Hydraulic Engineering
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Lecture # 63
Introduction to wave mechanics (Contd.)

Welcome back student to the last lecture of this module and also the last lecture of this course of hydraulic engineering, it has been a pleasure interacting with you. So, to go ahead and get started with this topic of pressure distribution under progressive waves.

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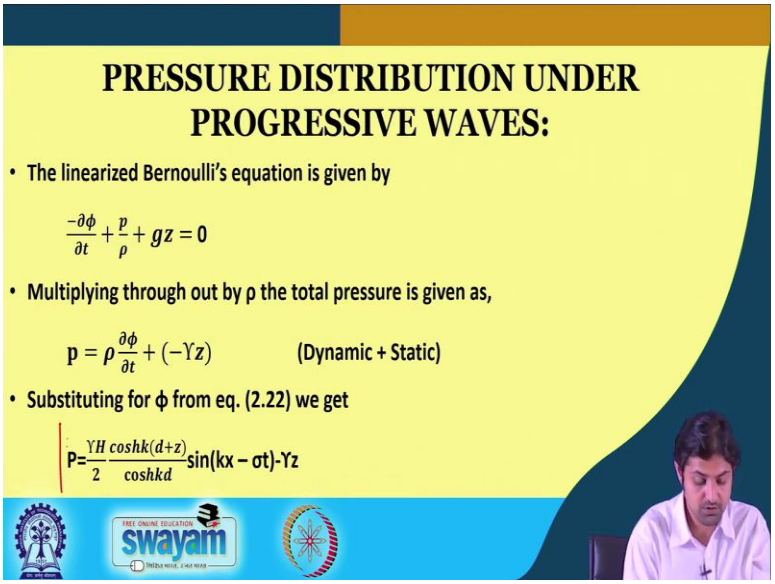

So, you remember we talked about linearized Bernoulli's equation, this equation is given by - del phi del t + p by rho + g z half rho squared + half u squared + w square we that term we avoided because of linearization. Now, if you multiply this throughout by rho the above equation, then the pressure can be given as you see, if we take beyond the other side it can be written as a rho del phi del t -.

So, let me take this so, we have multiplied by rho so it becomes - rho del phi del t + p because rho gets canceled here + rho g z that and this can be written as gamma. So p can be written as if this goes on the other side, this becomes rho del phi del t and this one becomes - gamma is it and this is the same equation here. So becomes p = rho del phi del t - gamma z or + of - gamma z said this is the dynamic pressure.

And this is the static component depending upon only the water depth now, if we substitute for phi was the velocity potential in this equation, what do we get? p will be so del phi del t will be written as gamma h by 2 into cos h k d + z divided by cos h k d into sin k x - sigma t - gamma z so this is the pressure equation.

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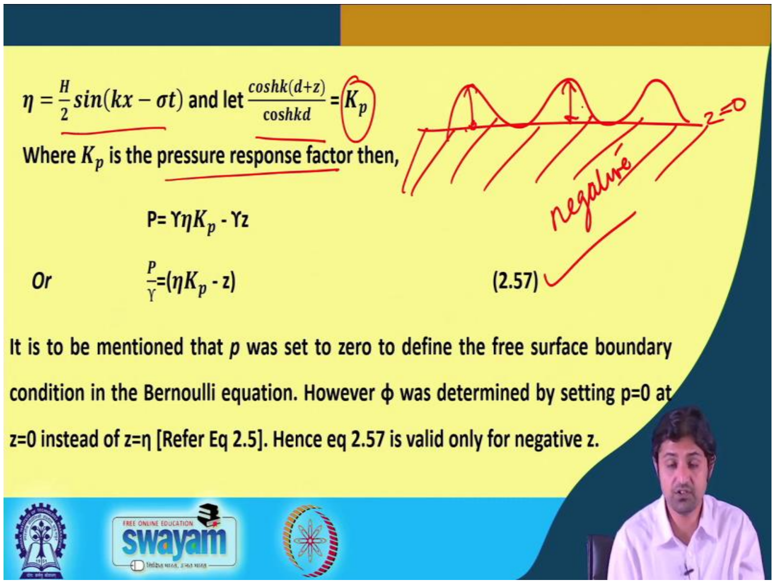

We already know that eta can be written as H by 2 sin k x - sigma t and let us assume that cos h k into d + z divided by cos h k d. If we write as K p substitute then the p this one can be written as see gamma H by 2 this will be K p or H by 2 sin k x - sigma t is eta. So, we can write and K p we say that K p is pressure response factor, then we can simply write p as gamma eta into K p - gamma z or p by gamma can be return as eta K p - z.

Now, it has to be mentioned that P was set to 0 to define the free surface boundary condition in Bernoulli's equation if you remember, however, phi was determined by setting p = 0 as is that = 0 instead of z = eta. Hence this means this particular equation is valid only for negative z that if you want to suppose this is the you know freeze this is the you know freezer. So, this is that = 0. So, the above equation is valid only in this region below. We cannot apply this here in this domain the pressure no, because it was because we had derived phi with certain approximations assumptions.

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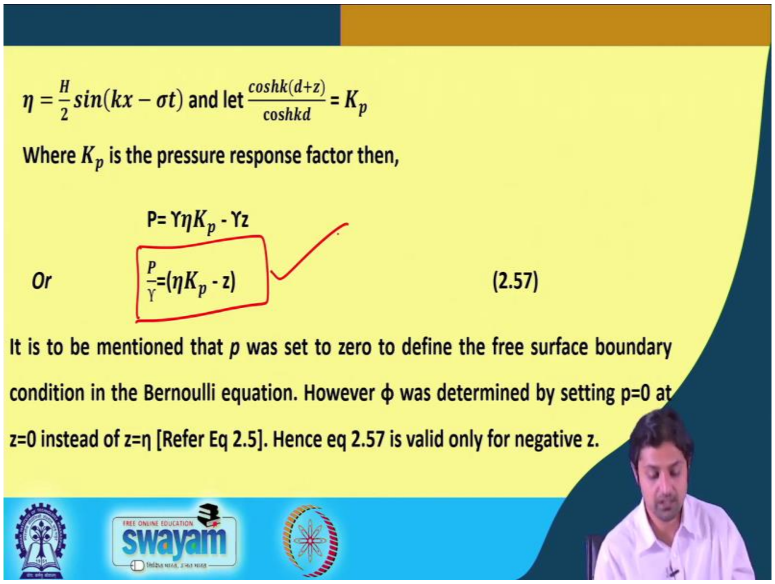

So, p by gamma is eta K p and I think this is one such equation which should be remembered K p is cos h k d + z divided by cos h k d and eta we already know it is a sin k x - sigma t or H by 2 sin k x - sigma t.

