Advanced Foundation Engineering
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Lecture-53
Pile Foundation: Under Lateral Load and Uplift-III

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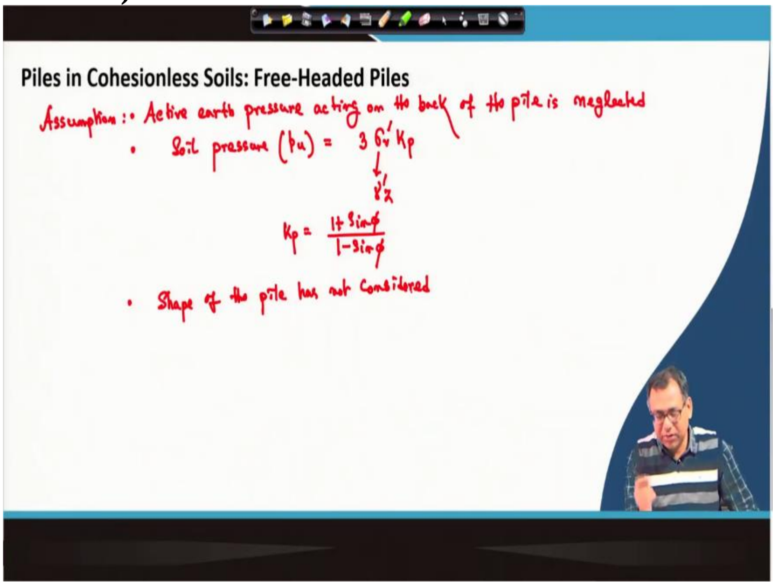

So, this class I will discuss how we can determine the ultimate lateral load for a pile if it is in cohesionless soil. So, in a cohesionless soil our main assumptions are also based on the Broms's theory. So, main assumptions are that the active earth pressure contribution we will not consider. So, the active earth pressure contribution acting on back of the pile is neglected.

So, this is one assumption. So, we will consider only the passive pressure which is acting along the pile. So, and we have considered that the soil pressure at a point, $p_u = 3\bar{\sigma}_v K_p$where, K_p is Rankine's passive earth pressure coefficient, which is $\frac{1+\sin\phi}{1-\sin\phi}$. This is also one assumption. At a point so $\bar{\sigma}_v = \gamma' z$, where, γ' is the effective unit weight of the soil.

So, this is another assumption the soil pressure at any point is 3 times of $\bar{\sigma}_v \times K_p$ and another assumption is the shape of the pile is not considered. So, shape of the pile effect is not considered. So, for the any shape, distribution will be same.

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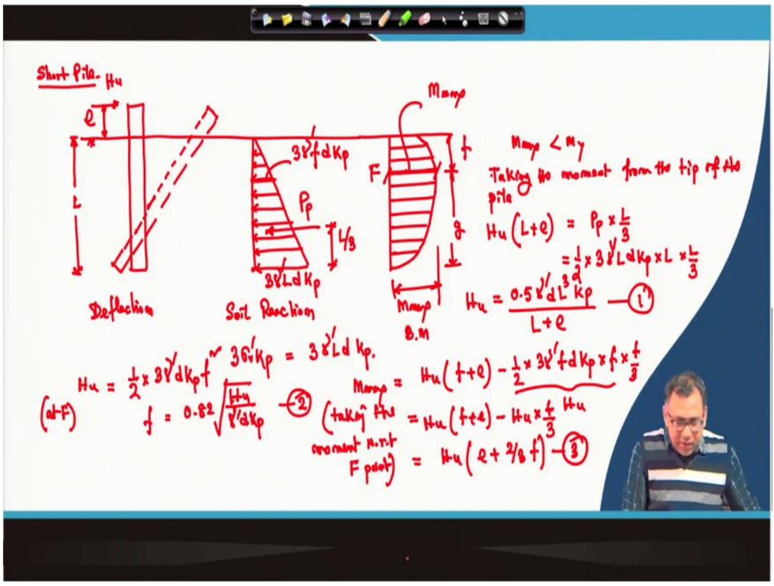

So, this way if I start with this whether it is a free-headed or fixed-headed pile, the assumptions are same. So, first one we will call for the free-headed piles and first one is for the short pile. So, remember that in case of cohesive soil we assumed the uniform distribution of the soil pressure. But in case of cohesionless soil the distribution is not uniform it is linearly varying with the depth.

