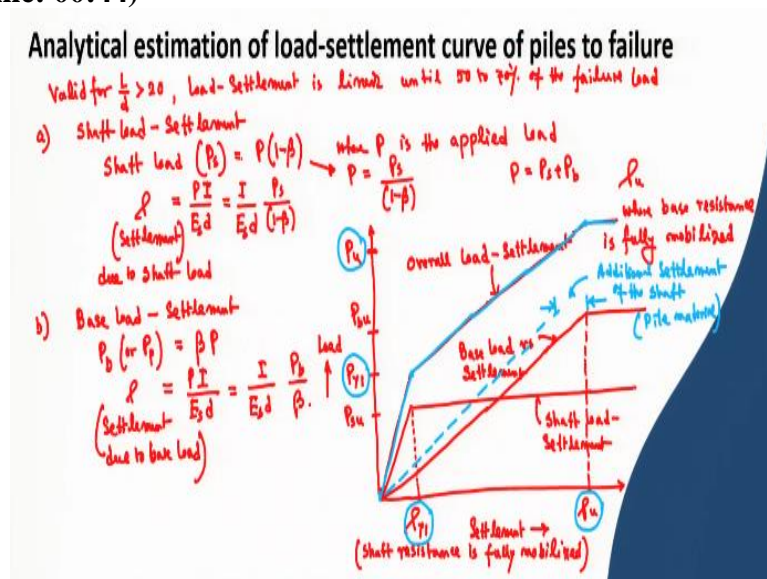


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Lecture – 46
Pile Foundation: Under Compressive Load - VI

So, last class I have discussed that how we can develop load settlement curve for pile analytically based on elastic analysis.

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Now, this is the procedure that I have discussed and these are the curves, so as I have discussed that first the frictional resistance will mobilize then that tip resistance will be mobilized and during the frictional resistance mobilization there will be also some amount of the tip resistance. So, this curve is the curve for frictional resistance or shaft load versus settlement. And this red curve is for tip resistance versus settlement and this blue curve is the overall load that means shaft load plus tip load settlement versus settlement.

And then I have drawn a blue dotted line, so as I mentioned that actually that tip load versus settlement that means settlement of the soil due to the tip load is represented by this blue curve blue dotted line. And this additional portion of the settlement because this x axis is the settlement. So, additional portion of the settlement is due to the settlement of the pile or pile material and that will happen due to the tip load.

And again, as I have already mentioned that this compression of the pile material will take place once the compression of the soil will be over. So, that means once the compression of

the soil will be over due to the tip resistance or tip load then only the compression of the pile will take place unless it is complete there is a slip between the soil and the pile or there is the settlement of the pile due to the deformation of the soil.

During that moment the deformation of the pile material will not take place. So, deformation of the pile material will take place once the deformation of the soil is over. So, that means we can say that up to this blue line this is the deformation of the soil due to the tip load then once it is done. Then the deformation of the pile material will take place and then the overall deformation of the pile material plus due to the tip load is the red line.

So that means the overall load settlement curve will be the deformation of the soil due to the shaft load plus deformation the soil due to the tip load plus deformation of the pile material. So, summations of these three components will give us the overall load settlement curve. And that means, as I mentioned when the shaft resistance is fully mobilized during that time also there will be some small amount of tip resistance, but that tip load will not contribute during the compression of the pile material.

So, when we calculate the compression of the pile material, so that portion of the tip resistance we have to subtract from the overall tip load. So, overall tip load will give us the deformation of the soil due to the tip resistance or tip load. But when you calculate the deformation of the pile material, then you have to subtract that portion of the tip load which is developed during the full mobilization of the shaft resistance or frictional resistance.

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Assume that pile material is perfectly elastic, additional compression of the shaft or pile material

$$\Delta l = \left[P_b - \frac{P_{su}\beta}{(1-\beta)} \right] \frac{L}{E_p A_p}$$