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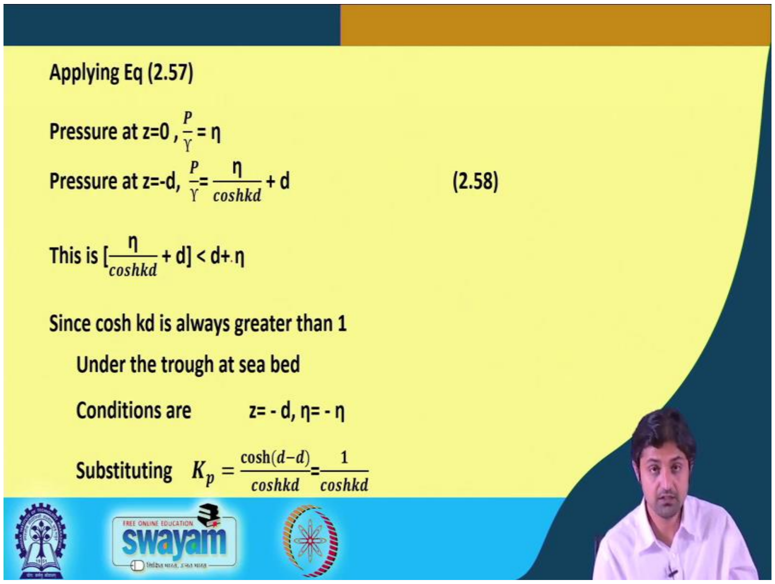

Now, if you apply this equation 2.57 so, pressure at z = 0 is going to be p by gamma = eta and pressure at z = - d so, this is free surface and this is at bottom z = - d at bottom. So, this means, if you n by cos h k d + d we know that it is definitely less than d + eta because cos h k d is always greater than one since cos so, under the trough at sea bed the conditions are z = at trough wave trough conditions are z = - d and eta = - eta. So, if we substitute that K p is going to be cos d - d divided by cos h k d or 1 by cos h k d.

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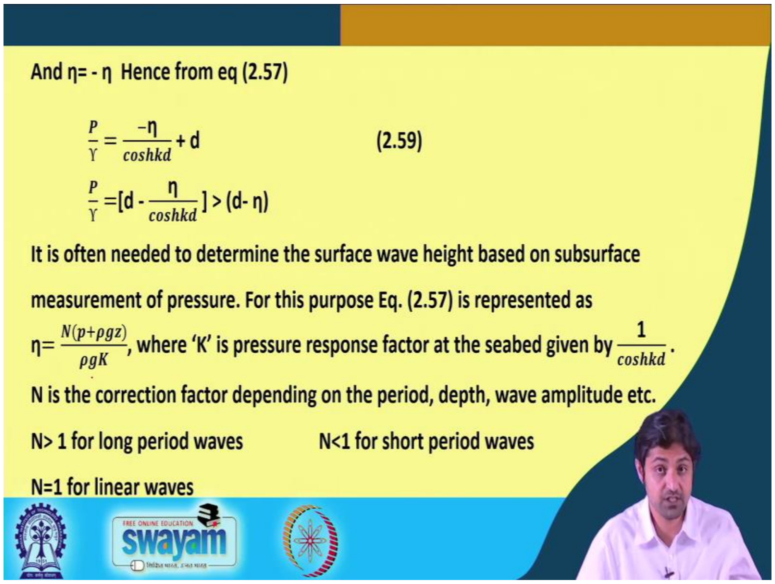

And n in eta = - eta therefore, we will get p by gamma = - eta by cos h k d + d which means that p by gamma is d - eta by cos h k d will always be greater than d - eta because this is greater than one it is often needed to determine the surface wave height based on sub surface measurement of the pressure. So, for this message for this purpose equation 2.7 can be represented as. So, if we have sub surface measurement techniques available.

So, we can actually you know calculate the n where n is the correction factor depending upon the period depth wave amplitude sector this is for the people who are interested more in practical. So, n is has been found that should be greater than one for long period waves and n should be less than one for short period waves and n = 1 for linear waves as we have seen already.

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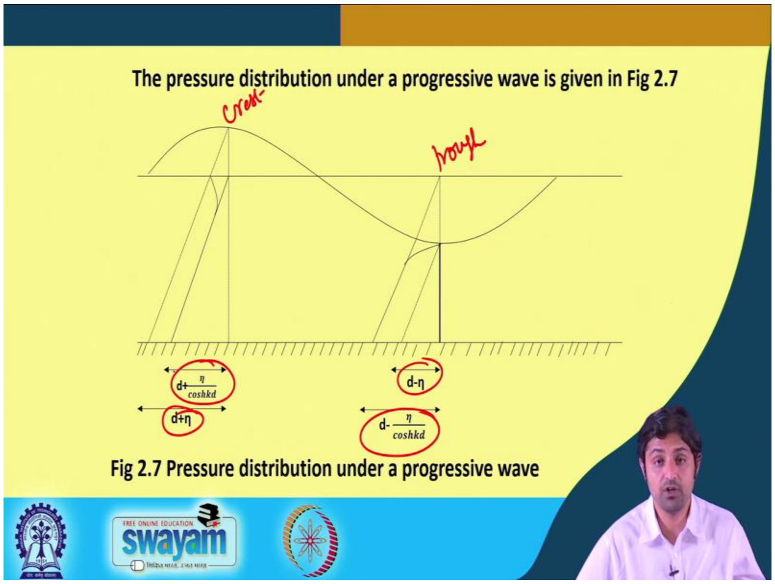

So, the pressure distribution under progressive wave is given by this is crest wave crest, so, pressure distribution under wave crest and pressure distribution wave trough to this is the pressure distribution under a progressive wave. So, these terms is important d + eta d - eta by cos h k d - eta and d +. So, in questions you can be given for example, this curve and asked to label water what is this quantity and other things.

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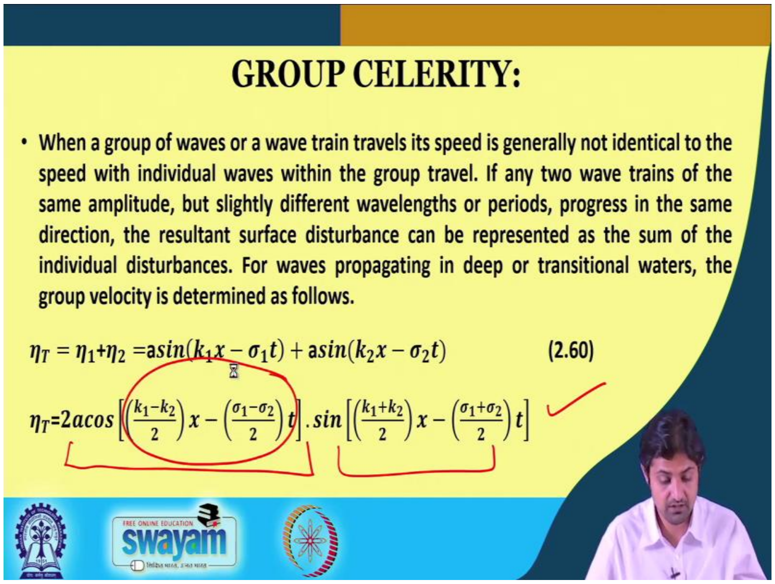

Now, there is something called group celerity, a very important concept. So, when a group of waves or a wave train travels its speed is generally not identical with the speed of the individual waves. So, if there is a group that is traveling, its speed is not going to be the same as it was traveling alone analogy is if you run alone and if you run along with the group of people taking everyone together the speed will be different.

