Advanced Foundation Engineering
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Lecture No -28
Beams on Elastic Foundation – II

So, last class I have discussed that how you will determine the subgrade modulus and what are the different corrections such as the size correction, shape correction and the depth correction.

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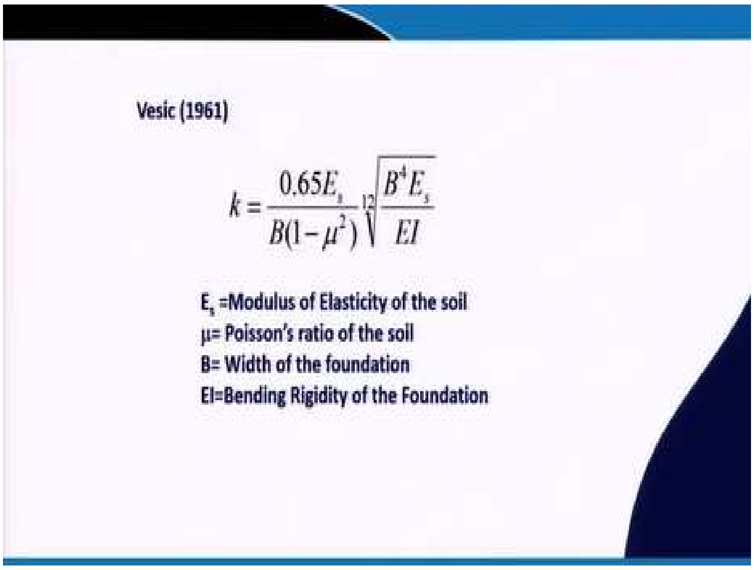

And now today I will discuss different correlations by which also you can determine the k value and then other methods by which you can determine the k value. Now, this is the basic correlation by which you can determine the subgrade modulus where E_S is the elastic modulus of the soil and B is the width of the foundation and EI is the bending rigidity of the foundation. That means E is the elastic modulus of the foundation material, I is the moment of inertia and μ is the Poisson's ratio of the soil.

So, if you use this correlation then also you will get the k value for a particular foundation or particular soil where the foundation and soil interaction both are considered.

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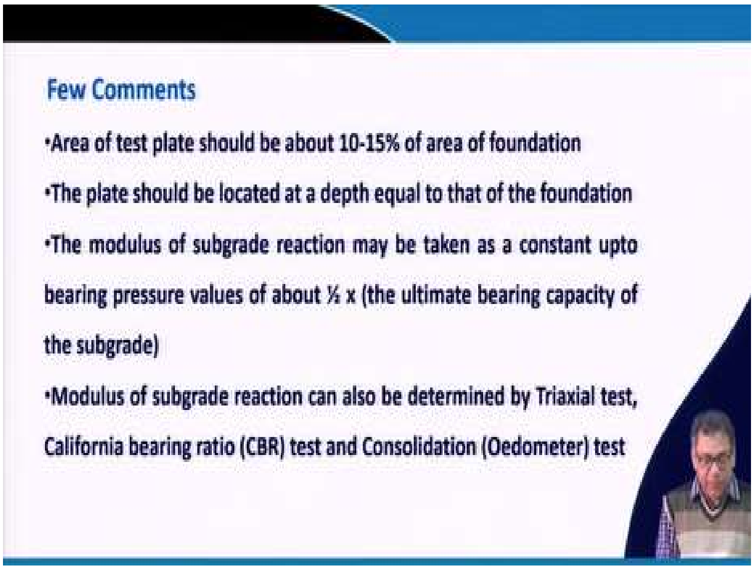

Now few comments when you do the plate load test, your plate sides should be taken such that at least 10% to 15% of the foundation area is covered by the plate size. Then plates should be located at a depth equal to the foundation depth, if not, and then you have to apply the foundation depth corrections. And sometimes it is given that the depth correction is given for surface plate that means the plate load test is done at the surface.

But sometimes it may be done at a certain depth, but that is less than the depth of the foundation. So, in that case also you have to apply the depth correction, but in that case the depth of foundation should be the difference between the depth of plate and depth of foundation, remember that. If it is done at the surface, then the depth of foundation is equal to the depth of plate where you will place your foundation.