$E = \frac{PL}{\Delta l}$
 $\Delta l = \frac{PL}{EA}$

$A_p = \text{area of the pile base}$

Total load at $P_{su} = P_{T1}$
 \downarrow
 ultimate shaft resistance

$\therefore P_{su} = P_{T1} (1-\beta)$
 $P_{T1} = \frac{P_{su}}{1-\beta}$

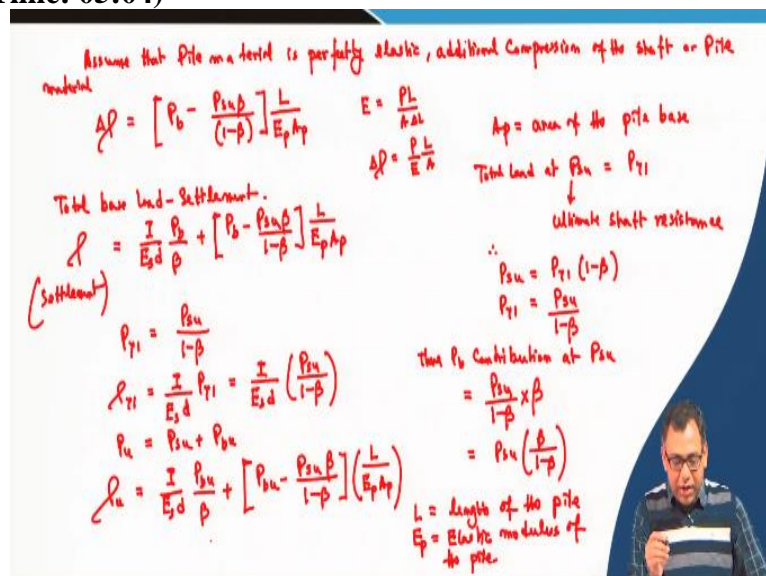
Thus P_b contribution at P_{su}
 $= \frac{P_{su}}{1-\beta} \times \beta$
 $= P_{su} \left(\frac{\beta}{1-\beta} \right)$

Total base load - Settlement.
 (Settlement)

$$s = \frac{I}{E_s d} \frac{P_b}{\beta} + \left[P_b - \frac{P_{su}\beta}{1-\beta} \right] \frac{L}{E_p A_p}$$

$P_{T1} = \frac{P_{su}}{1-\beta}$
 $s_{T1} = \frac{I}{E_s d} P_{T1} = \frac{I}{E_s d} \left(\frac{P_{su}}{1-\beta} \right)$
 $P_u = P_{su} + P_{bu}$
 $s_u = \frac{I}{E_s d} \frac{P_{su}}{\beta} + \left[P_{su} - \frac{P_{su}\beta}{1-\beta} \right] \left(\frac{L}{E_p A_p} \right)$

$L = \text{length of the pile}$
 $E_p = \text{elastic modulus of the pile}$



So, this is this amount, so that is done and then we should know the values of these four quantities, then only we can draw this overall curve or the other curves also. So, how we can determine these four quantities.

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Example.

$$\beta = \beta_0 C_k C_v$$

$$\frac{d_b}{d} = 1, \frac{L}{d} = \frac{15}{0.5} = 30$$

$$\beta_0 = 0.05$$

$$C_k = 1$$

$$C_v = 0.6$$

$$\beta = 0.05 \times 0.6 \times 1 = 0.03$$

$$K = \frac{E_p R_p}{E_s} = \frac{20 \times 10^6 \times 1}{70 \times 10^3} = 286$$

$$I = I_0 R_k R_q R_f$$

$$I_0 = 0.064, R_k = 1.6, R_q = 1, R_f = 1$$

$$I = 0.064 \times 1.6 \times 1 \times 1 = 0.1024$$

$$P_{u,c} = \frac{P_{u,c}}{(1 - 0.03)}$$

$$= \frac{825}{(1 - 0.03)}$$

$$= 850 \text{ kN}$$

$$P_{u,c} = A_s \alpha C_u$$

$$= \pi (0.5) \times 15 \times 0.35 \times 100$$

$$= 825 \text{ kN}$$

$$P_{u,c} = 9 C_u A_b = 9 \times 120 \times \frac{\pi (0.5)^2}{4}$$

$$= 212$$

Material Properties:
 $E_p = 20 \times 10^6 \text{ kN/m}^2$
 $E_s = 70 \times 10^3 \text{ kN/m}^2$
 $C_u = 100 \text{ kN/m}^2$ (Average along the pile shaft)
 $C_b = 120 \text{ kN/m}^2$
 $\mu = 0.5$
 $\alpha = 0.35$
 $C_k = 0.6$