So, this is the major difference between the earth pressure of cohesive soil and the cohesionless soil. So, cohesive soil we assume the uniform distribution of the earth pressure or the soil reaction, but here we will assume a varying distribution of the earth pressure or the soil reaction. So, for the short pile this is the ground surface. So, similar to our cohesive soil case here also for free-headed pile it will rotate about the point.

And as we have assumed and it is mentioned that active earth pressure contribution will not to be considered. So, the assumption is that the rotation is very close to the toe of the pile or the base of the pile. So, here H_u is acting here with the e value and this is the length of the pile, L. So, this is the deflection and the soil pressure will be so, it will act linearly.

So, it will vary and the assumption is that we will not consider the active earth pressure contribution and that is why we assumed that pile is rotating very close to the tip. So, that is why most of the portion of the pile is getting the pressure from the passive resistance of the soil. So, this is the soil reaction. And again, it is assumed that at any point the soil pressure or the soil reaction is 3 times $\bar{\sigma}_v \times K_p$.

So, here also this is 3 times and $\bar{\sigma}_v$ is $\gamma' z$. So, this will be $\gamma' L$ because here, $z = L$ at that point and we have to multiply. So, as I mentioned previous case also this value is in kN/m^2 if I multiply it with d it will be kN/m so, $3\gamma' Ld \times K_p$. So, that means, here also we assume that $\bar{\sigma}_v \times K_p$.

So, here s$\bar{\sigma}_v$ is $\gamma' L$ and then we will multiply with the d to make it kN/m $\times K_p$ and here this total passive force P_p will act and which will act at a height of $\frac{L}{3}$ from the base of the triangle of the pile and again here also the maximum moment will act at a distance f from the top but here no zero earth pressure condition is existing like the cohesive case where top $1.5d$ will be zero earth pressure, but here that type of condition is not existing.

So, the maximum moment will be here. So, there will be maximum moment. This is the bending moment diagram. So, this is f then this one is g and this is M_{max} which is acting at a depth of f from the ground surface. So, obviously for the short pile the M_{max} should be less than M_y yield moment that will not occur. So, now, if I write the equations for the short pile, what are the equations for this short pile that means, here that totally is passive resistance for the passive soil reaction and then this is the H_u.

So, if I take the moment from this tip of the pile then this will be $H_u \times (L + e)$ that is the moment and the counter moment will be equal to $P_p \times \frac{L}{3}$, where, P_p is the total passive force. So, I can write that P_p is nothing but $\frac{1}{2} \times 3\gamma' Ld \times K_p \times L$ that is the total $P_p \times \frac{L}{3}$. So, finally from here I can write H_u is equal to so these 3, 3 will cancel out.

So, $H_u = \frac{0.5\gamma' dL^3 K_p}{L+e}$. So, this is equation number 1. So, what are the unknowns here. All values are known. So, here H_u; I can calculate by using these equations, but if I want to determine what is the maximum moment as I mentioned that what are our interest. Our interest is the H_u and the maximum moment because we have to check that whether a maximum moment, M_{max} is less than M_y or not.

Because in that condition you understand whether it is intermediate pile or not and definitely for the short pile that is the must condition that the maximum moment, $M_{\max} \ngtr M_y$ and that means maximum moment should be less than the M_y. And for the intermediate case and that will occur for the fixed pile. So, that I will discuss later on, but again also for the short pile, free-headed pile also had the condition is that maximum moment should not be greater than M_y.

