Advanced Foundation Engineering
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Lecture-20
Shallow Foundation: Bearing Capacity XIV

So, last class I started on example problem for two-layered case. And I consider that the top layer is stiffer clay and the bottom layer is the softer clay.

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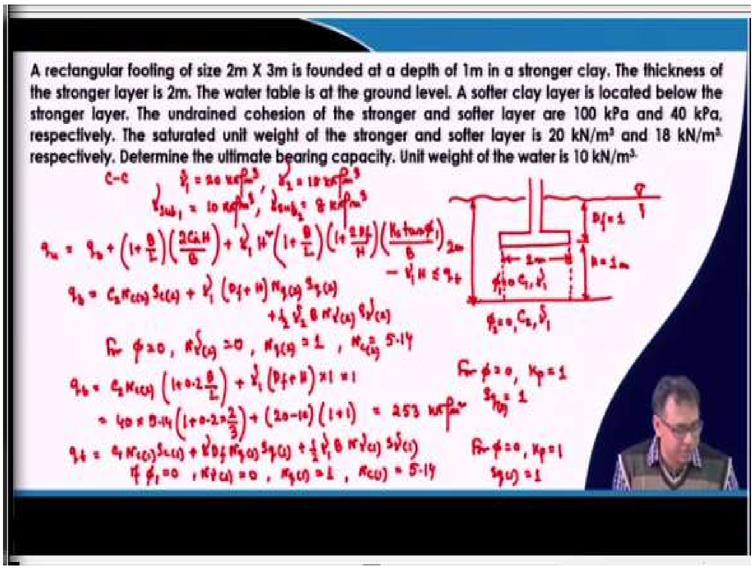

And where your ϕ value is 0 for both the cases, this ϕ value is 0 and this is also 0, there will be only cohesion and water table is at the ground surface and this is a rectangular footing of dimension 2 m × 3 m and the thickness, H is given as 1 m and then depth of the foundation, D_f is 1 m and saturated unit weight of the top layer is 20 kN/m^3.

And saturated unit weight of the bottom layer is 18 kN/m^3. So, submerged unit weight of the top layer is 10 kN/m^3, because submerged you know the saturated minus the unit weight of the water. And unit weight of the water is taken as 10 kN/m^3. So, submerged unit weight of the second layer will be 8 kN/m^3 and this is the basic equation.

Suppose, this is the basic equation for this rectangular footing and then now we have to calculate the bearing capacity. So, now you know that this should not be greater than q_t that we have mentioned. So, now q_b will be the bearing capacity of the bottom layer and that expression given as $q_b = c_2 N_{c2} \times s_{c2}$, s_{c2} is for the rectangular footing, for the strip footing these s terms are not present, there will be only $c_2 N_{c2}$ then $\gamma_1 (D_f + H) \times N_{q2}$ then $N_{\gamma 2}$ like this.

But as it is rectangular footing then you have to put this value also, this is $\gamma_1 (D_f + H)$, then N_{q2} then s then $+ \frac{1}{2}\gamma_2 B N_{\gamma 2} s_{\gamma 2}$. Now for this particular case my $\phi_2 = 0$, so my $N_{\gamma 2}$ will be 0 and $N_{q2} = 1$ and N_{c2} as per Meyerhof is 5.14 because we are using Meyerhof's theory.

So, my $q_b = c_2 N_{c2}$ and this s_{c2} is the shape factor and that is as per Meyerhof. So, these factors are s_{c2} and N_{c2} and all these things will be considered as per Meyerhof's recommendation. So that is equal to $\left(1 + 0.2\frac{B}{L}\right) + \gamma_1 (D_f + H)$ and $N_{q2} = s_{q2} = 1.$ Because if you see Meyerhof's table for $\phi = 0$, your $K_p = 1$, so my $s_q = 1$.

Now remember that, when you are applying these or converting the equation from strip to the rectangular footing then for your basic equation you use this shape factor always $1 + \frac{B}{L}$ in this term. But when you are calculating q_t and q_b, in that term because q_t is nothing but when your foundation is resting on the top layer only and q_b is when the foundation is resting on the bottom layer.

So, in that case you have to use Meyerhof's original recommendation, you have to use the shape factor which is proposed by Meyerhof in the original bearing capacity equation, that table I have given, you have to take the bearing capacity factor from there. So, all these things as per Meyerhof and another thing that is here you can ask that your depth effect why it is not considered?

Because this depth effect is already taken care during the derivation because this depth is already been taken care, so that is why it is not there. Because the original equation is developed basically for the surface footing, so that is why the depth corrections are required. But here it is not a surface footing, it has depth of D_f and then we have done the derivation and depth effect is taken.

