Advanced Foundation Engineering
Prof. Kousik Deb
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture No -33
Beams on Elastic Foundation- VII

So, last class, I was talking about the beam resting on two-parameter model. So, today, I will discuss that how we can solve this type of problem and what are the boundary conditions under different loading conditions when beam is resting on two-parameter model. And what is the difference between the beam resting of spring and beam resting on two-parameter model?

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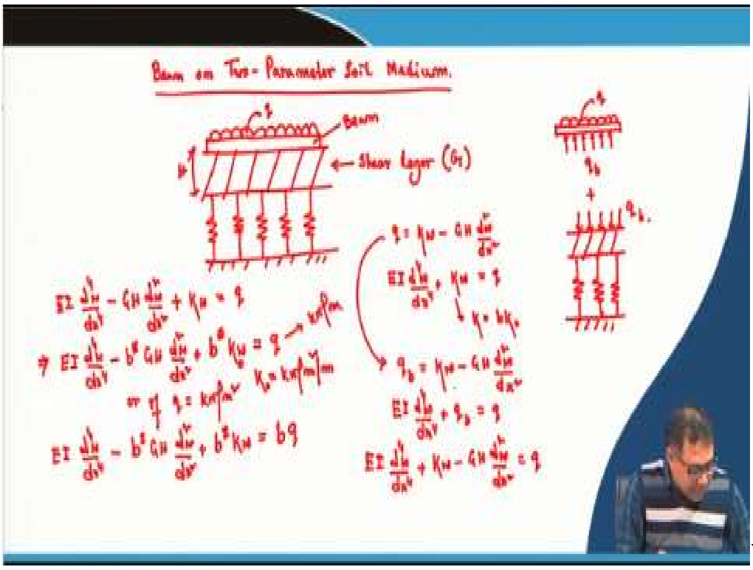

So, this is the problem that this is two-parameter model over that beam is resting and over that there is a UDL q is acting. So, now the expression for two-parameter model is given as $q = kw - GH\frac{d^2w}{dx^2}$. So, I am talking about the plane-strain condition. So, that means here $q = kw - GH\frac{d^2w}{dx^2}$. So, this is the expression for two-parameter model and the expression for the beam equation is $EI\frac{d^4w}{dx^4} + kw = q$.

So, this is the expression we are talking about for beam when it is resting on Winkler model. So, now, we have to combine these equations and then we will get the actual equation and here the $k = bk_0$. So, I can write that this is my beam equation and this is the two-parameter model equation. So, now if I divide it into two parts, so then I can write that this is my beam over there, there is a UDL q and below that a reaction is acting and that reaction is say q_b.

And this is q because this is a beam. I am, now taking this is equal on the two-parameter model and this is the two-parameter model. So, this reaction will act over the two-parameter model, so this is q_b. So, now I have separated this model that means only two-parameter model and the beam with the reaction. So, now if I write this equation then the two-parameter model, I can write this will be $q_b = kw - GH\frac{d^2w}{dx^2}$.

And for beam these equations will be now because this is the reaction which is nothing but the $k \times w$ in the actual beam problem I am writing $k \times w$ that is nothing but this a reaction q_b. So, this beam equation I can write $EI\frac{d^4w}{dx^4} + q_b = q$. If I replace this q_b here, then the equation will be $EI\frac{d^4w}{dx^4} + kw - GH\frac{d^2w}{dx^2} = q$.

So, I can write this equation as $EI\frac{d^4w}{dx^4} - GH\frac{d^2w}{dx^2} + kw = q$. So, in this equation as I mention, this $k = bk_0$. So, $k = bk_0$, but depending on this two-parameter model and depending upon the beam conditions, whether it is a finite beam or the infinite beam then your *b* value will change.

We can write this equation as $EI\frac{d^4w}{dx^4} - b^*GH\frac{d^2w}{dx^2} + b^*kw = q$. This equation is valid if your q is in kN/m. If your q is in kN/m^2, then this equation will be $EI\frac{d^4w}{dx^4} - b^*GH\frac{d^2w}{dx^2} + b^*kw = bq$. So, most of the cases q is kN/m so in that case, this is the equation.

Now, this is the equation so here this *k* is in kN/m^2/m. Because in this just look at this equation here, I have already multiplied the b with k_0, so that is why this equation is $k \times w$ but here I have multiplied b^* so that will be because as I mentioned different beam conditions this b^* value will change. So, that is why it is multiplied in this way so now if it is better for you then you can write this is k_0 also, which is the same as this one. So, k_0 is in kN/m^2/m.