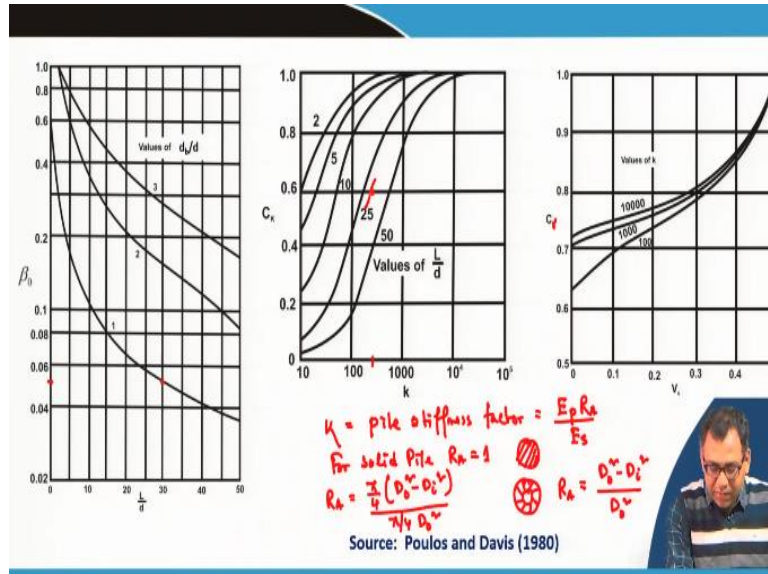
Diagram: A vertical pile of length 15m and diameter 0.5m. Labels include "Solid c/s", "R_p = 1", and "Floating pile".

So, now, today I will solve one example problem, the elastic modulus of the pile material is $20 \times 10^6 \text{ kN/m}^2$ and elastic modulus of the soil is $70 \times 10^3 \text{ kN/m}^2$, average undrained cohesion is 100 kN/m^2 that is average along the pile shaft that means along the length of the pile.

And cohesion at the base is 120 kN/m^2 then μ value of the soil is 0.5 it is the cohesive soil then adhesion factor, α is 0.35. So, I have already given you the table from which you can determine the adhesion factor for different soil or different cohesion values of the soil and so, adhesion is $\alpha \times$ cohesion. So, now it is a circular pile whose diameter is 0.5 m and length of the pile is 15 m it is in floating pile and we have taken the average cohesion. So, that means this is a floating pile.

So, now the portion of the load that will transfer to the tip that is equal to $\beta_0 C_k C_v$. So, what is β_0 , what is C_k , what is the C_v that I have discussed. So, now, we have d_b that means the diameter of the base that means the diameter of the shaft. So, both are same because it has a uniform cross section. So, that means we can write $\frac{d_b}{d} = 1$ and $\frac{L}{d} = \frac{15}{0.5} = 30$.

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So, now, if I go to that chart, so the $\frac{L}{d} = 30$ and $\frac{d_b}{d} = 1$, so β_0 value will be 0.05, so this is 0.04, 0.05, 0.06. So, that means, β_0 value corresponding to the $\frac{L}{d} = 30$ and $\frac{d_b}{d} = 1$ because this is $\frac{d_b}{d} = 1$ this is 2, this is 3. So, β_0 is 0.35 then we have two correction factors one is the correction factor for Poisson's ratio. So, that we have to take as 1 because this β_0 is developed for Poisson's ratio equal to 0.5 and our soil Poisson's ratio is also 0.5. So, no correction is required.

So, we have to take in the $C_v = 1$ because for 0.5 this value is 1. Now we have to take the value of C_v . So, now we can write that here corresponding to this value we have taken $\beta_0 = 0.05$. Now we know that our k value is $\frac{E_p R_A}{E_s}$, so what is k I have discussed, E_s is 70×10^3 kN/m², E_p is 20×10^6 kN/m² and this pile has a solid cross section or solid pile. So, that means R_A will be 1.

So, I have already discussed in the last class how we can determine the R_A for solid pile or for the hollow pile. So, it is for solid pile, so this is 1 and E_s is 70×10^3 kN/m². So, this value is 286. So, as I mentioned C_v will be 1 because the Poisson's ratio itself is 0.5, so this is C_v which is equal to 1. Now C_k we have to calculate for corresponding to this $k = 286$.

