Advanced Foundation Engineering
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Lecture-19
Shallow Foundation: Bearing Capacity XIII

So, last class I was discussing about the bearing capacity of two-layered soil.

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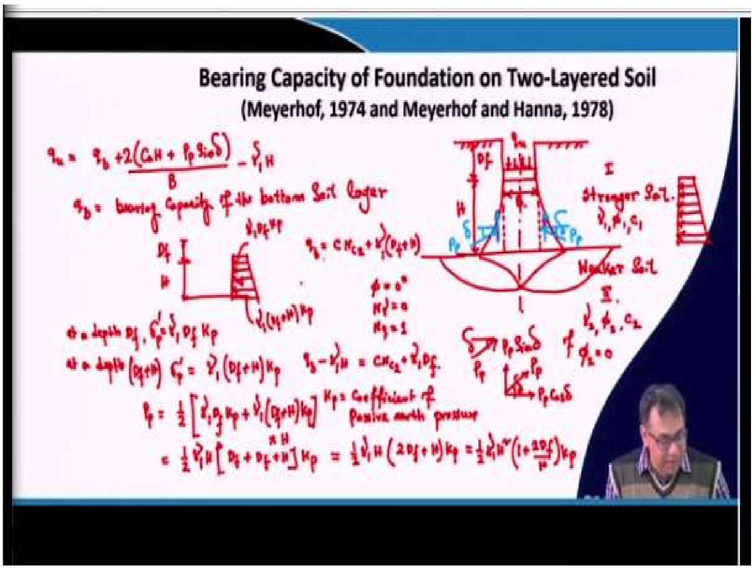

In the case 1, when the first layer is a stronger layer and the second layer is a weaker layer. And then I discussed that how the loading is acting and then because that means the bearing capacity you are getting from the contribution of the second layer that is the q_b. Then the frictional resistance that you are getting due to the movement of the soil within the first layer and that frictional resistance we are getting by using this equation.

And then we are subtracting $\gamma_1 H$, why $\gamma_1 H$? Because during the q_b calculation if we take the contribution of that D_f and the H as a surcharge, but as the H layer contribution or H portion contribution already been taken during the shear resistance calculation, then it is recommended to neglect that contribution during the q_b calculation. So, or it can be subtracted directly in the basic equation.

In such case we have to consider both H and D_f for q_b calculation. So, now if I further simplify this equation that now if I get the horizontal component of δ is acting here. Now, what is the passive pressure? So, now this is my say second layer, and this is the first layer. And the first layer is here and this is D_f and this portion is H, now I have to calculate the passive resistance at this zone.

So, passive resistance will be something like that, this will be the passive resistance at this zone. So, at a depth D_f what is the passive pressure? The passive pressure will be $\gamma_1 D_f K_p$, so D_f is the depth of the foundation, K_p is the coefficient of passive earth pressure. So, now at a depth of $D_f + H$ because here you are getting basically the passive resistance from here to here.

So, this passive resistance I am trying to calculate, this is at a depth of D_f, this is at a depth of $D_f + H$. So, that means my σ_P will be this value and here σ_P actually this is effective but here do not consider the water table, so I am considering effective and total are same. So, this value will be $\gamma_1 (D_f + H) \times K_p$, that is the pressure, so this is a trapezoidal.

So, here my contribution is $\gamma_1 D_f K_p$, and here it is $\gamma_1 (D_f + H) \times K_p$. So, now the total passive force will be $P_P = \frac{1}{2} \times [\gamma_1 D_f K_p + \gamma_1 (D_f + H) \times K_p] H$. Because into H because H is the thickness of this layer. So, finally I can write that this value will be half we can take H γ_1 and D_f outside**.**

And we can take γ_1 and then H outside let me first take that one. So, that will be this H I am taking, so that will be K_p also I can take outside. Because this is the $D_f + D_f + H$, so that is $\times K_p$, so that will be equal to $\frac{1}{2}\gamma_1 H(2D_f + H)K_p$. Now if I take H outside also, then this will be $\frac{1}{2}\gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) K_p$.

