

**Advanced Foundation Engineering**  
**Prof. Kousik Deb**  
**Department of Civil Engineering**  
**Indian Institute of Technology-Kharagpur**

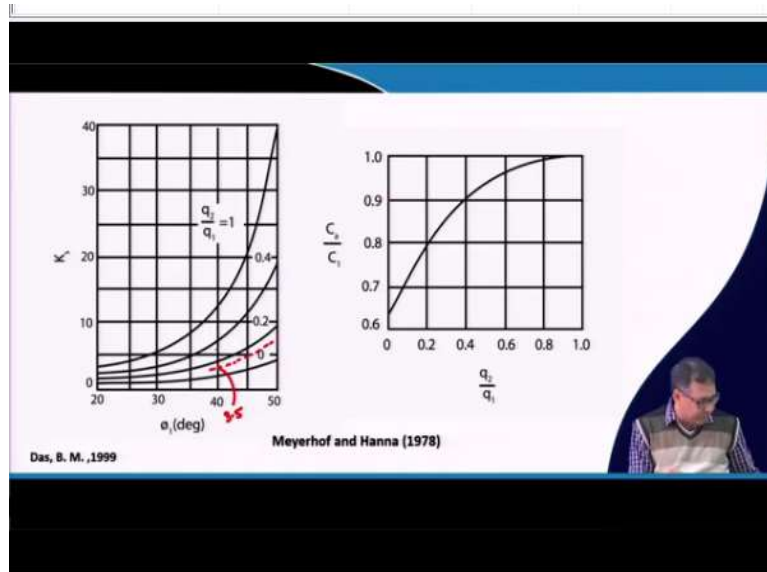
**Lecture-21**  
**Shallow Foundation: Bearing Capacity-XV**

So, last class I have discussed the bearing capacity of layered soil and then I was solving one example problem and I solve a case 1 where the top and the bottom both layers are considered as clay soil.

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So, now I started the second case that is the top layer is sandy soil and the bottom layer is a clay soil. So, as I have mentioned the theory that I have discussed is applicable if the top layer is stronger than the bottom layer. That means, the top layer is a strong soil and the bottom layer is weak soil. So, and the first case the case 1 I have discussed that the top layer is a strong clay or the I should say that stiff clay  $c$  value is 100 kPa or the strongest soil with  $c$  value is the 100 kPa stronger compared to the second layer. And for the second layer the  $c$  value is considered as 40 kPa.

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$$q_u = 5.14 c_1 \left(1 + 0.2 \frac{D_f}{L}\right) + \gamma_1 D_f = 5.14 \times 100 \left(1 + 0.2 \times \frac{3}{3}\right) + (20-10) \times 1 = 592.5 \text{ kN/m}^2$$

$$q_u = 253 + \left(1 + \frac{3}{3}\right) \left(\frac{2 \times 200 \times 1}{2}\right) + 0 - (20-10) \times 1 = 393 \text{ kN/m}^2 < 592.5 \text{ kN/m}^2$$

$$q_u = 393 \text{ kN/m}^2$$

Case 2:  $\phi = 40^\circ$ ,  $\gamma_{sat} = 20 \text{ kN/m}^3$ ,  $\gamma_{sub} = 18 \text{ kN/m}^3$ . Water table is at the base of the foundation.  $\gamma_{sub} = 10 \text{ kN/m}^3$ .

$$q_u = 40 \times 5.14 \left(1 + 0.2 \times \frac{3}{3}\right) + 18 \times 1 + (20-10) \times 1 = 261 \text{ kN/m}^2$$

$$\frac{1}{2} H \left[ \gamma_1 \left( \frac{D_f}{L} + D_f \right) + \gamma_2' \right]$$

$$\Rightarrow \frac{1}{2} H \gamma_2' \left[ 1 + \left( \frac{D_f}{L} + \frac{D_f}{L} \right) \right]$$

