

Advanced Foundation Engineer
Prof. Kousik Deb
Department of Civil Engineering
International Institute of Technology - Kharagpur

Lecture - 63
Well Foundation - IV

(Refer Slide Time: 00:39)

Lateral stability

Terzaghi's Analysis

Total resultant force (q_{imp}) (kN/m)

$$= \Delta ABC - \Delta BEF$$

$$= \frac{1}{2} \gamma' b^2 (k_p - k_a) - \frac{1}{2} \times 2 \gamma' b (k_p - k_a) D_1$$

$$q_{imp} = \frac{1}{2} \gamma' b (k_p - k_a) (D - 2D_1) \quad \text{--- (1)}$$

Taking moment about the base M_c

$$q_{imp} (D+H) = \frac{1}{2} \times \gamma' b^2 (k_p - k_a) \frac{D}{3} - \frac{1}{2} \times 2 \gamma' b (k_p - k_a) \times \frac{D_1}{3}$$

After solving $E_1(\omega)$ and $E_2(\omega)$

$$(D - 2D_1) H_1 = \frac{D^2}{3} - \frac{2}{3} D_1^2$$

Take $H_1 = D + H$

$$D_1^2 - 3D_1 H_1 + 1.5 D H_1 - 0.5 D^2 = 0$$

For light well $q_{imp} = k_{sp} H$

Active $k_p = \frac{1 + \sin \phi}{1 - \sin \phi} k_a$
 Passive $k_p = \frac{1 - \sin \phi}{1 + \sin \phi} k_a$
 (Rankine)

Rel. pressure distribution diagram F

So in this class I will discuss about the lateral stability of well, if it is subjected to lateral load and then first I will discuss about Terzaghi's analysis, then I will discuss about the codal provision so that Terzaghi's analysis suppose this is well and this is the central line and this is the scour depth. So, this one is D which is grip length, grip length is below the maximum scour level.

So, this is the scour level and this is the grip length and where W is weight and another horizontal force Q is acting. So, this is the condition where horizontal force Q is acting and W is also acting W is the downward force Q is the lateral force. Now, if the well is light well or I can write here for light well. So, this is the well centre line and supposes we are applying q_{max} .

So, first we are considering q_{max} what is q_{max} ? q_{max} is in terms of kN/m here Q is in terms of kN, but for this derivation purpose we are considering q_{max} later on I will discuss how I can convert q_{max} to Q . So, this is q_{max} is acting and this is the dredge level which will continue sorry this is the scour level which will continue. So, for the light well and this is the height H which is the distance from the scour level to the point where the lateral load is acting.

So because of this lateral force the well will rotate and the rotation will be with the points say O below the scour level. So, this is a point of rotation and assumes that say that, that point is at height of D_1 from the base of the well and total grip length is D . Now, when this is rotating, then again there will be an active zone here. So, that means there will be four zones so, there this will be active here this will be passive again this will be passive and this will be active.

So that means this will be passive, active, passive, active so, these are the four zones that will be present when this will rotate about a point O located above the base of the well. So, now if I draw the pressure distribution diagram for this particular case. So, this is our scour level, this is the centre line where this is the q_{\max} which is acting. Now there are four zones that means these zones are active, passive, active, passive.

So that means that there will be a net pressure distribution diagram if I want to draw the net pressure distribution diagram. So, if I draw the net pressure distribution diagram for both the side so, I am drawing the net pressure distribution diagram in the both the side. So, this is the net pressure distribution diagram this side and this is the net pressure distribution diagram in this side because when there is a passive then opposite side there is active also active.

So ultimately we have to take the net pressure distribution because we have active passive both. So, this is the net pressure distribution diagram. So, now, here this value will be $\gamma \times D(K_P - K_A)$ and this one will be also $\gamma \times D(K_P - K_A)$. So, because we are taking the net distribution so net is $\gamma \times D(K_P - K_A)$ at the base because this total grip length is D .

So where K_P is equal to Rankine's passive earth pressure coefficient $\frac{1+\sin\phi}{1-\sin\phi}$ and K_A is Rankine's active earth pressure coefficient $\frac{1-\sin\phi}{1+\sin\phi}$ or I can write that $K_P = \frac{1}{K_A}$ this is as per Rankine's theory. So, that means here above the O point so, your left side is the passive. So, that will be more so, that means the pressure distribution the net pressure will act left side to the well but once the O point is reached then it will shift to the right side because below the O point passive is in the right side.