So, this is the total passive force which is acting at an angle of δ. So, I should write in this way, so this is the total passive force and that I am calculating by using this expression.

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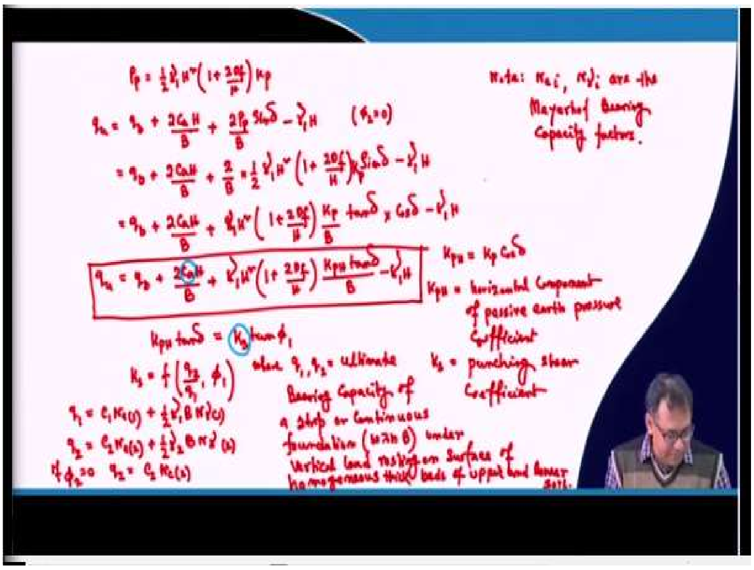

So, now if I put these values, so now I have calculated that $P_P = \frac{1}{2}\gamma_1 H^2\left(1+\frac{2D_f}{H}\right)K_p$. So, now if I put these values here, so I can now write $q_u = q_b + \frac{2C_aH}{B} + \frac{2P_P\sin\delta}{B} - \gamma_1 H$.

But remember that when I am writing $\gamma_1 H$ actually we are taking that $\phi_2 = 0$. Because for the time being we are considering that your $\phi_2 = 0$, that means it is a second clay layer. So, now, if I put this value $q_b + \frac{2C_aH}{B} + \frac{2}{B}\times\frac{1}{2}\gamma_1 H^2\left(1+\frac{2D_f}{H}\right)K_p\sin\delta - \gamma_1 H$. Now this K_p part, we are now converting and we are writing that $q_b + \frac{2C_aH}{B} + 2$, 2 is cancelled out, now this will be $\gamma_1 H^2$.

Then $\left(1+\frac{2D_f}{H}\right)K_p\sin\delta$ divided by B and then we are taking $\tan\delta$, so it is a $\tan\delta$. So, that means we have to multiply with $\cos\delta$, then $\gamma_1 H$. So, we are taking $\sin\delta$ and then that we are writing in this form. Now we are writing that $K_{pH} = K_p\cos\delta$. So, that means this is basically we are writing what is K_{pH}? K_{pH} is the horizontal component of passive earth pressure coefficient.

So, that means as I mentioned if you look at these equations over here, your P_P is acting with an angle δ. So, that means it has one horizontal component and on vertical component. Similarly the earth pressure is also represented in this way that the K_{pH} is the horizontal component of the earth pressure coefficient. Because K_p is the total component or the resultant component you can say which is acting at an angle δ.

Now if I take that $\cos\delta$ then that will give you the horizontal component. And so that K_{pH} will give you only the horizontal component not the vertical component remember that. It is at the resultant, K_p is the resultant but K_{pH} is the horizontal component of the earth pressure coefficient. Because all the charts are developed considering K_{pH}.

So, this way I can finally write that $q_b + \frac{2C_aH}{B} + \gamma_1 H^2\left(1+\frac{2D_f}{H}\right)\frac{K_{pH}\tan\delta}{B} - \gamma_1$. So, this is the expression, final expression will be this one, this is the final expression for case 2. So, now, we have represented them in this way and all the charts are produced, so that is why it is taken K_{pH} not the K_p.

