component H. So two components and this α is the angle which R is making with the vertical axis or the vertical load. So that means here α, i_c and i_q we will get from this equation and i_γ you will get from this equation if $\phi \geq 0°$.

If $\phi = 0°$, then i_γ will be 0 for any $\alpha \geq 0°$. So, α is the angle of resultant R measured from the vertical, remember that α is measured from the vertical. Now as I mentioned that in this equation, we can incorporate the eccentricity also, but you just note these two lines where it is written that in case of eccentric loading, one can directly use Meyerhof's equation with B' and L' to compute the shape and depth factor and B' in the term $\frac{1}{2}\gamma B' N_\gamma$.

What does it mean? As I mentioned I will discuss this eccentric loading effect in detail, but what happened due to the eccentric loading because now loading is not acting at the center of the footing. So, loading is acting outside the center of the footing. So, the effective area of the footing is now not being too well. So because as the loading is not acting in the center, it is acting away from the center, so our effective area of the loading will also decrease.

So, now the width will not be B, width will be B' because B' is the effective width in case of eccentric loading and L' is the effective length in case of your eccentric loading. So depending upon which way the eccentricity is, it is in L direction or it is in B or width direction or if it is in both the direction, so based on that we have to determine the effective width and effective length and then we have to calculate the effective area also.

So, that means now in case of eccentric loading when we calculate this shape factor and depth factor as well in Meyerhof's equation, that means you will not use B or L, we will use B' and L'. So, how we will calculate the B' and L' those things will be discussed later on, but now you remember that for eccentric loading we have to calculate B', L' and that B' and L' we have to replace in B and L okay.

So that means B or L or both have to be replaced by B' or L' in case of eccentric loading during calculation of shape factor and depth factor as per Meyerhof's equation, clear. So, now another thing is that here in the third term there is a term B. So in case of eccentric loading, we don't have to take B, we have to take B' okay, so this you have to remember. So, these two lines are very important.

So every theory, right now I will give this note and later on I will discuss how these things can be used. So, remember that in case of eccentric loading when you compute shape factor and depth factor you use B' and L' that means effective width and effective length and instead of B in the third term of the equation you use B' okay.

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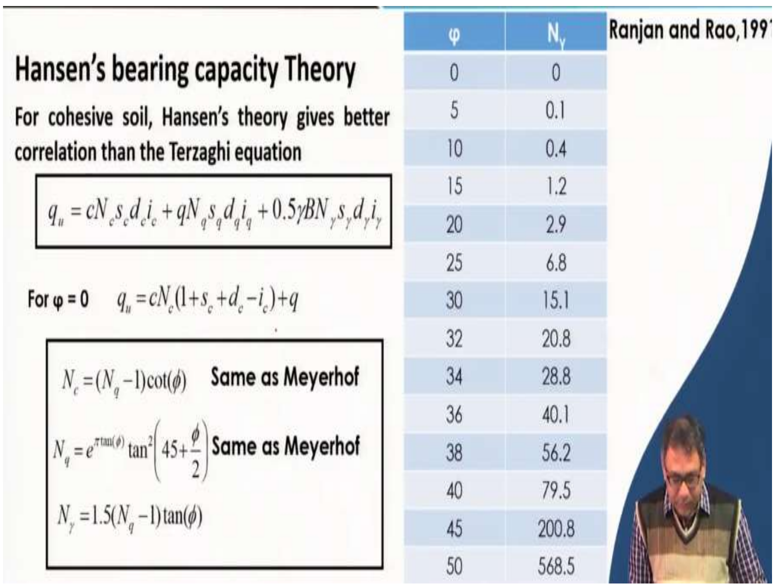

So, next one is Hansen's bearing capacity theory. So it is observed that for the cohesive soil, Hansen's theory gives a better correlation than Terzaghi's equation. So, this is a similar type of equation, as I mentioned this is a general type of equation. So, Hansen's bearing capacity theory, this is the general equation and the N_c and N_q values are same as Meyerhof, but N_γ value is different.

So, that N_γ value is given in this table and remember that for $\phi = 0°$ this is the equation okay. So, $\phi = 0°$ this will be the equation because $cN_c(1 + s_c + d_c - i_c) + q$ okay. So, in previous equation q_0 is mentioned, here it is q, but in both the cases either it is q or q_0 it is γD_f.