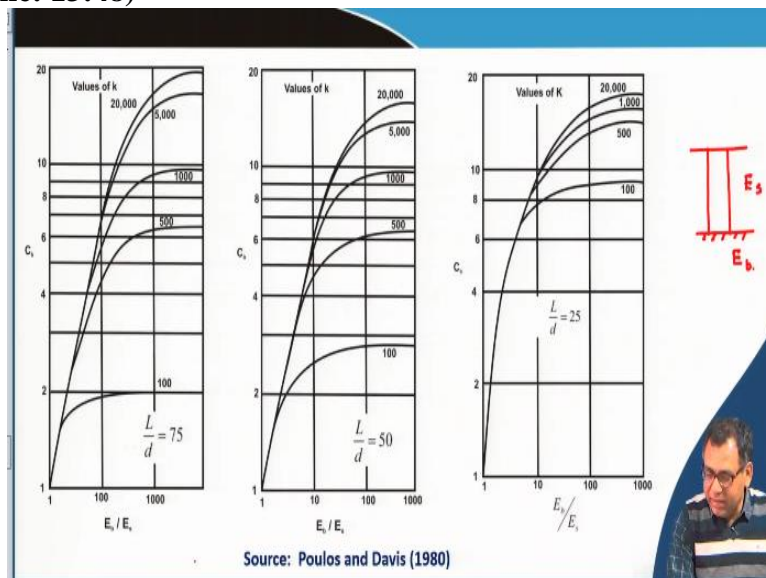
So, β_0 we have calculated this is 0.05 this is the value and for $k = 286$ and $\frac{L}{d} = 30$. So, this C_k value will be around 0.6. So, this is for 30 will be somewhere here and this is 286 will be here. So, this is the C_k value. So, the β_0 value is being calculated as 0.05 or it has been taken as 0.05

from the curve and C_k value for a correction factor due to pile stiffness and that is equal to 0.6 because $k = 286, \frac{L}{d} = 30$.

So, this will be around 30, 25 this is 50 this is 25, 10, 5, 2. So, 30 will be around here, so this is the value that is 0.6. So, we can write that $C_k = 0.6$, so finally, this β is equal to β_0 is 0.05, C_k is 0.6 and C_v is equal to 1. So, this value will be 0.03. So, now similarly we have to calculate I value also because in these expressions there is a term I , I and β these two we have to calculate and I have also discussed that what is I in the last class.

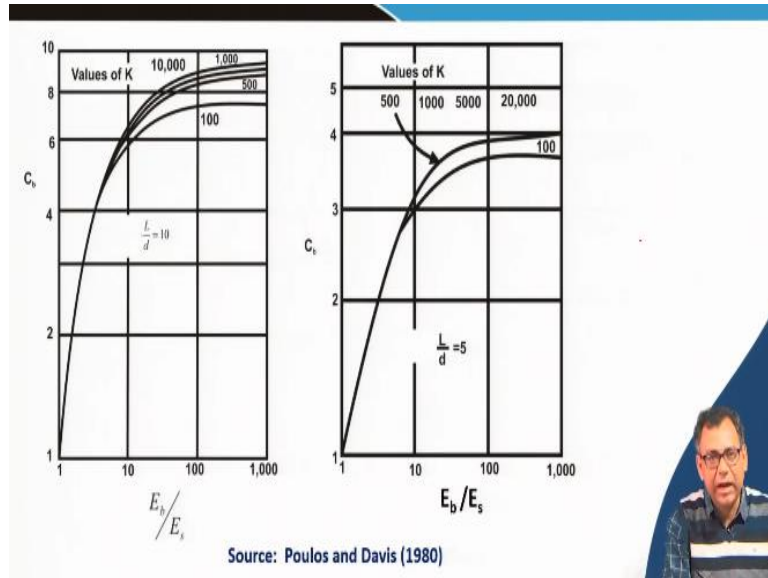
So that means $I = I_0 R_k R_h R_v$ and that is also for floating pile and for end bearing pile there will be different corrections. For end bearing piles there are R_b and similarly, C_b but here it is a floating pile. So, we are not taking those R_b or C_b value. So, again this I_0 we have to calculate. And I_0 values we can calculate from the charts that are given. So, here the I_0 chart, this is the I_0 chart because this is C_k and C_v for floating pile.

