Advanced Foundation Engineering
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Lecture No -32
Beams on Elastic Foundation- VI

So, last class I have solved two example problems, one is for infinite beam and another is for the semi-infinite beam. Now today I will discuss the other cases for semi-infinite beam and before I start the semi-infinite beam part, I am giving the expressions for few loading cases which are very common. So, these expressions I have already derived and you should also remember these expressions because these expressions are very common.

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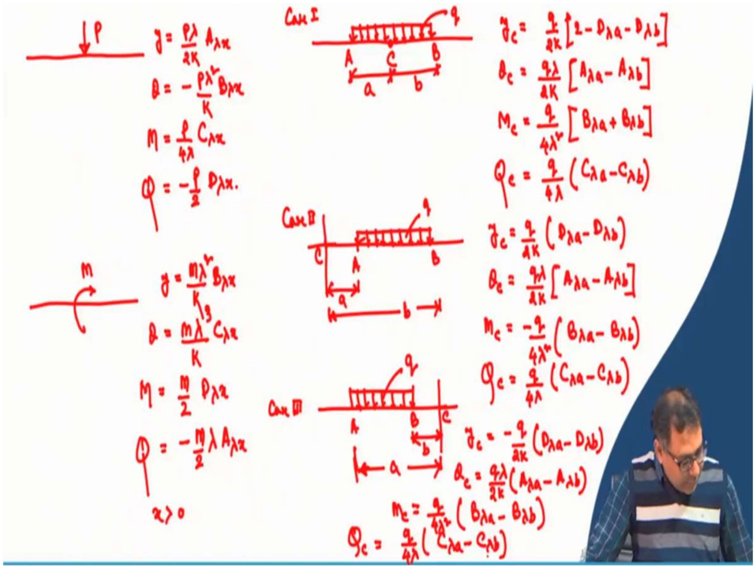

So, that is why I am giving these expressions or you can say these loadings are very common. So, this is the infinite beam subjected to P. So, the concentrated load P is acting on infinite beam. So, the deflection, $y = \frac{P\lambda}{2k} A_{\lambda x}$ and slope, $\theta = -\frac{P\lambda^2}{k} B_{\lambda x}$ and bending moment, $M = \frac{P}{4\lambda} C_{\lambda x}$ and shear force, $Q = -\frac{P}{2} D_{\lambda x}$.

Now for clockwise concentrated moment M the expression of deflection is $y = \frac{M\lambda^2}{k} B_{\lambda x}$, then $\theta = \frac{M\lambda^3}{k} C_{\lambda x}$, then moment is $\frac{M}{2} D_{\lambda x}$ and shear force is equal to $-\frac{M\lambda}{2} A_{\lambda x}$. So, these expressions I have already given. So, but I am writing all the important expressions under different loading conditions which are very common.

So, that is why I am writing all the expressions in one particular place. So, that means here all these expressions are valid for $x \geq 0$ that means for right side to the loading. Now for uniformly distributed load or UDL suppose this is infinite beam. So, this is the UDL whose intensity is q and one end is A and another end is B and I want to determine all these quantities at a point inside the UDL.

So, then the expressions will be the deflection at point C is $y_C = \frac{q}{2k}[2 - D_{\lambda a} - D_{\lambda b}]$ and slope at point C will be $\theta_C = \frac{q\lambda}{2k}[A_{\lambda a} - A_{\lambda b}]$. Similarly M_C is $\frac{q}{4\lambda^2}[B_{\lambda a} + B_{\lambda b}]$ and the $Q_C = \frac{q}{4\lambda}[C_{\lambda a} - C_{\lambda b}]$. So, these equations I have derived but I have derived only one case. When the point of interest is within the UDL but there may be a possibility that the point of interest may be outside of the loading.

