Advanced Foundation Engineering
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Lecture - 52
Pile Foundation - Under Lateral Load and Uplift - II

So, last class I have discussed how we can determine the ultimate lateral load for a laterally loaded pile of using the Broms's approach when the pile is within the cohesive soil and it is a free headed pile. So, we have discussed about both short pile and the long pile.

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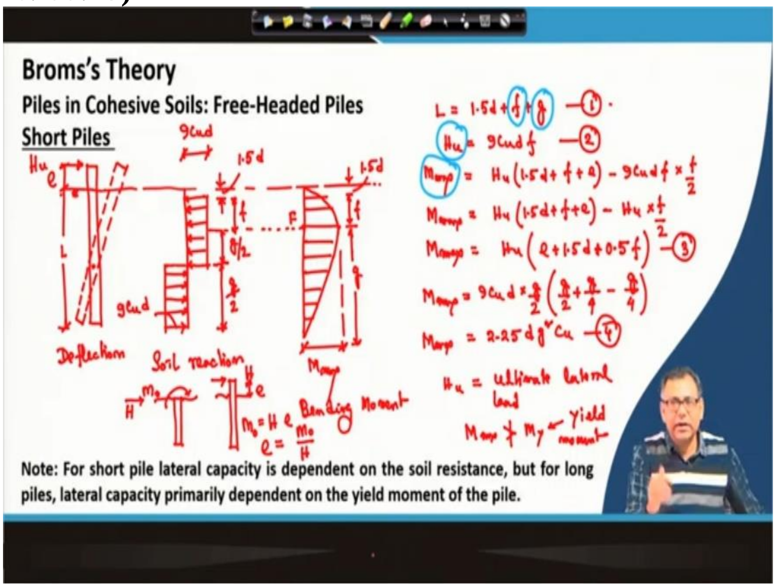

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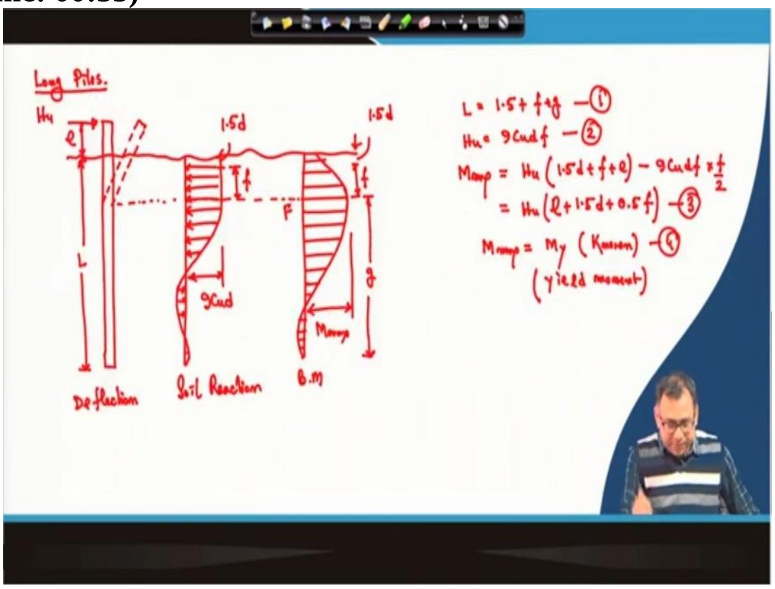

And now, these are the 4 equations for the short piles and these are the 4 equations for the long piles and for the long piles the maximum moment will be equal to yield moment and as I have mentioned that the short pile lateral capacity is due to the soil resistance or the soil reactions, but for the long pile primarily that is due to the yield moment of the pile.

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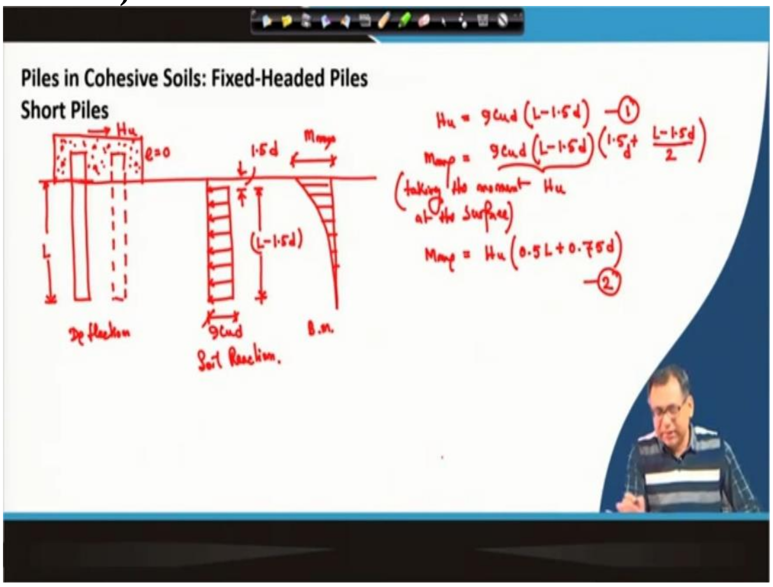