If you take K_p then it will be $\sin\delta$ if you take K_{pH} then it will be $\tan\delta$ so now it is K_{pH}. Now, as I mentioned δ is the angle between the soil versus soil. So, basically it will be $\sin\phi$. So that is why we can write that K_{pH} further as $K_{pH}\tan\delta$ is $K_s\tan\phi_1$. Because it is soil versus soil in this layer it is soil versus soil, so that is why the friction angle is ϕ_1, so that is ϕ_1.

And the K_{pH} is further represented as K_s, what is that K_s? That K_s is the punching shear coefficient, why a punching shear coefficient is considered? Because in horizontal direction, I mean in the vertical direction soil is punching basically. Because in the first layer you can see because of the bottom layer is very weak a punching of this foundation in the first layer that mean the soil is deforming along the horizontal line or along the dotted line and it is punching, so that is why it is considered as a punching shear coefficient K_s.

Now, q_b is the bearing capacity of the bottom layer which can be calculated by using the bearing capacity equation. And then finally I can write, that my K_s is function of your q_2, q_1 and ϕ_1. Now this K_s which is represented in a chart form where it is the function of ϕ_1 and your q_2 and q_1. So, obviously it is a function of ϕ_1, q_2 and q_1 also, what are q_2 and q_1?

The q_1 and q_2 are ultimate bearing capacities of a strip or continuous foundation with width B under vertical load resting on surface of homogeneous thick bed of upper and lower soil. So, what does it mean that? These q_1 and q_2 are the bearing capacities of a foundation resting on a homogeneous soil where the vertical load is acting and the foundation is strip foundation.

So, that means we consider these layers individually and then we place the strip footing on the surface. And then the soil is homogeneous, that means first we will consider suppose the one first layer or the top layer or the upper layer. And then we placed the foundation of strip footing with B and it is resting on the surface and the vertical, no inclined load and then the bearing capacity, that we will get that is q_1.

Similarly we will place the foundation on the surface of the second layer. And then vertical loads, strip footing with a width of B and the bearing capacity that we will get, that is q_2. So, I can write that my q_1 is the bearing capacity of the strip footing resting on the surface, so that q_1 will be c_1N_{c1} because it is your strip footing, so shape factor will be 1, and it is at the surface, so the second term will be 0, and there will be third term also.

And that is $\frac{1}{2}\gamma_1BN_{\gamma1}$, and q_2 is c_2N_{c2}, again is $\frac{1}{2}\gamma_2BN_{\gamma2}$. Now, I have considered the $\phi_2 = 0$ then automatically if the second layer is clay. Now if $\phi_2 = 0$ then q_2 will be simply c_2N_{c2}. But we are giving a general equation that is why I have written that c_2 I mean first term and the third term also as it is on the surface, so second term is neglected. So, that means if your $\phi_2 = 0$ then you will get q_2, and that will be c_2N_{c2}.

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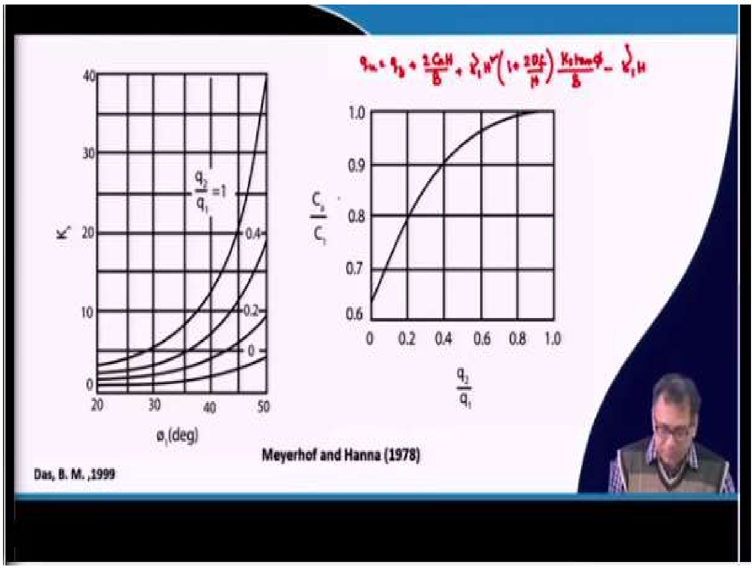

