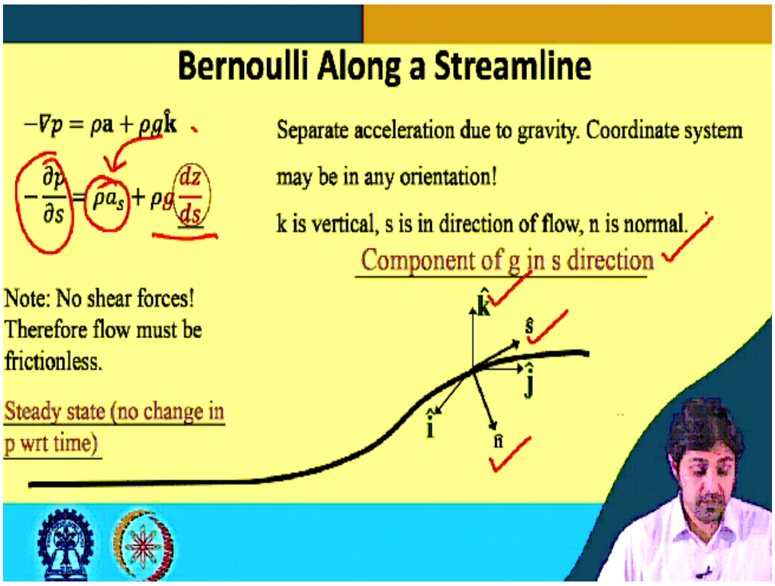
Hydraulic Engineering
Prof. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture – 09
Basics of fluid mechanics-II (contd.)

Welcome back in this lecture we are going to start elementary fluid dynamics or more commonly Bernoulli,s equation. So, we are going to cover some basic derivation in this regard and then we proceed further to some of the solved examples.

(Refer Slide Time: 00:43)

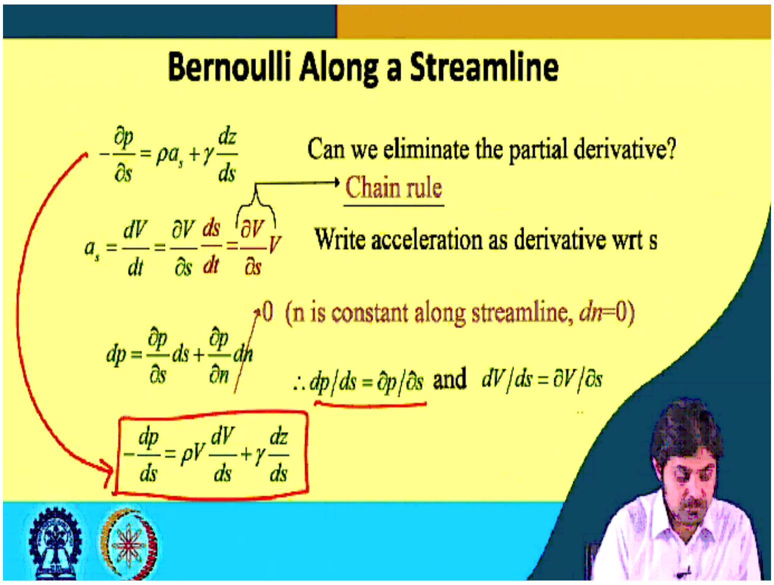


So, we are going to see the Bernoulli along a stream line. So, if you remember this equation we had got from the in the last weeks lecture that is on fluid statics. So, if you I am just going to, you know, draw a stream line this is i direction, this is j direction and this is k direction, it is better to take away all the ink I should and this is the s direction is the streamline, I mean, the direction of the streamline and this is normal to the stream line.

So, we have to separate the acceleration due to gravity and coordinate axis here can be in any orientation. This is quite important to understand, k is vertical, s is in the direction of the flow and is normal to the flow that is our standard assumption that we have been taking. So, this equation when we start writing down, you know, in the component form, so, delta p in the s direction will be $-\frac{\partial p}{\partial s} = \rho a_s + \rho g \frac{dz}{ds}$.

So, that is the component of g in s direction. So, we have to note that there is no shear forces there the flow must be frictionless. This is also a steady state so, there is no change in p with respect to time, p is pressure.

(Refer Slide Time: 02:49)



So, the equation that we have obtained

$$-\frac{\partial p}{\partial s} = \rho a_s + \gamma \frac{dz}{ds}$$

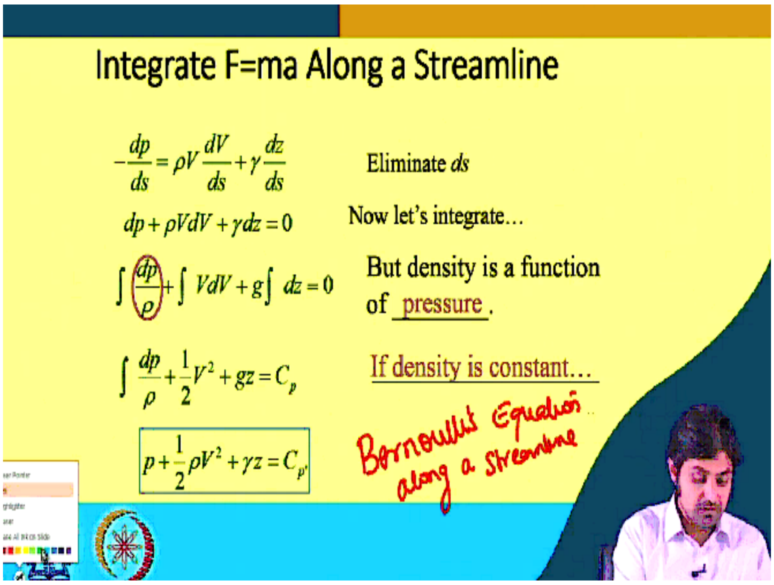
or

$$-\frac{dp}{ds} = \rho V \frac{dV}{ds} + \gamma \frac{dz}{ds}$$

And this is obtained through this way of doing this, you know, this differentiation is called the chain rule, write acceleration as a derivative with respect to s. So, the total change in pressure can be written as dp is delta p / delta s into d s + delta p / delta n into d n. So, n is constant along the stream line so, d n = 0. Therefore, this term goes so, dp / ds can be written as del p / del s, you see. Now, this is the total and this is the partial that is why we are able to write this and dv / ds is written del v / del s.