So, that means we should know the maximum moment. So, to determine the maximum moment the stress at this F point is similar to $3\gamma' f d K_p$ because now $z = f$ because at F the maximum moment is acting. So, this is the maximum moment actually this is $M_{\max}$ at F. So, I can write that H_u at F or I can write this point is F, H_u at F will be equal to or at that point this will be $\frac{1}{2} \times 3\gamma' f d K_p \times f$.

So, there will be f^2. This is the f. So, that f will come. So, that is the H_u. So, I can write that f in terms of H_u so, $f = 0.82\sqrt{\frac{H_u}{\gamma' d K_p}}$. Because in this case we take the moment with respect to F and this H_u is acting in this direction in f portion is acting in this direction. So, we can get.

So, once we get the H_u, we can get what will be the f value where the maximum moment is acting. So, once I get the f, so, then this is my second equation once I get the f then I have to calculate what is the maximum moment and that will be equal to so, I have taken the moment with respect to F so, taking the moment with respect to F point. So, that these will be $H_u \times (f + e)$ then the counter moment is $\frac{1}{2} \times f$ portion γ' this will be 3 because this is $\frac{1}{2} \times 3\gamma' f d K_p \times f$.

This is the force and then $\frac{f}{3}$. So, if I simplify these things and definitely this is nothing but H_u. So, this will be simple $H_u \times (f + e) - H_u \times \frac{f}{3}$, so simplify these I will get $H_u \times \left(e + \frac{2}{3} f\right)$. So, this is equation number 3. So, for this short pile free-headed pile, so, equation number 1 will give us the H_u, equation number 2 will give us the f because that f is required to determine the $M_{\max}$. So, once I get the f and the H_u then I will get what is the value of $M_{\max}$ based on the e value and then that you have to check definitely that $M_{\max}$ should be less than M_y.

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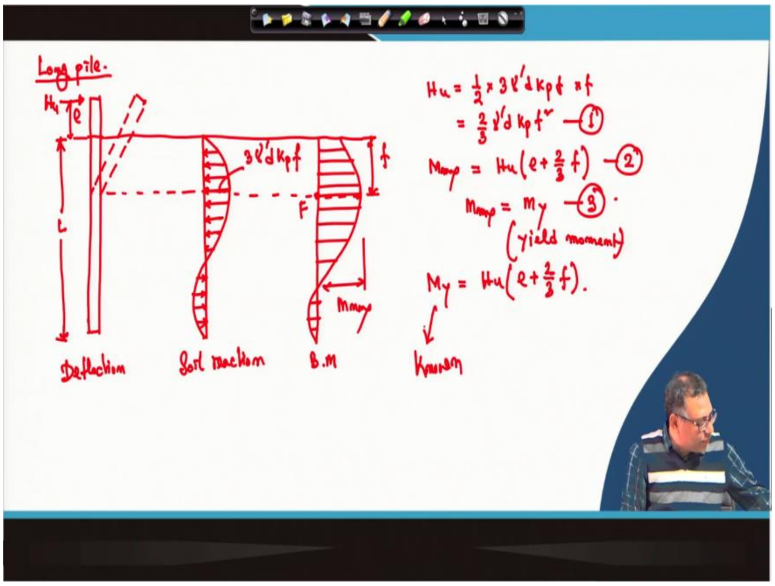

Now, I will go to the long pile case. So, I will go to the long pile case. So, long pile case this is the ground surface. So, again here there will be yield in case of long pile at a point. So, this is H_u and this value is e this is L fine but the distribution will be slightly different because here this is the point where maximum bending moment will occur. So, this is the point where maximum bending moment will occur then it will go like this.

So, this is the bending moment distribution this is the point which is again F and this value is $M_{\max}$ and the soil reaction will be something like that. This is deflection. This is soil reaction. This is bending moment fine. Now, for this particular case what are the valid equations. So, again we assume that the lower portion these negative positive they are balancing each other and this distribution is more or less linear in the top portion.