Now, the next one I will discuss the pile in cohesive soil. Because first part or the last class I have discussed piling cohesive soil but it is a free headed pile, but now it is a fixed headed pile and I will discuss for both short pile and the long pile. So, again, if this is the ground surface and this is the pile and it is a short pile and it is the fixed headed pile. So, when it is a fixed headed pile, so that means there will be not such rotation like the free headed pile, this type of rotation is not possible in case of fixed headed pile.

So, in case of fixed headed pile if it is a short pile the total pile will shift towards the lateral direction. So that means if we apply a lateral load to the pile that means total pile will shift in the lateral direction and this is the fixity or fixed headed pile. So, this is fixed headed pile. So, fixed headed pile means if we provide a rigid pile cap on the top of the pile then it is a fixed headed pile.

So that means here H_u will act here and there will be a moment also. So that means in case of fixed headed pile, the eccentricity value is equal to 0 because here we assume that the H_u is acting at the top of the pile, but in case of previous condition or that means where it is a free headed pile. So, where the H_u is acting with the height above the ground with the eccentricity of e. Sometimes we can represent that, suppose for the free headed pile, a moment is acting at the ground surface.

So that moment M_0 is acting on the ground surface that we can represent by converting this condition with this condition where H_u is acting with an eccentricity e, so that means M_0 will be $H_u \times e$ in that case. So, this way we can calculate this $\frac{M_0}{H_u}$. So, where H_u is, if H_u is known that means here I should write the H because it is not H_u and then some horizontal forces acting and some moment is acting at the ground surface. So that means here we are applying some horizontal force as well as some moment.

So, we can write in this way that H_u, M_0 is $H \times e$. So, eccentricity we can determine by $\frac{M_0}{H}$. So, this is also another way we can write, but sometimes the H can be applied with a certain height above the ground surface. In that case, directly e value will be known, otherwise, if we know what is the amount of H is acting, what is the amount of moment is acting at the pile top then we can calculate the eccentricity e value by taking the value of M_0 and H with the equation $e = \frac{M_0}{H}$.

Or otherwise, we can directly calculate the e value if the load is acting at a certain height above the ground surface. But in case of fixed headed pile, so e value is 0 that means for the free headed pile there can be different e values, eccentricity value but for the fixed headed pile only option is it will act at the top of the pile or the ground surface, so that is e value is equal to 0.

So, in this case, this is the deflection and we have a pressure distribution and the total length along the pile will be under passive reaction because the total pile is moving in one direction, in laterally. So, there will be a passive resistance. So, this is the passive resistance and again it is assumed that the top portion of the pile that means up to $1.5d$, no soil reaction is there. So, this is $1.5d$ no soil reaction is there again this uniform reaction is $9c_u d$, so this is soil reaction.

So, now, in this case the maximum moment will occur where because it is a fixed headed pile, so for the short pile, fixed pile maximum moment will occur at the ground surface. So, this is the maximum moment and that will occur at the top of the pile. So, this is $M_{\max}$. So, this is bending moment. So, what are the equations here that we can develop for the short pile. So, directly H_u, I can calculate here what will be the H_u?

Only there is a total reaction and that is what? Because if this is L and top $1.5d$ is not taking any or giving any soil resistance, so this will be $L - 1.5d$. So, H_u will be $9c_u d(L - 1.5d)$. So, this is equation number one for this particular case. Now, what will be the moment? So, we are taking moment at the ground surface, taking the moment at the surface and that moment will be this total moment this soil reaction is giving.

So, this will be $9c_u d(L - 1.5d)$, this is the total force on the lever arm will be because we are taking from the surface, so $1.5d + \frac{L-1.5d}{2}$. So, that is giving the total moment or maximum moment because in the previous case also we have taken the moment with respect to upper part of the point and the lower part of the point. Here we are taking the lower part because that is giving a total maximum moment because in the upper part this is fixed.