$$q_u = 18 \times 10 \times 1 = 180 \text{ kN/m}^2$$

$$C_2 = \gamma_{sub} \times H = 18 \times 10 = 180 \text{ kN/m}^2$$

$$\gamma = \gamma_{sub} - \gamma_w$$

$$\frac{C_2}{C_1} = \frac{180}{200} = 0.9$$

$$\therefore C_u = 0.9 \times C_1 = 0.9 \times 100 = 90 \text{ kN/m}^2$$

$$\frac{q_u}{q_1} = \frac{C_u \gamma_{sub} D_f}{C_1 \gamma_1 D_f} = \frac{C_u}{C_1} = \frac{90}{200} = 0.45$$

$$q_u = 393 \text{ kN/m}^2$$

Diagram: A foundation of width  $B_f = 3\text{m}$  and depth  $H = 3\text{m}$  is shown. The water table is at the base of the foundation. The soil above the foundation has  $\gamma_1 = 20 \text{ kN/m}^3$ . The soil below the foundation has  $\gamma_{sub} = 10 \text{ kN/m}^3$  and  $\gamma_{sat} = 20 \text{ kN/m}^3$ . The foundation is on a sand layer with  $\phi = 40^\circ$ .

And I got ultimate bearing capacity as 393 kN/m<sup>2</sup> and then in the second case that the top layer is a stronger sand layer and the bottom layer is the same soft layer with a weaker layer with a  $c$  value of 40 kPa. So, this is the second case problem where the passive pressure we have to calculate in a different way.

Because in the basic expression the unit weight throughout the layer is considered as the same, but because water table effect was not considered, but in the second case problem as the water table is considered at a depth of 1 m below the ground surface. That means it is at the base of the foundation, because our foundation base is also 1 m below the ground surface.

So, that means, in this case unit weight throughout soil is not same, because above the water table you have to consider the bulk unit weight and below the water table you have to consider the saturated or submerged unit weight. So, that is why we have to calculate this passive pressure or the force in first. Then we have to put this to the equation. So, let us derive that equation first if the unit weight is not same.

So, this is the case. So, as I mentioned this value is basically  $D_f \times \gamma_{\text{bulk}}$  because  $\gamma_{\text{bulk}}$  is unit weight above the water table and even the water table is not at the base, it is slightly above the base but below the ground surface then for some portion it will be  $\gamma_{\text{bulk}}$  and some portion will be  $\gamma_{\text{sub}}$  even within the  $D_f$  part.

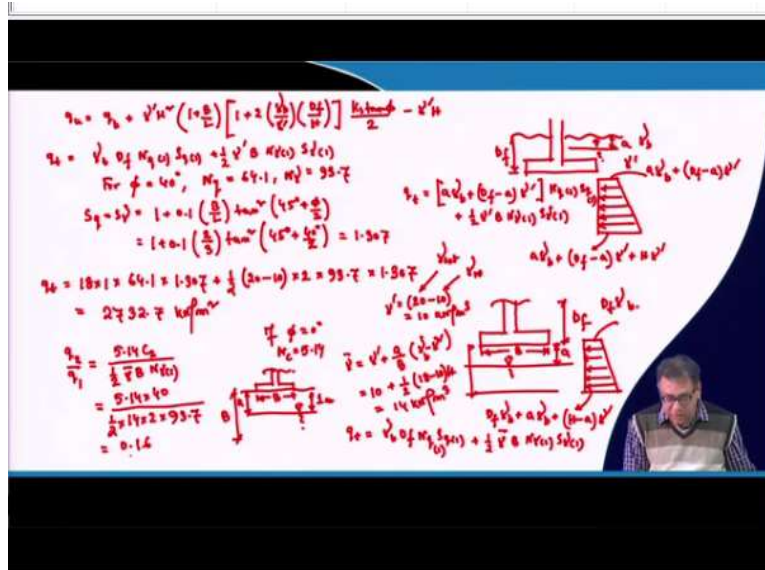
So, remember that because here total  $D_f$  part is the  $\gamma_{\text{bulk}}$  that is why it is taken  $\gamma_{\text{bulk}}$  and here the lower part is actually  $\gamma_{\text{bulk}} \times D_f + \gamma_{\text{sub}} \times H$ . So, the passive pressure I can write that the  $P_p$  will be or that force will be  $\frac{1}{2}$ , this I am only considering the effective overburden part, because that  $K$  already been multiplied in the general equation.

