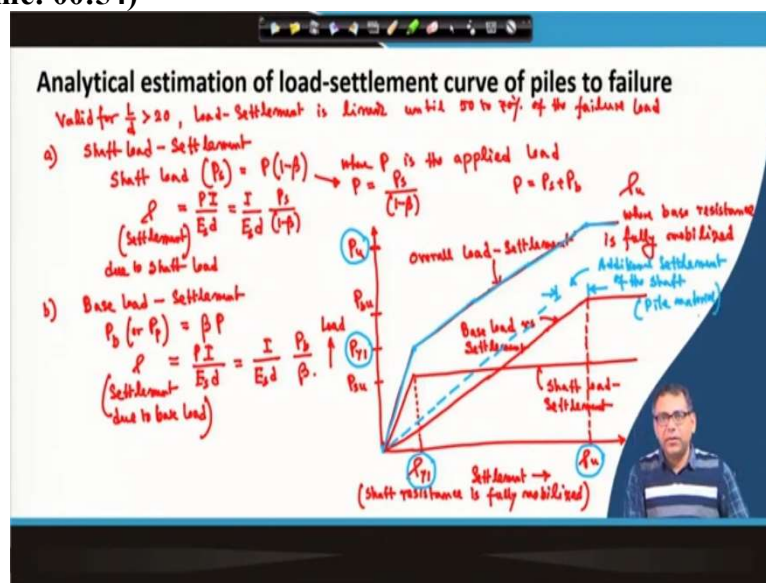


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**Lecture - 45**  
**Pile Foundation - Under Compressive Load - V**

So, last class I have discussed that how we can determine the portion of load which is transferred to the pile tip by using elastic analysis and then how we can determine the settlement of a single pile. And then we express the settlement of a single pile in terms of shaft load and in terms of tip or base load.

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So, this is an expression that we derived that this is the settlement due to the shaft load or in terms of shaft load and then this is the expression in terms of base load. And remember that in these two expressions let us assume that that load and settlement relationship is linear. So, now this is the settlement we have written in terms of base and the shaft load. So, now if I draw the settlement in terms of shaft and in terms of base then we can see that this is the settlement and this is the load.

So, as I mentioned that initially the frictional resistance will be mobilized then the tip resistance will be activated. And it is not that during the frictional resistance mobilization there will be no tip resistance, there will be tip resistance but that amount will be less compared to the frictional resistance. So, I can write that as these equations are developed considering a linear stress strain relationship. So, we can write that this is the load versus settlement for shaft resistance that in terms of shaft load this is the load settlement plot.

And this is the load settlement curve for shaft load because this is linear and this value is, say,  $\rho_{yI}$  that means when the shaft resistance will be fully mobilized. So that point the settlement is  $\rho_{yI}$  or  $\rho_{y1}$ . So, at this point the shaft resistance is fully mobilized. So, there will be a tip resistance versus settlement plot also, so that will also continue like this.

So, depending upon if it is a bearing pile or the friction pile this curve magnitude will change but here the tip resistance portion is more compared to the frictional resistance. So that means when the frictional resistance will be fully mobilized at that point, your tip resistance value is very small. At that point tip resistance value is very small. So that means at this point when the shaft resistance is fully mobilized the overall curve will be this total shaft resistance plus the tip resistance.

So, overall curve will be something like this at this point. So, this portion will be equal to the tip resistance part. So, I can draw that once the shaft resistance is fully mobilized then if we increase the settlement there will be no change in the shaft resistance part. So, we can draw that line like this. So, this line is basically the shaft load versus settlement plot and this plot is base load versus settlement.

So, here also once that, the base resistance is also fully mobilized and that settlement is  $\rho_u$ , where base resistance is fully mobilized. So, after that if you change the settlement there will be no change in the base resistance, so that also you can draw like this. So now, overall curve will be the overall load settlement curve, so up to this point, so, this contribution plus the base resistance contribution, this is the overall curve then after that point, after this point there is no contribution due to the shaft resistance.

Only contribution due to the base resistance. So, at this point the line will be parallel to the base resistance line and then it will go straight. So, now there are different points, so this is one point, this is another point and corresponding to this is another point and this is also another point. So, what are those points? So, these points are, this is the ultimate load because this is overall load settlement curve, overload settlement plot.

