Advance Foundation Engineering
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Lecture – 67
Well Foundation – VIII

So, last class I was discussing another method to determine or to check the stability of well. And that method was the ultimate soil reaction method. And then I have discussed the load factor and the load combinations.

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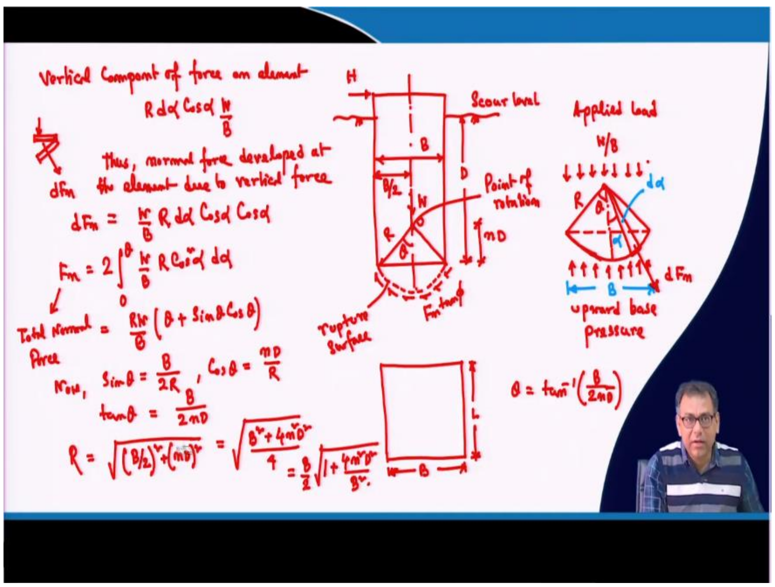

And then I was discussing about the normal force which is acting along this rupture surface because it is assumed that because of the rotation of that well about a point say O. So, this is the point about which this well is rotating. So, there will be a rupture surface below the well within the soil and the total normal force along that rupture surface is $\frac{RW}{B}(\theta + \sin\theta\cos\theta)$.

So, now if I look at this figure that $\sin\theta$ will be equal to so because this is θ this is nD this is R and this one is B, so this will be definitely $\frac{B}{2}$, so $\sin\theta$ will be $\frac{B}{2R}$. So, $\sin\theta$ will be $\frac{B}{2R}$, $\frac{B}{2}$ is this value, this is θ this is the R. So, this will be $\frac{B}{2}$ divided by R because in this triangle we can get that. Similarly, $\cos\theta = \frac{nD}{R}$.

So, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{B}{2nD}$ and what is R? Because we have to convert all these parameters. So, this is R. R will be nothing but $\sqrt{\left(\frac{B}{2}\right)^2 + (nD)^2}$ because this is R, so this is $\sqrt{\left(\frac{B}{2}\right)^2 + (nD)^2}$. So, we can write that this is equal to $\sqrt{\frac{B^2+4n^2D^2}{4}}$. So, finally I can write this is equal to $\frac{B}{2}\sqrt{1+\frac{4n^2D^2}{B^2}}$ so this is the equation of R.

Now if I put all these values in the F_n, so θ I can write that $\theta = \tan^{-1}\left(\frac{B}{2nD}\right)$. So, I can put all these values here.

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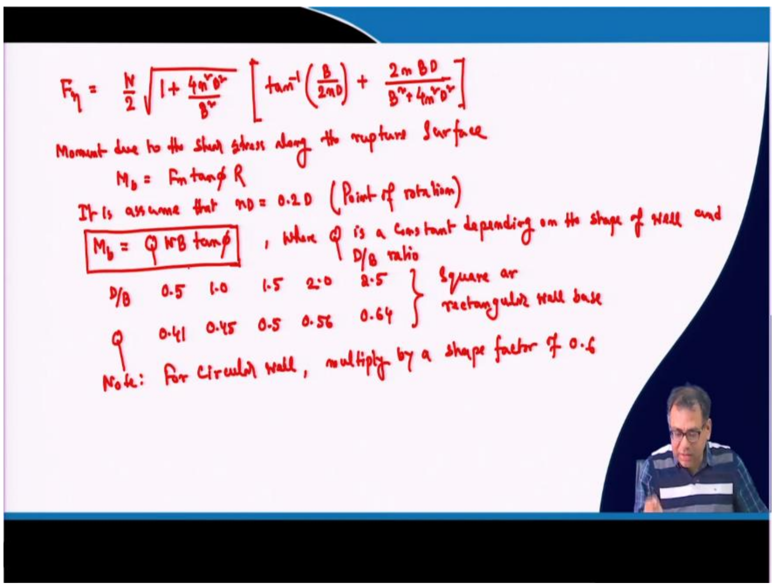