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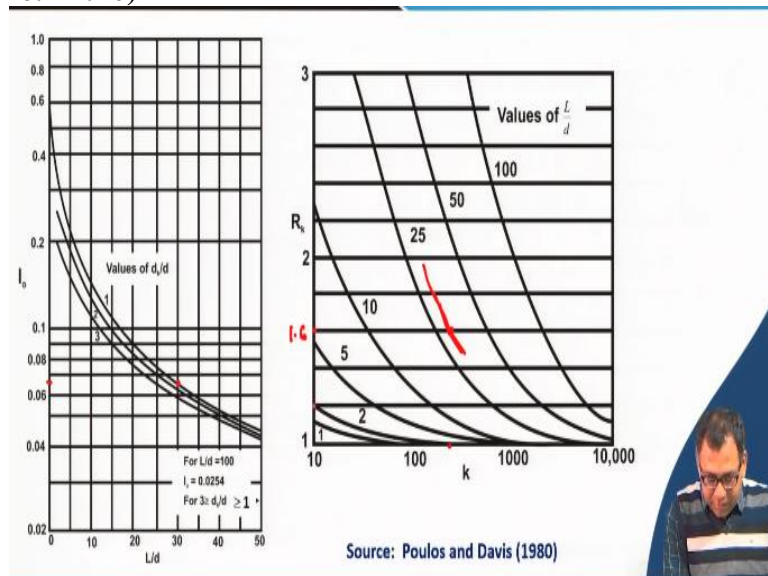
Similarly, for your end bearing pile, we have to introduce C_b but here it is not end bearing pile. So, that is why we are not taking the C_b value.

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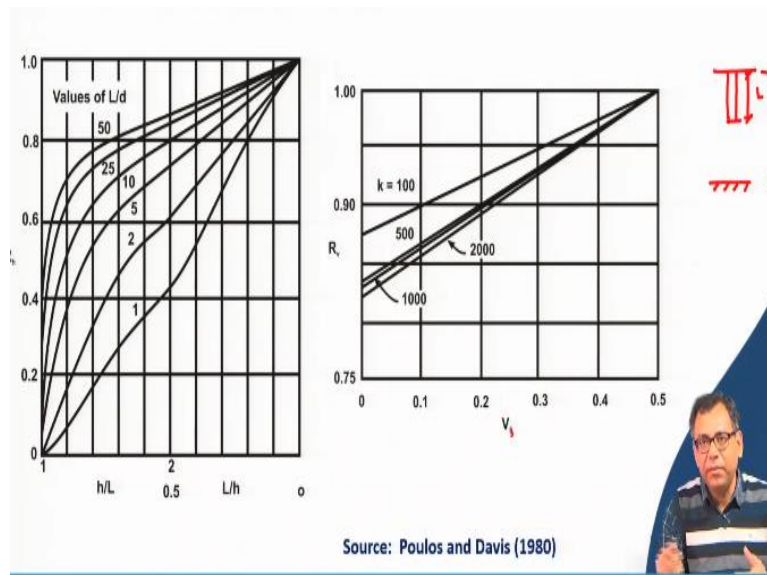
So, these are the C_b charts for different $\frac{L}{d}$ and $\frac{E_b}{E_s}$ ratio where, E_b is the elastic modulus of the base strata or the resisting strata.

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So, I_0 is for $\frac{L}{d} = 30$ and $\frac{d_b}{d} = 1$, so this is the value and I_0 will be 0.064, so here are I_0 values, this is 0.06 this is 0.07. So, it is in log scale, remember that is in the log scale. So, that means I_0 is 0.064. Similarly, R_v will be 1 again because Poisson's ratio is equal.

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And there is a term R_h , R_h is that if there is a resisting stratum and then within that soil this is the floating pile, pile is not resting on the strata but there is a resisting stratum and then the length of the pile is L and the thickness of that soil layer is h . In such case we have to use the R_h because this is an effect of finite soil layer but our case any that type of information is not given though there is a finite soil layer existing or not, it is a given soil layer whose properties are given.

So that means, we assume that there is no resisting stratum. So, it will be infinite, or the length of the soil layer is large enough, so that we will not apply that correction. So, that means, we will consider there is no finite strata. So, that means here that h value is not the finite value. So, that means the depth to this stratum is literally large. So, we can see in that case that $R_v = 1$ and $R_h = 1$ because we are not incorporating the effect of finite strata.

But there is R_k value is your k value is 286, so this is 286 and $\frac{L}{d} = 30$, so that will be again here, so 286 and this is 30, so 286 then these values will be around 1.6. This is 1.2, 1.4, 1.6. So, this is the value. So, this value is 1.6 because this is 1.2, 1.4, 1.6, 1.82. So, we have determined these values, so you will put these values here, so that means the I_0 is 0.064, $R_k = 1.6$, $R_h = 1$ and $R_v = 1$. So, we can say that I value is equal to $0.064 \times 1.6 \times 1 \times 1$ and that is equal to 0.1024 because this is a floating pile. So, based on the given information, we have determined these values.