So, now a next part this is the chart, by which we can determine these values. Now one thing is that how I can get the N_c , N_γ , N_{c2}, $N_{\gamma 2}$. So, these values I will get from the chart or the table given by Meyerhof's bearing capacity theory. So, note that these N_{ci} or $N_{\gamma i}$ are Meyerhof's bearing capacity factor. So, these bearing capacity factors I will get from the table given by Meyerhof in the original Meyerhof's bearing capacity equation, so that factors I can use here.

So, now we have to calculate few things, what are the things that we have to calculate? If you look at this equation then I can get q_b, from the bearing capacity equation. And then we have to calculate C_a and this K_s, what will be these two values? Other things you will get from the available values.

So, now final form of this equation is slightly changed, so that I am writing first. That final form of this equation is q_u or I should change the color of the pen. So, $q_u = q_b + \frac{2C_aH}{B} + \gamma_1 H^2\left(1+\frac{2D_f}{H}\right)\frac{K_s\tan\phi}{B} - \gamma_1$, so this is my equation. So, how will I calculate K_s? So, I will calculate K_s by using this chart, so for different ϕ values, if I know that q_2 and q_1, so I have already discussed what are q_2 and q_1.

So, and I will get the q_1 and q_2 by using these two equations, so I will get the q_1 and q_2. And if I know this $\frac{q_2}{q_1}$ are 1.0, 0.4, 0.2 and 0, and as the upper layer is stronger layer, so that means definitely $q_2 \leq q_1$. So, that is why this $\frac{q_2}{q_1}$ are equal or less than 1.0, so this is the equation and then I will get the K_s value corresponding to different ϕ_1 value.

And $\frac{C_a}{c_1}$, c_1 is the cohesion of the first layer. So, this is the c_1 similarly I will get $\frac{q_2}{q_1}$ and from this chart I will get the *c* value. So, this way I can get the C_a value as well as the K_s value, clear.

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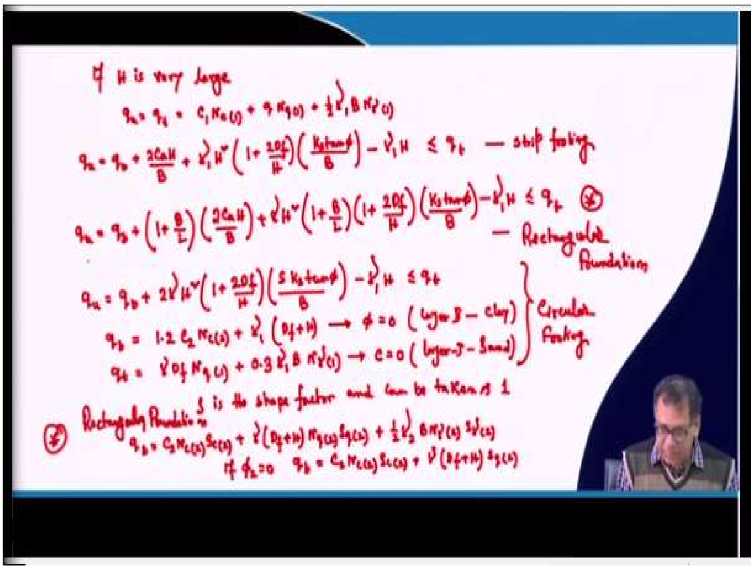

Now in next part, if *H* is very large, in such case that means here we have to put one limit, suppose if your *H* is very large that means all the failure zone is within that *H*. So, that means the bearing capacity of the foundation in such case will be the bearing capacity of the footing, if I put it on first layer itself where the depth of the layer is very high.

So, that means, in such case I will get a bearing capacity value if your *H* is very large, so, $q_{ultimate} = q_{top}$, where, q_{top} means the top layer bearing capacity. So, that I can calculate $cN_{c1} + qN_{q1} + \frac{1}{2}\gamma_1 BN_{\gamma 1}$. So, here I am not giving other factors, I am deriving Vesic's equation with only the coefficients and we assume that all the other factors are 1 and it is for strip footing, so that is why this is the equation.