So, this may give you the maximum load that is  $P_u$ , now this point is given, so I have to change because you are here particular this case, this is parallel to this line then it will go straight, here

also it will go straight. So, this is point I am removing, so this is the point corresponding to this point. So, now this curve is base versus settlement, this is overall load versus settlement.

So, I can write that  $\rho_u$ , where the base resistance is fully mobilized. So, this is equal to the  $P_u$  and this one is  $P_{bu}$  that means the ultimate base resistance this one  $P_{su}$ , ultimate shaft resistance and this one  $P_{y1}$ . So, now, I am giving one explanation that what are these  $P$  values?  $P_{su}$ , you can see this is the ultimate shaft resistance, then  $P_{bu}$  is the ultimate base resistance and  $P_u$  is the ultimate shaft resistance + ultimate base resistance that means the ultimate load of the pile.

The shaft resistance is fully mobilized at that point, this total load that means, the shaft resistance + the base resistance will be equal to  $P_{y1}$  and the corresponding settlement also  $\rho_{y1}$  and here also this is the ultimate,  $\rho_u$  which is same for the tip resistance and the ultimate, because as the tip resistance is more and these will mobilize late. So, after that there will be no settlement for the ultimate as well as the tip.

So, this is the load settlement curve that we are trying to reach here. So that means to draw this, ultimately, we will draw the overall load settlement curve. So, to draw these curves what are the points we should know or what are the values we should know? So, to draw this curve, we should know this value, we should know this value, we should know this value and we should know this value.

So, if I know these four values and I know the relationship is linear. So then, I can perfectly draw this curve this overall load versus settlement curve because if I know the settlement at this settlement and this point, so then we can look at this point and if I know this settlement and these overall then also, I can locate this point. Then what I will do? I will just join these points from this point to this point, another from this point to this point and the next one will be the straight line.

So, then I will get the overall curve. So, now, these things I have to derive that how I will get these points. So, now I have this settlement. So, next one that we will assume that pile or we will assume that pile material is perfectly elastic.

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Assume that Pile material is perfectly elastic, additional Compression of the shaft or Pile material

$$\Delta l = \left[ P_b - \frac{P_{su}\beta}{(1-\beta)} \right] \frac{L}{E_p A_p}$$

$$E = \frac{PL}{A\Delta l}$$

$$\Delta l = \frac{P L}{E A}$$

$A_p =$  area of the pile base

Total load at  $P_{su} = P_{T1}$

ultimate shaft resistance

$\therefore P_{su} = P_{T1} (1-\beta)$

$$P_{T1} = \frac{P_{su}}{1-\beta}$$

the  $P_b$  contribution at  $P_{su}$

$$= \frac{P_{su} \times \beta}{1-\beta}$$

$$= P_{su} \left( \frac{\beta}{1-\beta} \right)$$

Total base load - Settlement.

$$s = \frac{I}{E_s d} \frac{P_b}{\beta} + \left[ P_b - \frac{P_{su}\beta}{1-\beta} \right] \frac{L}{E_p A_p}$$

(Settlement)

$$P_{T1} = \frac{P_{su}}{1-\beta}$$

$$s_{T1} = \frac{I}{E_s d} P_{T1} = \frac{I}{E_s d} \left( \frac{P_{su}}{1-\beta} \right)$$

$$P_u = P_{su} + P_b$$

$$s_u = \frac{I}{E_s d} \frac{P_u}{\beta} + \left[ P_{su} - \frac{P_{su}\beta}{1-\beta} \right] \left( \frac{L}{E_p A_p} \right)$$

$L =$  length of the pile

$E_p =$  Elastic modulus of the pile.

Now, one thing I want to mention that when we are talking about the settlement, now this settlement is the settlement of the pile material + settlement of the soil. If the pile is incompressible then the settlement will be settlement of the soil itself. But if the pile is compressible then the settlement of the pile is due to the settlement of the soil and settlement of the pile material itself.

So, here when there is a mobilization of the friction that time also there is some tip resistance. So that tip resistance will not give any effect on the settlement of the pile material. So that means when there is a total tip resistance we have and due to that tip registers the settlement of the pile material will occur. So that means, the settlement of the pile material will occur due to the tip resistance at the tip, the load which is going to the tip.