In such case suppose this is my case 1, similarly for case 2 your point of interest so this is the same UDL this is also A this is also B now the point of interest is here at C which is at a distance of a from point A and b from point B. So, this is the same q. So, in such case I am giving the expressions only $y_C = \frac{q}{2k}[D_{\lambda a} - D_{\lambda b}]$ and then θ_C is $\frac{q\lambda}{2k}[A_{\lambda a} - A_{\lambda b}]$ then $M_C = \frac{q}{4\lambda^2}[B_{\lambda a} - B_{\lambda b}]$.

And $Q_C = \frac{q}{4\lambda}[C_{\lambda a} - C_{\lambda b}]$. So, this is the expression, now there is another case, case 3 when the point of interest is on the right side of the loading. So, this is the C and this is B, this point A and point C is at a distance of b and at a distance of a. So, this is q. Now in such case $y_C = \frac{q}{2k}[D_{\lambda a} - D_{\lambda b}]$ then $\theta_C = \frac{q\lambda}{2k}[A_{\lambda a} - A_{\lambda b}]$ then moment at C, M_C is $\frac{q}{4\lambda^2}[B_{\lambda a} - B_{\lambda b}]$ and shear force at C, Q_C will be $\frac{q}{4\lambda}[C_{\lambda a} - C_{\lambda b}]$.

So, these are the expressions. So, why I am giving these expressions only because these are very common loading conditions, the concentrated load, concentrated moment and the UDL. So, there are other loading conditions also for example the triangular load but here these are very common.

So, that is why if you know this expression that is good enough to solve problems. Because most of the cases loading conditions are concentrated load, concentrated moment and UDL.

So, that is why I am giving the expressions for all these loading conditions and you should remember these expressions or you should know so these are the expressions by which you can determine these values. Now I am going to the semi-infinite finite beam other cases because I have solved only the semi-infinite beam where it is a free end. Because now there are other end conditions also that mean the free end is one condition then the hinged end is another condition then there is a condition of a fixed end also.

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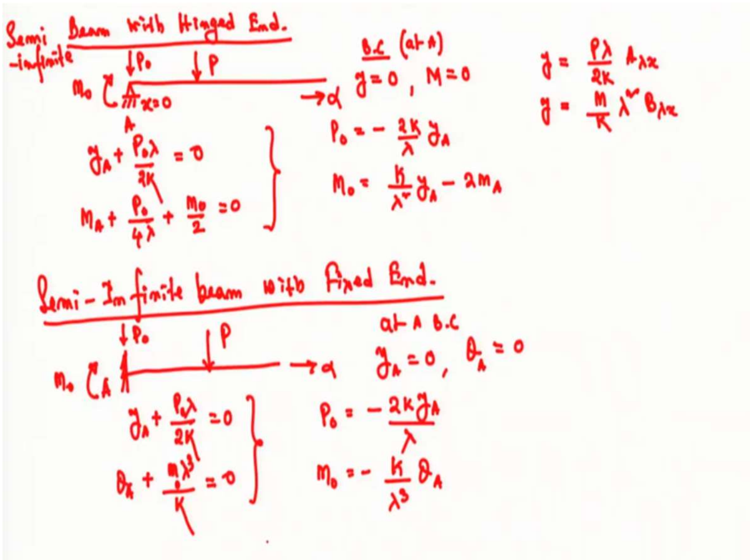

So, now I will consider the next condition that is beam with hinged end. So, that means in infinite beam this end is hinged and then a concentrated load *P* is acting. So, what is the boundary condition in such case, so the boundary conditions will be that deflection at point A will be 0 and it is a hinged end. So, moment will also be 0. So, we have to produce the end conditioning forces such that the net deflection and the net moment at hinged end will be 0.

Because same as the free end condition here also we will consider the beam as infinite beam and we will determine the deflection at point A and the bending moment at point A those will be induced due to the external load *P* and then we will apply the end conditioning forces and will make the deflection and bending moment at the hinge end is 0. So, we know that as I have already given this expression the deflection at any point due to load P is $\frac{P\lambda}{2k}A_{\lambda x}$.