So for the inclined footing you can put those recommendations which are given in the original bearing capacity equation by Meyerhof. So, that means that is why, here when I calculate q_b, we take the shape factor which is originally proposed by Meyerhof and which is given in the table, so remember that. And then put this value, $c_2 = 40$, $N_{c2} = 5.14$, then $1 + 0.2 \times \frac{2}{3}$, then + now it is the unit weight γ_1 and water table is at the surface.

So, γ_1 is basically will be 20 - 10, so now this will be γ_{sat} because water table at the surface, we have to take the effective pressure. So, $D_f = 1$, $H = 1$ and q_b is 253 kN/m^2. Remember that, here as I mentioned during the q_b calculations, we have taken the contribution of D_f and H both. So, we have to subtract that $\gamma_1 H$ in our basic equation.

But if during the calculation of q_b if you do not consider the H effect, you consider only $\gamma_1 D_f$. Then no need to subtract this $\gamma_1 H$, either you subtract from here during q_b calculation or you subtract from your Vesic's q_u equation and do not subtract twice, so this is 253. Now I will calculate the q_t, if the foundation is placed on the top surface and your q_t expression is $c_1 N_{c1} + \gamma D_f N_{q1} + \frac{1}{2}\gamma_1 B N_{\gamma 1} s_{\gamma 1}$.

But in the top layer, if your $c = 0$, now if $c_1 = 0$ because your case sorry $\phi_1 = 0$. So, again these conditions will be applied that $N_{\gamma 1}$ will be 0, so I can write that $N_{\gamma 1}$ will be 0, now N_{q1} will be 1 and N_{c1} will be 5.14.

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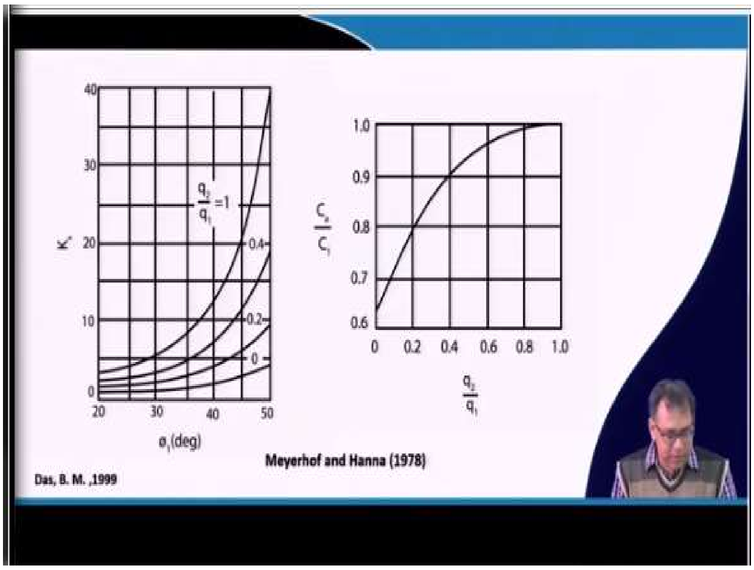

So, now I calculate the q_t in this condition.

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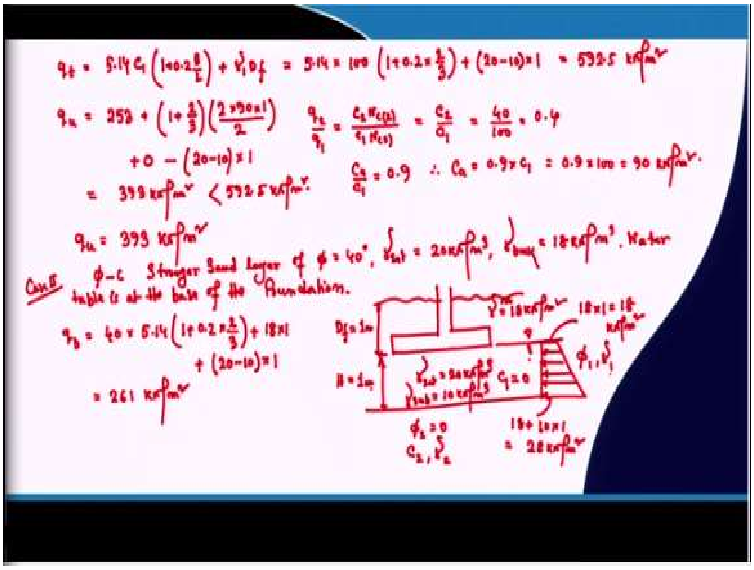

So, it will be $5.14 \times c_1$ then s_{c1} is same as Meyerhof's recommendation which is $1 + 0.2\frac{B}{L}$ then + here only the D_f. So, γD_f because it will be not $D_f + H$ because it is on the top layer, that is why D_f only, because that is the depth of foundation. If it is the bottom layer then the surcharge is considered because in the bottom layer depth of the foundation is considered.

