Advanced Foundation Engineering
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Lecture – 43
Pile Foundation: Under Compressive Load – III

So, this class I will discuss how we can generate a load settlement curve by using the theoretical approach.

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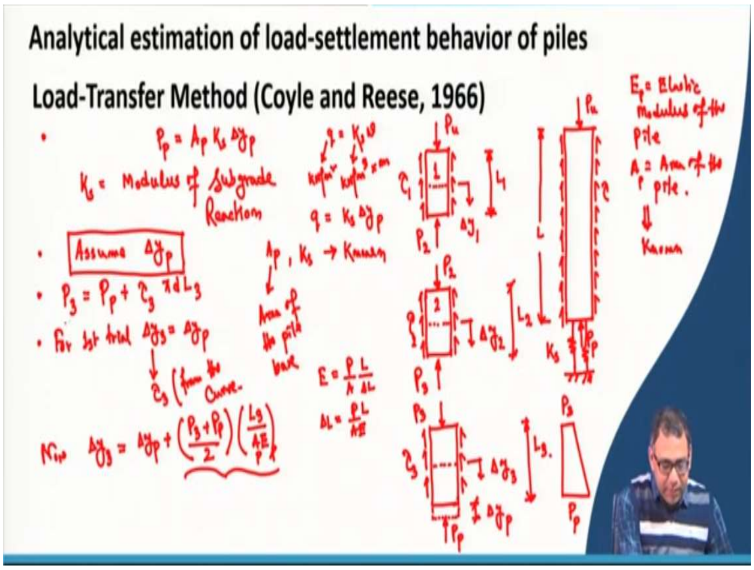

So, the first approach that we will discuss was proposed in 1966. So, that approach where we can discuss how we can propose the load transfer method. So, suppose if we have a pile. So, this is a pile, single pile whose length is L. And then when we apply a load on the piles, P_u is the ultimate load. Then we will have a tip resistance that is P_P and then we have frictional resistance. So, that frictional resistance you say this τ is the frictional resistance.

Now what we are doing that we are dividing this pile into number of segment. So, this is the first segment say and where this first segment, so there will be a P_u on the top and this is the segment number 1 and again in all the segments there will be a shear resistance. And that is τ_1 for the first segment. And then we have a reaction at the bottom that is say P_2 and then at the center, so this is the center of the element say at the center there is a deformation Δy_1.

So, Δy_1 is the deformation of the pile segment number 1 and the deformation is at the center. So, Δy_1 is the center deformation of the segment number 1. So, that segment length is say L_1. So, this is the segment 1. So, here three segments are taken. So, if you want you can take more segments also. So, this is the segment 2 the similar forces will act here. So, here reaction is P_2. So that will act here P_2 then there will be τ, now this will be τ_2, this is second segment.

This deflection at the center of the second segment will be Δy_2 then there will be reaction that will be P_3. And the thickness of the second segment is L_2. Now, there is a third segment. So, this is P_3 then this will be τ_3. So, now, this is acting on the tip this is the bottom most segment. So, there will be a deformation at the bottom also. So, now here we are taking the P_P because we are at the bottom and there is a deformation at the bottom that deformation is Δy_P that means Δy_P is that tip deformation.

And similarly, here also there will be a deformation at the middle portion and that is Δy_3 and then the thickness of this last segment is L_3. So, now for the bottom there is a deformation of Δy_P and the tip resistance is P_P. Now, if we take that these piles are resting on springs and then the spring constant is K_s. So, now we can write that P_P which is the tip resistance that will be equal to and as I have discussed during the beams on elastic foundation part.

That if at any point the stress $q = K_s w$. So, but that is a stress which is in kN/m^2, K_s is the spring constant which is in kN/m^3, so w is the deformation. So, here this is $q = K_s \times w$ is Δy_P, but here P_P is the force. So, we have to apply the area of the pile or base area of the pile. So, finally, I can write this as$P_P = A_P \times K_s \times \Delta y_P$, where K_s is the modulus of subgrade reaction.

So, this is modulus of subgrade reaction. So, if you know the modulus of subgrade reaction of that particular soil at that site then if I know the area and if I know the deformation or Δy_P then I can calculate the tip resistance. So, now for the first time what I will do? So, this is the first step we have to calculate the P_P and to calculate the P_P for the first trial we assume value of Δy_P.