And due to moment the deflection will be equal to $\frac{M_0\lambda^2}{k}B_{\lambda x}$. Because I have given this expression I am putting this as M_0. So, these expressions are given. So, this is the deflection $\frac{M\lambda^2}{k}B_{\lambda x}$. So, $\frac{M\lambda^2}{k}B_{\lambda x}$. So, these are the four deflections, will develop due to the P and any moment M. So, I can write the deflection at point A.

That is we have to apply end conditioning forces to make the deflection at point A is 0. So, that means this will be equal to $\frac{P_0\lambda}{2k}$. So, this is equal to 0 because here the y_A is the deflection due to the force P and P_0 is the end conditioning force and M_0 is also the end conditioning force. So, the deflection due to these three cases that means why I am not considering M_0 because it is $B_{\lambda x}$ which is 0 and this is the point $x = 0$.

So, that is why this M_0 part will not come into picture. Because we have applied the end conditioning forces P_0 and M_0 and then we will get the deflection due to P_0 and M_0 at point A as well as the deflection due to the P but here M_0 is not considered because M_0 part will be 0 because $B_{\lambda x}$ at $x = 0$ is 0. So, this is condition similarly now the moment is another condition.

So, M_A then the bending moment due to this P_0 will be $\frac{P_0}{4\lambda} + \frac{M_0}{2}$ that is also equal to 0. So, you can see the bending moment is $\frac{M_0}{2}$ and this is $\frac{P_0}{4\lambda}$. So, this is also $\frac{P_0}{4\lambda} + \frac{M_0}{2}$ and why I am not taking the coefficient part because coefficient part will be 1 because this is $x = 0$ this point is $x = 0$. So, where all coefficient will be 1 because $A_{\lambda x}$ will be 1, $C_{\lambda x}$ will be 1, $D_{\lambda x}$ will be 1.

But $B_{\lambda x}$ will be 0. So, this is the M part and then finally if I solve these two equations, I will get $P_0 = -\frac{2k}{\lambda}y_A$ and $M_0 = \frac{k}{\lambda^2}y_A - 2M_A$. So, this y_A and M_A are the deflection and the moment that will be induced in an infinite beam due to the external load P at the hinge condition or hinged end. So, similarly I have solved the free end semi-infinite beam problem.

So, this way we can get the expression of end conditioning forces P_0 and M_0. Similarly if we have a beam or semi-infinite beam I should write the semi-infinite beam. So, here semi-infinite beam with fixed end, this is the hinged end, this is fixed end. So, this is the end condition and here this side is infinite and here also this side is infinite. So, this P is acting this is the point A. So, what is the condition?

The condition is at A the boundary conditions are this is the fixed end. So, deflection at A will be 0 and slope at A that will be also 0. So, deflection at A will be 0 and the slope at A will be 0. So, the equation I will get the deflection due to the load P then again there will be end conditioning forces P_0 and M_0 and again M_0 part will not come into picture and then this will be the P_0 part $\frac{P_0\lambda}{2k}$ that is equal to 0.

And θ_A then plus here P_0 contribution will not come, because this θ is again $B_{\lambda x}$ and $B_{\lambda x}$ at $x = 0$ is 0. So, this will not come but θ contribution due to moment M_0 will come and that is $\frac{M_0\lambda^3}{k} C_{\lambda x}$ which is 1 here at $x = 0$. So, that means I can write that this is $\frac{M_0\lambda^3}{k}$ and that is equal to 0 because ultimately the net deflection and the slope will be 0.

So, after solving these two equations I will get $P_0 = -\frac{2k}{\lambda} y_A$ and $M_0 = \frac{k}{\lambda^3}\theta_A$ and remember that depending upon whether the point or your end condition is left side to the moment or right side to the moment or the concentrated load or any loading condition because I have discussed the three loading conditions.