So now the right side stresses will be more so, above O point the net pressure distribution will be left side to the well and below O point this will be right side to the well so, it will be shift basically. So, that means, if this is the O point, point of rotation this is the O point so, up to O

point there is no issue it will just if this is O point. So, up to this point, it will be the left side to the well because definitely up to O point this is the passive so this is the left side to the well.

But once the O point is crossed, then this net pressure distribution of the well will start to shift from left to the right because now the right-side force is more. So, it will start to shift from left to the right so, it will shift and this is the final distribution. So, this is the final distribution that we have drawn. So, this is the final pressure distribution diagram and here it is acting in the opposite direction this is the pressure distribution diagram.

Now we have to determine the forces so this is net pressure distribution diagram. So, I can write the total force or total resultant force is q_{\max} . So, this is q_{\max} , which is in kN/m I should write, so that will be equal to the resistance this soil is giving. So, how I will calculate that, first I will take the total left side triangle. So, if I take total left side triangle or if I give the name, so, this is say A, B and this point is C and this point is D and this point is or D I have already used so, this point is E and this point is F.

So, pressure distribution diagram will be if I take the total triangle A, B, C total triangle I have taken. But I have taken this portion within that total triangle, but that is not the pressure distribution. So, if I now subtract this triangle minus triangle B, E, F. Now if I subtract this triangle B, E, F so if I subtract this triangle then what will happen so, this portion will be subtracted and additionally this component will be added and these that means, we have taken net this portion.

And then this portion and these two things are acting in opposite direction so, that is why this minus sign is perfect. So, that means, basically what we are doing? We are taking this total triangle then subtracting this triangle. So, due to this subtraction we are neglecting this part which we have to do because this is not the pressure distribution part. So, we are neglecting these or we are removing this and then we are considering this portion and these both are acting in opposite direction.

So that means here we can write the pressure distribution due to this triangle this will be $\frac{1}{2}$ and this is if I take the effective unit weight so, $\frac{1}{2}\gamma'D^2$ then $(K_p - K_A)$. So, because this is $\frac{1}{2}$ this base \times this height is D which is D^2 then $-\frac{1}{2}$ and this portion F which is considered at the point

O is D_1 above the base, so, this is D_1 . So, I can write this is $\frac{1}{2}$ then this total base is the summation of these two parts.

So if I take the summations that mean this is $2\gamma'D(K_P - K_A)$ so, $\frac{1}{2} \times 2\gamma'D(K_P - K_A) \times D_1$. D_1 is the height of this triangle and this is the base. So, now it will be if I take this $\frac{1}{2}\gamma'D(K_P - K_A)$ outside then this will be D then these 2 2 is cancelled out and I have taken $\frac{1}{2}$ so, this will be $2D_1$. Now we are taking the moment about the base so, about C point. So, moment at C so, that means here q_{\max} which is acting at from the base is $D + H$ because this is H .

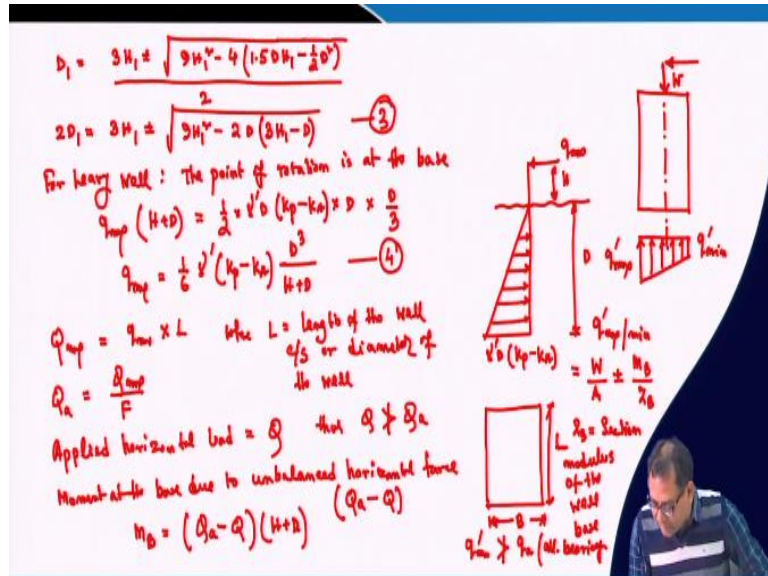
So this is $D + H$ then this net moment due to this pressure diagram again we will take the full triangle then we subtract this triangle and then so that means the full triangle this is $\frac{1}{2}\gamma'D^2(K_P - K_A)$. So that the lever arm is $\frac{D}{3}$ then minus this triangle, which is $\frac{1}{2} \times 2\gamma'D(K_P - K_A) \times D_1$ then that lever arm from the base will be $\frac{D_1}{3}$. So, this is my equation number 1, this is my equation number 2 because we are taking the moment from the base from the C point.