So, if any 2 wave trains have the same amplitude, but the same but slightly different wavelengths or periods progress in the same direction, the result in surface disturbance can be represented as the sum of individual disturbances based on the principle of superposition, so for wave train getting in deeper transitional water the group velocity is determined as follows. So, as I said, the superposition of phi I mean eta 1 and eta 2 says we can simply write combined eta, T is eta 1 + eta 2 = A sin k 1 x - sigma 1 t + a sin k 2 x - sigma 2 t.

Because we have assumed that the same almost same amplitude different phases. So, using the trigonometry we write it is 2 a cos of k 1 - k 2 by 2 x - sigma 1 - sigma 2 by 2 t sin of this term, this is what we get using the trigonometry. So the left hand side is k 1 - k 2 by 2 x - sigma 1 - sigma 2 by 2 into t.

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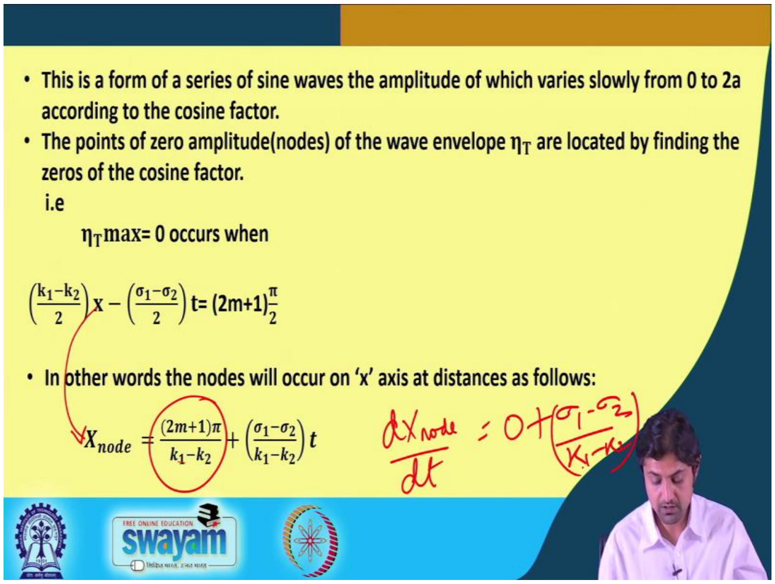

So, this is a form of a series of sin waves the amplitude of which very slowly from 0 to 2 way according to the cosine factor. So, we say this is the amplitude of the wave and this is the phase and the amplitude varies from 0 to 2 or depending upon the, what the phase is here. So the point of 0 amplitude or nodes, because it will very slowly from 0 to 2 and there will be 2 as well they will be 0 as well. So, the point of 0 amplitude are called nodes. They can be located by finding the 0s of the cosine factor.

Because if this is 0 that is eta T max = 0 occurs when k 1 - k 2 when this the term within the cos is pi by 2 or a multiple of pi by 2. In other words, the nodes will occur on x axis at a distance as follows. If you see you try to do that if you rearrange we can get x node here = x is here, just rearrange it will be 2 m + 1 pi by 2 2 will get cancelled from each side it will be k 1 - k 2 + sigma 1 - sigma 2 by k 1 - k 2 into T.

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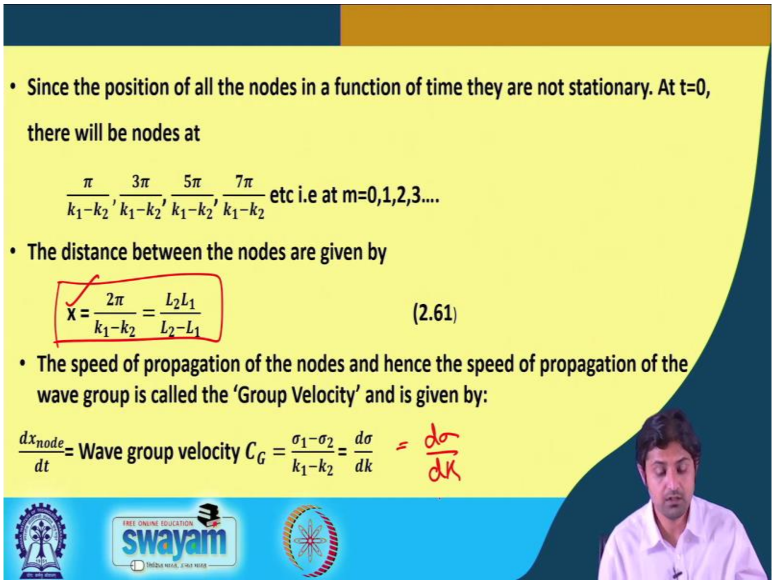

Since the position of all nodes in the function is a function of time and they are not stationary at t = 0 there will be nodes that so as we see the equate this at time t = 0. So, we will see there will be nodes at pi divided by k 1 - k 2 divided and then there will be node at 3 pi divided by k 1 - k 2 if we start putting m = 0123 and the distance between the node is given by 2 pi by divided by k 1 - k 2 or L1 L2 divided by L2 - L1.

So, you can be asked to find out the distance between the nodes and this is an important result that you can remember the speed of propagation of the nodes and hence, the speed of propagation of wave group is called the grief groups already so, what we have obtained is we have obtained in the distance between the nodes. So, now, if we find out the speed of these nodes, we can find out the speed of the propagation of the wave group and this particular velocity is called as group velocity.

So, if we find the x node by dt is the wave group velocity CG. So, if you go to see x node what was x node was here and if you differentiate the x node by dt, this term on differentiation will be 0 + sigma 1 - sigma 2 divided by k 1 - k 2. And this is what is told here sigma 1 - sigma 2 divided by k 1 – k 2 or d sigma by dk.