So, that means this is the bearing capacity if the foundation is resting on the top soil only, where the bottom soil influence is not there. So, that will be the maximum bearing capacity of this foundation. Now if the bottom soil influence is there, because as the bottom soil is the weaker soil, so then if the influence of the bottom soil zone increases your bearing capacity will reduce. So, the maximum bearing capacity you will get if the whole influence zone is within the first layer.

Because the first layer is the stronger layer compared to the second layer. So, you will get maximum bearing capacity if all our influence zone of the foundation is resting within the first layer or within the *H*, this is only possible if *H* is very large. So, that is the limiting value of your bearing capacity which cannot be greater than that value. Now, if the lower layer is the softer layer or weaker layer, so as more influence zone is covered within the lower layer, your bearing capacity will be decreased.

So, that is why a limit is placed, so finally we can write that $q_u = q_b + \frac{2C_a H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \frac{K_s \tan\phi}{B} - \gamma_1 H$ that should be $\leq q_t$. So, q_t I will get from this equation and because this is the limiting value this q_u cannot be greater than q_t, it can be equal to q_t and that is the limiting value. It can be either equal to q_t or less than that, it cannot be greater than that, so this is the limit which is placed.

Now this equation is for the strip footing. Now if I convert it for the rectangular footing, then $q_u = q_b + \left(1 + \frac{B}{L}\right) \frac{2C_a H}{B} + \gamma_1 H^2 \left(1 + \frac{B}{L}\right) \left(1 + \frac{2D_f}{H}\right) \frac{K_s \tan\phi}{B} - \gamma_1 H$, that should be less than q_t, so this equation is for the rectangular foundation or if you put $B = L$ then it can be used for the square foundation also.

Similarly, this type of foundation we can use it for the circular footing also. Now, how I can use for the circular footing? In such case q_u will be $q_b + 2\gamma H^2 \left(1 + \frac{2D_f}{H}\right) \frac{SK_s \tan\phi}{B} - \gamma_1 H$ that should be less than q_t. Now here your $q_b = 1.2 c_2 N_{c2} + \gamma_1 (D_f + H)$ for $\phi = 0°$, and q_t will be equal to that for the top layer, so $\gamma D_f N_{q1} + 0.3 \gamma_1 B N_{\gamma 1}$, so here $c = 0$.

So, for that means for your layer 2 or I should write this is the layer 2 which is clay, and layer 1 which is sand, so this equation is valid, now this is for circular footing. Now here there is a term *S* this *S* is a shape factor and can be taken as 1. So, that means we have given the equation for three cases, first one is the strip footing where the q_b equation is given. That q_b I can calculate by using this equation for clay soil and for general soil also you can calculate.

Now here this is for the rectangular footing where you have to calculate the q_b. And remember that when you calculate the q_b for rectangular and strip footing, you have to apply those shape factor corrections. But in the circular footing it is given q_b is $1.2c_2N_{c2} + \gamma_1(D_f + H)$. And q_t for the circular footing is also given and for circular footing assume that the top layer is sand and the bottom layer is the clay.

In other cases also we have considered that because $\phi_2 = 0$, bottom layer is equal to clay and the top layer is equal to sand. But here when I calculate q_b here I have considered strip footing. Now in the rectangular footing part there is also q_b, so remember that when you calculate q_b for rectangular footing, you have to apply the shape factor, in this equation, so remember that.

And the circular one it has been already given. So, these are the three equations for strip footing, rectangular footing and circular footing. For two-layered soil and mainly the first layer is stronger which is sand and second layer is weaker which is clay, so this is the condition we have discussed. Now, what we will do? We will now solve one problem and we will see that how we can calculate the value?

And again as I was talking about q_b calculation for rectangular footing, and the q_t calculation, because this q_t is calculated without considering shape factor, so this is also valid for the strip footing. So, now let me write these things that for the rectangular footing, so as I mentioned that this q_t will be for the strip footing.