So, now here I can write that, because when we are multiplying this is bq so this bq we are basically multiplying by b^*. So, this is also in kN/m, so this is basically I am multiplying b^* with the k, I am multiplying b^* with the GH also. Now why it is b^*? Let me explain that so now this b^* value for different conditions. Now what are that conditions? Now, let me explain that thing first.

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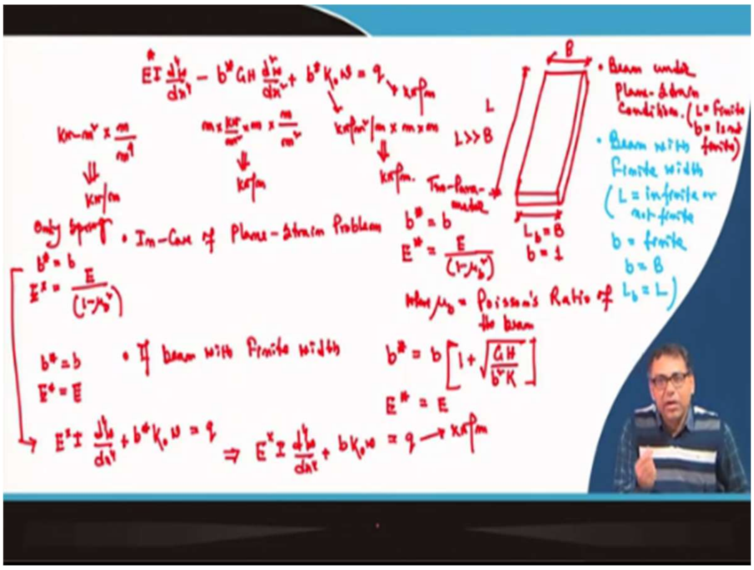

So, that means the final expression is $EI\frac{d^4w}{dx^4} - b^*GH\frac{d^2w}{dx^2} + b^*k_0w = q$. Now, let me check whether this dimension is matching or not. So, as I mentioned these expressions are valid if q is in kN/m. If it is not in kN/m then you have to use the second equation. So, it is in kN/m. So, here b is meter and this is in kN/m^2/m so meter × meter. So, this is also kN/m.

Now this comes, this is meter then kN/m^2 and G is in kN/m^2, H is in meter, then this is $\frac{d^2w}{dx^2}$, w is in meter, x^2 is in m^2. So, that means this will be kN/m. So, again, this will be kN/m. Here, this is kN/m × m × m because b^* and the w, so this is kN/m. Now come to this part, EI is in kN/m^2 then d^4w that is in m and dx^4 is in m^4. So, that means here this is m^3.

So, ultimately, this will be kN/m. So, that means we have to multiply this b^* in second term as well as the third term. Because third time it is obvious, because in our original beam equation this b is already multiplied. But that is if beam is resting on spring but if beam is resting on two-parameter model, then it is always not b, so there will be something else. So, that is why it is written b^*. It is not b and that we have to multiply with the second term also.

So, now this b^* condition that in case of plane-strain problem $b^* = b$ that means the width of the beam and E should be replaced by $\frac{E}{1-\mu_b^2}$. So, here not only b^*, I have to write E^* also. So, that means here what is E^*? In case of plane-strain problem $b^* = b$ and $E^* = \frac{E}{1-\mu_b^2}$ where μ_b is Poisson's ratio of the beam or beam material.

Now if beam has a finite width then $b^* = b\left[1 + \sqrt{\frac{GH}{b^2k}}\right]$ and $E^* = E$. Now what is the difference between these two cases? In case of beam under plane-strain condition and beam with finite width. So, suppose if you have infinite beam or you have strip footing kind of foundation. So, this strip footing kind of foundation you can analyze in two ways. One is beam under plane-strain condition and beam with finite width.

So, if beam with finite width in that case length will be infinite. And if the beam is under plane-strain condition, then your length will be finite. For example first case I am considering that beam under plane-strain condition or plane-strain problem. So, that means in such case this will be the length of the beam. So, if this is a strip footing kind of problem where this is the length, L and this is the width B and definitely your L is much, much greater than B because this is strip footing kind of problem.

So, if this beam is under plane-strain condition, then this will be the length of the beam. So, length of the beam will be now the width of the footing because it is under plane-strain condition. And in such case your width of the beam, B will be unity that is 1. And this is one condition and the next condition, if I consider that beam with finite width. So, previous case when I analyze the beam as a plane-strain condition that means this width of the beam is not finite because this is the width of the beam which is not finite, which is very large infinite kind of thing but it has a finite length.