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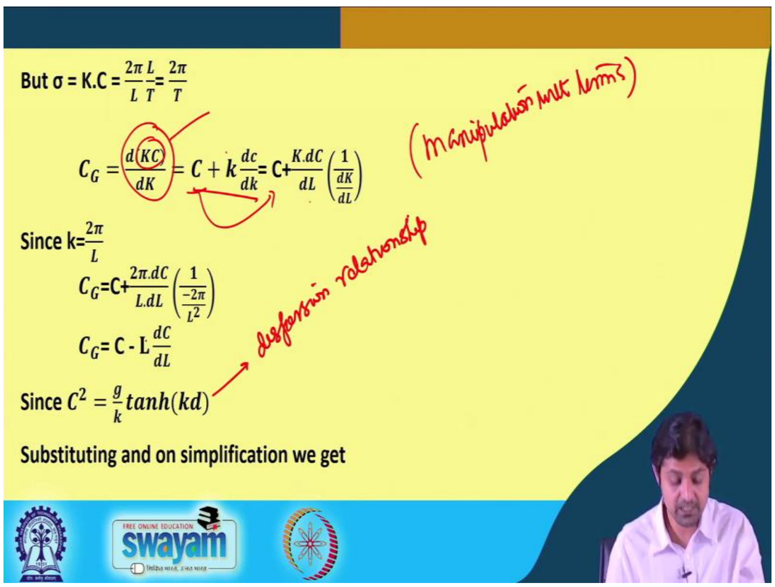

But, sigma is what K into C, KC, sigma can be written as K into C 2 pi by L into L by T. So, CG will be d instantly instead of sigma we write K into C and if we differentiate this we are going to get so differentiation by no I mean if there are differentiation of a b a b is ad db dt + b + a + b da dt. So, we have used this rule change rule and we often c + k dc dk C remains C here and K in So, what we do we write k remain same instead of dc dk we write dc dl into 1 by dK dL.

So, we are actually doing some manipulation here with terms here. And since k = 2 phi by L CG will be C + kV write to phi by L dC by dL into 1 -, so, into 1 divided by - 2 phi by L Squire or CG = C - L dC dL. Since C squared = g by K tan hkd from dispersion relationship and if we substitute this C here and do dC by dl as well.

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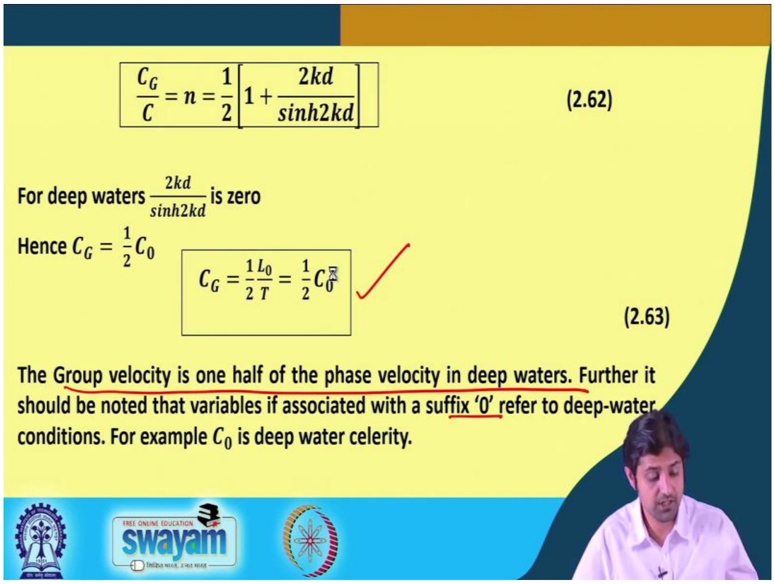

We will get CG by C = n, where n is half into 1 + 2 kf divided by sin h2kd. So the derivation you do not have to worry about that much but this equation is important or CG can be written as group velocity will be C by 2 into 1 + 2 kd divided by sin h2kd this equation is very important. Do remember you must remember this equation so for deep water 2 kd divided by sin h2kd is 0. Therefore CG group velocity is C not by 2 C not is, these were solidity of a single wave. An important result that the group velocity in deep water is wave velocity of similarity by 2.

So, group velocity is one half of the phase velocity in deep waters. Further it should be noted that the variables associated with suffix 0 refers to deep water condition, wherever we have ref put 0 that means deep water condition. Andy Porter celerity you already remember 1.56 times the time period.

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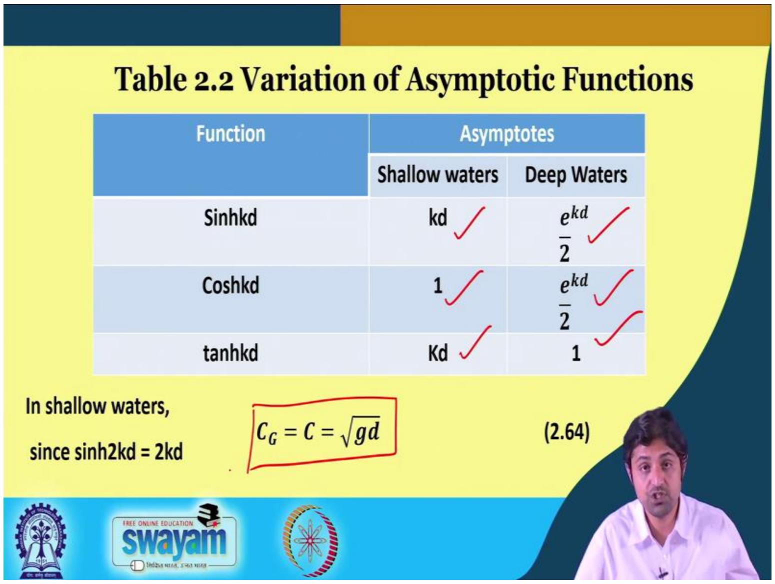

There is a table that shows the sum total variations so sin hkd in shallow water this kd whereas in deep water it is e to the power kd by 2 cos hkd in shallow water is none and cos hkd in deep water e to the power kd by 2 tan hkd kd in shallow water and deep water it is one this we have already discussed once before also in a different format but yes this is a thing so in shallow water sign h2kd becomes 2 kd. Therefore CG is going to be you look at this equation sin h2kd will be 2 kd so this 2 kd gets canceled. So it will be 1 + 1 to 2 into half of so CG will become CG will become C and that equal to under route gd.

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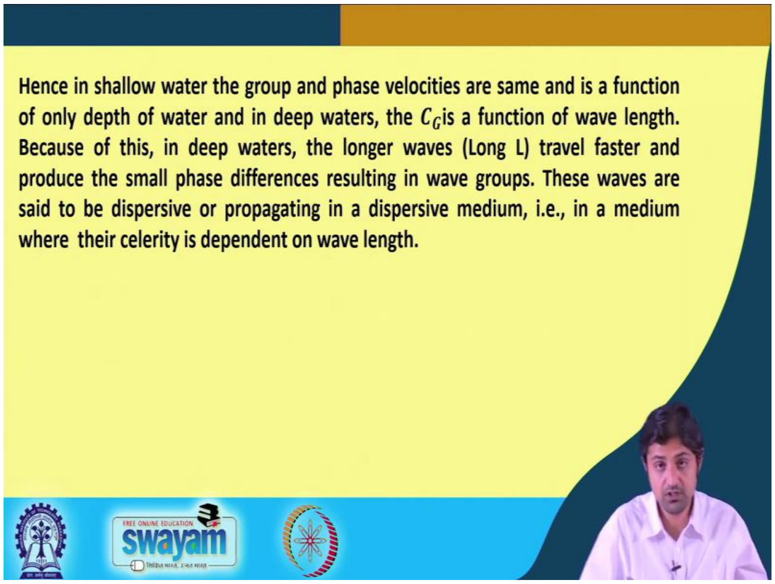

So in shallow water the group velocity same as the face velocities all this celerity of the wave. So deep water it is C not by 2 whereas CG = C shallow waters and in for the normal for the intermediate water you can use this equation C = CG = C by 2 into 1 + 2 kd / sin h2kd. This is actually for all the waters but intermediate waters, because deep water in shallow water we have some approximations, but intermediate waters we can use the same formula.