I am just calculating the force. So, without considering  $K$  only that overburden vertical force part I am considering, have to multiply with the  $K$  to get the lateral pressure. So, that means this will be  $\frac{1}{2} \times H$  is the thickness  $\times$  that because this is a trapezoidal  $\times A + B$  and  $A$  is  $\gamma_{\text{bulk}} \times D_f + \gamma_{\text{sub}} \times H$  where the  $\gamma_{\text{sub}} = \gamma_{\text{sat}} - \gamma_w$ .

Here the  $\gamma_w$  is given as  $10 \text{ kN/m}^3$  and  $\gamma_{\text{sat}}$  is  $20 \text{ kN/m}^3$ . So,  $\gamma_{\text{sub}}$  or  $\gamma'$  is  $10 \text{ kN/m}^3$ . So, I can write this equation further  $\frac{1}{2} \times \gamma'$  outside then this  $\frac{1}{2} \times \gamma'$  this will be  $+ 1 +$  there will be  $\frac{\gamma_{\text{bulk}}}{\gamma_{\text{sub}}}$  and this is a part term 2 and  $\frac{D_f}{H}$ .

So, this will be  $H^2$ . So, what I have done I have taken  $\gamma' H$  common. So, then this part will be your 1, then  $\gamma_{\text{bulk}} D_f + \gamma_{\text{bulk}} D_f$  this will be twice, then  $\frac{\gamma_{\text{bulk}}}{\gamma_{\text{sub}}} \times \frac{D_f}{H}$ . But our actual equation you can see it is  $\gamma_1$ . So, that if there is a different unit weight or due to the water or in the soil then we have to incorporate these effects. So, that means we have to modify these equations slightly.

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So, now, the modified equation will be that  $q_u$  will be  $q_b + \gamma'$  because this is  $\frac{1}{2}$  then these forces are acting in both directions. So, this twice  $\times \frac{1}{2}$ . So, this part will cancel. So, it will be  $\gamma'H^2$ , then if I consider the shape factor then  $1 + \frac{B}{L}$ , then that will be  $\left[1 + 2 \left(\frac{\gamma_{\text{bulk}}}{\gamma_{\text{sub}}}\right) \times \left(\frac{D_f}{H}\right)\right] \times \frac{K_s \tan \phi}{2} - \gamma'H$ .

Now, remember that this equation is applicable not in general again this equation is applicable if water table is at the base of the foundation, remember that. So, now, the water table is not at the base of the foundation, it can be anywhere. For example, the water table is here with a distance of  $a$  and this is the  $D_f$ . Now, this is  $\gamma_{\text{bulk}}$ , this is  $\gamma'$ . So, now what will be the pressure distribution?

So, this portion will be  $a \times \gamma_{\text{bulk}} + (D_f - a) \times \gamma'$  and this portion will be your  $a \times \gamma_{\text{bulk}} + (D_f - a) \times \gamma' + H \times \gamma'$ . So, then you determine this trapezoidal area and then multiply with  $\frac{1}{2} \times H$  and then we will get the force, because you have to multiply with the  $K$  because that  $K$  part is already incorporated in the equation.

So, that means again you have to modify this equation that I have derived. So, the equation that I am using now is because the water table is at the base of the foundation but if water table is at any position then you have to modify it accordingly, remember that. In another

case suppose the water table is below the base of the foundation. So, suppose the water table is here with a distance of  $a$ , this is  $D_f$ .

And then this portion is  $H$ , then what would be the pressure distribution? So, this will be the pressure distribution, this one will be  $D_f \times \gamma_{\text{bulk}}$  and this one will be  $D_f \times \gamma_{\text{bulk}} + a \times \gamma_{\text{bulk}} + (H - a) \times \gamma'$ , then you have to again modify these equations because ultimately remember that we are basically calculating the area of these trapezoidal and the sides  $A$  and  $B$  can be calculated in different way depending upon the position of the water table.