And finally, I will get the expression of F_n so that will be $\frac{W}{2}$ then R is there. So, if I put this all θ $\sin\theta$ $\cos\theta$ value and R value in terms of nD and B and simplify the equation then I will get the F_n expression is like this. So, $F_n = \frac{W}{2}\sqrt{1+\frac{4n^2D^2}{B^2}}\left[\tan^{-1}\left(\frac{B}{2nD}\right) + \frac{2nBD}{B^2+4n^2D^2}\right]$.

So, this is the expression so this B will cancel out in the R term there is a B so this will be simple $\frac{W}{2}$ and this term. So, and then if I put θ, $\cos\theta$ and $\sin\theta$ then I will get this equation. So, this is the F_n. So, now F_n is acting along this surface, now what is moment due to this F_n. So, $F_n \times \tan\phi$ is acting around the surface.

So, the moment due to this $F_n \tan\phi$ because now F_n is the normal and if I multiply that F_n with $\tan\phi$ then that will give you the shear stress which is acting along the rupture surface. Now the moment due to the shear stress will be how much, so the moment due to the shear stress along the rupture surface will be how much? $F_n \tan\phi \times R$. R is the radius and that radius is same for along the rupture surface.

So that will be the moment so this moment M_b that will be a $F_n \tan\phi \times R$, now it is assumed that $nD = 0.2D$. So, it is observed that this point of rotation is around $0.2D$. So that above of the base of the well, so that means it is assumed that $nD = 0.2D$. So, it is recommended by observation that the point of rotation is 0.2 times of D above the base of the well. So, this is the point of rotation.

So, now we can put this value here and then I will get the M_b, so M_b is $F_n \tan\phi\, R$. Now R also I can put this value which is also in terms of nD and B. So, now if in place of nD I put $0.2D$ and then I simplify this equation, so I put F_n here $\tan\phi$ also I can replace by this equation. So, sorry ϕ is the friction angle so $\tan\phi$ should be there. So, $\tan\phi$ will be there R I can replace with this equation.

So, R I can replace with this equation then F_n I can put with this equation I can put $0.2D$ in place of nD and then if I simplify that then M_b will be $QWB \tan\phi$, now where Q is a constant depending on the shape of well because and $\frac{D}{B}$ ratio. So, this Q is also determined from this equation and for different $\frac{D}{B}$ because in this equation, now this M_b equation what are the terms present only the $\frac{D}{B}$ because nD is replaced by $0.2D$ so there will be only D and B.

So that means and there is a W term also so WB and D. So, this Q is basically function of $\frac{D}{B}$ and the shape of the well. So, and then these Q values are given in terms of $\frac{D}{B}$ values. So, $\frac{D}{B}$ value and this is the Q value, so if $\frac{D}{B}$ is 0.5 then Q is 0.41, if $\frac{D}{B}$ is 1 then Q is 0.45, if $\frac{D}{B}$ is 1.5 then Q is 0.5, if $\frac{D}{B}$ is 2 then Q is 0.56, if $\frac{D}{B}$ is 2.5 then Q is 0.64 and these values are for square or rectangular well. Now note if it is a circular well we have to multiply by a shape factor of 0.6.

That means first for circular well we will get the Q value from this table and then we have to multiply by 0.6 with these values we can directly use for square and rectangular well base but for circular well we have to multiply by 0.6 with these values. So, this is M_b we can determine from this equation so that means M_b now we can determine. So, this is the M_b which is the moment due to the shear stress acting along the rupture surface, so this M_b we can determine.

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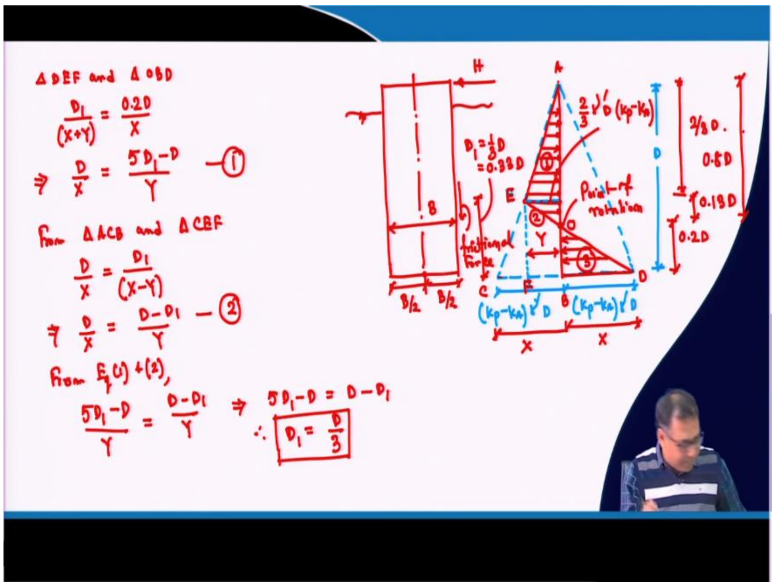