And then that means, here q_{\max} is acting and remember that q_{\max} is in kN/m here also it is kN/m because, this stress or this force you can see, this is γ' is $\text{kN/m}^3 \times \text{depth}^2$, so, this is kN/m. So, now, after solving equation 1 and equation 2 what we will get so, if because we have q_{\max} here we have q_{\max} here also because this is q_{\max} .

So, if I put these q_{\max} value in equation 1 and then simplify then putting the q_{\max} value in equation from equation 1 to equation 2 and then simplify then you will get $(D - 2D_1)H_1 = \frac{D^2}{3} - \frac{2}{3}D_1^2$. Now, where $H_1 = D + H$. Now, if I further simplify this equation, then this will give me $D_1^2 - 3D_1H_1 + 1.5DH_1 - 0.5D^2 = 0$. So, here H_1 is known because we know the total depth of the well and the height of the well H where the force is acting but D_1 is unknown in this particular equation.

Because we do not know where the rotation of the point O and so, that means, we have to determine the D_1 . So from this equation we have to determine the D_1 .

(Refer Slide Time: 17:51)



So, that means from this equation $D_1 = \frac{3H_1 \pm \sqrt{9H_1^2 - 4(1.5DH_1 - \frac{1}{2}D^2)}}{2}$. So, finally, we can write that $2D_1 = 3H_1 \pm \sqrt{9H_1^2 - 2D(3H_1 - D)}$. If I take 2, so, this will be 2 outside that means I am dividing it this is 2 I am taking $\frac{1}{2}$ outside.

So, if I take $\frac{1}{2}$ outside so and the D so this will be $3H_1 - D$ so, this way we can determine the D_1 . So, from equation 1 we can determine the D_1 because on the right-hand side all quantities are known. So, we can determine what would be the D_1 value from equation. So once I get the D_1 then I will put this D_1 value in equation number 1 because here only D_1 is unknown then I will get what is the q_{max} .

So, I will get the q_{max} from equation number 1 by putting the value of D_1 that I will get from this equation or now for heavy well. So, this is for the light well now for heavy well the point of rotation is at the base. So, now here this O point and the C point both are same, because it is rotation with respect to base. So, that means, here the total left side will be passive and total right side will be the active.

So, the net pressure diagram will be total in the left side of the well so, this is $\gamma' D(K_p - K_a)$ and diagram will be totally on the left side, so, here q_{max} is acting this is H and this one is D for heavy well. So, now here I can calculate what would be the q_{max} so, q_{max} will be this is if I take the moment from the base so, again I can write $q_{max}(H + D) = \frac{1}{2} \times \gamma' D(K_p - K_a) \times D \times \frac{D}{3}$.

So, finally $q_{\max} = \frac{1}{6} \gamma' (K_P - K_A) \times \frac{D^3}{H+D}$. So, this is for heavy well and then I have discussed how we can calculate the q_{\max} for the light well also so, that the heavy well H is known D is known. So, directly we will get what is the q_{\max} value. So, this is my equation number 4 and this is equation number 3 so, now, what I will get the q_{\max} .

And then now in the well what is the total force Q_{\max} which is nothing but that $q_{\max} \times L$ what is L ? So, this L is the width or length of the well cross section or the diameter of the well for example, if well has a cross section of this. So, this is say B and this is say L so, that means here in the section we have written this is the width of the well. So, then the perpendicular direction of this width that will be length is L so, this is the cross section.