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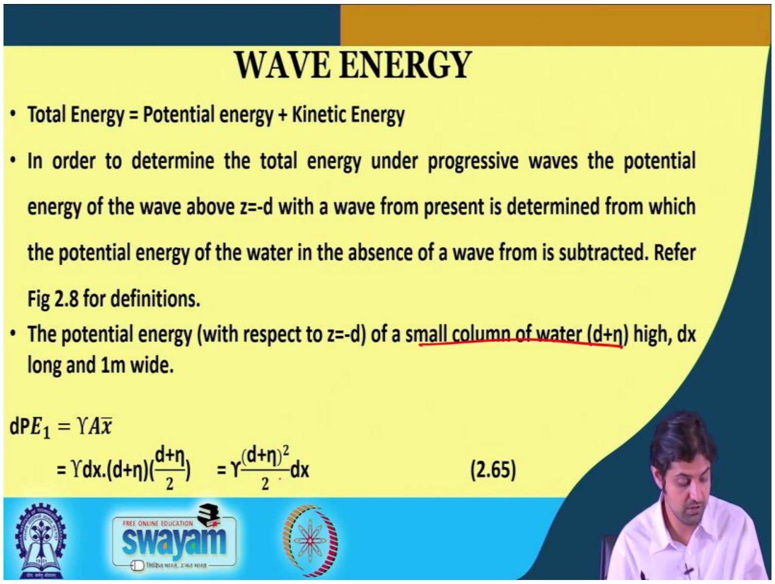

Now what is wave energy? We have discussed about almost everything. So, we have discussed about the particle trajectories then we have discussed about you know this group velocity now we must go towards wave energy. So, total energy potential energy + kinetic energy. So, in order to determine the total energy and a progressive wave the potential energy of the wave above z = -d with a wave from present is determine from which the potential energy of the water in absence of the wave is subtracted.

So, how do we determine total energy and the way we determine the total energy and we total energy or the way when we subtract it with the potential the water in the absence of the waves so, we get the energy in presence of waves only. So, the potential energy with respect to z = -d of a small column of water d + Eta high dx long and 1 meter wide can be written as, so if we have a small column of water d + eta high, where dx is length and 1 meter wide potential energy will be gamma a x bar area will be d + eta into d + eta by 2 into dx.

So, the potential energy will be gamma d + eta whole squared by 2 multiplied by dx. So, the potential energy of a small column of water.

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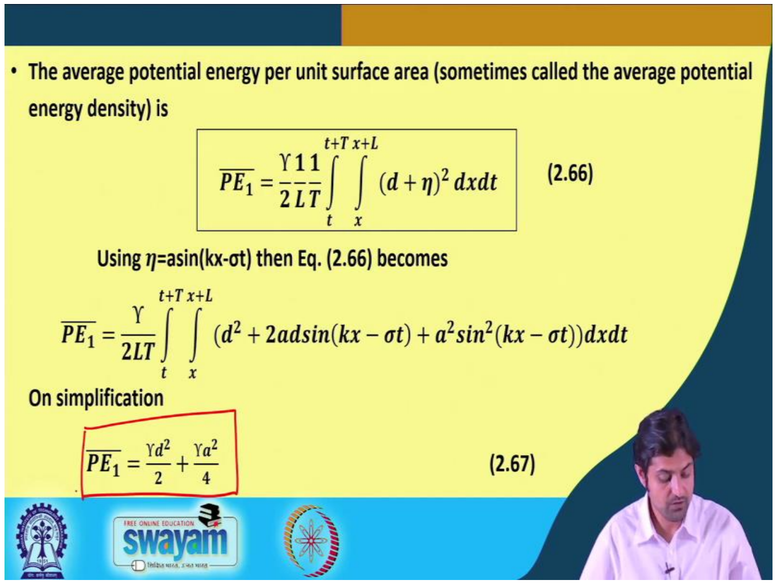

The average potential energy per unit surface area is going to be integration of gamma by 2 into so, we have to average over the whole wave period and also average over the entire wavelength. And using the equation below you see, we integrated and after integration if we use n = a sin k x - sigma t we and then we integrate and simplification we will get potential energy total potential energy is going to be gamma d squared by 2 + gamma a squared by 4.

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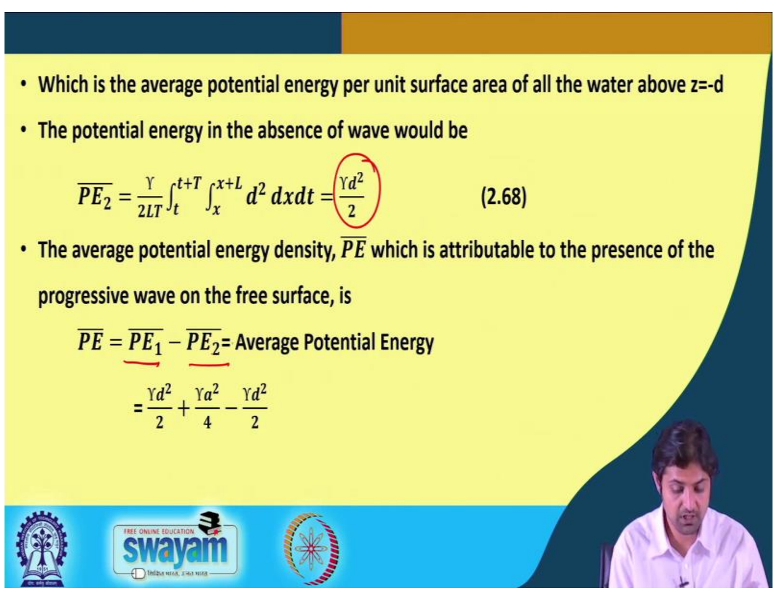

However, this is the average potential energy per unit surface area of all the water above z - z = -d. Now, we should also calculate the potential energy and absence of the wave and the wave is not there, when the wave is not there then there will be no eta in this term you see there was eta here. So, now, this would not be there when there is no wave and we do the same procedure for finding the total potential energy and this comes to be gamma d square by 2.

And the total energy due to the presence of the wave will be the total energy - the energy that are - the energy when the wave is not there. So, this will become the average potential energy this one came to be gamma d squared by 2 + gamma a squared by 4 and this one came to be gamma d squared by 2.