And the derivations are given only for the concentrated load and it is not true that, only concentrated load will act. No, there can be a UDL also there can be a concentrated moment also for any loading condition you can determine the deflections or the bending moment or shear force or the slope but here I have derived for concentrated load but that can be any loading condition and the equation I have already given.

So, that means at any loading conditions you can derive the expressions for P_0 and M_0 for different boundary conditions by the same procedure that I have discussed for the concentrated load P. Similar way you can do it for the concentrated moment you can do it for the UDL, because I have now given the equation if the point is outside of the loading which may be the UDL may be the right side or the left side. So, that is the equation.

Now another thing I want to mention here when I have given the equations for the UDL for this first case if you put $a = 0$ and $b = L$ suppose this is the length of the UDL, L now at point A if you want to determine all these quantities then put $a = 0$ and $b = L$ and at point B your $b = 0$ and $a = L$ because now your B and C will be the same point.

So, that your $b = 0$ and $a = L$. So, remember that now if you put $b = 0$ and $a = L$ in these equations you will get the deflection, shear force, bending moment and slope at the two edges of the loading. And now I want to mention that these equations whether it is moment or it is load I mean concentrated load or it is UDL depending upon whether the point of interest or the end is right side to the loading or the left side to the loading you have to use the proper sign.

Because these equations are given for $x \geq 0$ if it is on the right side of the loading and if it is on the left side to the loading, then you have to modify these equations that I have discussed. But in this UDL all the three cases are given separately. So, you have to use the three cases depending upon its requirement or your position of the end or based on your point of interest.

So, whether it is left side to the loading or right side to the loading or within the UDL whatever it is. So, now one more thing I want to discuss that once you, get P_0 and M_0, you have to consider these two end conditioning forces and the external force or load or moment or UDL then you can determine these slope, deflection, bending moment and the shear force at any point you want, the way I solve the example problem for free and semi-infinite beam condition the similar way you can do that. So, now next one is that beam with finite length.

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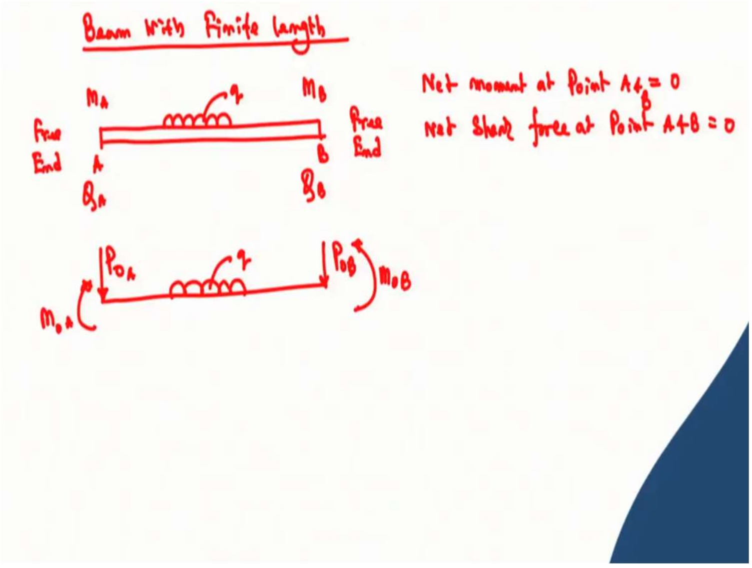

So, this is beam with finite length. So, similar type of procedure you will use for beam with finite length that we use for semi-infinite beams. Suppose here also you have UDL q and you have A point and B point. So, these are free ends. So, this is also free end. So, because of this UDL the moment and shear force will develop and here also moment will develop and the shear force will develop.

And the boundary condition for now both the boundaries are free end. So, that means we can write that the net moment at point A will be equal to 0 and the net shear force at point A and B, I should write is equal to 0 here also A and B because both the boundaries are free end conditions. So, that means here because of this external UDL the moment and shear force will be induced at point A and B if I consider the beam as an infinite beam.