So, this equation turns out to be – dp / ds because this is what we have proved from here, and dv / ds as del v / del s or the opposite. So, we can write rho V dv / ds + gamma dz / ds, a s is V del v del s and instead of del v del s we can write dv ds. So, this is the equation.

(Refer Slide Time: 05:41)



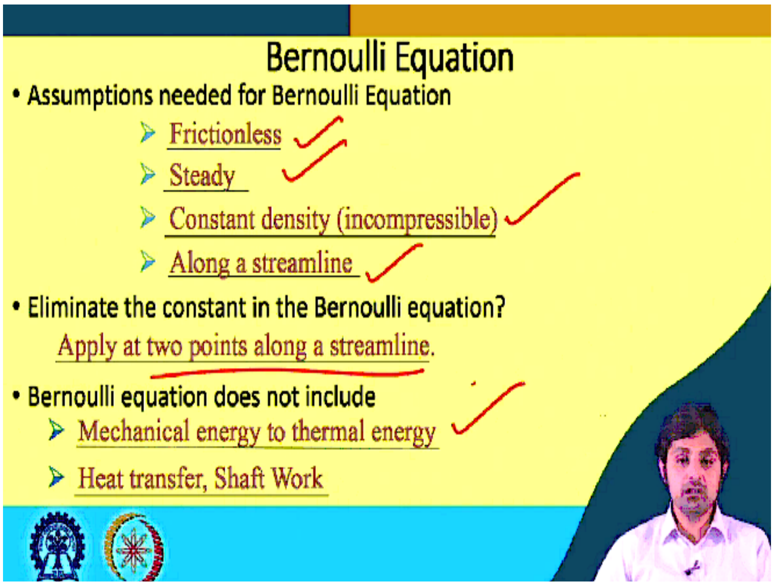
Now, we have to use force is equal to mass into acceleration along a streamline and integrate it. So, this is the equation that we have, so, what do we do? We integrate it, first eliminate ds because it is common so, we can write dp + rho V dv, because dp will come this side + gamma dz = 0 and if we integrate, this will become dp. So, what we do is we divide by, you know, rho so that this gamma becomes z it will become dp / p + V dv + g integral dz = 0.

Here, the density is a function of pressure so, we have to keep it inside so, we write integral dp / rho + V square / 2 and gz = constant. If density is constant we can write

$$p+\frac{1}{2}\rho V^2+\gamma z=C_{p'}$$

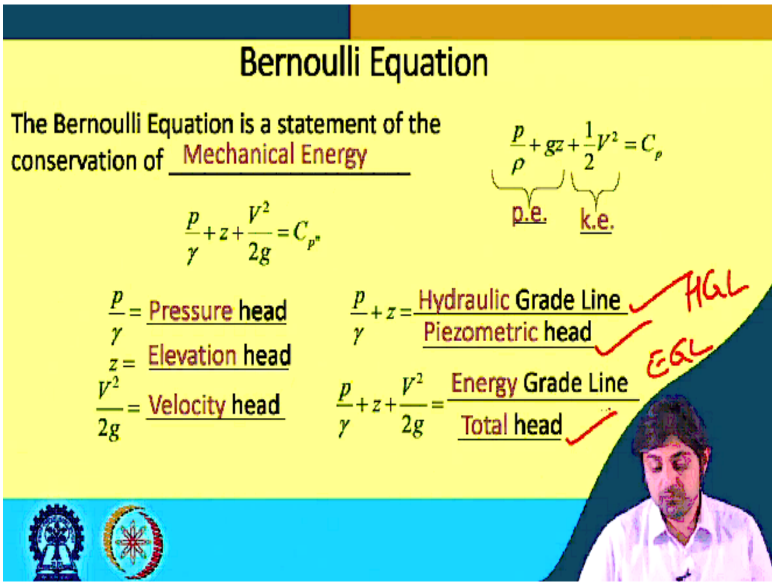
So, what we have done so, we have written the equation along the stream line, applied force is equal to mass into acceleration along the streamline integrated it and we have obtained Bernoulli's equation along a stream line. This is one of the ways of the derivation.

(Refer Slide Time: 07:57)



Now, the Bernoulli equation, the assumptions that are needed for Bernoulli's equation, what have we assumed, the flow was frictionless, the flow was steady, the final equation that we have derived we have assumed, constant density that means the flow was incompressible and we have done it for along a streamline. Now, can we eliminate the constant in Bernoulli's equation? Yes, if we apply the Bernoulli's equation at 2 points along a streamline, this is important so, the streamline should be the same and if we apply this equation along two streamline, I mean, to 2 points along a streamline the constant would be the same and therefore, it can be eliminated. Bernoulli equation does not include mechanical energy to thermal energy conversation, heat transfer or shaft work. So, this is some general information about the Bernoulli equation that you might be aware of from before.