If there is no water then the first equation that I have derived can be used directly and that is you have to do the modification if the top layer is sand, because top layer is clay then these forces will not be required because  $\tan \phi = 0$ . So, total part will be 0. So, this exercise is not required. This is required if the top layer is sand. And then remember that do not always use these equations for the water table case because we have to modify the equation and we have to calculate this area depending upon where your water table is resting or the position of the water table.

So, depending upon the condition you have to modify these. Because our case now the example problem that I have considered water table is at the base of the foundation. So, I can use these equations. So, now we can calculate this one  $D_f \times \gamma_{\text{bulk}} + a \times \gamma_{\text{bulk}} + (H - a) \times \gamma'$  this is the water table and here this is the water table. So, now first we have already calculated the  $q_b$  which is the clay part and it is  $261 \text{ kN/m}^2$ .

Now, I will calculate the  $\gamma$  of the top which is this clay layer. So,  $\gamma$  of top layer the initial portion suppose it is now resting in the soil where this is the case. So, the unit weight soil above the base level is bulk. So, I have to consider because first time will be 0 because there is no cohesion of the top layer. So, this will be  $\gamma_{\text{bulk}} D_f N_{q1} S_{q1}$  because it is a rectangular footing  $+ \frac{1}{2}$ . Now, it will be  $\gamma_{\text{sub}}$  because your water table is below the base of the foundation. So, this will be  $\frac{1}{2} \gamma_{\text{sub}} B N_{\gamma 1} S_{\gamma 1}$ .

Remember that again this equation will be different if the position of the water table is different. So, for example, if the position of the water table is like this then these equations

will be for the first case and in this equation  $\gamma_t$  will be  $a \times \gamma_{\text{bulk}} + (D_f - a) \times \gamma_{\text{sub}}$ . Then this will be  $N_{q1}$ , then  $s_{q1}$  then  $+\frac{1}{2} \gamma' B N_{\gamma 1} s_{\gamma 1}$ .

Now, if this is the second case, then what would be the top layer bearing capacity. In this case the  $q_t$  will be the soil above the base of the foundation is  $\gamma_{\text{bulk}}$ , then this is  $D_f$ , then  $N_{q1}$ ,  $s_{q1}$  then  $+\frac{1}{2}$ , this one you consider  $\gamma'$ , because I am calculating  $\gamma'$  then  $B$ ,  $N_{\gamma 1}$  and  $s_{\gamma 1}$ .

Because how you calculate the  $\gamma'$  because I am basically doing the same process that I have discussed during the effect of water table on bearing capacity in Terzaghi's bearing capacity theory. So, that I have discussed in one of my previous lectures. So, how we can incorporate the water table effect in Terzaghi's bearing capacity equation that I am doing here.

So, now, because that we have to do for every bearing capacity equation, if you want to incorporate the water table effect that I have discussed except the IS code method where the water table effect is directly incorporated in the equation and you have to determine that  $W'$ . So, now, what is  $\bar{\gamma}$ ?  $\bar{\gamma} = \gamma' + b$ . Now, if you consider this is  $a$  or  $b$  or you can write  $a$  itself also no problem.

So, then this will be  $a$ , then this is  $B$ , then  $\gamma - \gamma'$ . So, now, this is the general equation where  $B$  is the width of the foundation, because now it is top layer the total bearing capacity is resting on the top layer. So, this  $H$  part will not be there because there is no thickness of the soil layer because total layer is the one same layer. So, that means it will be  $a$  in this equation.

Where  $a$  is the position of the water table below the base of the foundation. So, in this way you can calculate  $q_t$ . So, again this  $q_t$  top equation is not general for the water table case depending upon where your water table is located you has to modify this equation also. So, now finally for our case if I calculate  $q_t$ . So, I have to calculate this shape factor also.

Now, first we take  $\phi = 40^\circ$ . Now, in such case for Meyerhof's bearing capacity,  $N_q$  is 64.1 and  $N_\gamma$  is 93.7. So,  $s_q = s_\gamma = 1 + 0.1 \left(\frac{B}{L}\right) \tan^2 \left(45^\circ + \frac{\phi}{2}\right)$ . So, if I put  $B = 2$ ,  $L = 3$ , then  $\tan^2 \left(45^\circ + \frac{40^\circ}{2}\right)$ .