So, the fixed moment will act on the surface. So, this is the $M_{\max}$. So, if I simplify that, then the $M_{\max} = H_u(0.5L + 0.75d)$ because we have to replace this portion by H_u because this is nothing but $9c_u d(L - 1.5d)$ this is H_u. So, this will be $H_u\left(1.5d + \frac{L-1.5d}{2}\right)$, this is $1.5d + \frac{L-1.5d}{2}$. So, if I simplify that, so this will be $1.5d - 0.75d$. So, this will be $0.75d$ and this is $\frac{L}{2} = 0.5L$. So that will this is equation number 2.

So, what are the number of unknowns here? So, number of unknowns is only two, H_u and $M_{\max}$. So, H_u directly I can calculate by equation number one and $M_{\max}$ if I know the H_u value I can calculate what will be the maximum moment that will develop at the top of the pile or the ground surface. So, this is for the short pile where I can calculate the H_u and the maximum moment. Next, I will go for the long pile. So, before I go for the long pile there is an intermediate state, so that I am writing here.

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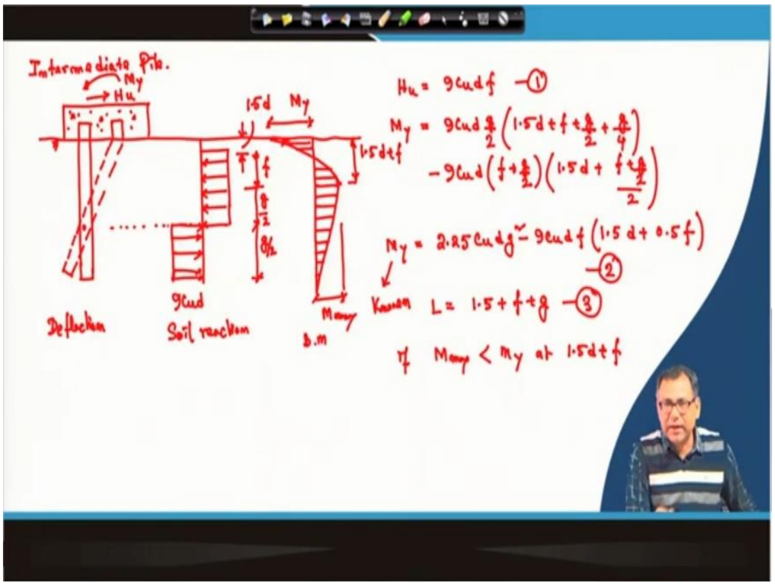

So, this will be the intermediate pile where the total pile will not shift in lateral direction. So, this is the pile cap and this is pile and this total pile will not shift because the intermediate state between long and the short. So, this will be something like this, so that means there will be a rotation at some point within the pile, so this is the point. And then this is the fixity and the soil reaction will be similar to the short pile, free head condition, with this deflection is similar to that.

So, this is the deflection, so again initial $1.5d$ there will be no reaction, but after that because the upper portion is giving the passive force, the lower portion is also giving the passive force. So, this is the passive reaction or the soil reaction. So, then this is $9c_u d$, this portion is how much? This portion is $1.5d$ again but here the bending moment distribution will be, there will be a maximum moment.

And that maximum moment is this one and maximum moment will be at a distance, f from this point, that we have already discussed, so there will be a moment at the top also. So, this will be the distribution. So, I should draw slightly higher value here. So, this is the bending moment diagram for the intermediate pile and this is the $M_{\max}$. So, this is $M_{\max}$ and this one is M_y, yield moment. So, for the intermediate pile the yielding of the pile will start or the yield moment will develop first at the top where the pile is fixed.

So, the fixed intermediate pile first the yielding of pile will occur at the head. So, now how I can calculate the values? So, this is the soil reaction and this is the bending moment. And the soil reaction will be because this is the f and this one $\frac{g}{2}$ like the previous case, this is also $\frac{g}{2}$, so $\frac{g}{2}$, $\frac{g}{2}$ portion positive, negative will cancel out, only the f distance or f portion will give us the H_u contribution.

So, H_u will be equal to $9c_u d \times f$. So, this is equation number one and as I mentioned that yield moment will first occur at the pile head. So, we are calculating the yield moment value, so we are taking the moment from the ground surface, so what are the forces? So, first we are taking this portion moment. So, here for the free headed pile will take from upper part, lower part, both, but here you have to take only the lower part moment.