(Refer Slide Time: 09:11)

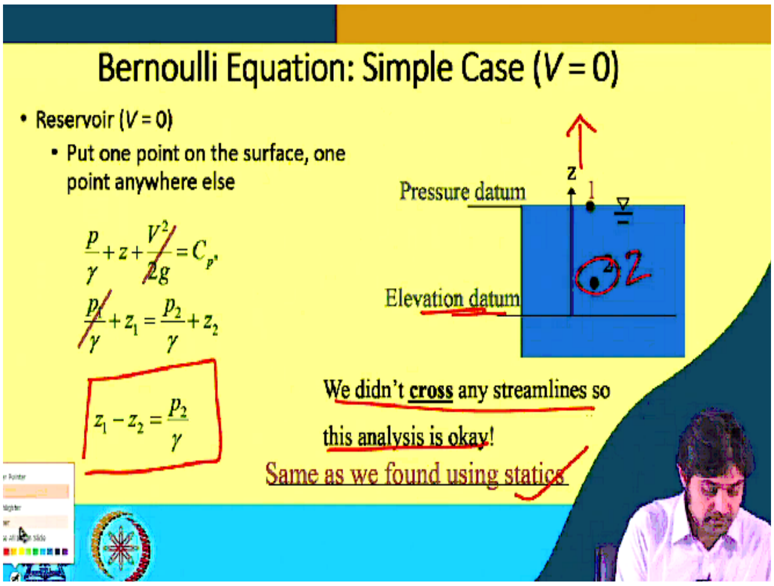


Now, the Bernoulli equation is a statement of conservation of mechanical energy. So, you see

$$\frac{p}{\rho} + gz + \frac{1}{2}V^2 = C_p$$

So this is potential energy. So, let me take the eraser. And this is let it this is kinetic energy. I will take down the ink again. So, P / gamma + z + V square / 2g = C p, where p / gamma is the pressure head, z is the elevation head and V square / 2g is the velocity head. We can also say that p / gamma + z is hydraulic grade line HGL, very common concept in your fluid mechanics class, or piezometric head, whereas, p / gamma + z + V square / 2g is called energy grade line EGL or total head. This is something that you can be asked, you know, by anyone whether you are giving, I mean, a gate exam or an interview sitting for a company which works in hydraulics. So, you must be remembering what hydraulic grade line is, what is energy grade line, what is piezometric head, what is total head.

(Refer Slide Time: 11:20)

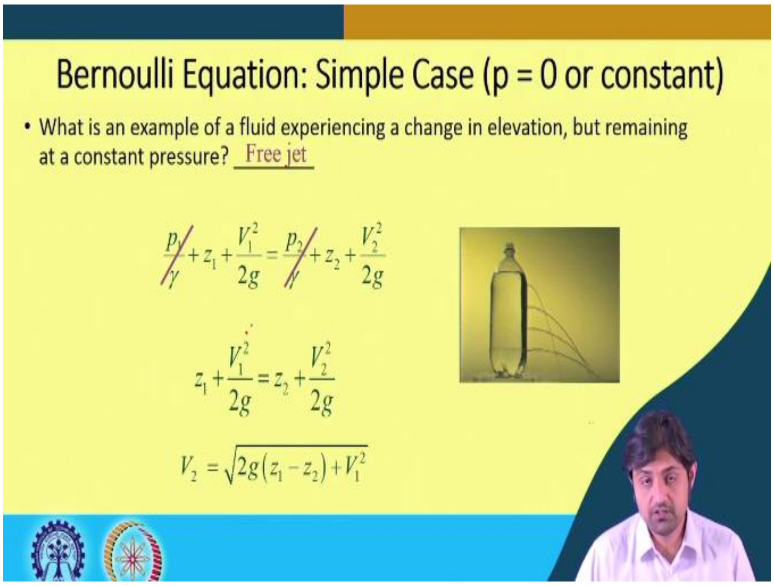


So, now, proceeding forward first simple case, very simple case, where V = 0. So, there is point 1 and there is point 2, z direction is shown as above, you know, this is pressure datum and this is elevation datum. Reservoir means V = 0. So, now, as I said this point 1 has been put on surface and we can put point 2 anywhere we have decided to put at this random point, point 2, point number 2.

So, we apply the Bernoulli equation between these 2 points. So, in at in the reservoir the velocity is 0. So, V square / 2g is going to be eliminated here. So, the between these 2 points we can write p1 / gamma + z1 = p2 / gamma + z2, whatever it is going to be, we also know that p1 is a gauge pressure and this is a pressure datum atmospheric pressure, so, p1 will be 0. In our second case, so, we can write z1 – z2 is = P2 / gamma.

Because elevation datum is here, so, 2 is anywhere. Here, we did not cross any stream line so this analysis is okay. If we keep crossing the stream lines, then what happens is this equation is valid for 2 points on the same stream lines; you cannot cross the stream lines. So, this analysis z1 - z2 = P 2 / gamma. This is exactly the same analysis, which we have found using the statics lecture, if you remember from last week.