But if the plate load test is done at a certain depth, but less than the depth of foundation then depth of foundation in the equation given in the depth correction factor will be the difference between the depth of foundation and the depth of plate. So, that means the modulus of subgrade reaction may be taken as a constant up to bearing values or about half the ultimate bearing capacity of the subgrade.

So, that means generally the k value is taken as constant up to stress which is equal to the half of the bearing capacity of the subgrade soil. Now I have discussed the determination of modulus of subgrade reaction by using plate load test. I have given some correlations also. But the modulus of subgrade reaction can also be determined by Triaxial test, then CBR test and consolidation (Oedometer) test also.

And we have the correlations between this CBR value and the subgrade modulus values. So, you can do the CBR test and then you can use those correlations to get the subgrade modulus. But here you know for foundation engineering we prefer to conduct the plate load test and from that plate load test we try to determine the subgrade modulus.

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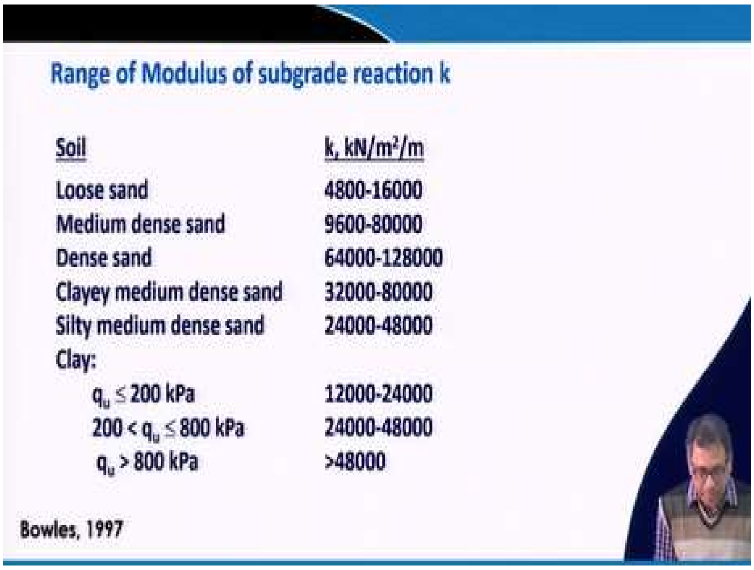

Now, these are some ranges of subgrade modulus values for different types of soil. So, loose sand to clay, this is the range that also you can use for your analysis purpose if you do not have the actual subgrade modulus value for a particular soil. The unconfined compressive strength, q_u is two times of undrained cohesion, c_u that means, $q_u = 2c_u$.

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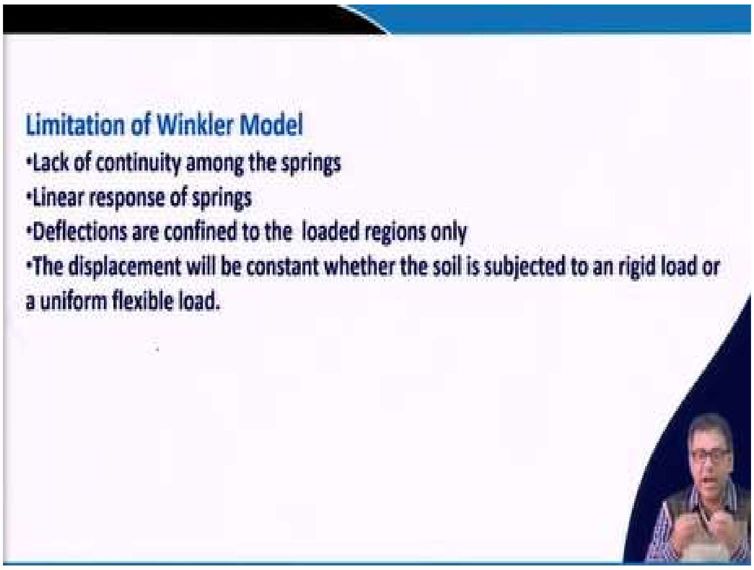

Now we are talking about the Winkler model. So, Winkler model has few limitations. What are those limitations? One limitation is there is no connectivity between the springs, but in actual foundation these springs are I mean, this soil layers are connected. That means once you apply a load on a foundation, the soil layers will deform.