But actually this is a finite beam with free ends. So, we have to apply the end conditioning forces $P_{0\text{A}}$ and $M_{0\text{A}}$ here also $P_{0\text{B}}$ and $M_{0\text{B}}$ such that the net bending moment at both the ends are 0 and the net shear force at both the ends are also 0. So, similar way you have to solve these equations. So, now I have discussed already the semi-infinite beam solution.

So, here I am not discussing this in detail this beam with finite length conditions. But the procedure is similar type the semi-infinite beam but only difference is in semi-infinite beam that is only one end is finite and another end is infinite, but here both the ends are finite. Similarly if you have both end hinge condition then you have to apply the same boundary condition if you have a hinge condition beam that means the deflection at the hinge will be 0 and moment at the hinge will be 0.

And then if it is a fixed end condition then the deflection at the fixed end will be 0 and the slope at the fixed end will be 0. So, similar way you have to solve this beam with finite length conditions. Because these conditions we will discuss when we will model pile as finite length beam and then this concept will be used. So, that time I will discuss that using this concept how I will get the expression for laterally loaded pile to determine the deflection.

But now the beam whether it is the semi-infinite beam or finite beam or infinite beam. So, these problems can be solved in general way by numerical tool. So, I will discuss also that this beam with finite length how you can solve this problem by using finite difference technique.

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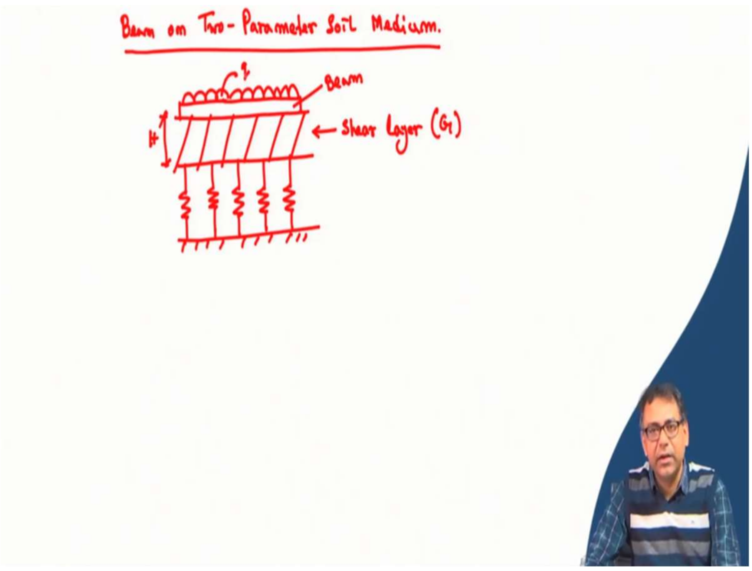

So, next part is that beam with two-parameter model. So, that means here beam on two-parameter soil medium that means in previous cases beam was resting on the springs only and as I have discussed that there will be a connectivity between the springs as the beam is resting on the spring but again the deflection you will get within the beam region only. So, deflections are confined within the beam region only.

So, we will not get any deformation outside the beam region but actually if I model a foundation as beam there will be a deformation beyond the foundation also in the soil. So, to get that deformation we have to place or you have to model the soil as two-parameter model and then over that two-parameter model we have to place the beam. So, that means here we have a two-parameter medium.

So, this is the two-parameter soil, this is shear layer with shear modulus G and which has thickness of H and then this soil is replaced as spring. Now over this two-parameter model the beam is resting and over the beam there is the UDL say q. Now this beam can be infinite beam this can be semi-infinite beam. So, this is the problem. So, in the next class I will discuss or I will give you the expressions of this beam on two-parameter soil medium and then I will discuss that how these problems can be solved by using finite difference technique, thank you.