(Refer Slide Time: 13:29)



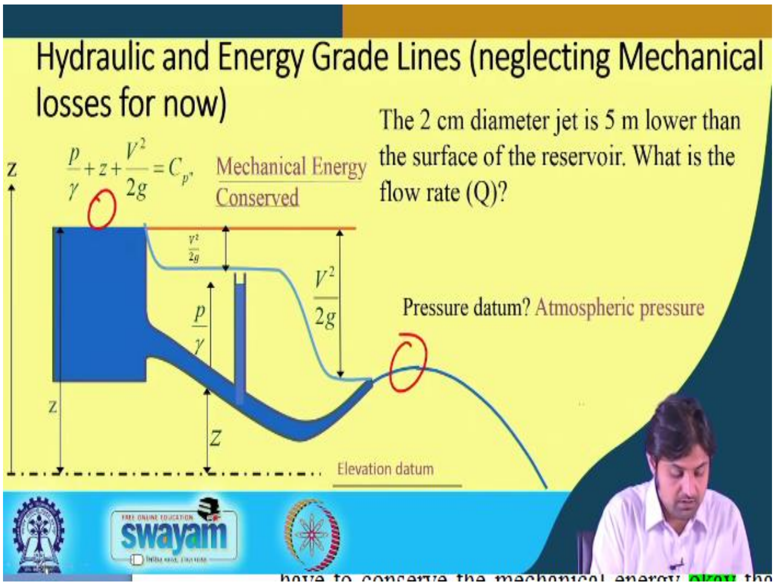
So, proceeding forward, so, we take another simple case, where pressure is 0 or constant. What is an example of fluid experiencing a change in elevation but remaining at a constant pressure? It is an example is a free jet. What is a free jet? This can be taken as a free jet, you fill a bottle with water or any fluid and make many holes, we saw similar type of figure last week as well and you see there are, you know, jets of, I mean, the free jets, this one, this one, this one, this one as well. I will delete that.

So, the equation, Bernoulli's equation along 1 streamline will be p1 / gamma + z1 + V 1 square / 2g will be = at any point two same, p 2 / gamma + z 2 + V 2 square / 2g. Because this is atmospheric pressure at 0.1, we assume, the 0.1 to be the free surface there. So, pressure at 0.1 will be atmospheric therefore, 0. Similarly, if we consider any point along the stream line what is going to happen is p 2 also exposed to atmosphere and that is also 0.

So, we can write z 1 + V 1 square / 2g = z 2 + V 2 square / 2g and therefore, the velocity at point 2 will be $V_2 = \sqrt{2g(z_1 - z_2) + V_1^2}$

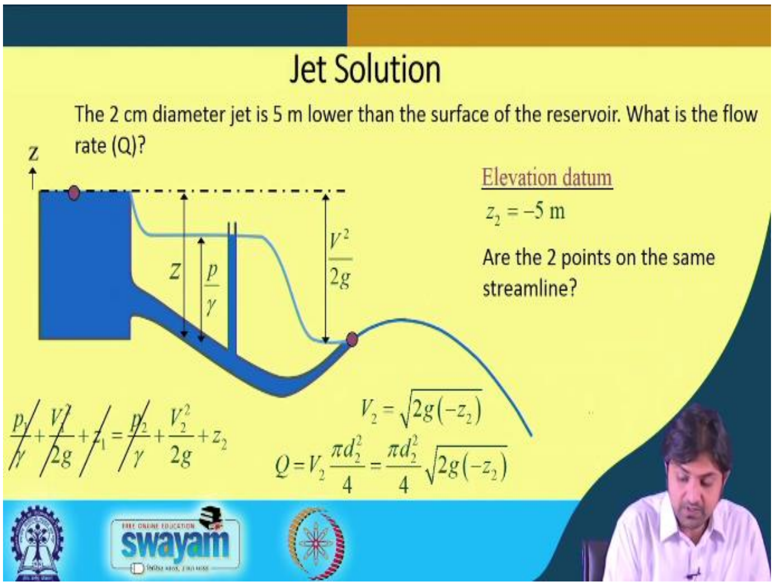
where V 1 can be the velocity for velocity of the surface.

(Refer Slide Time: 15:34)



So, we talked about but if we talk about hydraulic and energy grade lines, the question is the 2 centimeter diameter jet is 5 meter lower than the surface of the reservoir, what is the flow rate Q? Let us draw a figure like this. So, this is the elevation datum here. So, the equation Bernoulli equation is p / gamma + z + V square / 2g = constant. We this is we have to conserve the mechanical energy that is the energy conservation. So, to make this less cluttered will delete these annotations. So, pressure datum is atmospheric pressure, very simple, both at the out at here or here.

(Refer Slide Time: 16:37)

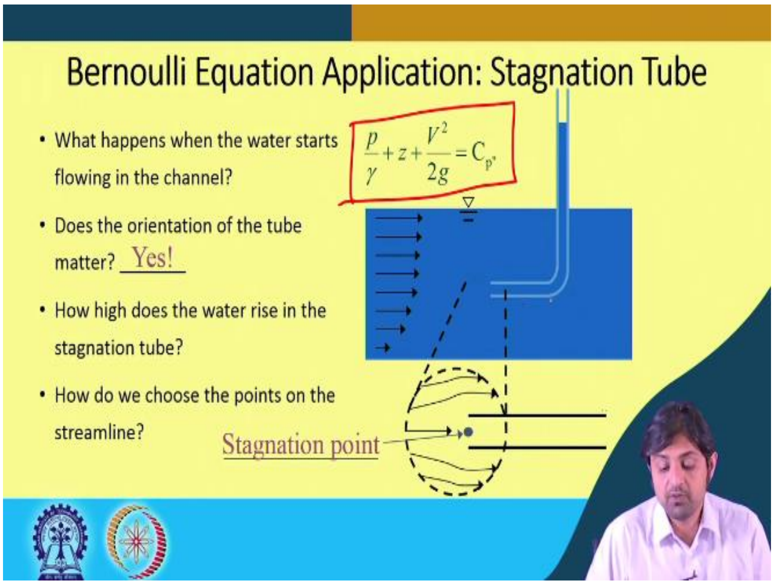


So, the jet solution, so, the question again coming back to the question, the 2 centimeter diameter jet is 5 meter lower than the surface of the reservoir. What is the flow rate Q? Again drawing the same figure, this is the elevation datum, therefore, z 2 is 5 meter below that is what it says, 5 meter lower are the 2 points on the stream streamline? Yes. So, we can write p 1 / gamma + V 1 square / 2g + z 1 = so, these are the 2 points just that if you are confused.