Because the load which is going to the friction part that will not influence the settlement of the pile material because that is going to the friction part. So that will not create any effect on the settlement of the pile material. But that will create the effect of the settlement of the soil, but that the settlement of the pile material that friction part component will not make any effect. The settlement of the pile material will be due to the load which is transferred to the tip of the pile or the base of the pile.

That is one point, another point is that even if the total base load will not create effect on the settlement of the pile material. Because once the settlement of the soil is completed, then the settlement of the pile material will start. So, remember that, once the settlement of the pile

material is completed, settlement of the soil is completed, then the settlement of the pile material will start.

So that means, settlement of the soil will be completed at the end of the fully mobilization of the base settlement or base load. So, after that the settlement of the pile material will start. So that means, even if the total tip load will not create effect on the pile load settlement, pile material settlement because the load which is transferred during the fully mobilization of the shaft, so that will not create any effect on that pile material settlement because that time soil settlement is going on.

So, as I mentioned the when the soil settlement will stop then the pile material settlement or pile settlement will start. So that, mean the tip resistance that is during the friction resistance mobilization will not create any effect on the pile material settlement. So that means we have to subtract that component from the total base load, such that the actual load contribution to the pile material settlement we can determine.

So, now once, so that means, here I can draw another line and then we will get the real picture that this is the dotted line actually thus the dotted line actually the soil versus when soil settlement due to the pile base load, this dotted line and this additional line is the pile material settlement, once the slip or the total shaft or your soil resistance, soil settlement is over. So, this is the additional settlement of the shaft. Shaft means the pile material.

After the full slip or you can say the full settlement is over. So, this is the shaft or pile material or pile I should say, this is the pile settlement or pile material settlement. Because the pile is incompressible it is a pile settlement, pile material settlement this is clear. So, now what we have to subtract that tip component, which will go to the base during the fully mobilization of the shaft load.

So that means, we can write the additional component of the additional compression or additional settlement of the shaft or the pile material. So that will be  $\Delta\rho$ . So that will be as I discussed that total base load is  $P_b$ , so during the full mobilization of the shaft resistance the tip load which is transferred to the tip that we have to subtract during the additional compression of shaft calculation.

So that load is how much? So, you can see that during this total load is  $P_{y1}$ . So, total load at  $P = P_{y1}$ , what does it mean? That total load when the ultimate shaft resistance is  $P_{su}$ , in that case total load is  $P_{y1}$ . So, when the total load is  $P_{su}$ ,  $P_{su}$  means ultimate shaft resistance. So, I can write that during this total load is  $P_{y1}$ . So, how much load is transferred to the tip. So, if total load is  $P$  then shaft portion is  $P(1 - \beta)$  and base portion is  $\beta P$ .

So, I can write that the total load is this one or I can write that load in the shaft portion,  $P_{su} = P_{y1}(1 - \beta)$ . So, total load is  $P_{y1}(1 - \beta)$ , so the total load  $P_{y1}$  will be in terms of shaft resistance  $\frac{P_{su}}{(1-\beta)}$ . Now this is total load,  $\beta$  portion is transferred to the base. So, I can write thus,  $P_b$  contribution at  $P_{su} =$  this is the total load  $\frac{P_{su}}{(1-\beta)}$  then  $\times \beta$ . That is the  $\beta$  portion is transferred to the tip.

So, I can write this is  $\frac{P_{su}\beta}{(1-\beta)}$ . So, clear that. So that means when the total shaft resistance is fully mobilized that is the ultimate shaft resistance at that time, total load is that means shaft resistance plus tip resistance is  $P_{y1}$ . Now when the total load is  $P_{y1}$  in that case the shaft resistance will be  $P_{y1}(1 - \beta)$ . So, the total load I can express in terms of ultimate shaft resistance is  $\frac{P_{su}}{(1-\beta)}$ . And from this the total load contribution to the base will be this total load  $\times \beta$ .