So that moment first we are taking the moment for this portion and this is the lower part below the ground surface. So that means here, I should write that portion moment $9c_u d$ and total forces $\frac{g}{2}$ and lever arm will be from the ground surface $1.5d + f + \frac{g}{2} + \frac{g}{4}$ that is the lever arm and this the other portion is in the opposite then that will be the negative part and the negative part will be $9c_u d$.

So that portion is $f + \frac{g}{2}$ this total is $f + \frac{g}{2}$ then the lever arm is $1.5d + \frac{f+\frac{g}{2}}{2}$. So, if I simplify this equation, I will get that $M_y = 2.25c_u dg^2 - 9c_u df(1.5d + 0.5f)$. So, this is equation number 2 for this case. Now, what are the unknowns? Unknowns are H_u, f and g. So, I need another equation because M_y is not unknown this is known value. So, another equation is same as $L = 1.5d + f + g$.

So, this is the third equation. So, now, we have 3 unknowns f, g and H_u and we have 3 equations. So, from these 3 equations I will get the value of H_u and the M_y definitely the known value. So, now, for the long pile what will happen, I am drawing the long pile distribution.

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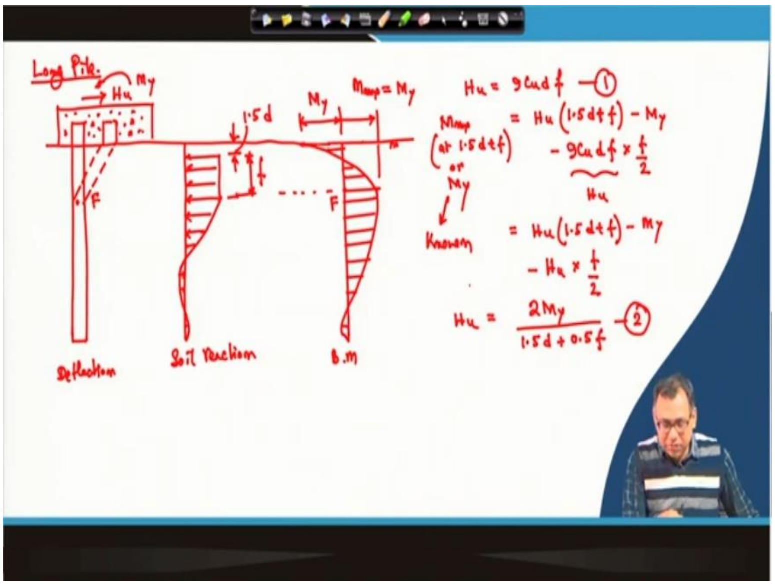

So, for the long pile, so here, so for the short pile you can see the total pile is shifting for the intermediate state it is rotating about the point and for the long pile there will be a yielding. So that means that is the long pile we can say this is the ground surface and this is the fixity. This is the pile and it will bend or there will be a yield of the pile with respect to a point. So, this is the point. So, this is the fixity of the pile and we will get the soil reaction.

So, soil reaction here up to this, this will be uniform after that, this is the soil reaction. And again, this portion is $1.5d$ there is no reaction and this is equal to f and at that point, definitely at F point there will be a maximum bending moment. So, maximum bending moment cannot be greater than yield moment. So, intermediate state already yield moment is developed at the pile top.

So, maximum moment at F point also there will be yield moment because maximum cannot go beyond the yield moment. So that mean this is the maximum moment at point F, this is F point, so this is one yield moment this is one yield moment. So, this is the bending moment diagram for this long pile. So, I can write, this is ground surface, so this is the maximum moment and this is the yield moment for that head and this is also $M_{\max}$ which is equal to M_y.

So that means, the yield moment will generate first at the head of the pile for intermediate stage then it will go towards the maximum moment point and when the maximum moment which is developed at the yield point is equal to the yield moment then or if it is beyond that then the yielding will start. So, you have to consider $M_{\max}$ up to equal to M_y. So, now for the long pile what are the equations?

So, for the long pile what are the equations? You should know that here the H_u is acting here. So, previous case also H_u is acting here, and there will be yield moment also, M_y and this case also H_u and there will be yield moment M_y. Because which is developed at the pile head. So, for this particular case what are the equations? So, for the intermediate case we have to check that what is the difference this distribution of intermediate state and the long pile state?

Because if you see the bending moment distribution that if $M_{\max}$ for intermediate state which is this value is less than M_{F} at $1.5d + f$, this is $1.5d + f$ from the ground surface then we will call it is an intermediate state. So, this condition you have to satisfy that if the moment which is developed at a depth of $1.5d + f$ from the ground surface is less than the yield moment then it is intermediate pile and if the $M_{\max} = M_y$ then it is or greater than that then it is a long pile.