So,  $\frac{40^\circ}{2}$ , that will be  $20^\circ$ . So, now, this value will be 1.307. So, finally, if I calculate then  $q_t$  is  $\gamma_{\text{bulk}}$  is 18,  $D_f$  is 1,  $N_{q1}$  is 64.1, then  $s_{q1}$  is  $1.307 + \frac{1}{2} \gamma'$  is  $20 - 10$ , then  $B$  is 2, then  $N_{\gamma1}$  is 93.7, then a factor of 1.307. So, it is 2732.7. So, this is the limiting value of the bearing capacity for this layered soil and it should be within 2732.7 kN/m<sup>2</sup>.

Now, we know  $q_b$ . So, now we have to calculate because I have not considered that  $C_a$  part in the basic equation, because in this equation you can see there is a  $C_a$  part,  $C_a \times H$  because as the cohesion is 0, so this part will be automatically 0. So that is why I have not considered.

So, because already remember that in this equation there is a friction part and the cohesion part. So, as the cohesion is 0, that is why I am considering only the friction and then we have to calculate the  $K_s$ . Now to calculate the  $K_s$  you should know  $\frac{q_2}{q_1}$  and what is  $q_2$ ?  $q_2$  is the bearing capacity of a strip footing of width  $B$  resting on the surface of the bottom layer.

Now, if a strip or continuous footing is resting on the surface of the top layer, when the width of the footing is  $B$  then that is called  $q_1$ . So, it is a strip footing if I put on the surface, so, second term is 0 and this bottom layer is cohesive soil. So, third term is also 0, only there will be first term and that first term,  $N_c$  is taken as 5.14. Now, if your  $\phi = 0^\circ$ , so  $N_c$  is 5.14 as per bearing capacity factor table.

So, I can see this is  $5.14c_2$  divided by it is strip footing for the top layer is the sand. So, first term is 0, second term is 0, because it is resting on the surface. So, it will be  $\gamma_{\text{bulk}}$  because why  $\gamma$  actually I should write it is also  $\bar{\gamma}$ , I am writing  $\bar{\gamma}$  because in this case the water table is here at a depth of 1 m and for the bottom layer does not matter because it is a clay soil.

So,  $\gamma$  is not important in this calculation, but at the top layer suppose we are placing this foundation at the surface. So, that means water table will affect up to a depth of  $B$  from the base of the foundation, here base of the foundation and the surface both are same, because it is a surface footing. So, now we have to calculate the  $\gamma'$  value and  $\gamma'$  value is given like this.

So, now if I put the  $\gamma'$  then,  $\bar{\gamma}$  can be calculated like this where  $a = 1$  m and  $B = 2$  m. So, now, if I put  $\gamma' = 10$ ,  $\gamma_{\text{sat}} = 20$  and unit weight of water = 10. So, this is in  $\text{kN/m}^3$ , because this is  $\gamma_{\text{sat}}$  and this is  $\gamma_w$ . So, this is  $10 + a$  is 1,  $B$  is 2, this will be bulk,  $\gamma_{\text{bulk}}$  is 18, then  $\gamma'$  is 10.

So, this value is coming out to be  $14 \text{ kN/m}^3$ , for this particular case. So, that means I am writing here these why I am taking  $\bar{\gamma}$  because the water table is below the base of the foundation at a distance of 1 m from the base. So,  $\bar{\gamma}$  and then this will be  $B$ , then  $N_{\gamma_1}$  and why I am not taking the shape factor because it is strip footing. So, shape factor will be always 1.

So, now this is the equation. So, if I put 5.14,  $c_2$  is  $40 \text{ kN/m}^2$ ,  $\frac{1}{2} \times \bar{\gamma}$  is 14,  $B$  is 2 and  $N_{\gamma_1}$  is how much?  $N_{\gamma_1}$  is 93.7. So, this is 93.7. So, this value is roughly 0.16. Now, let us go to the chart. Now, this is 0.42 this is 0. So, 1 will be here, 0.1 will be here, 0.16 will be here.