For rectangular footing foundation now $q_b = c_2N_{c2} \times s_{c2}$ shape factor we have to apply, + $\gamma(D_f + H)N_{q2}$ +, then $s_{q2} + \frac{1}{2}\gamma_2 BN_{\gamma 2}s_{\gamma 2}$. Now, this is the general equation, now if your $\phi_2 = 0$

then q_b will be $c_2 N_{c2} s_{c2}$ then + your this is $\gamma(D_f + H) \times N_q = 1$, so this will be your s_{q2}, so because your $N_q = 1$, in such case, so s_{q2} that also you have to use, and then $\gamma(D_f + H)$.

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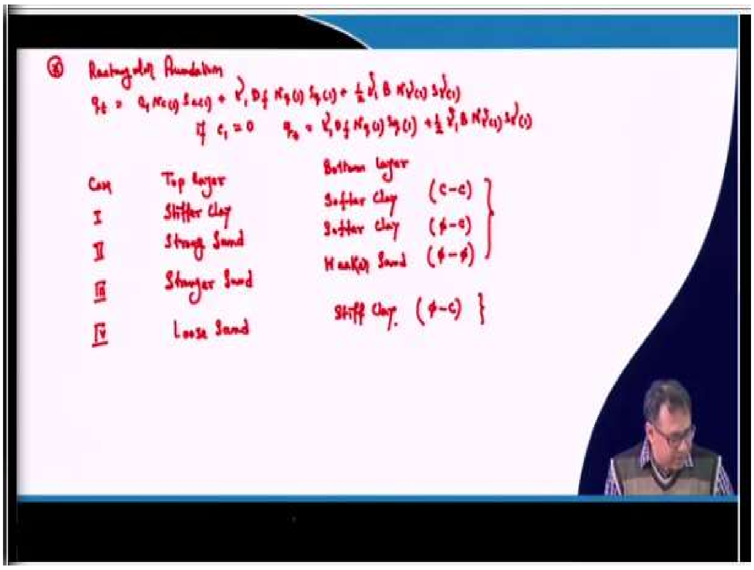

So, now for the rectangular foundation again, how we can write q_t? That q_t again I can write as $c_1 N_{c1}$, then shape factor we have to apply s_{c1} then + $\gamma_1 D_f$ then $N_{q1} s_{q1} + \frac{1}{2}\gamma_1 B N_{\gamma 1} s_{\gamma 1}$, 1 is for first layer, 2 is for second layer. Now, if your $c_1 = 0$, then q_t will be $\gamma_1 D_f N_{q1} s_{q1} + \frac{1}{2}\gamma_1 B N_{\gamma 1} s_{\gamma 1}$, so this is the expression.

So, now I will take a different example problem and then what are the cases possible? So, the cases that are possible are that if I take this is my top layer and this is my bottom layer. So, first case, the case 1, the top layer that I will discuss is the stiffer clay because I have considered the top layers are the stronger compared to the bottom layer. This top layer is the stiffer clay, bottom layer is the softer clay this is my first case.

Second case I can write that this is stronger sand and this is softer clay. So, that means in case 1, both are cohesive soil, here in case 2, first layer is sandy soil and second layer is the cohesive soil. In the first case both the soils are cohesive soil, and in the third case I can write that the stronger sand and weaker sand that means both are cohesionless soil.

So, these two cases, I have already derived, because I have considered this derivation this $\gamma_1 H$ which is applicable if it is a strip footing resting on the clay soil, so you know that. Then stronger sand and weaker sand is also possible. And I will discuss another case, fourth case which is not in these cases. So, that fourth case I will discuss where the top layer is loose sand and the bottom layer is the stiff clay.

So, this is again c-ϕ, but this will not be similar to the cases 1, 2, 3. So, cases 1, 2, 3 is one group and case 4 is another group, because in cases 1, 2, 3 the lower soil layer is weaker than the upper stronger soil layer, ok. So, the derivations that I have done is valid for first three cases only where the weaker soil is in lower layer 2 and layer 1 is the stronger soil.