But the deformation will not be only in the loaded region. The deformation will be beyond the loaded region also. And deformation will be within the foundation also. But as there is no connection between these springs, so deformation will be only in the loaded region, it will not be beyond the loaded region. But actually in foundation, deformation will be in the loaded region as well as outside the loaded region. For example, if we have a foundation and if we place it on the soil. So, the deformation will be something like this.

Suppose this is your say ground surface so that means that deformation beyond the loaded region is observed. But as there is no connectivity between the Winkler springs, so in such case if this is your foundation or this is the loaded region, your spring will deform only within the loaded region. So, you will not get any deformation outside the loaded region, so outside the loaded region, there will be no deformation. This is one of the limitations of the Winkler model. Then this is the linear response, that means $q = kw$.

That means as you increase the stress, the settlement will increase, so it is linear. But in actual case this stress-strain hysteresis versus settlement response is not linear. It is linear up to a certain stress level or strain level but after that it will become non-linear. But that effect is not incorporated in the Winkler spring. So as I mentioned deflection is confined within the loaded region. And another limitation is we know that for flexible footing and the rigid footing both the cases deformation patterns are not same.

Because for the rigid footing your deformation is more or less uniform type. And for the flexible footing, the deformation will be more at the center and less at the edges. But for Winkler spring whether your foundation is a flexible or the rigid, your deformation will be uniform, unless you do something else. For example, if you take the stiffer spring at the edges and weaker spring at the middle then you will get less deformation.

Because if you increase the k value definitely, the w value will decrease. So, if you put the stiffer spring at the bottom of the edges and weaker spring at the bottom of the middle, then definitely at the middle you will get more deformation compared to the edge of the foundation. That can be achieved, but in normal case you consider the uniform k value, along the length of the foundation or width of the foundation.

Then whether it is a flexible foundation or the rigid foundation, your deformation will be constant or uniform so that is another limitation. So, now we have to remove all these limitations.

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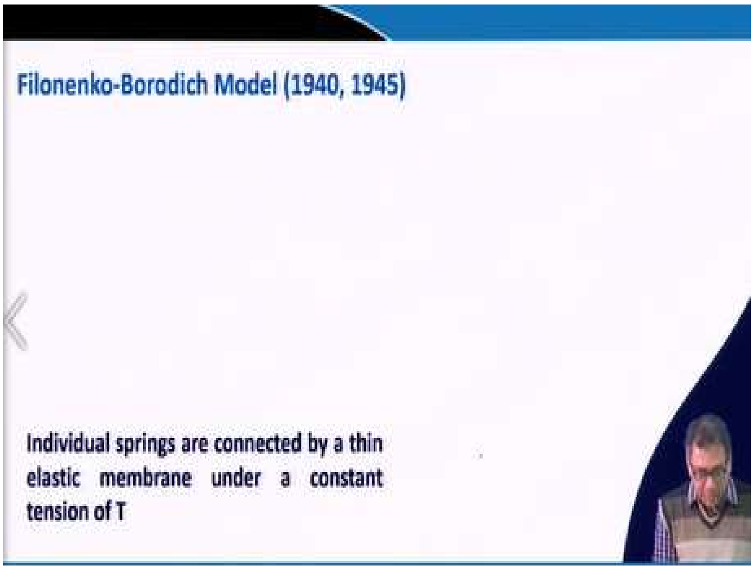

So, to remove the connectivity issue Filonenko-Borodich proposed a model where these springs are connected by a stretched membrane. Suppose these are the springs. So, these are connected by a stretched membrane, so this is equal to *T*. Now if you apply the load, suppose these are the springs. Now if you apply the load then what will happen then? Then not only that spring, then the outside springs will also be deformed.

There will be deformation of the outside spring also but not that equal amount at the middle spring. But there will be some influence to the outside springs or the other springs due to the application of the load as now the springs are connected with this stretched membrane. So, now I can give the equation in such case, suppose for a *X*-*Y* plane if it is, we are applying a stress on *X*-*Y* plane, so this is *Y* this is *X* and this is the loaded region and below that we have idealized the soil as spring.

In such case this is equal to kw. So, this was the equation for the Winkler model. But now for this particular model this is equal to $T\nabla^2 w(x,y)$, where ∇^2 is $\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$. So, finally, this equation will be $q(x,y) = kw(x,y) - T\left\{\frac{\partial^2 w(x,y)}{\partial x^2}+\frac{\partial^2 w(x,y)}{\partial y^2}\right\}$. And now for the strip loading or the continuous loading, strip loading means where the length of the loaded region is much longer than the width of the loaded region.