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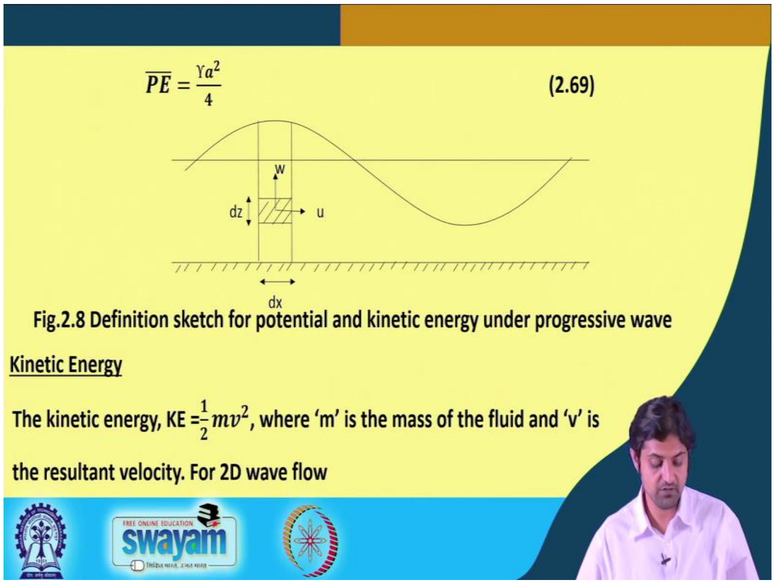

And this gives us the potential energy due to waves is gamma a square by 4 where a is the wave amplitude? You have seen how it is derived, but results are more importantly you must remember the potential energy due to the waves is written as gamma a squared by 4. So, this was the assumption we have assumed, using this small particle we and by integrating over the entire wavelength and entire wave period we have obtained the potential energy.

So, the method was total potential energy with wave - the energy water potential energy without waves gave us due to waves alone. Now we will also need to find kinetic energy it is very simple kinetic energy is half mv squared, where m is the mass of the fluid and v is a result and velocity.

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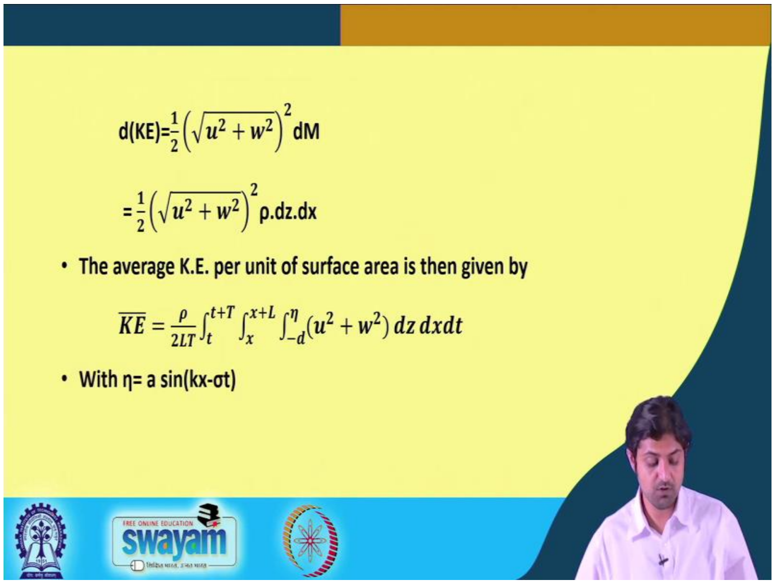

For 2 dimensional wave flow, we know it is half under route u squared + w squared into dM can be written as rho into dz dx for that definition here,, these are dx and under route u squared + w squared, so the average kinetic energy per unit of surface area, if we integrate it over the entire wavelength and wave period will come out to be it is a complex into I mean.

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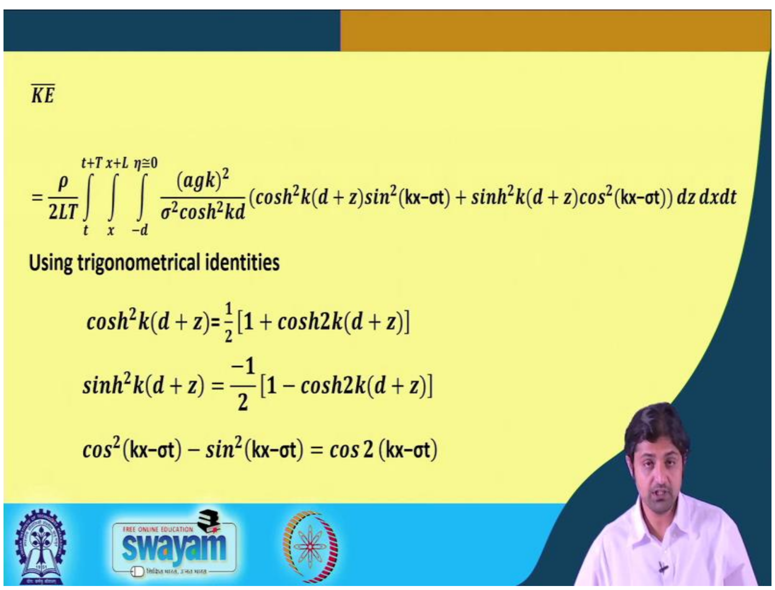

It is so many terms, but on you know, applying the simplification of trigonometry, we can get if we apply these simplifications, you will get these slides so you will have more you know, it will be easy to understand. But more important are the results.

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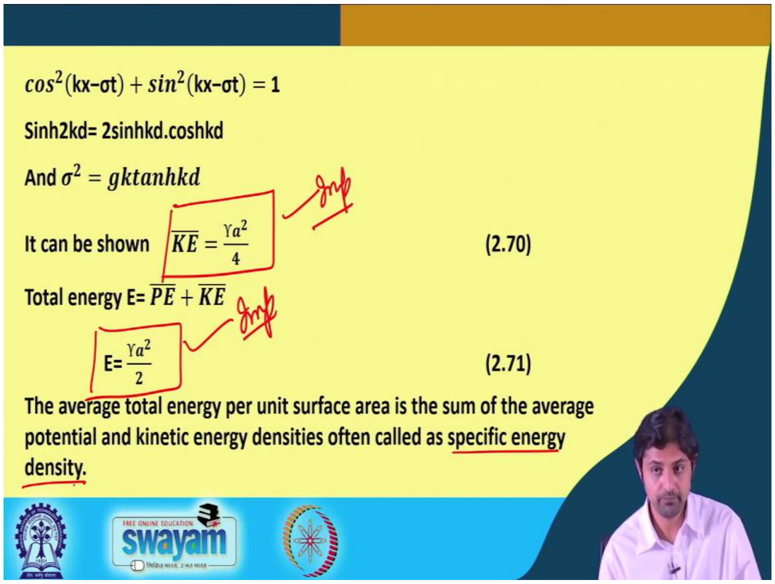

So we can show that kinetic energy is gamma a squared by 4 potential energy was also gamma a squared by 4. Therefore, the total energy is going to be due to the waves alone is potential energy + kinetic energy, the total energy is gamma a squared by 2. So, the average this is an important result again so the kinetic energy value the potential energy and the total energy these equations you must remember gamma a squared by 4 and total energy is gamma squared by 2.