So, the  $\phi$  value of the top layer is  $40^\circ$ . So, now if I draw a chart which is like this. So, that means this value is around this is 2.5. So, actually it will go further like this because this distance is something like this. So, this is 5, this is 10. So, this value is roughly 3.5, but this is 5, this is 10 roughly 2.5 will be something here, it will be in between around 3.5 and so.

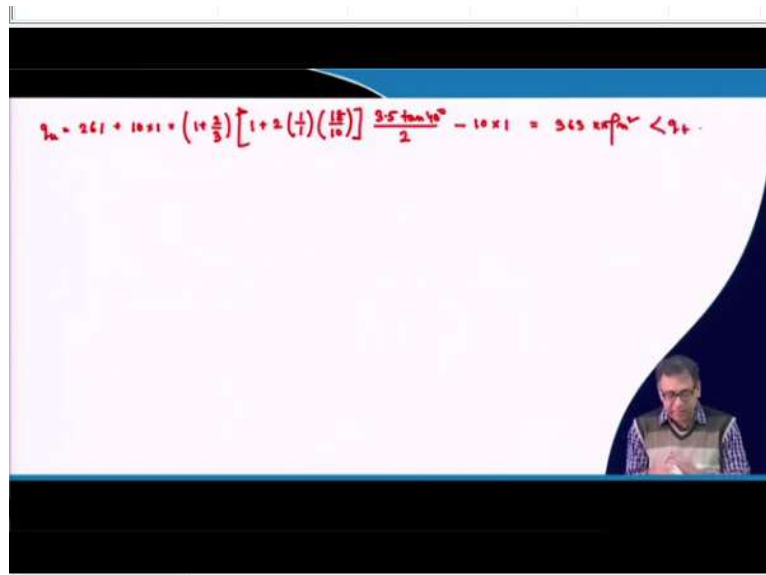
As it is 1.6, so, it will go slightly up so, it is 3.5. So, now, the 0.16 the  $K_s$  value is roughly 3.5 or you can take because when you are taking values from the chart slightly these values may vary because you are taking by eye estimation by your own and I am also taking. So, there may be some variation, but that is not the issue. But that variation should be very small.

And in the exam problem definitely these  $K_s$  values or the values those are required will be given in the exam because, those value we have to take from the chart or those values you have to take from the tables so those values will be given, because you will not be allowed to take any chart or table inside the exam hall.

So, in the exam problem, definitely these values are required to take from the chart or table which will be given. So, in the assignment problem also I will try to give these values, but if it is not given you can take the  $K_s$  value, but the variation will not be that much significant. So, your answer and the answer that I will give will be very close, this may not be an issue. So,  $K_s$  value is 3.5. So, now, I put these values.



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So, this value will be  $q_t$ . Finally, I can write that  $q_b$  is 261 and  $q_u$  value is how much?  $q_u$  value is 261 + your this equation is  $\gamma'$ ,  $\gamma'$  is 10  $H^2$  is 1,  $a^2$  is  $1 \times 1 + B$  is 2,  $L$  is 3, then  $1 +$  this is  $2 \frac{D_f}{H}$ ,  $D_f$  is 1,  $H$  is 1, then  $\frac{\gamma_{bulk}}{\gamma'} = \frac{18}{10} \times K_s$  is  $3.5 \times \tan 40^\circ$  divided by 2 because  $B$  value is 2 -  $\gamma' \times H$ ,  $\gamma'$  is 10 and  $H = 1$ .

So, this is 363 kN/m<sup>2</sup>. So, which is less than  $q_t$  is fine. So, that means you can determine the ultimate bearing capacity of the layered soil if the top layer is a stronger soil and the bottom layer is weaker clay. Because that means we have discussed two cases. So, first case is the stronger clay and the weaker clay and then the second case the stronger sand and the weaker clay.

Now, in the next class I will discuss the third case when the stronger sand versus weaker sand. So, that is the third case which I will discuss in the next class, thank you.