Next is that suppose this is the well position and again this well is rotating with a point. So, one side will be active and another side will be passive and I have discussed that what is the pressure distribution because it is not acting at the base because if it is rotating at the base then one side will be totally passive and another side totally active because elastic theory analysis so that is the case because one side is totally active one side is totally passive.

But for Terzaghi's analysis this point of rotation for light well is slightly above the base of the well so that means one side will be active and then another side would be the active and that thing will change above the point of rotation and below the point of rotation. So, here also the point of rotation is above the base of the well. So that means here also the active passive zone will change based on the point of rotation above the point of rotation and below the point of rotation the active and passive zones will change.

So, but in Terzaghi's analysis I have shown how we can get the net pressure distribution in such case but here this pressure distribution is assumed. So assumed pressure distribution is that again so if I draw that net pressure distribution, so again one side is active one side will be passive so there will be a net pressure distribution. So that is the net pressure distribution.

And this is equal to $(K_P - K_A)\gamma'$ and we can say this is D because this is the depth of the well and this is also $(K_P - K_A)\gamma' D$. So, this is also depth of the well but here the net pressure distribution is drawn or assumed in such a way that this is a point of rotation. So, O is a point of rotation, so that point of rotation is assumed at a distance of $0.2D$ from the base.

Now the pressure distribution is assumed such that at point of rotation the net pressure is 0. So, this will be the net pressure distribution of the well. So, this is the net pressure distribution or assumed pressure distribution for the well and obviously if the horizontal force is acting in this other direction or because the horizontal force can be at any direction. So, this figure I have drawn this is acting from left to right but it can be from right to left.

So, it is not a problem but according to that I have to draw the pressure distribution. So, here the pressure distribution I have drawn in this direction means the horizontal load is acting in the other direction. So, but you can draw it in the other side also if the direction of the horizontal force is different so that is not an issue. So, for this particular case so definitely the horizontal force is acting in this direction.

But this can be other direction also then this figure will be totally the opposite this side will be the when the figure will just be shifted. So, this is the scour level so now this is $0.2D$ and this value is same as D_1 and this is total is D. So, now if I write this is A this one B this is one C then this is D and then if I draw one line here, so this is say, E and this is F if I write this point this point is E and this point is F.

And this is the center line of the well this is the width and definitely this will be $\frac{B}{2}$ this will be $\frac{B}{2}$ fine. Now from this particular case if I consider triangle DEF and triangle OBD so we are considering DEF this triangle and DOB this triangle and if I take these distances Y are these pressure these values Y and this value is X say this is X and this one is Y then from the similar triangle because these two triangles are considered as similar triangle one is this bigger one another is the smaller one.

So, this means similar triangles that means these bigger one and the smaller one the similar triangle I can write that $\frac{D_1}{X+Y}$. So that will be equal to this height is $0.2D$. So, this is $\frac{0.2D}{X}$. So, now if I simplify this equation so I can write after simplification this equation I can write $\frac{D}{X} = \frac{5D_1 - D}{Y}$. So, this is equation number 1.

Now again from another similar triangle ACB and the similar triangle CEF. ACB and then CEF So, these bigger triangle ACB and this smaller triangle CEF, so these from these two similar triangles also I can write that $\frac{D}{X}$. So that will be this distance then if this is X, this one Y, so these will be definitely $X - Y$. So that will be D_1 because this height is D_1 and this distance will be $X - Y$, so this will be $X - Y$.

And then if I simplify this equation so then it will be $\frac{D}{X} = \frac{D - D_1}{Y}$. So, now if I compare equation number 1 and 2 then I can write from equations 1 and 2 that $\frac{5D_1 - D}{Y} = \frac{D - D_1}{Y}$ and these Y, Y will cancel out. So, if I simplify this equation then I will get that $5D_1 - D = D - D_1$ therefore, $D_1 = \frac{D}{3}$.