So, I can write that equation 2 is valid here. So, I can write H_u will be equal to $\frac{1}{2} \times 3$ because here we assume the distribution so, this value is $3\gamma' dK_p f$. So, I can write this is $3\gamma' dK_p f \times f$, so, $H_u = \frac{2}{3}\gamma' dK_p f^2$. So, this equation is valid here because, why it is valid because we assume that this portion is more or less linear and these positive and these negative, they are balancing each other.

So, this is one equation and then equation 3 is also valid. So, equation 3 for previous case, so, this is the equation 3 that we can write here. So, that we can write here because equation 2 is also valid, equation 3 is also valid. So, we can write here this equation this is $\frac{2}{3}\gamma' dK_p f^2$ and in equation 3 I can write here equation 3 is what equation 3 is $H_u\left(e + \frac{2}{3} f\right)$.

So, $H_u\left(e+\frac{2}{3}f\right)$ why equation 2 is also valid, because we are taking moment at this point F where the maximum moment is acting and as we assumed this is more or less in a linear distribution. So, which is same as this point for the short pile. So, that means here also we assume this linear distribution the same way we can calculate the $M_{\max}$ also here. So, equation 1 and 2 are valid, but what are the unknowns here.

So, here $M_{\max}$ is unknown H_u is unknown if *f* is unknown, but we have 3 equations we have 3 unknowns, but we have 2 equations. So, for the long piles third equation will come that maximum moment $M_{\max}$ should be the yield moment of the pile. So, this is third equation. So, if I replace this will be $M_y = H_u\left(e+\frac{2}{3}f\right)$ and this M_y is known. So, for using equation number 1 we will get the H_u.

Now, what are the unknowns H_u is unknown and *f* is unknown and we have 2 equations. So, we will get the *f* and we will get the H_u we will get where the maximum moment will occur and what will be the value of H_u. So, that this is the long pile and short pile case. So, now, that means if in case of short pile if you have done the analysis for the short pile and then once you calculate the $M_{\max}$ you find that $M_{\max}$ value is greater than M_y.

So, that means that will be a long pile. So, in that case you have to design it for the long pile considering $M_{\max} = M_y$ and then you have to find all the other values. So, this is the $M_{\max}$ and $M_{\max}$ will be replaced by M_y and this way we can calculate the long pile and short pile case. So, now I will go for the fixed-headed pile.

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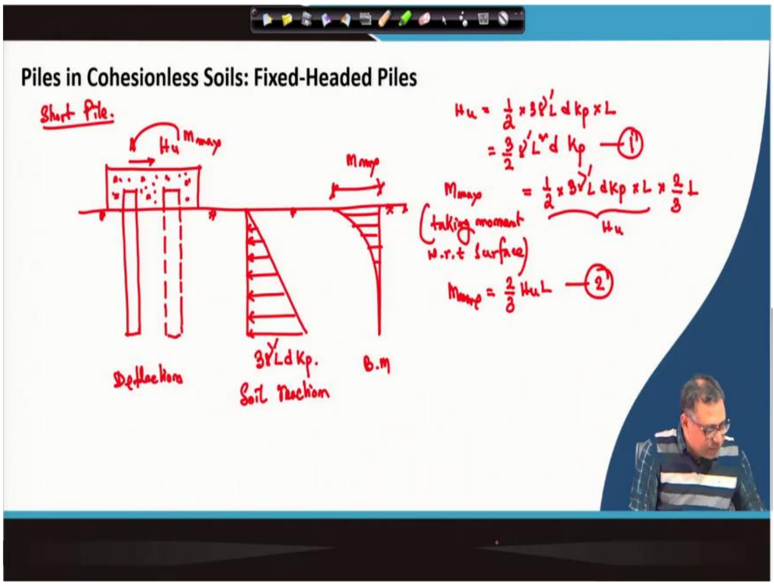

So, this is the free-headed pile. Now we will go into the fixed-headed pile and pile in cohesionless soil. So, for the fixed-headed pile, first I will go for the short pile, then I will go for the intermediate pile, then I will go for the long pile. So, again this is the ground surface and this is the fixity of the pile or the pile cap and the short pile. So, again here also the total pile will move laterally because it is a short pile and it is the fixed-headed pile.