In such case only x direction will be influenced and the vertical direction will not be influenced that is called that splint string condition. So, that means in such case $q(x) = kw(x) - T\frac{d^2w(x)}{dx^2}$. Or simple I can write $q = kw - T\frac{d^2w}{dx^2}$. Or finally I can write this equation. But this is for strip loading. But for this rectangular loading this is the equation.

So, now this connectivity issue has been solved by using this membrane tension which is *T*. So, that means the individual springs are connected by a thick elastic membrane under the constant tension of *T*. So, this membrane is under tension of *T*.

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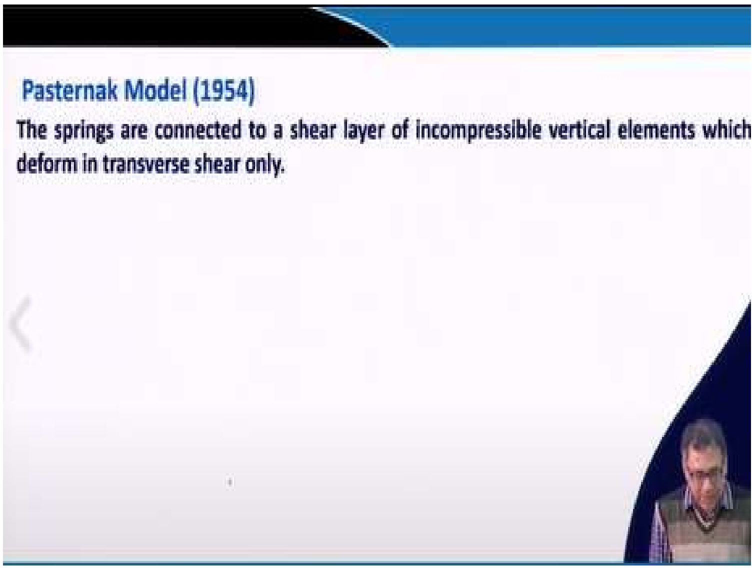

Now the next model is proposed by the Pasternak where the springs are connected with the shear layer. Because in the initial model the first model or the second model that Filolenko-Borodich model the determination of this *T* is very I mean confusing that which value we will consider for *T*. So, to represent the real soil condition, this model is more suitable or it can be applicable for this real-life problem also.

For example, suppose we have soft clay layer and it is obvious that if the soil is very soft, then we have to put some sand layer over the soft clay layer. So that we can construct or do the construction or we can increase the bearing capacity of the soil by putting that sand layer. Sometimes the sand layer and the soft clay layer or the placing of sand layer over the soft layer is very common to increase the load carrying capacity.

So, now if we do that kind of situation that means this is a sand layer. And then we place the foundation over there. So, this situation can be represented by the Winkler model. So, what is that model, Winkler springs are connected with a shear layer. So, these are springs with k as the spring constant or sub grade modulus, so this is a shear layer. So, this i is the sub grade modulus, G is the shear modulus and this H is the thickness of the shear layer.

So, in this way, the connectivity issue has been solved, because all the springs are now connected with the shear layer. Now, when we apply the stress over this spring or over this layer, so there will be a deformation like this. So, you can see that not only the loaded region, the region beyond the loaded zone is also deformed because of this connection between the springs with the shear layer.

And this is representing this condition. That means the sand layer, which is represented by the shear layer and the soft layer which is idealized by the spring. So, that means we can now determine the shear layer also for sandy soil and I know the shear layer for this sand, that we can use other shear layer for this G value for the shear modulus of the shear layer and the properties of the subgrade modulus for this soft clay can be used as the properties of this spring constant or the subgrade modulus.

Now here the equation will be for xy plane = $kw(x,y) - GH\nabla^2 w(x,y)$. So, finally, I can write that $q(x,y) = kw(x,y) - GH\left\{\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right\}$. Again, for strip loading equation will be simple $q(x) = kw(x) - GH\frac{d^2 w(x)}{dx^2}$ or simple $q = kw - GH\frac{d^2 w}{dx^2}$. So, this is the equation for the strip loading.