So that means in case of long pile this $M_{\max} = M_y$ at this point then it is a long pile. So, this way we can distinguish what is the intermediate pile? What is the long pile and then now for the long pile what are the values? So, H_u will be equal to, this is the top portion as I mentioned that the bottom plus minus, we are not considering that both are equating each other. So, the top portion $H_u = 9c_u df$.

This is equation number one. And the yield moment, now the maximum moment, $M_{\max}$ that will act at $1.5d + f$. So, I can write the maximum moment that will be equal to H_u then $1.5d + f$ because in previous case I took the moment from the ground surface where the H_u is acting. So, the point where the H_u is acting at the same level, we are taking the moment, so that means H_u contribution within that moment was not considered.

Because H_u is acting at that point, but when we took it for the free headed pile, we consider the H_u contribution during the calculation of moment because this H_u is acting at a height of *e*. So, now if I consider this $M_{\max}$, now $M_{\max}$ I am taking at a depth of $1.5d + f$, so I have to consider H_u contribution, so $H_u(1.5d + f)$. So that is the moment due to H_u then there will be a M_y, yield moment on the ground surface, so that is $-M_y$ then the lower portion contribution because we are taking from the *f*.

So, we are taking the upper part of the *f*, so *f* then this contribution that is also negative because H_u is acting from left to right and this reaction is acting right to left because this is deflection.

This is soil reaction; this is bending moment. So that will be $-9c_u df$ then that will be $\frac{f}{2}$. So, now, this portion this is nothing but it is equal to H_u. So, finally, I can write that $H_u(1.5d + f) - M_y - H_u \times \frac{f}{2}$.

So, finally, the H_u will be equal to $2M_y$ and now, we can replace this $M_{\max}$ by M_y because, as I mentioned that $M_{\max}$ cannot be greater than M_y because then there will be a yielding, so we have to restrict it up to M_y. So that means here we can write $M_{\max}$ or M_y. So, we can write this is $\frac{2M_y}{1.5d+0.5f}$ this is $f - 0.5f$, so this will be $0.5f$.

So, this is equation number 2 for this case. Now, what are the unknowns? Here M_y is known, so M_y is known for a particular pile. So, M_y is known, so only unknown are H_u and f. So, if I solve these two equations then I will get the H_u and f then I can determine what would be the H_u value and where are the maximum bending moment will occur, so that also we can determine.

So, then f and this is what are the unknowns, f and then H_u and we have two equations. So, this way we can determine what will be the H_u value and what is the f value that means, where the maximum moment will occur. So, we have discussed the five cases the first case short pile free headed pile then long pile free headed pile then short pile fixed headed pile then short pile short intermediate pile fixed headed pile then the long pile fixed headed pile.

So, we have given all possible equations for all the cases the first case 4 possible equations and then second case 4 possible equations and third case 2 possible equations, fourth case 3 possible equations or 3 required equations and the fifth case also 2 required equations. So, now we have all the equations. So, depending upon our case, we can consider the respective equation and then we can put the values, unknown values so for different condition different e value and considering all these equations all these five cases.

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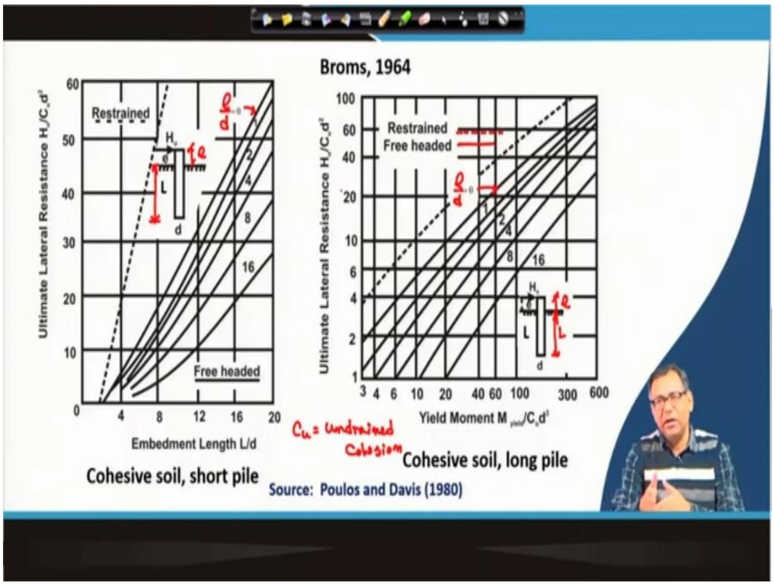