So, p 1 / gamma + V 1 square / 2g + z 1 = p 2 / gamma + V 2 square / 2g + z 2, p 1 / gamma will be 0, there is no velocity and, I mean, this is at rest. So, the velocity is also 0 and then the datum is also 0 because we have set the elevation datum here. The pressure at 2 will also be 0 atmospheric pressure, now, there will be a velocity and because there is z 2, so V 2 will be under route 2g – z 2 and the discharge will be pi d 2 square / 4, if d is the diameter 2 centimeter diameter jet it says, under root 2g – z2 very simple applications, but it is important to, you know, apply to these cases.

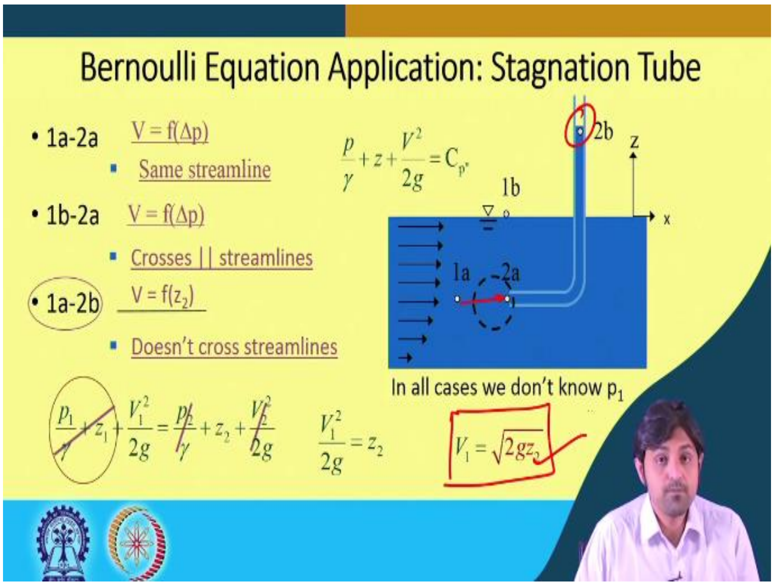
So, this is our 2 results, you know, v 2 and z 2 is – 5, so, you can obtain the, it will be approximately 10 meters per second. For example, if you assume g as, if you take 10 meters per second square, v 2 will be under root 2 into – 5, so, that means, +5 into 10. So, v will come out to be 10 meters per second.

(Refer Slide Time: 18:53)



So, one of the other applications of Bernoulli equation is the stagnation tube. What happens when the water starts flowing in the channel? For example, this and so, the equation along the stream line by Bernoulli equation will be can be given as p / gamma + z + V square / 2g = constant. Here, the orientation of the tube is like this, but does the orientation of the tube matter? Yes, because it can either lower up, I mean, it can change the different value of z so, yes. How does high does the water rise in this stagnation tube that we have to calculate. How do we choose the points on this timeline, another important question that we will solve?

(Refer Slide Time: 19:46)

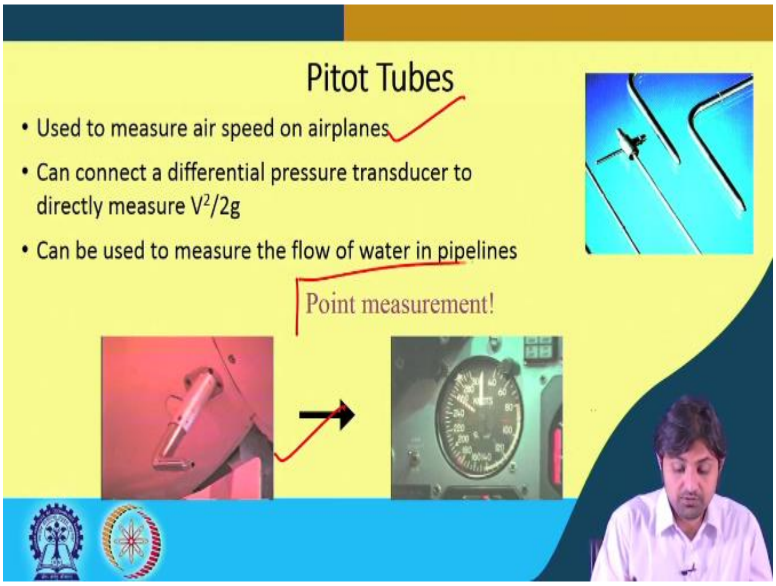


So, we assume, points 1a, 2a,1b and 2b like this, this point, this point, this point and this point. The equation, Bernoulli equation will be the same p / gamma + z + V square / 2g = constant. So, if you go from 1a to 2a, velocity will be a function of pressure, it is the same stream line; 1a is going to 1a to 2a, if you go from 1b to 2a it crosses perpendicular stream lines. If you go from 1a to 2b, it does not cross the stream lines, going from 1a to 2a does not cross the stream line.

In all cases, what we do not know is p1. So, the equation, Bernoulli equation p 1 / gamma + z 1 + V 1 square / 2g is constant. So, p 1 / gamma + z 1 is going to be, we can set it as 0 at 0.1 and because this is exposed to atmosphere we can assume to be 0 and because this has reached at the top what happens is, there will be no velocity because the water will have high rise until the maximum, you know, that it can and there the velocity will be 0.