So this is the length because your small q_{\max} is acting per length if I multiply it with the length of this well, then I will get the total q_{\max} or Q_{\max} . So, that is the $q_{\max} \times L$ and then for in case of diameter I can put D or the outside diameter of the well in place of L also that is the length of the well cross section that is for rectangular or a diameter of the well that is for circle. So that means for the circle we have to consider the diameter and that we can put as a length of the well.

So now the allowable load, $Q_a = \frac{Q_{\max}}{F}$, where, F is a factor of safety so this way we can calculate that how much Q or lateral force we can apply on a well. So, now, we have Q_a well but it will not provide exactly that amount of horizontal force into the well because horizontal force should not be greater than Q_a , that means suppose you will apply force of Q . Now applied horizontal load is equal to Q , thus that Q should not be greater than Q_a .

So, it should be less than the Q_a , the Q is the load which is applied and Q_a is the load which can be applied and we cannot apply load beyond Q_a . So, that means definitely there will be unbalanced moment because Q_a we are getting from the soil resistance and we are applying Q . So, there will be a unbalanced moment at the base suppose, if your $Q_a = Q$ I mean you are applying exactly the same amount of load which you are allowed to apply then there will be no unbalanced horizontal force.

But actually it will not happen so, we will apply horizontal load which is less than allowable well load to that when there will be unbalanced moment. So, that unbalanced moment or I should say the moments due to the unbalanced horizontal force. So, that is the moment at the base due to unbalanced horizontal force. So, what is that horizontal force Q_a well you can apply and you have applied Q . So, that is the unbalanced horizontal force.

So, that means, this moment $M_B = (Q_a - Q)(H + D)$ again because that Q is applied at the top so, this is the unbalanced force that is acting at the base. So, that means, we are up to acting at the base because of that suppose, if we have a well like this. So, because of this unbalanced force that means now, the pressure distribution will be something like that at the base of the well because here this unbalanced force is acting so, that means, this is the pressure distribution.

So, this will be q_{\min} and this will be q_{\max} so, now, my q_{\max} or q_{\min} or I should write ' here because I have already used the q_{\max} . So, q'_{\max} , q'_{\min} will be equal to how much that will be the total vertical force which is acting divided by the area of the base of the well $\pm \frac{M_B}{Z_B}$. So M_B equation is given this is an unbalanced moment due to the unbalanced force and Z_B is what?

Z_B is the section modulus of the well base and I have already discussed how we can determine the section modulus for different sections, rectangular circular then hollow section. So, all this section modulus I have discussed in my previous lectures, so, that way you can determine the stress which is acting at the base of the well and that q'_{\max} that you are getting that should not be greater than the q_a well so, that means the allowable well bearing pressure.

So, that means how you will calculate the allowable well bearing pressure? So, that I have already discussed that you can calculate what would be the allowable load carrying capacity or of the well that I discussed in my previous lecture. So, from there you will get this is my allowable well bearing capacity and then these allowable well bearing capacity and this q'_{\max} you have to compare. So, q'_{\max} which is acting on the base of the well or on the soil that should not be greater than q_a well, that means allowable well bearing pressure of the soil I should write allowable well bearing pressure of soil at the base of the well.

(Refer Slide Time: 31:14)

To find the point of the maximum B.M. thus, at that point Shear force = 0

$$\frac{1}{2} \gamma' x (K_p - K_A) x - q_{\max} x = 0$$

$$\frac{1}{2} \gamma' x^2 (K_p - K_A) - q_{\max} x = 0$$

$$x = \left[\frac{2 q_{\max}}{\gamma' (K_p - K_A)} \right]^{1/2}$$

$q_{\max} = q_{\max} \times L$
 $q_{\max} = q_{\max} \frac{L}{F}$
 $q_{\max} = q_a \frac{F}{L}$
 q (applied force)

$$B.M. |_{\max} = q (H+x) - \frac{1}{2} \gamma' x^2 (K_p - K_A) \frac{x}{F} \times \frac{x}{3}$$

$$= q (H+x) - q \times \frac{x}{3}$$

$$M_{\max} = qH + \frac{2}{3} x q$$

So now, next part that I will discuss that where there will be maximum moment in well so, in this case the last point that I will discuss that to find the point of the maximum bending moment. So, where the maximum bending moment will occur in the well suppose, if this is the well this is HFL and this is the scour level and assume that at a distance of x from the scour level where at this point there will be maximum bending moment in the well.