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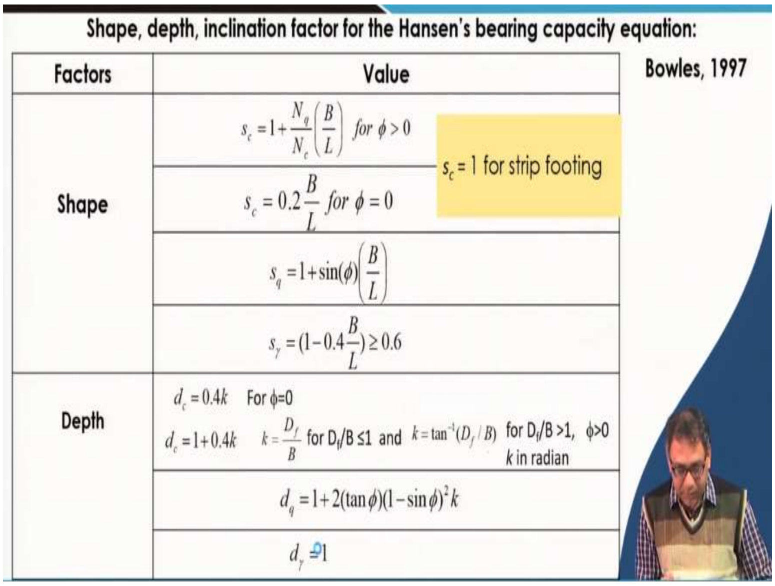

Shape, depth, inclination factor for the Hansen's bearing capacity equation:

Bowles, 1997

Factors	Value
Shape	$s_c = 1+\frac{N_q}{N_c}\left(\frac{B}{L}\right)$ *for* $\phi > 0$
	$s_c = 0.2\frac{B}{L}$ *for* $\phi = 0$
	$s_q = 1+\sin(\phi)\left(\frac{B}{L}\right)$
	$s_\gamma = (1-0.4\frac{B}{L}) \geq 0.6$
Depth	$d_c = 0.4k$ For $\phi=0$ $d_c = 1+0.4k$ $k = \frac{D_f}{B}$ for $D_f/B \leq 1$ and $k = \tan^{-1}(D_f/B)$ for $D_f/B > 1$, $\phi>0$ k in radian
	$d_q = 1+2(\tan\phi)(1-\sin\phi)^2 k$
	$d_\gamma = 1$

$s_c = 1$ for strip footing

So, now here we can determine the shape factor, depth factor and inclination factor as per Hansen's bearing capacity equation. So, shape factor you can get by this equation. Now if $s_c = 0.2\frac{B}{L}$ for $\phi = 0°$, for $\phi > 0°$ we will use this equation okay. Similarly, we can calculate s_q and s_γ, remember that this $s_\gamma \geq 0.6$. So, if s_γ is coming say less than 0.6, then we have to use 0.6, remember that.

If it is coming more than 0.6, then the actual value we have to use but if it is coming less than 0.6 then you have to use 0.6. And for $\phi = 0°$, $d_c = 0.4 \times k$, what is k? k is $\frac{D_f}{B}$, if $\frac{D_f}{B} \leq 1$, and if $\frac{D_f}{B} > 1$ then $k = \tan^{-1}\frac{D_f}{B}$, remember that k is in radian okay. And similarly d_q also we can calculate by using this expression.

Similarly, we can calculate k if $\frac{D_f}{B} \leq 1$ and if $\frac{D_f}{B} > 1$. Based on that we have to select the k. similar to d_c and d_q, d_γ is always 1 for any ϕ value.