So, we can simply write, V 1 square / 2 g, so, the velocity that is there will be = 2g z 2 or v 1 is 2g z 2. So, this is the velocity or if we know the velocity there we can estimate z 2 that is how the water is going to rise in the stagnation tube.

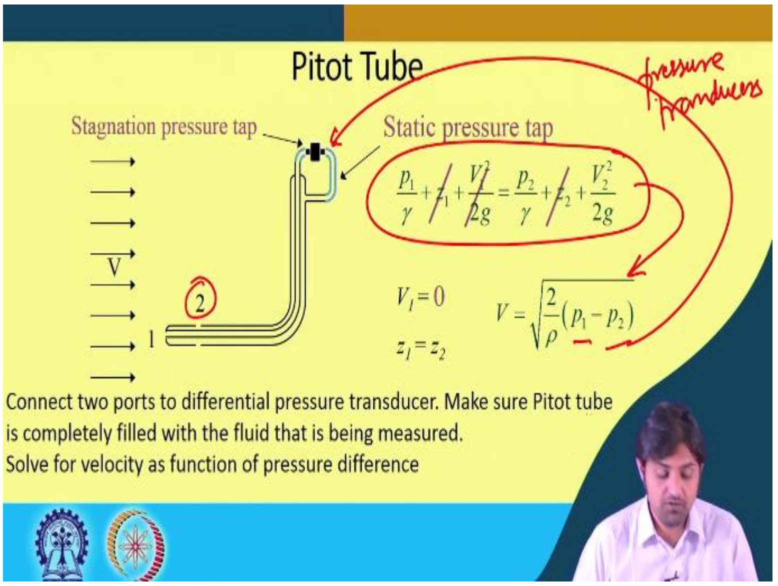
(Refer Slide Time: 21:53)



Now, there is another thing pitot tube, which is used to measure the flow velocities in an aero plane as well. So, this is how a pitot tube looks like. It is used to measure air speed on aero planes. It is very old technique but still very successfully this is used for measuring the air speed on an airplane. It can connect differential pressure transducers to measure the V square / 2g because Bernoulli equation has V square / 2g. So, what it does is measures V square / 2g and therefore tells the airspeed.

It can be used to measure the flow of water in pipelines also pitot tube. So, this is something like this, it looks something like this, in an aero plane, so this is fixed in the aeroplane. So, this is, it shows this how many knots are there, I mean, the typical panel of the aero plane speed. An important thing to note is, this is a point measurement because Bernoulli principle is applied at different points along a stream line this is important to note.

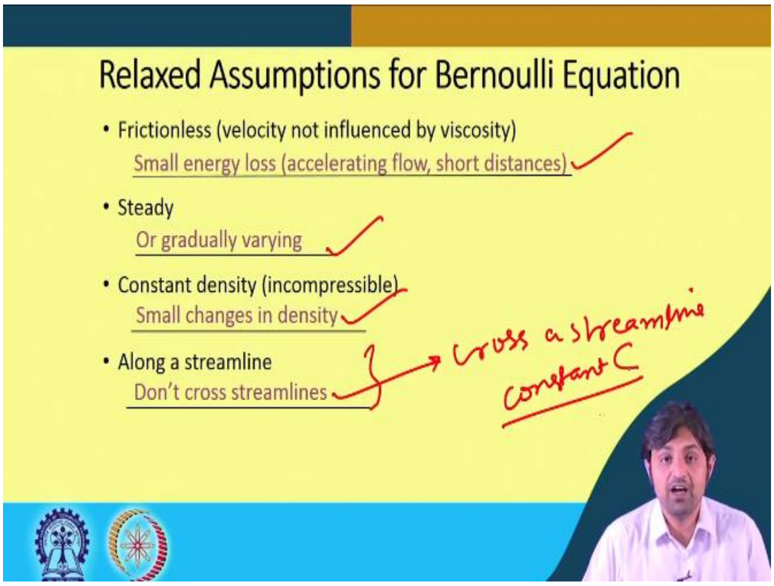
(Refer Slide Time: 23:28)



So, pitot tube, so, there is a velocity V, there is 0.1 here and 0.2 here. So, connect 2 ports to a differential pressure transducers. Make sure that the pitot tube is completely filled with the fluid that is being measured. Solve for velocity as a function of pressure difference. This is stagnation pressure tap, this is static pressure tap. So, what it does is it measures pressure at 0.1 at 0.2. And so, the Bernoulli equation says p 1 / gamma + z 1 + V 1 square / 2g is constant. So, here, z 1 is, you know, is 0 at 0.1.

We assume, that is the datum and because it is a stagnation point, the velocity at 1 will also be = 0. Because this is also at the point 2 is also here, where datum is 0, that z 2 is also = 0. So, V 1 is 0 and z 1 = z 2 that is why we have done this. So, V that is measured here is a difference of pressure and is equal to V is equal to using this equation we get this and p 1 and p 2 are measured using this pressure transducers. So, this is how the pitot tube works or functions.