So, the average total energy per unit surface area is the sum of average potential and kinetic energy density is often called as a specific energy density. You can also be asked what is specific energy density for example.

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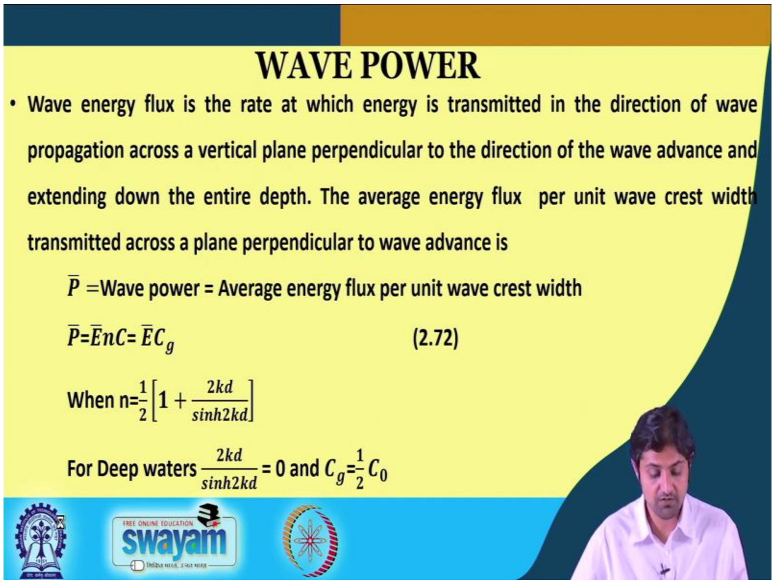

So, now we have studied wave energy it is very obvious that we study wave power wave energy flux is the rate at which energy is transmitted in the direction of wave propagation across a vertical plane perpendicular to the direction of wave advance and extending down the entire. So, the average energy flux per unit wave crest with transmitted across the plane perpendicular to the wave advances is wave power. And it is given as e into CG.

This is important CG is group velocity and e is the energy that we just derived. So, the power is given as e into CG or in if you want to write it in terms of celerity. It is e bar n into CG u did you saw that n was this so, the wave power is nothing you reject. You just need to remember the formulas e bar into CG where e bar was gamma a Squared by 2. So, for deep water you know that CG was half c not.

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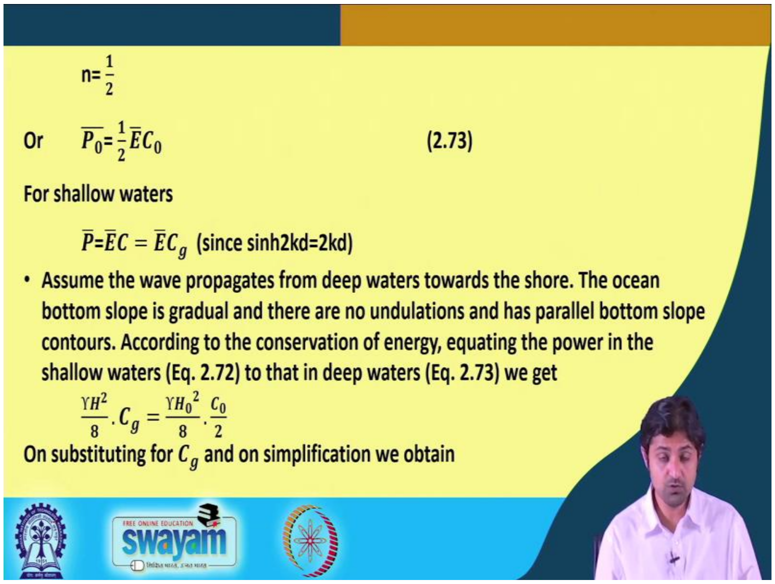

n = half so, power is going to be half into C not for shallow water it will e into CG because CG is C only if you assume that the wave propagates from deep water towards the shore and the ocean bottom slope is gradual and there are no undulations and as parallel bottom slope because, you see the wave power is going to be conserved if the wave moves from 1 depth to the other. Because you see this if there is no loss, it is average energy flux per unit wave crest rate.

And if we apply the conservation of wave power. So, you see this is any depth. So, e is gamma a Squared by 2 = gamma is h by 2 whole squared by 2. So, this becomes gamma h squared by 8 and CG V not. So, with the wave propagates from deep water to let us say any depth whether intermediate or shallow. So, wave power is going to be conserved correct. So, in deep water, it will be gamma h not squared by 8 h not indicates the wave height in deep water.

multiplied by we have already seen that this is going CG is C not by 2 and this will be equal to gamma h squared by 8 this is the way high at any depth where we want to calculate multiplied by CG, this is what this equation is all about this equation so we are conserving power from deep water to any water depth. And if we substitute for CG as C by 2 into 1 + 2 kd by sin h2kd.

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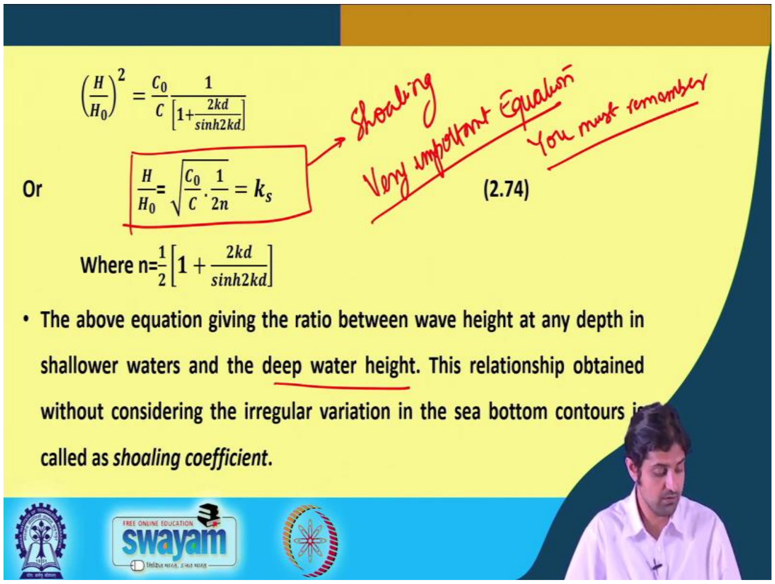

We are going to obtain h by h not = C not by C into 1 by 1 + 2 kd divided by sin 2hkd or we can obtain h by h not = under route of because this is n this is n by I mean this is a 2n. Therefore, h by h not = under or under route of C not by C into 1 by 2 n this is called Ks where n = half into 1 + 2 k d divided by sin h2kd. So, the above equation is very important again this represents of phenomena which is called shoaling. And this gives the ratio between wave height at any depth in the shallower waters compared to the deep water wave height.