So, in the Winkler model the only parameter involved was k that means the modulus of subgrade reaction. But for the Filolenko-Borodich model two parameters were involved one is T and another is k. But here also in Pasternak model also two parameters are involved one is k that is the subgrade modulus and another is G, the shear modulus of the shear layer. H is the thickness it is not a soil parameter.

So, that is why these models are called two-parameter models. And the Winkler model is one-parameter model. So, that means this connectivity things are solved then this non-linearity is also incorporated in these equations because in this particular course or these classes, I will consider only the linear part. But non-linearity can be incorporated, then the time dependent because here as I mentioned only *k* and *G* values are given.

But no time dependent phenomena are incorporated here. But that can be done and it is already been done by the researchers to incorporate the time dependent effect into this model by replacing this spring with spring and dashpot. That means the spring will give you that settlement and the dashpot will give you the time dependent behavior. So, that means if you replace with the spring and dashpot then you can incorporate the time dependent effect into these models also.

So, that means the connectivity issues can be solved then your deformation is not only confined into the loaded region. You will get the deformation outside the loaded region then you can incorporate the non-linearity effect. And then you can incorporate the time dependent effect. So, all these effects are now been incorporated in these models.

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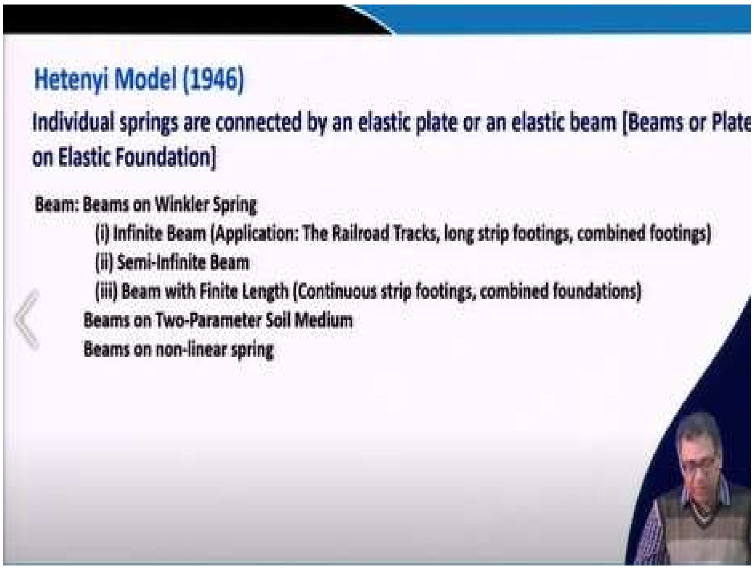

Now next model that I will discuss is Hetenyi model, where the individual springs are connected by an elastic plate or elastic beam. So, now here our beams on elastic foundation concept will start. That means here these springs are all connected with the elastic beam. For example, now these springs are connected with a beam. So, this P is the force which is acting.

So, this is a beam and this can be a plate also. So, now in the previous module the interaction between the foundation and soil was not considered. It is only the soil. But now in this case the foundation is idealized with beam or plate and then the soil is idealized with these mechanical elements. That means either spring or spring dashpot and now there is an interaction between this plate element or the beam element with this mechanical elements.

Now here also this connectivity issues can be solved. And now the loaded region will not be confined only within the; I mean settlement will not be confined within the loaded region. Now again the non-linearity and time dependent effects can be incorporated. Now the question is as I mentioned that settlement cannot be confined within the loaded region. But if this beam is resting on a spring only then again, this settlement will be confined within the loaded region.

But if you want to get the settlement beyond the loaded region, which is the actual case, then you have to place this beam on two-parameter model. So, that means this is the beam on Winkler model or Winkler spring. Now if you place the beam on two-parameter model, so these are the two parameters your shear layer and your spring, now you place the beam over here. So, this is basically beam and a load is applied this is shear layer whose shear modulus is G, this is the thickness of the shear layer and this is the k.

And beam has an EI which is the flexural rigidity. So, now if you use, we place the beam on two-parameter model, then you will get the deformation beyond the loaded region also. So, I will discuss both the cases where, the beam is resting on only spring and resting on two-parameter model. So, this is on spring then this beam is on two-parameter model. So, I will discuss both the cases and now this beam can be infinite, it can be semi-infinite, it can be finite also.