Let we assume that. So, unless it is on the rock that means the pile is resting on rock there will be a small value of Δy_P. So, that we have to assume that means we have to assume a small value of

Δy_P and we know the A_P, we know the K_s. So, that means the A_P is the area of the pile base. So, that is a known and then the K_s is also known because this is a soil property. So, these are known.

So, these two values are known. So, that unknown is the value of Δy_P because A_P is the area of the pile base which is known, but we do not know Δy_P. So, first we assume a very small value of Δy_P. We assume that we know K_s, we know A_P so we will get the P_P. So, P_P is now known. Now let us for the second or the next step that we have to compute the P_3.

What is P_3? P_3 is basically $P_P + \tau_3$, where, τ_3 is the frictional resistance that means the friction τ_3 and that is acting on the periphery of this small element 3 and the area of the periphery of this small element 3 is $\pi d L_3$. So, again now what are the unknowns here because we know P_P from the initial step and we know L_3, because L_3 is the segment because we know the length of the pile, we have divided it number of segments.

So, that L_3 is also known because we have divided it. Now, diameter of the pile is also known. So, that means here unknown is τ_3, so how I will get the τ_3 value. So, this τ_3 we have to again assume, now for the first trial, we assume that $\Delta y_3 = \Delta y_P$ because we have assumed Δy_P. Now, we are taking $\Delta y_3 = \Delta y_P$. Now, what is Δy_3? Δy_3 is the deformation at the center of the element 3 and $\Delta y_3 = \Delta y_P$ is the tip deformation. So, now first all we assume the Δy_3.

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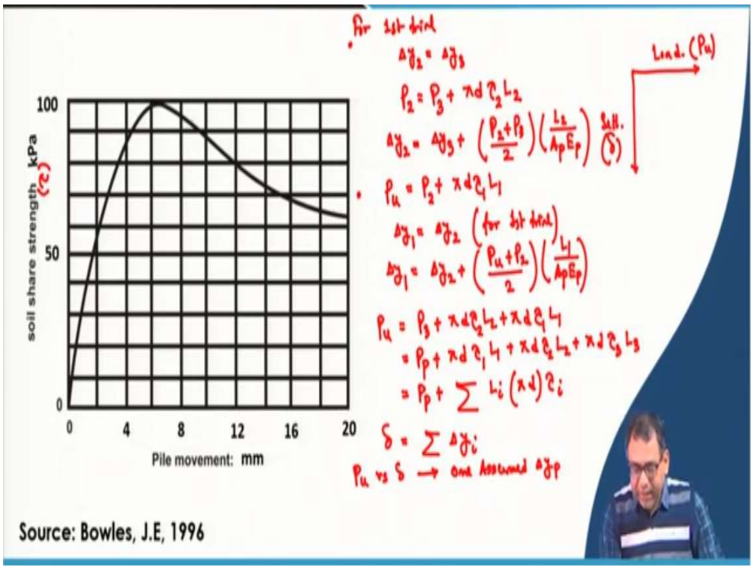

Now, we will take the help of this particular curve. So, this curve is given for different pile movement and the soil shear strength. So, this soil shear strength is nothing but the τ. Now, how this pile curve is generated? So, that means to use this approach, so, you should have this type of curve that means the slip of pile and the soil and when a deformation of the pile corresponding to the shear stress or the stress that pile is getting for that particular movement.

So, for example, the movement of 2 mm, the pile shear stress is 50 kPa that means, for movement up to 2 mm movement, there will be a shear resistance of 50 kPa. So, that is the idea and for 4 mm movement of the pile we will get a shear resistance of around 90 kPa or 80 kPa 50, 60, 70, 80, 90 kPa was around 6 mm of movement will give shear resistance of 100 kPa. So, these types of charts we should know and we should have.

So, if you do not have any chart, we can use this curve but it is better that you should have your own curve. So, how this curve is generated? So, that means by field instrumentation, so the pile deformation at different points is measured and the strain gauge are placed along the length of the pile. And based on that strain gauge data, the stress on the pile for a certain deformation and other values are calculated.

So, that when strain gauges are attached, strains are calculated for the pile for different deformation, then for that strain the stresses which is acting on the pile or the piles surface for certain deformation is calculated and then this chart is produced. So, for this particular case, we will use this chart. But if you can generate this type of curve, then that will be better for your own soil, but otherwise you can use these curves.