So, this is the fixed-head or pile cap. So, we are applying H_u and definitely there will be moment. So, this is deflection now, the reaction again here total will be the passive reaction. So, soil reaction is the passive. So, this is again $3\gamma'$ or if it is a dry soil, I can write only γ or that is the effective unit weight, $3\gamma' L d K_p$ and here for the fixed-head there will be a maximum moment.

So, bending moment diagram will be something like this and this will be the maximum moment $M_{\max}$ because now it is a fixed-headed pile. So, this M will be $M_{\max}$ which will be generated at the pile head and that is the same as the pile in cohesive soil. So, now what are the equations now the short piles we have to determine again H_u. So, H_u will be equal to this total resistance.

So, this will be $\frac{1}{2} \times 3\gamma' L d K_p \times L$. So, this will be H_u we have a $\frac{3}{2}$ or 1.5. So, I am writing $\frac{3}{2}\gamma' L^2 d K_p$. So, this is equation number 1 and $M_{\max}$. If I take the moment with respect to surface, so, $M_{\max}$ taking moment with respect to surface and that is why this H_u contribution we will not consider because H_u also acting at the surface.

So, $M_{\max}$ will be this total passive soil reaction contribution and this is bending moment. So, total is H_u because this is $\frac{3}{2}\gamma' L^2 d K_p \times \frac{2}{3} L$ because it is from here from the base it is $\frac{1}{3}$ from the top it is $\frac{2}{3}$. So, that is the moment and this is nothing but H_u. So, I can write that $M_{\max}$ will be equal to $\frac{2}{3} H_u \times L$. So, we have calculated H_u by using equation 1 and you will calculate maximum moment using equation 2 and you have to check whether the maximum moment is less than the yield moment or not.

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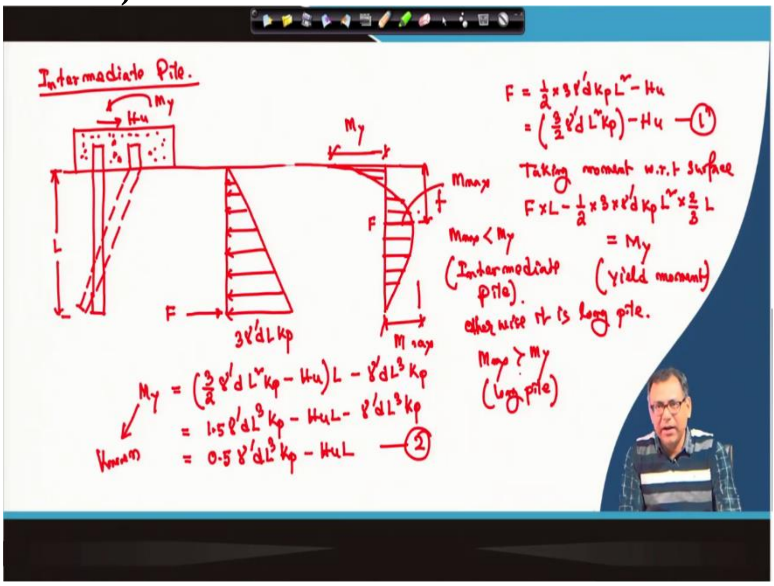

Now, we will go for the intermediate pile. So, for the intermediate pile again this is the ground surface and this is the fixity and this is the pile and again like the cohesive soil it will rotate with respect to a point. So, with respect to a point and then again, we are considering here the all passive resistance only and not considering our rotation is very close to the tip.

So, the resistance will be passive resistance is this one and where we will be the maximum bending moment. Maximum bending moment will be at a distance *f* from the top like the previous case, but yield moment will start at the pile top or pile head. So, these will be the diagram. So, this is the bending moment diagram. So, this value is $M_{\max}$. So, this value is $M_{\max}$ basically and this distance is *f* from the surface and this value is M_y and this is obviously again $3\gamma' L d K_p$ because this is L.