So, these charts are developed by Broms. So, the first chart is for short pile and the second chart is for the long pile. And L is this value, this is the L, it is the length of the pile and e is this one this is $\frac{e}{d}$ and this is the 0 then 1, 2, 3, 4, 5. So, different $\frac{e}{d}$, d is the diameter of the pile from 0 to, this is 0, 1, 2, 4, 8, 16 these firm lines are for the free headed pile and dotted lines are for the restrained or the fixed headed pile.

So, if I know the $\frac{L}{d}$ value for a particular pile, if I know where the pile forces are acting then for the fixed headed pile as I mentioned there will be one case though the e will be always 0. So that means, it is only one case for the fixed headed pile you can see but the free headed pile there will be number of cases based on the e values and then I will get the $\frac{H_u}{c_u d^2}$, c_u is the un drained cohesion of the soil.

And you can see that $\frac{L}{d}$ is taken up to 20. So, when I will say this is a short pile that if $\frac{L}{d} \leq 20$ then I can say roughly it is a short pile and if it is more than that then it will be long pile but if it is the borderline case then we can consider this intermediate pile also. But if $\frac{L}{d}$ say, 20 to 30 we can consider the intermediate pile then above that we can consider a long pile. But if $\frac{L}{d} \leq$ 20 then it is short pile and above it is a long pile.

But sometimes above some value also we can consider the intermediate pile and then we consider that as a long pile. So, we will get the $\frac{H_u}{c_u d^2}$. So, we know the d value, we know the c_u

value we can calculate what will be the H_u value. Similarly, for this long pile case also, but here we should know what is the yield moment? So, yield moment, this is the undrained cohesion, c_u and d is the diameter and here also we will get the firm line is for free headed and the restrain or the fixed headed is the dotted line.

So, $\frac{e}{d} = 0$ is this line, this is zero this is 1, 2, 4, 8, 16 again this is the L and this one is the e and for the fixed headed pile only the dotted line and firm line is for the free headed pile with different e value or $\frac{e}{d}$ value from 0 to 16 and here if we know the yield moment and cohesion and diameter of the pile then corresponding $\frac{H_u}{c_u d^2}$ you will get. From there also c_u and d are known, so you will get the H_u value.

So, either we can use those equations or we can use these charts. So, these charts are basically developed based on those equations, those 4 cases basically here the short pile fixed headed and free headed and long pile fixed headed and free headed these four cases are given in this chart form. And there will be intermediate case as you know, I have discussed that intermediate case if the moment which is developed at a depth of $1.5d + f$ from the surface is less than yield moment then this will be an intermediate pile.

So, we can develop what is the moment at that point because in the intermediate pile we can determine what is the moment at this point, we can take the moment at the point F also. And we can check that whether that moment is within the yield moment or not because in the long pile we have determined the moment with respect to F point and then intermediate pile also you can check with respect to F point that moment is within the yield moment or not.

If it is within the yield moment, maximum moment is within the M_y then it is the intermediate pile, otherwise it is the long pile. So, these charts are given for short pile, long pile, free headed and fixed headed. So, one thing that as I mentioned that if $\frac{L}{d} \leq 20$ we consider it as a short pile otherwise we can consider it as a long pile, but, we have to check whether it is the intermediate pile or not by considering that condition that the maximum moment that will develop at a point of $1.5d + f$ is within the yield moment or not.

If it is within the yield moment then it is the intermediate pile otherwise, it is a long pile. But generally this $\frac{L}{d}$ term this is the more or less guidelines 20 and sometimes as you can see the some cases that up to say 30 also you will get the intermediate case, but that you have to check that most of the cases if it is more than 30 then you will get the long pile behavior but some cases before that also you will get that long pile behavior. So, long pile, intermediate pile that behavior we have to check based on that moment condition.

Otherwise, most of the cases if $\frac{L}{d} \leq 20$ go for the short pile or more than that I will go for the long pile and then I have to check whether it is a long or intermediate pile. So, with this I am finishing today's lecture and in the next class, I will discuss the pile in cohesionless soil because up to this I have discussed pile in cohesive soil. So, next class I will discuss pile in cohesionless soil for different conditions free headed, fixed headed, short pile, long pile, intermediate pile. Thank you.