So, I will use this curve to explain this particular problem. So, that means, by field instrumentations this movement versus shear strength curve is generated. Shear strength means that τ which is the frictional resistance that pile is getting for a particular movement. So, that means this case first trial we have taken $\Delta y_3 = \Delta y_P$ and Δy_P we have assumed small value. So, now that values are taken using Δy_3.

Once we know the Δy_3, so that Δy_3 means the deformation then corresponding τ value we will get from this curve. So, once we get the Δy_3 value, then corresponding τ we will get from this curve. So, then I will get the τ_3 that is from the curve. So that I will put here in this equation. So, now P_3 is known. Now, once you get the P_3 then what will be the value of Δy_3. Now, we have to calculate the Δy_3, Δy_3 is what? Δy_3 is the deformation of the pile center portion.

So, that will be equal to the deformation of the tip that is Δy_P + deformation of the pile itself. So, deformation of the tip is that deformation of the soil. So, this deformation of the pile material how I will get that? So, we assumed that the load is changing linearly along the depth. So, the average load which is acting at the center. So, P_3 is acting at the top of the element 3 and P_P is acting bottom of the element 3 and they are changing linearly.

So, the average load that will act at the center will be $\frac{P_3+P_P}{2}$. So, we know that $E = \frac{\frac{P}{A}}{\frac{\Delta L}{L}}$. So, $\Delta L = \frac{PL}{AE}$, so that P is this average $P \times L = L_3$ here and then that will be AE, A is that cross-sectional area of the pile at element 3 and E is the elastic modulus of the pile. So, E is the elastic modulus of the pile and A is the area of the pile or we can write this is $A_P E_P$ also. So, then area of the pile and elastic modulus of the pile.

So, that means you can see this second portion, this portion will give me the deformation of the pile and that means that is equal to the total deformation at the center is the deformation of the tip or the soil plus deformation of the pile. So, deformation of the pile material is the second part and Δy_P is the deformation of the soil and that is Δy_P. So, that will give me the deformation at the center of the element 3. So, that is here.

So, now first we have assumed that Δy_P and then we calculated P_P then we assume $\Delta y_3 = \Delta y_P$ and then determine the τ from the chart once I get the τ from the chart, then I will update the P_3. So, now P_3 is known P_P is known, L_3 is known, A_P is known, E_P is known. So, this A_P, E_P all are known, these are known values. So, we will get a new value of Δy_3.

Because initial trial we have done now, we have a new value of Δy_3, because now we put those values here then Δy_P we have assumed already. So, we will get a new value of Δy_3. Now that Δy_3 we will check whether the previously assume Δy_3 and this Δy_3 are within the tolerance limit or not? They should be same or very close to each other. So, previous assume Δy_3 and now new Δy_3 if they are within that tolerance limit then you will stop.

Otherwise, now, in the next trial using the new Δy_3 we will again calculate τ_3 and then again, I will put the τ_3 and then you will get the P_3 and then once you get the P_3 then again you will get the another Δy_3 and then again, we will check it and it will continue, unless the previously assumed or calculated Δy_3 is equal to or very close to new calculated Δy_3. So, once we get the Δy_3 updated one after iteration, then we will get the final P_3.

So, now P_3, P_P are known and Δy_3 is also known and Δy_P is assumed. So, you may remember this Δy_P is always is assumed but others values are calculated based on these assume Δy_P. So, once we get the Δy_3 and the P_3 the next trial will be then again, we assume for the second segment and then we assume $\Delta y_2 = \Delta y_3$, because now we know the Δy_3 for the second segment, we assume for the first trial the first value we assume Δy_3.

Again we will calculate the P_2 now, $P_2 = P_3 + \pi d \tau_2 L_2$, where, P_3 is known. So, πd is known, L_2 is known again once we assume $\Delta y_2 = \Delta y_3$. Again, by using this chart we can you use the τ_2 for deformation and then that τ_2 we will put here we calculate the P_2 and then we will calculate Δy_2 which is equal to $\Delta y_3 + \frac{P_2+P_3}{2}$, now for the second segment, this will be L_2 and A_P, E_P the same like the third segment. So, here also for the second trial.