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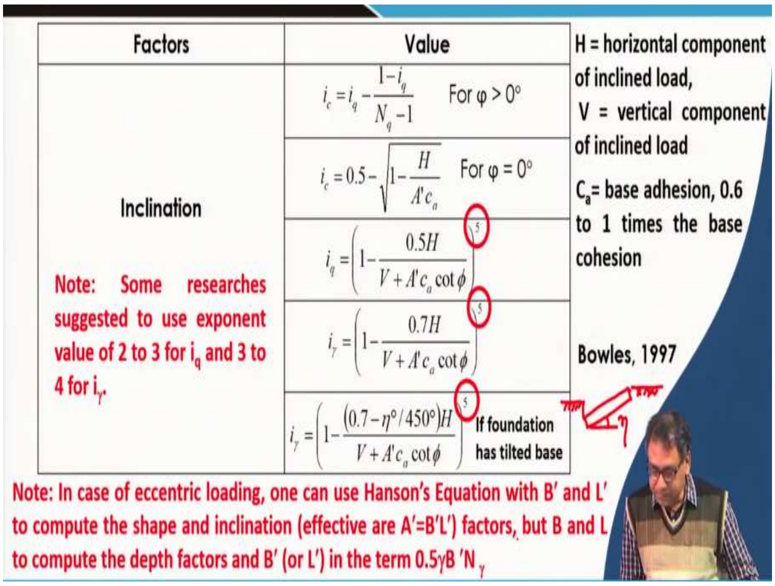

Now, this is the inclination factor, we will calculate the inclination factor. So inclination factor i_c, i_q we will get by using this equation, i_q, N_q I will get from the chart because N_q is similar to the Meyerhof's N_q. So from that chart we can get the N_q value and similarly here also N_c and N_q also we will get from the chart because N_c and N_q both the same as suggested by Meyerhof.

If $\phi > 0°$, then i_c will be this equation and if $\phi = 0°$, then i_c will be this equation, where H is the horizontal component of inclined load because inclined load has two components. One is vertical, one is the horizontal. So, H is the horizontal component of the inclined load and then i_q can be determined by using this expression and i_γ also can be determined by this expression.

And here two i_γ equations are given, now first i_γ equation if foundation base is not tilted. Suppose if foundation base is tilted because there is a tilt, suppose if the foundation base this is my foundation okay, so this is a foundation and this is the surface and this foundation is tilted. Suppose it is tilted at angle of η with the horizontal. So that means the foundation base is not perfectly horizontal.

So, first i_γ expression we can use this foundation base is horizontal, but second i_γ equation we will use if foundation has tilted base with an angle η okay, so remember that. So, these are the differences between two i_γ expressions. So, second one you will use for if foundation has tilted base, which means foundation is not perfectly horizontal, foundation has a tilt, okay. So, if the tilt is there, then we will use this equation.

So, later on I will discuss in details about the tilted base foundation, so this time we will use the second i_γ expression. And then a few things you have to note, one that there is a term A', so A' is the effective area. So, if your loading is not eccentric, then obviously $A = A'$, A' is the effective area, B' is the effective width, L' is the effective length.

So if loading is acting perfectly at the center, then $A = A'$ or $B = B'$ or $L = L'$. But if there is an eccentricity, then you have to calculate the L', B' and then we have to calculate the A'. A'will be obviously equal to $B' \times L'$ okay. So that A' we have to use. And there comes C_a. If there is cohesion in the soil, then C_a basically the adhesion, okay.

So, that adhesion I can calculate by multiplying adhesion factor 0.6 to 1 with the base cohesion. Suppose base cohesion has a value of 20 kPa so, the C_a value will be if I take in between 0.6 to 1 that means 0.7 say, so that means the C_a will be 0.7 times of that cohesion 20, so that will be 14. So C_a value will be 14 if I take adhesion factor as 0.7 and the cohesion value as 20 kPa.

So, you can take any value in between 0.6 to 1. If anything is mentioned in the question then you have to use that value, otherwise during design you can use any value between 0.6 to 1 time the base cohesion, the cohesion which is acting at the base of the footing. Another one is as I mentioned H is the horizontal component of the inclined load, V is the vertical component of the inclined load.

And one thing is mentioned the original expression given by the Hansen, this exponent was given 5 okay. So, later on many researchers have observed that it is a very higher value. So, now, they have suggested lower value of this exponent and that value is suggested as 2 to 3 for i_q and 3 to 4 for i_γ. Instead of using 5 you can use any value between 2 to 3 for i_q, you can use 2 also, you can use 2.5, you can use 3 also in case of i_q or in case of i_γ you can use 3, 3.5 or 4, okay.