So, there will be H_u and M_y. So, I can see that with all the cases in *M* value properly or not. So, let us see for the previous cohesive soil, short pile there is no *M*. So, this is for the short pile fixed-headed pile. So, there will be one moment here. So that moment is $M_{\max}$. There will be one moment for here this will be a M_y here again this will be M_y.

So, now I have to consider here this is $M_{\max}$ fine. This is M_y. So, now if I take one reaction because your tip is moving so, if I take one reaction force F at the tip of the pile because the pile is rotating as close as tip of the pile. So, now I can write in case of intermediate. So, intermediate case that definitely $M_{\max} > M_y$ for the intermediate case if $M_{\max} < M_y$ then it will be a long pile case I have discussed.

So, this is the $M_{\max} > M_y$. So, this force F I can write this reaction force $F = \frac{3}{2}\gamma' L^2 d K_p - H_u$ because, H_u is acting in the opposite direction. So, that will give me the F. So, I can write $= \frac{3}{2}\gamma' L^2 d K_p - H_u$. So, this is equation number 1.

Now, this yield moment will be at the top. So, taking moment with respect to surface. So, what will be that moment? So, this F is acting, so, $F \times L$ length - this contribution because H_u we will not consider because it is also acting at the surface. So, that contribution is nothing but $\frac{3}{2}\gamma' L^2 d K_p \times \frac{2}{3} L$ from the top $\frac{2}{3} L$. So, this is nothing but this contribution or I should write that total one here the replacement scope is not there.

So, this moment we are taking at the surface and surface moment is equal to M_y because at the surface this moment is M_y. So, at M_y is the yield moment at the surface. So, now, we have to replace this F by this equation number 1. Now, if I replace this equation number 1, so, I can write that M_y is equal to I can replace these, this equation number 1 and that is $\left(\frac{3}{2}\gamma' L^2 d K_p - H_u\right) L3$ - these 2 2 2 3 3 will cancel out $\gamma' L^3 d K_p$ because these 3 these 3 these 2 these 2 will cancel.

So, this will be equal to $1.5\gamma' L^3 d K_p - H_u L - \gamma' L^3 d K_p$. So, finally, it will be this is one $0.5\gamma' L^3 d K_p - H_u L$ and this is known. So, by this equation 2, I can calculate what will be the value of H_u because M_y is known and all the other factors are known except H_u. So, if you want you can calculate what will be the $M_{\max}$ at a distance *f* from the top.

And then you can check whether that $M_{\max}$ is greater than or less than M_y if it is less than M_y

then definitely intermediate pile if it is greater than M_y, then it will be long pile. So, $M_{\max} > M_y$ then intermediate pile if $M_{\max} < M_y$ then it will be a long pile that means, if M_y is greater than the or less than the moment which is induced at point F then it will be a long pile.

So, that means, if this is the case then intermediate piles or if $M_{\max}$ is less than I should write that this point this should be less than if the moment which is induced because here also you can see that for this case for the intermediate piles that $M_{\max} < M_y$. So, here also that $M_{\max}$ should be less than M_y. So, if $M_{\max}$ should be less than M_y then it is intermediate pile. Otherwise it is long pile.

So, that means if $M_{\max} > M_y$ which was the condition previously written in such case it is a long pile. So, it will be a long pile but generally we restrict it to $M_{\max} = M_y$for the long pile and that I will discuss in the next class. So, that means here from equation 2 we can determine what would be the value of H_u because M_y is known and then we can calculate the $M_{\max}$ which is acting.

Which will induce at a depth of f from the surface and then you will check whether it is satisfying the intermediate pile condition known that means the $M_{\max}$ should be less than M_y then it is an intermediate pile condition otherwise, if $M_{\max} > M_y$ or if it is not less than M_y then it a long pile, but in case of long pile it is considered $M_{\max} = M_y$ and that I will discuss in the next class. Thank you.