So, this D_1 is the point where the stress is changing from this side to that side this E point is at $\frac{D}{3}$ distance above the base of well. So, now we have determined what is the value of D_1 in terms of D? So, now we have to take the moment so what are the moments we will consider?

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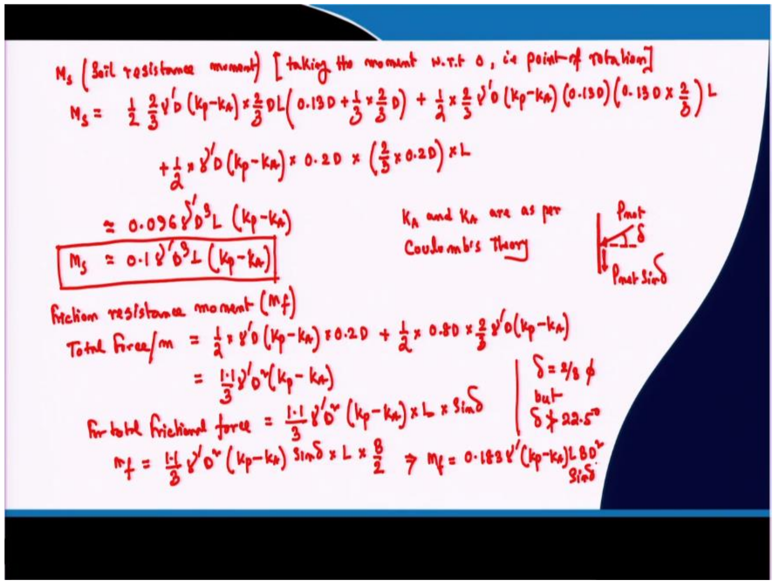

So, first we will consider the moment M_s so this M_s is due to soil reaction or resistance. So, M_s is the moment due to the soil resistance or soil resistance moment. So, the soil resistance moment we are taking with respect to O because this is the point of rotation, so we are taking the moment with respect to O that is point of rotation. So, we are taking the moment with respect to O.

Now if I take the moment with respect to O then what are the moment so this moment above the point O and moment due to the soil resistance below the point O, both will act in the same direction. So, we have to add them and then we have divided into 3 parts this is the part 1, this is part 2 these 3 triangles. So, this is the triangle one then there is another triangle that we have considered. So, this is the triangle 2 and this is the triangle 3.

So, we have taken three triangles now I will take the moment for the soil resistance with respect to O, so now if I take the moment then the force will be for the triangle one half. Now if this is $D_1 = \frac{D}{3}$ then this point this value will be $\frac{2D}{3}$. At this the distance will be equal to because this is 0.3 this one is $\frac{D}{3}$ means point $0.33D$ so this is $\frac{D}{3}$ but D_1 is equal to I should write that $D_1 = \frac{D}{3}$ which is nothing but $0.33D$.

So, this is $0.2D$ so this will be $0.13D$ so this should be $0.13D$ because this is $0.33D$. So, now for the first triangle, triangle one the total force will be $\frac{1}{2} \times \frac{2}{3}$ of and then the net pressure because this net pressure at this point E point will be definitely $\frac{2}{3}\gamma' D(K_P - K_A)$ because if this is at a depth of D if this is $(K_P - K_A)\gamma' D$, then at $\frac{2}{3}D$, this will be $\frac{2}{3}\gamma' D(K_P - K_A)$.

Because that is the point which is that $\frac{2}{3}D$ from the top because of at D it is $(K_P - K_A)\gamma' D$ so at $\frac{2}{3}D$ you have to replace D by $\frac{2}{3}D$ that I have done. So, this is the pressure distribution at this point. So, I can say the force for the triangle 1 will be then $\frac{1}{2} \times \frac{2}{3}\gamma' D(K_P - K_A) \times \frac{2}{3}D$. So that is the force then lever arm will be because we are taking moment so the lever arm will be then we are taking from point O.

So, the lever arm will be $0.13D + \frac{1}{3} \times \frac{2}{3}D$. So, that we can write here that lever arm is $0.13D + \frac{1}{3} \times \frac{2}{3}D$. So, this is for triangle 1. Now for the triangle 2 this small triangle the base is $0.13D$ height is $\frac{2}{3}\gamma' D(K_P - K_A)$. So, I can write this point small triangle is $\frac{1}{2} \times \frac{2}{3}\gamma' D(K_P - K_A) \times 0.13D$ then the lever arm will be $\frac{2}{3} \times 0.13D$.