Because if you look at the original derivation of these stresses are going like this, like a straight line. It is assumed that these stresses or the soil are moving along a straight line or a straight zone or strip where the dotted lines are there. So, that means all the stresses actually are acting on the bottom of the bottom layer exactly within the same width equal to the width of the foundation.

So, you can question that when you apply the load, then the stresses will be distributed, so the loading area on the second soil or the second layer may be more. But actually, as it is assumed that there will be a punching kind of failure, so the total soil along with that foundation will deform within a zone given by these dotted lines with a straight line. So, that means this total soil along with that foundation.

So, that means these stresses will be exactly on the bottom layer where the width of the stress which is acting on the bottom layer will be equal to the width of the foundation that is why always we are taking B either it is a top layer or the bottom layer. So, that is what during q_b calculation also we have taken bottom layer and as I mentioned that is H, that is why when I calculate the q_b bottom layer as it is acting at a depth of $D_f + H$.

Because now your footing depth will be $D_f + H$, but for the top layer your footing depth or depth of footing will be D_f, but bottom layer it will be $D_f + H$, that will be the footing depth for the bottom layer. For the bottom layer it is $D_f + H$and for the top layer this is only D_f and we have subtracted H contribution twice that is why you have to subtract once that one either during q_b calculation or from Vesic's equation.

So, now this is the equation, so finally I can write this is the shape factor and then $\gamma_1 D_f$. Because again the s_{q1} will be as for $\phi = 0$, so your $K_p = 1$, so $s_{q1} = 1$, similarly $s_{q2} = 1$ also, so I can write D_f × that part is 1. So, and then $5.14 \times c_1$where, $c_1$is 100 kPa. And then $1 + 0.2 \times \frac{2}{3} +$ again it is at the ground surface, so $D_f = 1$.

So, this is 1, this is 592.5 kN/m^2. So, your bearing capacity of this layer soil cannot be more than 592.5 kN/m^2, it has to be less than that, let us see whether it is coming less than that or not. And another thing I want to mention that here the water table is considered at the ground surface, if it is not at the ground surface slightly below that but within the ground and the base of the foundation.

Then these surcharge or effective overburden pressure you have to calculate accordingly. So, now I can put this value in my original equation, so q_b is 253. Now this is $1+\frac{2}{3}$, now we have to calculate C_a and K_s otherwise we cannot calculate the value, let us calculate this first C_a and K_s. Because here K_s will not be required, you can see that your $\phi_1 = 0$, so $\tan\phi_1 = 0$, so this total term will be 0.

As $\phi_1 = 0$, so $\tan\phi_1 = 0$, so this total term will be 0. So, you only have to calculate the C_a, so let us see, how to calculate the C_a. For C_a calculation we should know $\frac{q_2}{q_1}$, now what is $\frac{q_2}{q_1}$? So, as it is mentioned that $\frac{q_2}{q_1}$ is when the foundation is a strip footing resting on a soil at the surface. So, now and for individual layers, so if I place the foundation on the second layer as if there is no other layer is present, only the second layer is your soil layer.

And if I place the foundation over there, and it is the surface footing, so second term will be 0. Now here in this case $\phi = 0$, so your third term will be also 0. So, only the first term and the first term will be what? First term will be it is a strip footing, so shape factor also we will not consider, so that will be c_2N_{c2}. So, that is the only q_2 value, second term will be 0, third term will be 0, first term it is a strip footing.

So, you will not consider and even if you consider the shape factor it does not matter because it is the ratio, it will cancel out. So, now for the q_1 first layer if I put on first layer the same thing will happen it will be c_1N_{c1} but for both the cases $\phi = 0$. So, $N_{c1} = N_{c2} = 5.14$ for both the cases, so it will cancel out. So, if both the layers are cohesive soil then it will be simply the ratio of the cohesion of each layer.

So, $\frac{c_2}{c_1}$, now here your c_2 is 40 kPa, c_1 is 100 kPa, so this is 0.4. So, now let us see if $\frac{q_2}{q_1}$ is 0.4, then, and $\frac{C_a}{c_1}$ is 0.9. So, I can write that my $\frac{C_a}{c_1}$ is 0.9, so C_a will be $0.9 \times c_1$, so 0.9×100, so it will be 90 kN/m^2. So, now if I put this value, that my original equation I am now putting these values, so this term is 0, as I mentioned your $\phi_1 = 0$.