And this relationship is obtained without considering the irregular variation in the sea bottom. And this ratios under root of C not by C into 1 by 2 n is called shoaling coefficient or chaos. So, until now, you saw that we had been able to obtain relationship between the water velocities and only kinematics, but this is the only equation that we have studied that if we know the wave height in deep water we will also be able to calculate the height it any depth to z.

So, a very important equation which you must remember the shoaling coefficient and the root C not by C into 1 by and where C not is the, speed in the deep water which is 1.56 times t. So, there are some, you know formulas to remember, but, remember only the simple ones, the ones that I have already told you. So, by equating the wave power, we have been able to obtain the wave height the wave height transformation if the wave propagates from 1 depth to the other depth.

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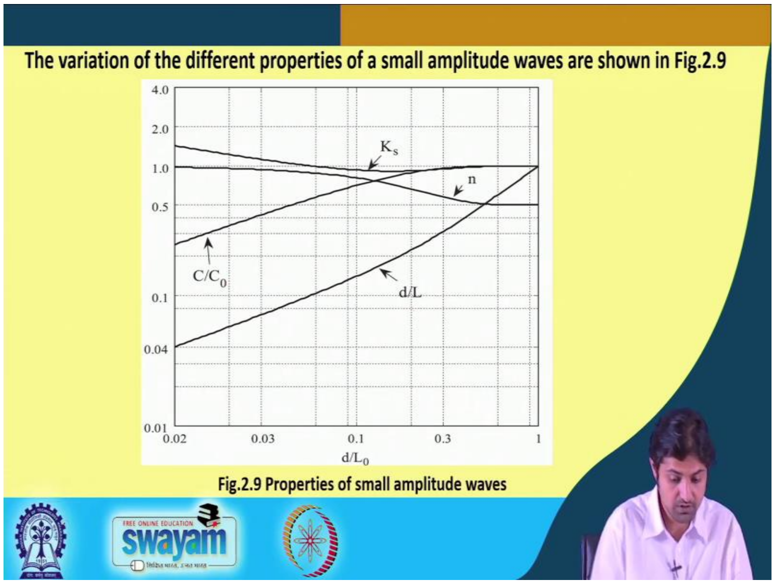

So, the variation of different properties of a small amplitude waves are shown in this figure you do not need to pay too much attention just we have plotted d by L C by C not k s and n which you already know chaos is the shoaling coefficient.

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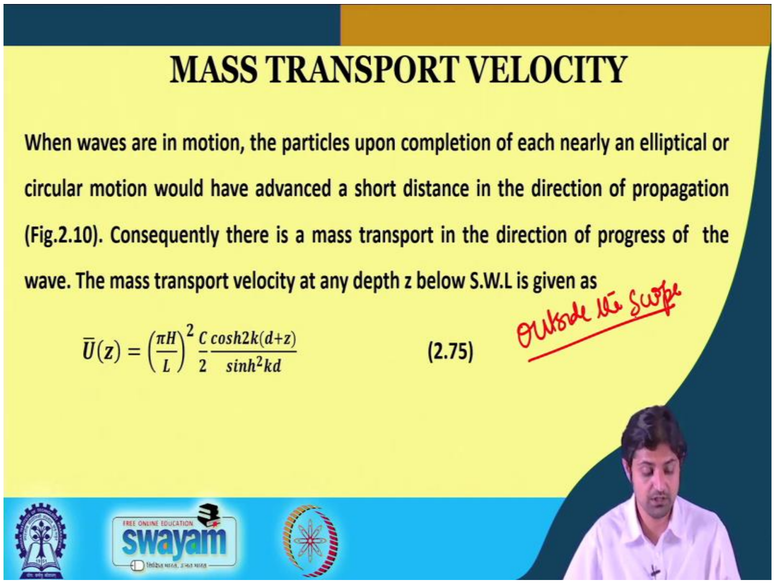

So, this is the last topic the derivation is outside the scope, but what you must understand is that when waves are in motion, the particles upon completion of each nearly an elliptical or circular motion would have an advanced a short distance in the direction of propagation of waves, you can check it from those equations as well. Therefore, if they move ahead that means that the mass associated with them has already moved forward correct. This mass transport is happening in the direction of the progress of the wave.

And the mass transport velocity is given by phi h by L whole squared into C by 2 cos h2kd + z divided by sin h squared kd. So, this is outside the scope formula I do not expect you to you know, remember this particular equation, but more importantly the message is to convey messages that you must understand that the mass transport happens and then the wave motion is there. Of course, you must remember that it is a function of edgy Squared for example, mass transport is the function of h squared or you know is related to how I mean the end of the vein which is proportional to one quantity.

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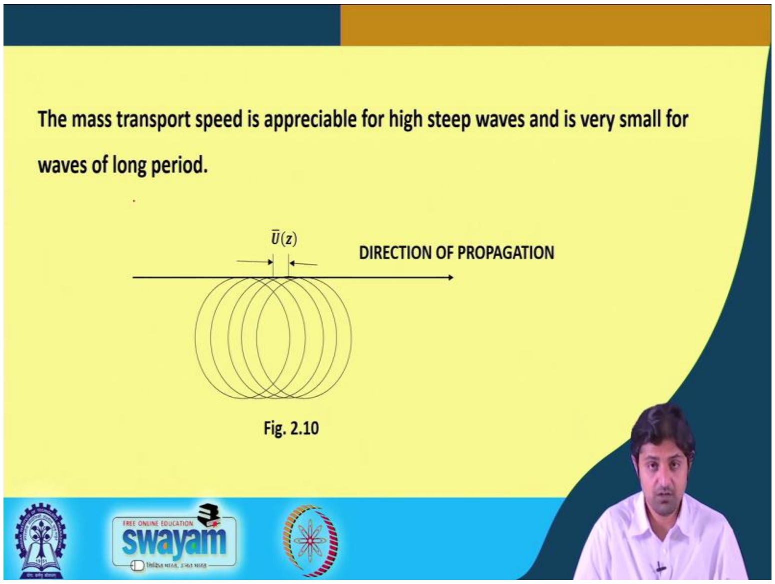

So, the mass transport speed is appreciable for high steep waves and very small for waves of long period you can see this equation. So, if it is a long wave, the mass transport is very less because, so if period is long the length is long. It is the high steep wave which means which is directly proportional to h squared. So, the mass transport will be high for long the higher waves.

So, this was the last topic of hydraulic engineering which we have finished now, and with this I would like to close the lecture and also this course. If you have further doubts, please do contact us and the forum or I will be appearing in some live sessions. Also, I do not know if you have something after the this lecture, but if you have you can send email to my teaching assistants, or also post the question and forum. I wish you good luck for you are assignment for this particular week. And also the final exams. Thank you so much for listening. Have a great year.