Now we have to calculate these four quantities and equations are given for these four quantities and I have derived all these equations in the last class. So, directly I will use these equations or

these four values. So, first one is P_{y1} and that is equal to $\frac{P_{su}}{1-\beta}$. So, we can calculate that P_{su} is the ultimate frictional resistance of the pile and that we can calculate as $A_s \alpha C_u L$ that is the ultimate frictional resistance of the pile.

So, we can write that $A_s = \pi d L$, $d = 0.5$ and $L = 15$ and the α value is 0.35 and average C_u along the length of the pile or along the shaft of the pile is 100 kPa. So, this is the value. So, P_{su} is 825 kN, so ultimate frictional resistance of the pile.

Similarly, P_{bu} is the ultimate base resistance of the pile or the tip resistance of the pile that is equal to $9 C_u A_b$. So, this is equal to $9 \times C_u$ is C_{ub} at the base is $120 \times$ the area $\frac{\pi d^2}{4}$, d is 0.5, so this is equal to 212. So, we know the ultimate frictional resistance and the ultimate tip resistance also. So, P_{su} is 825 then $1 - \beta = 1 - 0.03$. So, this is equal to 850 kN.

So, that means that when the frictional resistance is fully mobilized that means 825 kN in that time the tip resistance contribution will be $850 - 825 = 25$ kN. So, that means here when the frictional resistance is fully mobilized that time also there will be some tip resistance and that tip resistance is 25 kN and total is 850 kN. So, tip resistance contribution is 850 kN - 825 kN is equal to 25 kN. So, now, we will calculate what is the deformation at this load when the frictional resistance is fully mobilized.

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$$s_{y1} = \frac{I}{E_s d} \left(\frac{P_{su}}{1-\beta} \right) = \frac{I}{E_s d} (P_{y1}) = \frac{0.1024}{70 \times 10^3 \times 0.5} (850) = 2.49 = 2.5 \text{ mm}$$

$$P_u = P_{su} + P_{bu} = 825 + 212 = 1037 \text{ kN}$$

$$s_u = \left(\frac{I}{E_s d} \right) \left(\frac{P_{bu}}{\beta} \right) + \left[P_{bu} - \frac{P_{su}}{(1-\beta)} \right] \left[\frac{L}{A_p E_p} \right]$$

$$= \frac{0.1024}{70 \times 10^3 \times 0.5} \left(\frac{212}{0.03} \right) + \left[212 - \frac{825 \times 0.03}{1-0.03} \right] \left[\frac{15}{7(0.5)^2 \times 20 \times 10^6} \right]$$

$$= 20.7 + 0.71$$

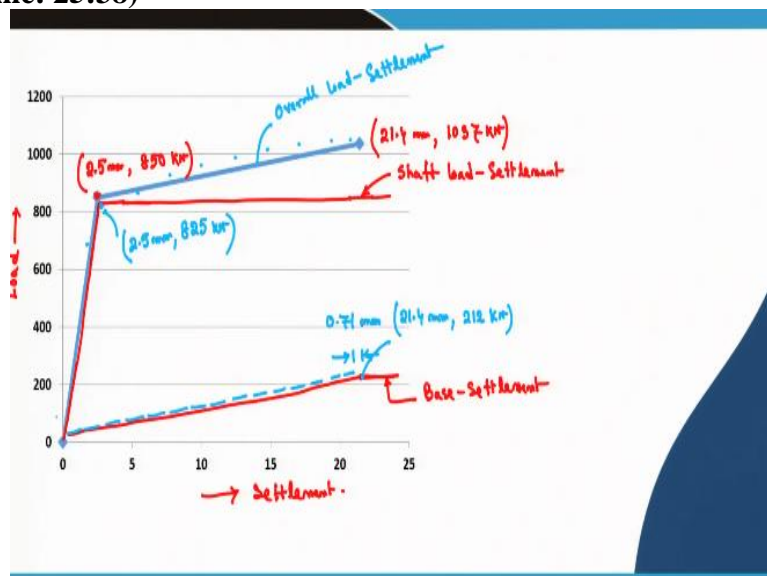
$$= 21.4 \text{ mm}$$

So, at that point the deformation is equal to that expression is also given and this is the expression, so that expression is given as $\frac{I}{E_s d} \left(\frac{P_{su}}{1-\beta} \right)$. So, we will write that. So, $\frac{I}{E_s d} \left(\frac{P_{su}}{1-\beta} \right)$ or directly I can write $\frac{I}{E_s d} \times P_{y1}$. So, I value is how much? I value is 0.1024. So, I value is 0.1024, E_s is 70×10^3 , then $d = 0.5$ and P_{y1} is 850. So, at that point settlement is 2.49 or roughly 2.5 mm.