But these equations are not valid for the fourth case, the fourth case I will discuss in detail later on, let me first solve the problems for the first 3 cases, and then I will discuss the fourth case.

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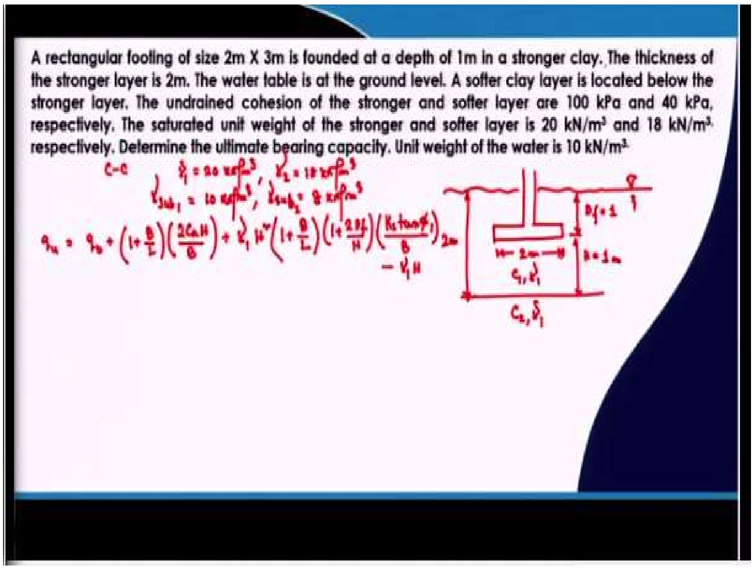

So, now let me discuss the first problem that rectangular footing of size 2 m × 3 m is placed at a depth of 1 m in stronger clay. So, first one is I mean it is resting on a stronger clay, the thickness of the stronger clay is 2 m, the water table is at the ground level, a softer clay layer is located below the stronger layer. The untrained cohesion of the stronger and the softer layer are 100 kPa and 40 kPa, respectively.

The saturated unit weight of the stronger and the softer layer are 20 kN/m^3 and 18 kN/m^3 respectively. Determine the ultimate bearing capacity if unit weight of the water is taken as 10 kN/m^3. So, now your problem is when both the soil is cohesive, ok.

So, now your foundation is resting at a depth of 1 m**.** So, your depth of foundation D_f is 1 m, the width of foundation is 2 m and the thickness of the stronger layer is 2 m. That means this is my stronger layer whose thickness is 2 m, so my H value will be 1 m, the thickness of the stronger layer below that foundation will be 1 m and there is a weaker layer at the bottom.

So, this is c_1 and γ_1, this is c_2 and γ_2 because ϕ_1 and ϕ_2 both are 0 and water table is considered at the ground level. So, now let me solve this problem, the saturated unit weights for the stronger and softer layer are 20 kN/m^3 and 18 kN/m^3 respectively, so both are different. For stronger layer it is 20 kN/m^3 and softer layer it is 18 kN/m^3, so submerged unit weight for layer 1 i.e. γ_{sub1} will be 10 kN/m^3 .

And for layer 2 submerged unit weight will be 18 -10 = 8 kN/m^3, that means γ_{sub2} = 8 kN/m^3. Now first I mean it is rectangular footing, let me write the general equation, so then general equation is this one for the rectangular footing, this is the general equation. So, let me write that general equation, that q_u will be $q_b + \left(1+\frac{B}{L}\right)\frac{2C_aH}{B} + \gamma_1 H^2\left(1+\frac{B}{L}\right)\left(1+\frac{2D_f}{H}\right)\frac{K_s\tan\phi_1}{B} - \gamma_1$.

So, in all these equations remember that this is tan ϕ_1 because this is in the first layer. So, in all these equations it is tan ϕ_1, I do not know whether I have written or not, so it is tan ϕ_1. So, I may miss that, but you remember that this will be tan ϕ_1, so this is the basic equation. So, next class I will solve this problem, because I have taken Vesic's equation, all the properties are given. So, next class I will first solve this problem and then I will discuss about the other cases that I have discussed in this class, thank you.