So that you have to subtract this  $P_b$  contribution at  $P_{su}$  we have to subtract because that will not create any effect on the pile material compression or the additional compression. So that you have to subtract here. So, this is we have to subtract  $\frac{P_{su}\beta}{(1-\beta)}$ . So, again we know that is  $E = \frac{PL}{A\Delta L}$ . So,  $\Delta\rho = \frac{PL}{EA}$ . So, this  $P$  is this one, this will be  $\frac{L}{E_P A_P}$ , where  $E$  is the elastic modulus of the pile,  $A_P$  is the area of the pile,  $L$  is the length of the pile.

So, the total load, now the total base load versus settlement will be that mean the total settlement will be the settlement due to the soil and settlement due to the pile material. So, settlement due to the soil will be  $\frac{I}{E_s d}$  and that is  $P$  and we will convert it to in terms of base. So, this is the base. So, this base is  $\frac{P_b}{\beta}$  then + this settlement then  $\left[ P_b - \frac{P_{su}\beta}{(1-\beta)} \right] \frac{L}{E_P A_P}$ .

So, now as I mentioned we have to determine these four quantities. So, now we can write this way,  $P_{y1} = \frac{P_{su}}{(1-\beta)}$ . So,  $\beta$  I have discussed how I will get the  $\beta$ ? By elastic analysis we will get  $\beta$ . I will calculate this is the total shaft resistance,  $P_{su}$  by using the static equations that I have discussed. So, I will get the  $P_{y1}$ . So, once I get the  $P_{y1}$  then I will get the  $\rho_{y1}$ .

Which is nothing but  $\frac{I}{E_s d} P_{y1}$  and I can write  $P_{y1} = \frac{P_{su}}{(1-\beta)}$ . So, this also if I know  $P_{y1}$ , I can get the  $\rho_{y1}$  because I have discussed how I can get the  $I$  value. So, remember that due to the shaft resistance the settlement is due to the soil and for that tip resistance the settlement is the summation of the soil contribution plus the pile material contribution. So, the ultimate load or  $P_u$ , I have to calculate.

So, this  $P_u$  will be the  $P_{su} + P_{bu}$ , so these  $P_{su}$  and  $P_{bu}$  I will calculate by using the static equations and then the ultimate settlement you can see from this curve, the ultimate settlement is this one and this ultimate settlement is equal to the settlement of the soil contribution or the tip soil contribution and the additional contribution. That mean due to the ultimate settlement we will get when the total tip resistance is mobilized. So that means that tip contribution is the ultimate settlement.

And in the tip contribution there will be a contribution from the soil and the pile material. So that means I can write this is the expression here. So, ultimate settlement,  $\rho_u$  will be  $\frac{I}{E_s d} \times$  now,  $P_{bu}$  ultimate because it is up to the ultimate, this is  $P_{bu}$  ultimate corresponding to this is the  $\rho_u$ . So,  $\frac{P_{bu}}{\beta} + \left[ P_{bu} - \frac{P_{su}\beta}{(1-\beta)} \right] \left( \frac{L}{E_P A_P} \right)$ . So, now, these four quantities I have determined. So, now once I know these four quantities, I can draw the overall load settlement curve.

Or I can draw the individual curves also because I know the  $P_{su}$ , I know the  $P_{bu}$ , I know the settlement, so I can draw the individual curve or I can draw that total load settlement curve also or overall load settlement curve. So, here I can write that  $L$  is equal to length of the pile,  $E_P$  is the elastic modulus of the pile,  $A_P$  is the area of the pile base. So, remember that this  $P_{su}$  and  $P_{bu}$ . We calculate by using the static expressions. So, this way we can develop theoretical load settlement curve.

So, in the next class, I will solve one example problem related to this topic. So that you can get an idea that how we can develop a load settlement curve by using some numerical values and you will get your settlement, ultimate settlement. What is the settlement when the frictional resistance is fully mobilized and then we will get the overall load settlement curve of the pile. And then after that we will discuss the group action of the pile. And then we will discuss about the other methods.

By which we can determine the load carrying capacity of the pile. That means a dynamic equation and the correlation by the penetration and then we will discuss about the negative skin friction that will reduce the pile load carrying capacity. So, those things we will be discussing in the coming classes. And the next class, first we will discuss a numerical problem related to this topic then I will slowly discuss the other topics. Thank you.