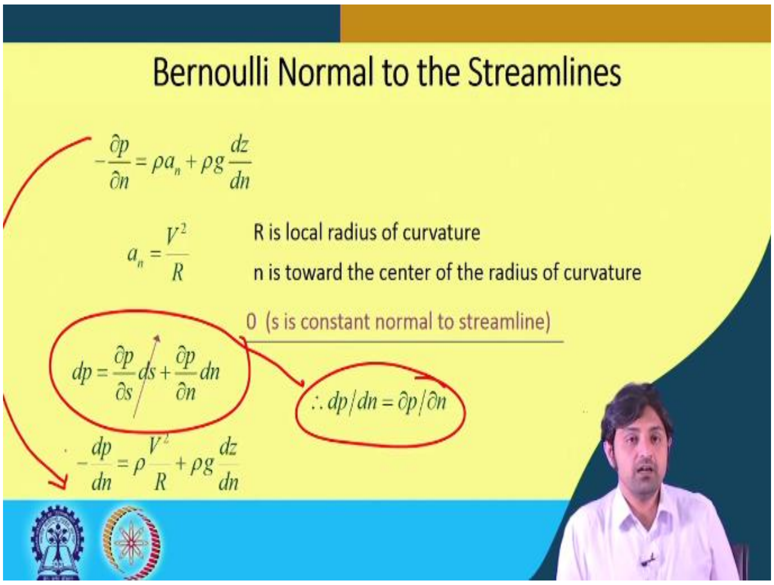
(Refer Slide Time: 25:28)



So, there, I mean, there are some relaxed assumptions for Bernoulli equation, I mean, ideally but Bernoulli equation should be applied for the assumptions that we had discussed a couple of slides ago. But there could be some relaxed assumptions for Bernoulli equation. For example, frictionless, that the velocity is not influenced by viscosity, so, there is if the flow is not frictionless there will be some energy loss accelerating flow.

For example, at short distances there we can apply a steady flow or we can also use gradually varying flow. Constant density incompressible, so, if we have very small changes in density then also we can apply Bernoullis equation, it will not be the exact, but approximately equal and we have to use it along a stream line and but this cannot be relaxed, we cannot cross a stream line because the constant C will change and therefore, Bernoullis equation will no longer be valid.

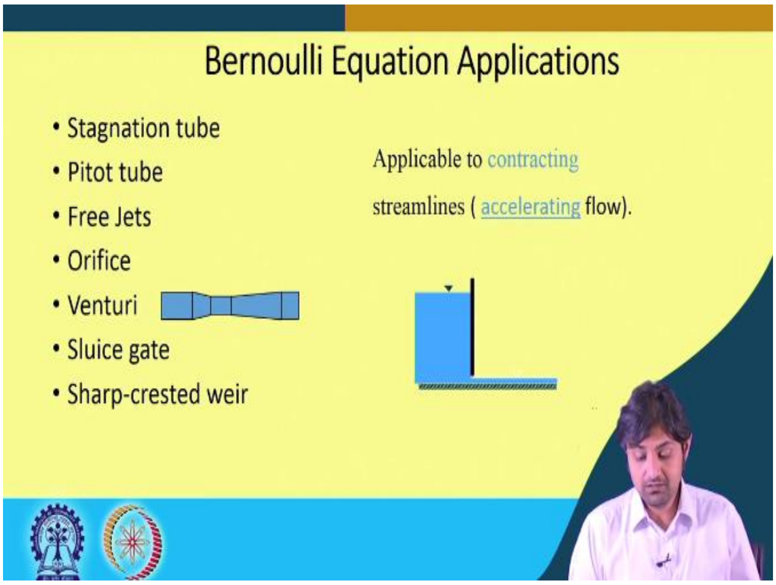
(Refer Slide Time: 26:54)



So, another concept is Bernoulli's equation normal to the stream lines. We have seen along the streamline, now the other is normal to the stream line. So, we have seen this in the previous, you know, when we are dealing with the Bernoulli's equation along the stream line. So, similarly, we can write minus the force equation we can write, - delta p / delta n + rho a n + rho g dz dn. Here, a n is V square / R, where, R is the local radius of curvature, n is towards the center of the radius of curvature and this is equal to 0, s is constant along the stream line.

So, therefore, proceeding in a similar way, we obtain dp = del p del s into ds + delta p delta n into dn, you remember, we had the same along the stream line ds = 0 because and we can write dp dn = del p del n. So, what happens is we get - dp dn = rho V square / R + rho g dz dn because we can write delta p delta n is equal to and we obtained this from this, so, this equation becomes this, the same procedure, no changes whatsoever.

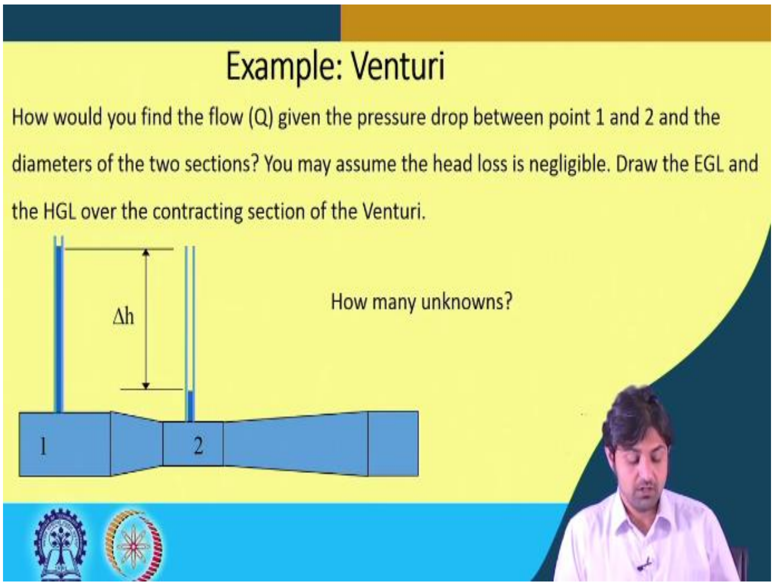
(Refer Slide Time: 28:45)



So, this is the equation we are not going into too much detail about this, but this is how you can have the Bernoulli equation normal to the streamlined. So, you see when you integrate this, this will have a different consequence. So, now the Bernoulli's equation applications; 1 is stagnation tube that you have already seen, the other was pitot tube which we discussed, the free jets equation we have seen.