So where the moment is maximum the shear force will be 0 so, thus at that point shear force is 0. Now, what is the shear force at that point because we have a distribution like this so say suppose that point is here this is x because this is also scour level because remember that whenever you will consider the soil that is below scour level in the grip length portion only we will consider the soil resistance. So, this is x so, we have net bearing pressure.

So, net shear force will be shear force for that portion is the $\frac{1}{2} \times \gamma' x (K_p - K_A) x$. So, that is a shear force at that point and that is minus so, this is the shear force at this point then minus this q_{\max} . So, that is $-q_{\max}$ so, that should be equal to 0 and the similarly these were also in laterally loaded pile we determine the shear force or the maximum bending moment point where the shear force is 0.

Because in laterally loaded pile also determine the maximum moment point we have to identify the maximum moment point and we did the same thing where the shear force is 0 and we consider this concept. So, that mean here you can see at this point the shear force will be 0. So, this is the q_{\max} and this is the contribution so, this soil contribution you should be equal. So, at that point the shear force will be 0 so, that mean the shear force = 0.

So, now I can write that $\frac{1}{2} \times \gamma' x^2 (K_P - K_A)$ now, this Q_{\max} , I can convert it to $Q \times \frac{F}{L}$ how I can convert? Because I know that $Q_{\max} = q_{\max} \times L$ and $Q_a = q_{\max} \times \frac{L}{F}$ thus small q_{\max} is $Q_a \times \frac{F}{L}$. So, that I am writing here, but here, I am not writing Q_a well I am writing in place of Q_a well I am writing Q , because ultimately that is the load which is acting.

So, as I mentioned we will not exactly apply the Q_a well we apply less than Q_a well and that Q is the applied force so, we will write Q so, that is equal to 0. So, now, from here I will get that $x = \left[\frac{2QF}{\gamma' L (K_P - K_A)} \right]^{\frac{1}{2}}$. So, this is the point where shear force will be 0 and bending moment will be maximum. So, the maximum bending moment at this point so, that maximum bending moment because this is equal to H from the scour level so this is equal to H .

So, this is basically from scour level this is equal to H so, I can write the maximum bending moment that will be now $Q(H + x) - \frac{1}{2} \times \gamma' x^2 (K_P - K_A) \times \frac{L}{F} \times \frac{x}{3}$. So and then the moment due to that contribution that is $\frac{1}{2} \times \gamma' x^2 (K_P - K_A) \times \frac{L}{F} \times \frac{x}{3}$ that is the moment. Now, this total part is how much? This total part is nothing but so, from this equation I can write that $\frac{1}{2} \times \gamma' x^2 (K_P - K_A) = Q \times \frac{F}{L}$ so, that I am writing here.

So that I am writing here or in other way also you can write that means, when you were writing these things we can apply now in here because this is the bending moment and we have written the F . So, that also we have to write we have to multiply L because we are writing because here the unit is kN/m, but here it is kN. So, you have to write the L and then the factor of safety then it is $\frac{x}{3}$.

So finally, I can write from these equations that $Q = \frac{L}{F} \times \frac{1}{2} \times \gamma' x^2 (K_P - K_A)$. So, that total term I can replace with Q so, that means it will be $Q(H + x) - Q \times \frac{x}{3}$. So, that $M_{\max} = QH + \frac{2}{3}xQ$. So, x I will put from here and I know H I know Q .

So remember that is why I have multiplied with the L because, you see the moment is taken with respect to Q not the q_{\max} . So, q_{\max} is kN/m, but Q is kN. So, this is in terms of kN so that is why we have to multiply the L with the force that I am getting for due to the soil contribution

and then we applied the factor of safety because here in Q also applied factor of safety is applied.

So to match this Q and this soil contribution, we applied the L and divided by the factor of safety then we take the moment of $\frac{H}{3}$ and this is equal to F . So, this is total one is equal to Q because from this equation you can see, so, then we can replace with the Q . So, this will be $QH + \frac{2}{3}xQ$. So, this way we can determine the position of the maximum bending moment and the maximum bending moment at the well.

So, in the next class I will solve one example problem on Terzaghi's analysis, then you will find that how we can determine all these quantities by putting some numerical values. Thank you.