So, under this condition, beam is under plane-strain condition, I can write that length is finite but the b is not finite and that is why we considered during the analysis b because your beam is not finite. So, during the analysis if beam is under plane-strain condition, then length will be equal to width of your foundation that is b and the b as it is not a finite length or finite width we have to consider it as unity that is 1.

And in the second case if your beam is with finite width in such case your L is infinite because beam has a finite width so L is infinite or not finite but width is finite. So, in such case your b will be equal to finite width so width of the foundation and L_b will be equal to length of the foundation. So, that means the length is infinite or not finite because it is very large compared to the width and b will be the finite which is width of the foundation.

So, under these two conditions if beam is under plane-strain condition, then your $b^* = B$ and generally it is considered as unity and $E^* = \frac{E}{1-\mu_b^2}$ and, if beam is finite width even for infinite. So, that means it is infinite beam with finite width, for example the railway track were the length is infinite that means beam with finite width case. Second case where the beam has a finite width but the length is not finite that is a second case that means I have written in the blue color. So, that means the length is not finite but your width is finite.

In such case, $b^* = b\left[1 + \sqrt{\frac{GH}{b^2k}}\right]$ and $E^* = E$. So, this is for the two-parameter model. So, now if I go for the same thing for only spring so, this is for two-parameter model, and this is for the only spring or one-parameter. So, this case also the $b^* = b$. And if your spring problem is there, so in that case you expression will be I am just first writing the expression if it is only spring then the expression will be $E^*I\frac{d^4w}{dx^4}$ this part will not exist because this is one-parameter model it will be $+b^*k_0w = q$.

So, now if it is only spring under plane-strain condition then $b^* = b$ and $E^* = \frac{E}{1-\mu_b^2}$. And if it is finite width second case because in such case $G = 0$ or $H = 0$ this is the one-parameter model again $b^* = b$ and $E^* = E$. All the problems that I have solved previously is for beam resting on spring and beam has finite width. So, that is why I have written b^*, b^* is always b and E^* is always *E*.

But, there is another condition you can analyze the beam under plane-strain condition also. In such case if it is on spring then b^* will be the *b* but E^* will change. So, that means, if your beam is resting on spring then whether it is a beam under plane-strain condition, or beam has finite width, then always $b^* = b$ and that is why our general expression is always $E^*I\frac{d^4w}{dx^4} + b^*k_0w = q$ where remember that q is kN/m.

So, now that is why this is always *b* but in previous expression I am not using E^* I have used only the *E* because most of the problems or all the problems that are solved I have discussed the beam resting on spring is for beam with finite width that means infinite beam. So, that is why width is finite and length is infinite or not finite. That is why I have written always $E^* = E$. But remember that if you are analyzing the beam under plane-strain condition, that means your length is finite, but width is not finite in such case you have to consider $b^* = b$ but in such case *b* will be equal to unity.

So, ultimately when you multiply this b with k_0, so that means your b will be unity in that time. So, you have to multiply with k_0. So, basically in such case your $k_0 = k$ because *b* is unity. Only the k_0 and the k unit will change k_0 unit will be kN/m^3 and k unit will be kN/m^2 because as we have multiplied with the one meter unit it with the k_0. But for all the cases if you beam is under plane-strain condition.

Remember that you have to replace *E* also by $\frac{E}{1-\mu_b^2}$ whether beam is resting on spring or beam is resting on two-parameter model. So, if beam is under plane-strain condition, then you have to replace each star under both the conditions. That means beam is on spring or beam is on two-parameter model. So, that means here these equations you have to remember that this is the general equation for beam resting on two-parameter model and then how to convert E^* and b^* depending upon under which condition you are analyzing your beam.

So, now next one that I discuss, another plane-strain condition all these things are discussed.

Now, I want to discuss that these problems that means here these problems are general equation. I mean these equations are general equations. Here, I have discussed that means you can use them with infinite length you can use them with finite length also. Similar way the equations of this spring case that means this equation also this is also general equation you can use them it for finite beam where you can use them for infinite beam also.

But, I have discussed that how you can solve that when beam is resting on spring for infinite case as well as a semi-infinite case and the similar way you can do it for the beam with finite length case also. But now I will discuss or in the next class I will discuss that how I can solve this general equation for different loading conditions and different beam conditions. I mean, whether it is a finite beam or infinite beam by using numerical technique.

And I will concentrate on finite difference technique only. So, in the next class I will discuss that how these equations can be solved by using the finite difference method.