Slightly lower value, but as the original equation is given 5, so in all the example problems I will use 5, but in your question if it is mentioned then you have to use this exponent value. If nothing is mentioned in the question, then you can use the 5 directly okay. If nothing is mentioned you use 5, if any value is mentioned within this given range, then you have to use that value.

In your design purpose, you can take any value in between the given range or you can use the 5 also, but as it is suggested these ranges so you can use this range. But in my example problems I will use 5 and if nothing is mentioned in the assignment problem or the exam question you have to also use 5, but if any value is mentioned in the question then that value you have to use.

And note another important thing is that in case of eccentric loading, again like Meyerhof that one can use Hansen's equation with B' and L' to compute the shape and inclination factors. So, remember that here because different theories have suggested different approach to take care of the eccentric loading effect as in Meyerhof's approach, what was the thing that you use B' and L' to compute shape factor and depth factor okay.

But as per Hansen's theory that you use B' and L' to compute the shear factor and inclination factor, but for depth factor calculation even if for eccentric loading you use B and L, not B' and L' okay. So, that is mentioned here because inclination there is A', so that you can use as B' and L', but B and L to compute the depth factors okay. So that means we have to use B and L to compute the depth factor as per Hansen's bearing capacity equation for eccentric loading.

But to compute shape factor and inclination factor you have to use B' and L' or effective areas. But in the third term of the equation, you have to use B' okay or L', later on I will discuss why I have mentioned L' okay because Meyerhof's equation I have given only B', but here I have given B' or L', why it is B' or L' because in Hansen's equation there is a provision that you can incorporate the effect of horizontal loading direction also.

What does it mean that in Meyerhof's equation the only angle is given, that means this load can act towards the width direction and towards the length direction also. For example, if loading is acting parallel to the width okay, horizontal load because inclined load has 2 components, but in Meyerhof's equation that angle is only given, so you can calculate the angle and you can put it.

So, there is no provision to incorporate that which direction the horizontal load is acting, vertical load will always act vertically, but horizontal load will act horizontally but there is a provision that it can act parallel to the length or it can act parallel to the base or width also. So that provision can be incorporated in Hansen's equation and Vesic also. So, remember that it is also advantage that if you have a loading direction specifically mentioned, then you can use Hansen's equation, you can use Vesic's equation.

You can also use Meyerhof's equation because if the vertical load and the horizontal load are given, you can calculate the resultant and you can calculate the angle at which the load is acting with the vertical and you can use Meyerhof's equation. But in Hansen's equation that specifically if the horizontal load is parallel to length or parallel to width that can be incorporated into the equation.

So, that is why if the loading is parallel to L, then we have to actually check both the conditions that means taking B' as well as L'. So, I will explain these things what I have discussed right now through an example problem, then you will find that how this thing can be incorporated in Hansen's equation. So, that means these things the only difference is that depth factor when you calculate, you put B and L, not B' and L'.

But for other factors and the third term, use the effective length or effective width or effective area in case of eccentric loading in Hansen's equation.

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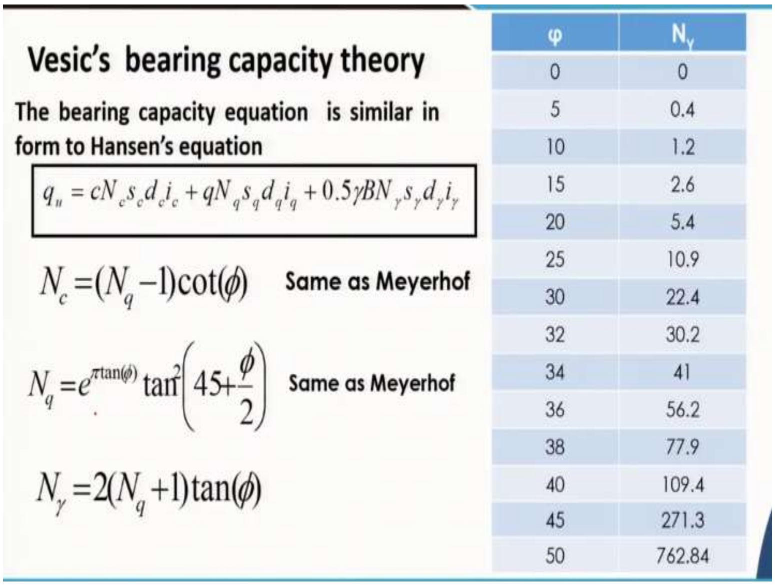

Now, this is Vesic's bearing capacity theory which is similar to Hansen's bearing capacity theory. This is again the general equation, but the N_c value and N_q value we will get for the Meyerhof's equation, but again N_γ is different. So, this is the N_γ table. From here I will get the value of N_γ corresponding to different ϕ value.