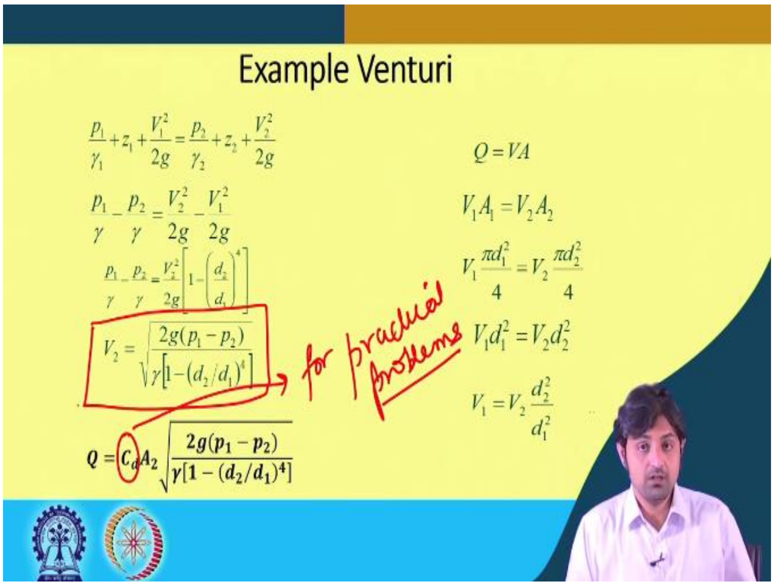
We have not seen orifice, but orifice is one of the other areas where Bernoulli's equation is there. Venturi meter, the Bernoulli's equation is applied across sluice gate and sharp crested weir, this is how a Venturi meter looks like, this is how a sluice gate. So, these actually are all, you know, practical places where Bernoulli's equation can be applied. So, I will take this one and these are applicable to the contracting streamlines accelerating flow.

(Refer Slide Time: 30:24)



One of the examples as I said, we had not seen was venturi meter, how would you find the flow Q given the pressure drop between point 1 and 2 and the diameter of the 2 section? You may assume the head loss is negligible. Draw the EGL and the HGL over the contracting section of the venturi metre? So, this is the venturi meter, the water will rise depending upon the pressure. How many unknowns what equations are you going to use? Those are the questions.

(Refer Slide Time: 30:54)

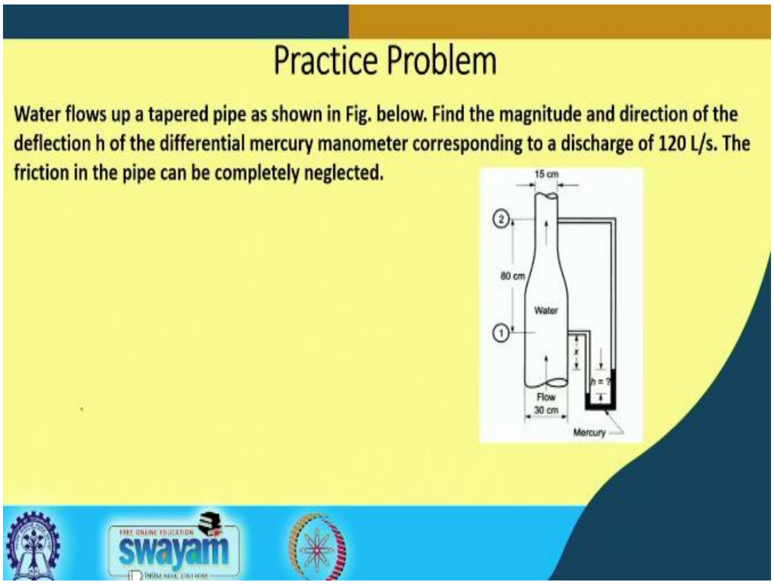


So, the first thing that we can do is, we apply at points at 2 points we apply the Bernoulli's equation. So, we take the pressure p 1 p 2 on one side z 1 = z 2, so, that will be p 1 / gamma - p 2 / gamma = V 2 square / 2g - V 1 square / 2g. As you see the point 1 and 2 the datum is same that is why we were able to cancel this 1 out and Q = velocity into area V 1 A 1 = V 2 A 2. So, V 1 pi d 1 square / 4 = V 2 pi d 2 square / 4. So, V 1 d 1 square = V 2 d 2 square just continuing with this thing, so, V 1 = V 2 d 2 square / d 1 square.

Normally, we know d 1 and d 2. So, there is a relationship obtained between V1 and V2. So, putting V1 in terms of V2, so, we can instead of V 1 obtained from here, we can put here and therefore, we can get this equation here, you know. Therefore, on writing V 2 we can get 2g p 1 - p 2 gamma 1 - d 2 / d 1 to the power 4 and Q simply we multiply this with V 1 A 1.

We have this account this term Cd, that is, for, you know, practical problems and I know practical problems. There is a coefficient that you determine in experiments but anyways this is the theoretically this is the main method p 1 and p 2 you can obtain from how much the pressure water has risen into the.

(Refer Slide Time: 33:03)



So, this is the question, water flows up a tapered pipe as shown in figure below. Find the magnitude and direction of the deflection h of the differential mercury manometer comprising to a discharge of 120 liters per second. The friction in the pipe can be completely neglected. I think we should stop the class for today now, and resume our next class by solving this practice problem. Thank you so much. See you in the next class.