So, this y_3 is the deformation at the center for this segment 3 and then we get the deformation of Δy_3 plus the deformation of that segment. So that means the Δy_P that we are talking about this is the deformation of the tip and then we are calculating the deformation of the pile material that will give you the deformation at the center of any segment. So, that mean the deformation of the pile tip plus deformation of the segment 3 will give us the deformation of the center of this third tip.

And for the second tip deformation of the center of the third tip plus the deformation of the pile of the second tip. So, that we have to add because we have added the deformation of the tip we have already considered now every time we have to add the deformation of the pile segment. So, that means first one when we have to add the deformation of the pile tip plus the deformation of the third segment of the pile or the pile material. In the second one.

So this deformation you can see this deformation is the summation of the deformation of this Δy_3 in that deformation also we can replace this. So, now if I keep on adding so that we can have the deformation of the tip plus summation of the deformation of the pile material for each segment. So, now we have this deformation, now we have initially assumed that $\Delta y_2 = \Delta y_3$ then you will calculate the τ_2 then you will calculate the P_2. And now we put these values P_3 we have already calculated in previous segment.

Now, again you will get a new Δy_2 and then again, we compare the previous Δy_2 and the new Δy_2. And if it is within tolerance limit then we will stop otherwise this process will continue. Now, we will go for this next trial or the next segment and that segment also we have Δy_2. Now for similar way we can have that segment also and that segment we can put that this is P_u because this is for the top segment that is $P_u = P_2 + \pi d \tau_1 L_1$. And then again, we will continue by considering that $\Delta y_1 = \Delta y_2$ for the first trial.

And then we will continue the same thing here also. So, and then based on that we will calculate the Δy_1. So, this $\Delta y_1 = \Delta y_2 + \frac{P_u + P_2}{2}$, then L_1, A_P and E_P. So, we will continue this thing. So, and then based on that, we will finally calculate the P_u and other values and the deflection also. So, finally, what would be the P_u here? The P_u, I can replace by $P_3 + \pi d \tau_2 L_2 + \pi d \tau_1 L_1$and P_3 also I can replace by this equation. So, this is $P_P + \pi d \tau_1 L_1 + \pi d \tau_2 L_2 + \pi d \tau_3 L_3$.

So, this equation will be $P_P +$ if the pile has a uniform diameter or you can keep it inside this is L_i then $\pi d \tau_i$. So, by the trial if I know the L_i, πd also is known, So, only we have to know the τ_i. So, for every segment if I know the τ_i then if I add them then you will get the P_u. So, we will get the P_P by using these equations. So, if I assume this value of Δy_P we know the case, we know the A_P we have the P_P.

Now, that P_P I will put here and then we have to calculate the τ_i for every segment by using this chart by trial-and-error method. So, once we get that value we will get the ultimate load carrying capacity of the pile. And what will be the deflection of the pile? The total deflection of the pile is equal to$\sum \Delta y_i$. So, $\sum \Delta y_i$ will be the deflection of the pile.

So, if I add the summation of center point deflection of each segment that will give us the deflection of the pile. So, that means by assuming one particular Δy_P I will get 1 set of one P_u versus δ. So, this δ is nothing but that deflection of the pile top. So, this δ is the deflection of the pile top why should I show you that this is the δ part. So, that means, we will get P_u versus δ for one assumed Δy_P. Still Δy_P we assume because all these values are calculated based on the assumed Δy_P.

If I change this Δy_P value, then these P_u and δ all values will change. So, that means, we have to assume now different Δy_P and you will get different P_u versus δ value. So, that means you will assume different Δy_P value and we will get one particular P_u and one particular δ. So, finally, we will plot these load versus settlement curve. So, this is my or we can say that, this is the load and settlement.

The settlement is δ, load is P_u. So, once we assume different Δy_P we get one set of P_u and δ. So, that we will plot here, then we will assume another Δy_P and we get another settlement and the P_u. And then we will plot these points over this load versus settlement plot. And then you will get the load versus settlement curve for the pile. So, this way we can generate our load versus settlement plot theoretically, but remember that in this method, we have to take the help of this particular graph.

So, in the next class, I will discuss another approach which is called elastic approach. So, I will discuss how by using elastic approach also we can generate theoretically the load settlement curve of pile up to failure also. Thank you.