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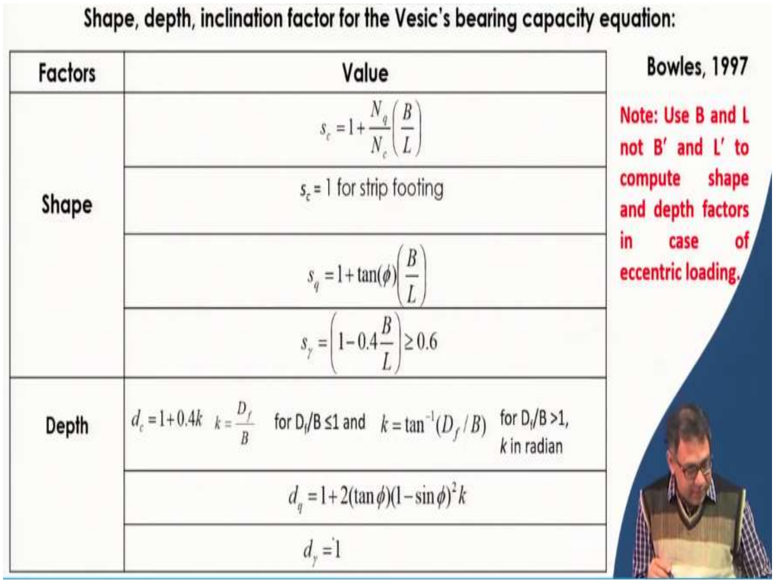

And this is the table which is similar to Hansen's theory, but remember that in case of Hansen's if $\phi = 0$ our particular equation is given, but here it is not given. So that means if you use Hansen's equation for $\phi = 0$, then the table that is given, so you have to use that table, then you have to use that equation which is given for $\phi = 0$, but here in Hansen and Vesic you can see that type of equation is not given for $\phi = 0$.

It is a general equation where you can use these factors. So, that means s_c value you can calculate by using this value, then s_c is 1 for strip footing because for all the values shape factors are 1 for strip footing and s_q we will get by using this equation, s_γ we will get by using this equation, and again s_γ should not be less than 0.6 okay. And d_c because as there is no option for $\phi = 0$ case, so it is a general equation for any ϕ.

And that is similar to Hansen that ϕ for d_c we can use by using this expression for any ϕ. Two cases, one is if $\frac{D_f}{B} \leq 1$, another if $\frac{D_f}{B} \geq 1$ okay. And then d_q I will get from this equation and d_γ is 1 like Hansen. But again, when I calculate these factors for eccentric loading you can see but here as per Vesic's recommendation you use B and L when you calculate shape factor and depth factor both for eccentric loading okay.

So that means you have to use B and L, not B' and L' to compute the shape factor and depth factor in case of eccentric loading in Vesic's bearing capacity equation, remember that.