So, it is only this term and then $-\gamma_1 H$, because we have taken H contribution during q_b calculation, so we have to subtract. So, this is $\left(1+\frac{2}{3}\right) \times 2 \times 90$ $D_f = 1$ m and $B = 2$ m then + this old term is 0, then $-\gamma_1 D_f$ or $-\gamma_1 H$ and this is effective unit weight we have to consider $H = 1$. So, finally the value will be 393 kN/m^2 < 592.5 kN/m^2, it is good.

So, finally q_u = 393 kN/m^2, so this is the value that we are getting if both the soils are cohesive soil. And then we have considered that, obviously these equations that I have derived is valid if the lower layer is the softer layer compared to the upper layer. So, this is my case 1 where we consider both the soils are clay upper layer is stiff clay and the lower layer is the weaker or the softer clay.

Now the case 2, I will go for the case 2 where this is a c-ϕ soil, the same problem I have only changed the properties of the first layer. So, the first layer is obvious the stronger sand layer of ϕ = 40° and γ_{sat} = 20 kN/m^3 and γ_{bulk} = 18 kN/m^3, because here I am considering the water table not at the ground surface like the previous case.

It is below the ground surface just to show you how we can solve the problem in such case. And water table is at the base of the foundation, so let us see how we can do that? So, before I go to the calculation here, you cannot use the basic equation that I have given directly because here you have to do some slight modifications. Because in earth pressure there is only γ_1 term, but now here the γ will be the different because water table is present.

So, let us first calculate earth pressure and then we will put that value in your equation. So, I will show you how we can calculate the earth pressure. So, this is your ground surface obviously the same problem we are taking. So, your $D_f = 1$ and this is the first layer and the second layer interface. So, where H = 1 m and water table is at the base of the foundation. And here the unit weight is 18 kN/m^3, and here the saturated unit weight is 20 kN/m^3, so γ_{sub} will be 10 kN/m^3. Because unit weight of water is 10 kN/m^3, so this is the earth pressure distribution.

So, for this portion it will be 18 × 1, this will be 18 kN/m^2 and here it will be your 18 + 10 × 1, so this will be 28 kN/m^2. So, the water table effect we will not use to calculate the earth pressure because we will consider only the soil part. Because water will just increase your bearing capacity, so that is not recommended because water may not be present sometimes.

That is the only but during your earth pressure calculation, you have to consider the water pressure because that will reduce your factor of safety. But here if you consider the water that will increase your factor of safety, so that is not good thing. So, that is why we will consider only the soil contribution. So, you can ask why I am not using that expression directly here. Because if these are same for top and bottom of this zone, then you can directly use that expression.

But here the γ part and here the γ part are different because of the water table position that is why we have calculated separately. If the γ is same throughout the soil then you can use your basic equation that is given by using the same γ but here the γ value is different, so that is why you are calculating in this way. So, this is my passive pressure zone.

So, now I will calculate the γ_b, which will be of the same value because it will change slightly, because previously water table was at the ground surface and now the water table is at the base, so at the base of the foundation slight change will be in the q_b value. So, in my previous example you can see q_b is basically this is the equation, so here only this part will change. So, now initially it is $D_f + H$ both are same γ but is a different γ now, so only this is the change.

So, q_b will be $40 \times 5.14 \times \left(1 + 0.2\frac{2}{3}\right)$, now the change is there, this portion is same $40 \times 5.14 \times \left(1 + 0.2\frac{2}{3}\right)$. Now this portion will slightly change for the top one that is $\gamma \times D_f$, that is 18 × 1. Because that is above the water table, γ_{bulk} means above the water table and then the saturated one that is submerged (20 – 10) × 1. So, that will give you 261 kN/m^2.

So, I have calculated the q_b part in this case and I have also shown that how I will calculate the passive resistance part. So, next class I will complete this case 2 problem where I will also show how I can calculate the q_t then how I can calculate the K_s? And then because again C_a will not be required here. Because here you can see that c_1 value is 0. So, it has ϕ_1 and γ.

And here the second layer your $\phi_2 = 0$, it is c_2 and the γ_2 or γ, here also γ_1. So, C_a will not be required but only the K_s will be required, so that I will discuss. And then in the next class we will go to the few more cases because I have discussed four cases. So, I have finished the example for first case, I am doing the second case, that I will finish in the next class, and then I will discuss the other two cases, thank you.