Now we will calculate the ultimate settlement, because we know that tip resistance and base resistance. So, total load carrying capacity of the pile is summation of the frictional resistance and the base resistance. So, that will be $825 + 212$ that will be 1037 kN. So, at that moment the ultimate settlement, $\rho_u = \frac{I}{E_s d} \left(\frac{P_{bu}}{\beta} \right) + \left[P_{bu} - \left(\frac{P_{su}}{1-\beta} \right) \beta \right] \left[\frac{L}{A_p E_p} \right]$.

So, now I will put these values $I = 0.1024$, E_s is 70×10^3 , diameter is 0.5 then P_{bu} is 212, β is 0.03 then $+ P_{bu}$ is 212 - P_{su} is $825 \times \beta$ is 0.03 divided by $1 - 0.03$. Then length of the pile is 15, then area of the pile is $\frac{\pi \times 0.5^2}{4}$ and E_p is 70×10^6 . So, from there I will get the value as 20.7 + 0.71 that is 21.4 mm. So, from here you can see the soil deformation is 20.7 mm and the deformation of the pile material is 0.71 mm. So, overall is 21.4 mm, so finally, we know these all four values.

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So, finally, we have drawn this load settlement curve, so this is the settlement and this is the load. So, you can see that from here that when the total load when frictional resistance is fully mobilized is 850, then settlement is 2.5. So, the total load is 850 and settlement is 2.5. Similarly,

the total pile load carrying capacity at tip resistance plus base resistance is 1037 and that total settlement is 21.4. So, 1037 total settlement is this is 1037 and total settlement is 21.4.

So that means here this is 1037 or I should write that x coordinate is 21.4 mm and 1037 kN. And this is 2.5 mm and 850 kN. So, similarly, I can draw the other two curves also. So, the other two curves are if I draw all curves like this one then the values will be, so for when the frictional distance is fully mobilized, so at that time also this will be 2.5 but the value is 825. So, this is 850 this will be the value around something like this. So, 825 is the value.

So, I can draw this is the curve because there is only slight extra value due to the tip resistance and this is also 2.5 this curve is also this is also 2.5. So, I should draw like this and then it will be parallel to x axis. So, this is the shaft load settlement curve, so in this condition the tip resistance is only 25 at the settlement only 25 this is the tip resistance you can say and that will continue like this up to this value and total tip resistance is 212, so 212 means 212, 212 will be somewhere here.

So, I can draw this is like this, and then I can extend that. So, now this is the curve for base load and settlement of pile. And then I have discussed that there will be a portion that is due to the pile material settlement and that value is less than 1 mm. So, I can draw something here, so this will be here, so I can extend that curve also, but that difference will be very small as the deformation is only 0.71 mm which is less than one mm.

So, if I write those values, so this coordinate will be 2.5 mm and y axis load will be 825 kN, this coordinate the values will be again 21.4 mm and this is 212 kN. So, this additional deformation due to the pile material is 0.71 mm. So, this is the curve, so I can draw the overall curve like this. So, this I will add and then it will be parallel to this base load settlement curve. So, this one is the overall curve, this is the overall load versus settlement.

This one is a shaft load versus settlement and this is the base load versus settlement and this one is the additional settlement due to the pile material deformation or pile deformation. So, in this way, we can determine the load settlement curve theoretically for a pile and with definitely with some assumptions and that I mentioned previous class. So, here I should say that actual deformation or the load versus deformation curve may be the nonlinear in pattern.

So, but here it is represented by linear curves or in linear form, so which is almost closely we will get the actual wave here because in actual curve there will be points around this curve. So, this may be the actual behaviour, so which is not exactly this binary linear type of deformation pattern, but by linear type of deformation pattern or simplified approach we can approximately obtain the load settlement curve of a pile by using theoretical approach.

So, in the next class I will discuss the next topics related to the piles for example, the other different ways by which we can determine the load carrying capacity of the pile. And then I will discuss how we can calculate the consolidation settlement for a group pile and how we can determine the group efficiency for the pile. Thank you.