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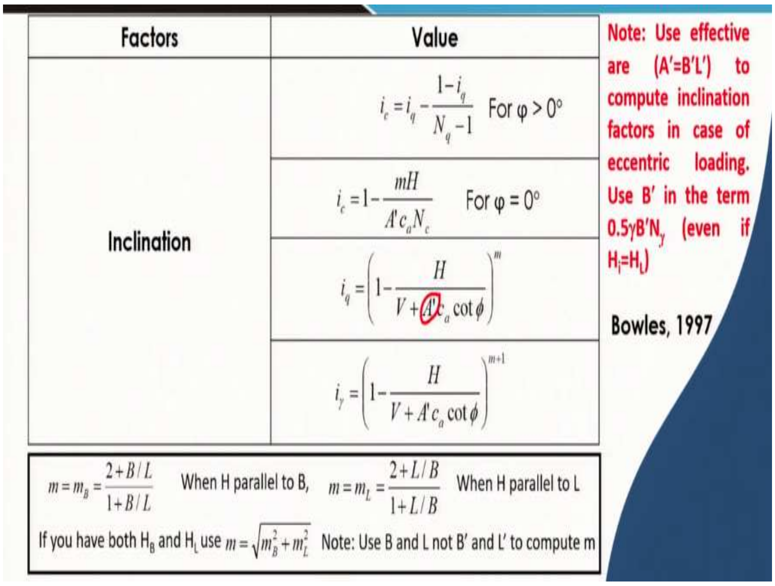

Factors	Value
Inclination	$i_c = i_q - \frac{1-i_q}{N_q - 1}$ For $\varphi > 0°$
	$i_c = 1 - \frac{mH}{A' c_a N_c}$ For $\varphi = 0°$
	$i_q = \left(1 - \frac{H}{V + A' c_a \cot\phi}\right)^m$
	$i_\gamma = \left(1 - \frac{H}{V + A' c_a \cot\phi}\right)^{m+1}$

$m = m_B = \frac{2 + B/L}{1 + B/L}$ When H parallel to B, $m = m_L = \frac{2 + L/B}{1 + L/B}$ When H parallel to L

If you have both H_B and H_L use $m = \sqrt{m_B^2 + m_L^2}$ Note: Use B and L not B' and L' to compute m

Now, this is the inclination factor, so this is $\phi = 0$ case and this first one is the $\phi > 0$ case and then i_q, i_γ. So, we can use these two equations A' is same as effective area c_a is same as 0.6 to 1 time the base equation c_a value and ϕ is a friction angle, but m value you will calculate by using these conditions. So, again like Hansen's here also if the horizontal component of the inclined load is parallel to B or L or both that effect can be incorporated here okay.

Now, m will be m_B when H horizontal component of the inclined load is parallel to B okay. If it is parallel to width, then we will use this equation m will be m_B. Now, if it is parallel to L, then m will be m_L okay. If it is parallel to both, then you have to calculate m_A and m_L and m_B, then you use the $\sqrt{m_B^2 + m_L^2}$ then that will be the m, so this clear. Another thing that again you use B and L, not B' and L', to compute m okay.

So that means in Hansen's expressions if you see Hansen's expression that parallel to B or L case is not directly incorporated in these factors, not in the shape factor, depth factor, not in the inclination factor, it is mentioned H okay. So that can be either parallel to B or parallel to L, but specifically it is not mentioned. So later on, I will explain how that effect can be incorporated okay.

But in case of Vesic that effect is incorporated in inclination factor determination okay because when you calculate the m, it will be m_B if H is parallel to B, m will be a m_L if H is parallel to L. If they are parallel to both, then both H_B and H_L, H_B means horizontal load parallel to B, H_L means horizontal load parallel to L, both are acting then your m will be $\sqrt{m_B^2 + m_L^2}$.

I mean you have to calculate m_B and m_L and then you calculate the m part. Again, remember that when you calculate these things for eccentric loading, do not use B' and L' during calculation of m, you use B or L. So, again you have to use B and L during calculation of m, not B' or L', but you have to use effective area A' okay that will be effective area A'during calculation of i_q and i_γ.

So, A' will be the effective. So that means when you calculate depth factor and shape factor as per Vesic use B and L, when you calculate m use B and L, but when you use this A you use the effective area and in the third term of the equation you use B' okay. So, you use B'. So, this is the difference in all these three cases, the three theories that I have discussed that in the third term you have to use B',that means effective for all the cases.

But for Meyerhof, all factors will be calculated by using B' and L'. But Vesic I mean factors will be calculated considering B and L except the A where you have to consider the A'. For Hansen you have to calculate all the factors by considering effective length or width or area except the depth factor, clear okay. So, now the in the next class, I will discuss the first IS method, what is our IS code recommendation.

Then I will solve few example problems and other effects like if the loading is inclined, if the loading is eccentric or a compressibility effect in the bearing capacity equation. Thank you.