So, this is the force and lever arm is $\frac{2}{3} \times 0.13D$ and when we are talking about these moments, so if I look at these forces, these forces will be this is kN/m^3 and this will be kN/m. So, these forces is kN/m. So, now if I want to convert them in kN then these forces are in kN/m. So, if I convert them in kilonewton you have to multiply with that L.

Because this is the L so that long distance side of the well, so this is the side of the well and these forces are in kN/m we have to multiply with L. So that is why we will multiply with L. So, here also we will multiply with L clear. Now it is kN/m. Now for the 3rd triangle the force will be $\frac{1}{2}\gamma' D(K_P - K_A)$ then the height is $\frac{1}{3}$ of so this height is $0.2D$ this side for the 3rd triangle is $0.2D$ this is the base.

So, this will be $0.2D$ and then from here the lever arm will be $\frac{2}{3} \times 0.2D$. So, if I simplify this equation then it will come roughly as $0.096\gamma' D^3 L(K_P - K_A)$ again we have to multiply with L

here. So, M_s is roughly taken as so M s is taken as $0.1\gamma' D^3 L(K_P - K_A)$. So, M_b we have calculated M_s also we have calculated. Now next one is the frictional resistance moment which is M_f.

So, what is friction resistance moment? That means there will be a soil reaction because of the soil reaction a friction force will generate along the side of the well. So, because of the friction how much will be the moment, so that means that we have to calculate, so to calculate that so what is the total amount of force which is acting? Now for that purpose the total force per meter will be equal to now we have these two triangles.

But we are taking the three triangles but now for this calculation purpose now we are taking two triangles this the bigger one and one triangle above the point O and one triangle below the point O. So that two triangle reactions is given the total soil reaction, so that total force will be equal to for the upper triangle that force is for the lower triangle first we are calculating. So, for the lower triangle $\frac{1}{2} \times \gamma' D(K_P - K_A) \times 0.2D$.

So that is for the lower triangle this is $(K_P - K_A)\gamma' D \times \frac{1}{2} \times 0.2D$ and for the upper triangle this is the total here base and this is the height. So, total base will be for the upper triangle this is $\frac{1}{2} \times 0.8D$ why $0.8D$? Because this is $0.2D$. So that means this total 1 will be $0.8D$ or the total 1 will be $0.8D$ or I should write this because from base it is $0.2D$. So, from A to O from top to point O will be $0.8D$ from base B to O point $0.2D$ point A to O is $0.8D$.

So, I can write that this is $0.8D$ and then the height will be $\frac{2}{3}\gamma' D(K_P - K_A)$. Now if I simplify this equation then this will be $\frac{1.1}{3}\gamma' D^2(K_P - K_A)$. So, the total friction force, so this is kN/m. So, this will be you have to multiply by L, so this is $\frac{1.1}{3}\gamma' D^2(K_P - K_A) \times L$. Now one thing I want to mention that as I have already discussed that these K_P and K_A are as per Coulomb's theory.

So, again there will be a component of earth pressure as per Coulomb's theory. So that means P is acting or P_{net} is acting at an angle δ and the horizontal force was acting with the value of $P_{net} \times \cos\delta$ but the friction force will act with the value of $P_{net} \times \sin\delta$. So, because this moment is due to the friction that is acting along the side of the well. So that means we have to multiply with our $\sin\delta$. Now here δ is taken as $\frac{2}{3}\phi$ but δ should not be greater than 22.5°.

So, it will be taken as $\frac{2}{3}\phi$ or it should not be greater than 22.5°. So, you can write here $M_f = \frac{1.1}{3}\gamma' D^2(K_P - K_A) \times \sin\delta$. So, this is total frictional force. So, the moment will be $\times \sin\delta \times L \times$ this is the force this is $\frac{1.1}{3}\gamma' D^2(K_P - K_A) \times \sin\delta\, L$ is the force $\times \frac{B}{2}$ because this is acting at the side because frictional forces are acting, so at moment we are taking from this point. This moment we are taking from the center.

So, we have to multiply with the $\frac{B}{2}$ that is the lever arm. So, it will be $\frac{B}{2}$. So, finally $M_f = 0.183\gamma'(K_P - K_A)LBD^2 \sin\delta$. So, these equations are for rectangular footing. So, in the next class I will discuss that how I will get this equation of M_f for the circular footing circular well. So, this is for the rectangular well and the next class I will discuss the circular well.

And then how I will set the condition that this summation of these moments because these are the resisting moment should not be less than the moment which is applied. So, those things will be discussed in next class. Thank you.