So, infinite beam can be applied by railroad tracks, long strip footing and combined footing. For example, the rail track can be idealized by an infinite beam. Suppose you have a long strip footing that means this is the, footing where this is the B, I am drawing the plan and this is the length L. And your L is much larger than B then you can idealize this footing as an infinite plate.

Suppose, you have a combined footing suppose you have a column. And those columns are provided on a single foundation again this is equal to B. And this is equal to L then again L is much larger than B then also you can idealize this foundation as an infinite beam. So, in such case, there will be multiple numbers of concentrated loads coming from the column. So, that means in such case your foundation will be that means here, this is the beam and there will be n number of concentrated load.

So, that can be idealized as an infinite beam. Then there will be semi-infinite beam where one side is fixed or hinged or free and another side is infinite. And there is a finite length beam also, that is for your strip footing or combined footing. So, where it is a finite length that means any foundation, length is finite. So, this is the length of the beam which is finite and if I draw say your cross section of this beam, so, I will get this type of cross section.

But one thing I want to mention if the beam is infinite, definitely it has a finite width. So, infinite beam then your width is finite. But if you have a finite width beam, then there is a possibility that your length of the beam can be the distance I mean, this is the length of the beam and the length of the beam or the dimension of the beam perpendicular to this direction can be infinite.

For in such case, what we will do? We will consider the width of the beam as a unity. For example, if your length of the beam is finite then there is a possibility that beam will be in plane-strain condition. Then beam may be in plane-strain condition, which means the dimension of this beam along the perpendicular direction is very much longer than the length of the beam.

So, in such case this will be treated as finite beam in plane -strain condition where your width of the beam will be taken as one or unity. So, remember that if this is the infinite beam then you have a finite width of the beam. So that, if it has a finite length, then there is a possibility that you have beam which will be in under plane-strain condition and we have to consider B as a unity.

So, that means beam can be under plane-strain condition, also. In such case the beam width we have to consider as unity. If beam has a finite width like this then there is no issue if beam has a finite width, then you will get the cross section. So, cross section is simple like this will be the equal to B and this is equal to h. But if this is like this, so that means here your beam is like this and if you cut this section there will be a strip.

So, this beam is under plane-strain condition. So, in that case you have to consider your B as unity. That means this will be your section where in this direction B will be one and h will be the thickness of the beam. So, here also h is the thickness of the beam. So, that means you remember that beam can have finite width and infinite length or it can have finite length and a unit width. That means it is under plane-strain condition where length of the beam is finite.

But the dimension along the perpendicular distance is very large where it is under plane-strain condition and we have to consider $B = L$. So, that means beam can be under plane-strain condition also. Then what is the application area? So, in an infinite beam with finite width, this is the rail track can be modeled like this. But if you have a long strip footings like this case long strip footing.

This is long strip footing, which can be modeled in two ways. One is suppose it can be considered as the length of the beam, in that case it is an infinite beam and the width of the foundation this is one way. Another way that it is modeled such that this width of the beam is equal to length of the beam. And in such case the beam will be under plane-strain condition.

And in such case, you have to consider width is equal to 1. That means this type of situation I can model in two ways, one is infinite beam and finite width. Another way the beam under plane-strain condition because you can model this type of problem in two ways. So, in first case if it is infinite beam the length is equal to the length of the foundation and we told you width of the foundation simple.

In second case your width of the foundation will be the length of the beam. But as it is under plane-strain condition, in that case your distance perpendicular to the length is much longer. So, it is plane-strain condition. So, we have to consider a unit width $B = 1$. So, that means here length B = to length of the footing and $B = 1$ and in this first case your $L = B$ means the width of the foundation.

So, now here I should write that the length of the beam is equal to B and width of the beam is equal to 1. And here width of the beam will be equal to B width of the foundation and length of the beam will be equal to length of the foundation. So, in this way on the different condition, we can use infinite beam we can use semi-infinite beam or we can use beam witth finite length also. Then we can know beam can be on two-parameter model or the beam can be on non-linear spring also.

So, as I mentioned I will consider only the beam on linear spring in this particular course. So, in the next class I will start the derivation, I will give you that a first case as I have discussed. I will discuss the two cases one is beam on spring another beam is on two-parameter model. So, in the next class I will discuss the beam resting on Winkler spring, then I will discuss about the beam resting on two parameter model. Thank you.