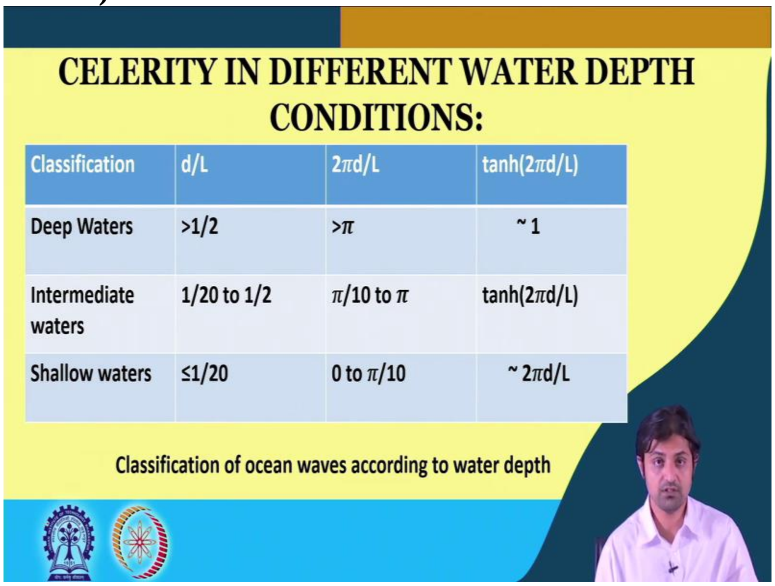
Hydraulic Engineering

Prof. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology-Kharagpur

Lecture # 62

Introduction to wave mechanics (Contd.)

(Refer Slide Time: 00:31)



Welcome back student to the fourth lecture of this final module about inviscid flow or we are going to derive we everywhere actually deriving the equations of velocity potential here. So, to proceed forward, we are going to see we have already derived the dispersion relationship. So, one thing you must remember that the complex derivation you do not need to remember the only thing that you need to remember are the end results.

For example, the velocity potential phi and the dispersion relationship. Now, whatever results we have obtained, we are going to apply in certain situations such as different water depth. So, this is a table, but first without going into too much you know detail first I would like to say there is something which we have said deep waters, we have said intermediate waters and we have shallow waters.

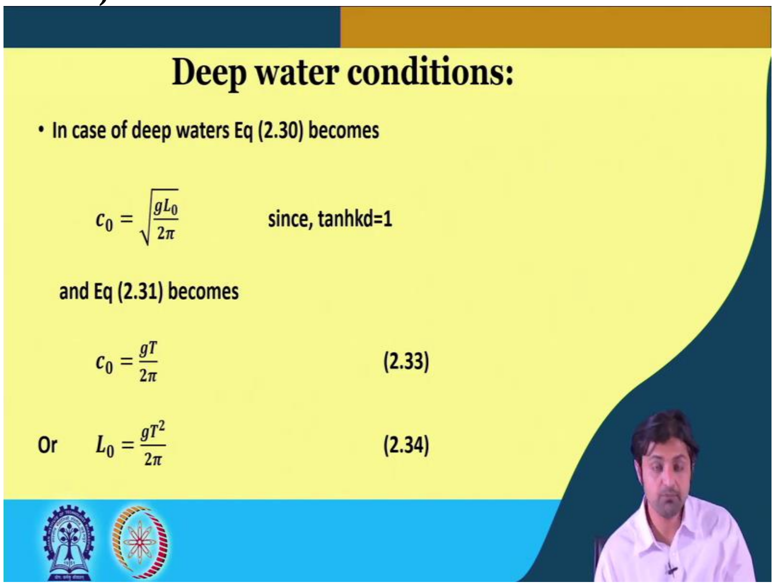
So, what are the definition of deep water deep water means that the depth of the water divided by the wavelength in which the you know the wavelength of the wave if it is greater than half. That means it is deep water if this ratio d by L is less than 1 by 20 or 0.05. Less than 0.05 that the depth of the water and the way the ratio of the depth of the water to the wavelength of the wave it is less than 1 by 20. That means we are in the shallow waters. And what the value in between 0.5 and 0.05 we call it intermediate waters, that is our definition that is how we have defined what a deep water is, and water shallow water and what are the intermediate waters are.

So, if in case of deep water, if d by L is greater than half, then we will have 2 pi d by L will be greater than pi, what you do simply is 2 pi just the d by L is greater than half. So, 2 pi d by L will be greater than 2 pi into half. So, this for Deepwater 2 by l will be greater than pi. Similarly, for intermediate waters this value is going to be 2 pi d by L will be between pi by 10 and pi and for below that in shallow waters.

It will be from 0 to pi by 10 very simple to calculate yourself. So, for this condition when in 2 pi d by L is greater than by then tan h of 2 pi d by L can be approximated as 1 you can use you are trigonometric calculators to find that for shallow waters if 2 pi d by L is 02 pi by 10 we can approximate that is 2 pi d by L So, tan h will go away whereas, in case of intermediate it will remain the same 2 tan h 2 pi d by L.

So, these are 3 values of tan h 2 pi d by L that we get and this is the classification of ocean waves according to the water depth deep water Intermediate water in shallow waters.

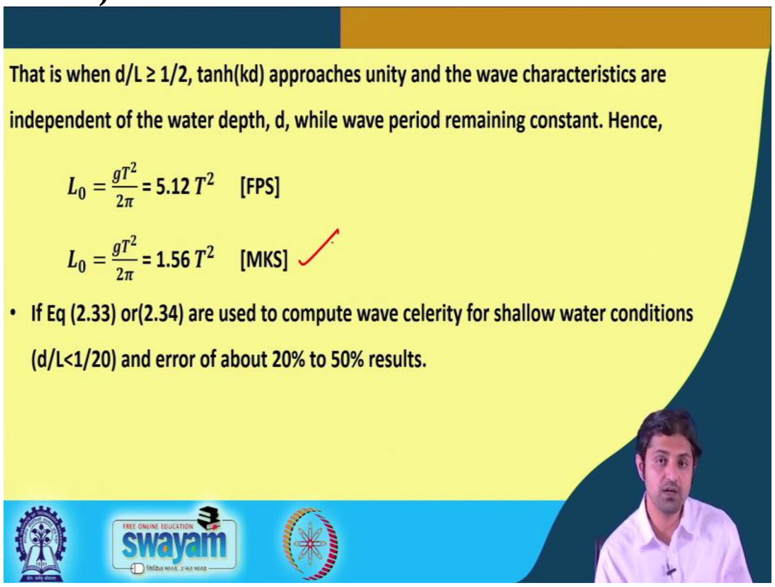
(Refer Slide Time: 04:02)



Let us say the deep water condition in case of deep water equation 2 point 30 so, what was equation number 2 point 30. So, equation 2 point 30 was C squared = g by k tan hkd now we go back to this. So, c not because since tan hkd = 1 c not will become under root gL not by 2 pi. And equation 2 point 31 was we see not becomes gt by 2 pi or a lot can also be written using this equation here, g by 2 pi T squared or we can also write L not is g by 2 pi is 9.8 divided by 2 into 3.14 into T squared are lot can be written as 1.56 T squared.

So, in deep water if we know the time period of the wavelength can be simply derived as 1.56 times T squared a very important result also the wave speed c not gT by 2 pi 1.56 into it T. You see how we have applied the findings into case of deep water.

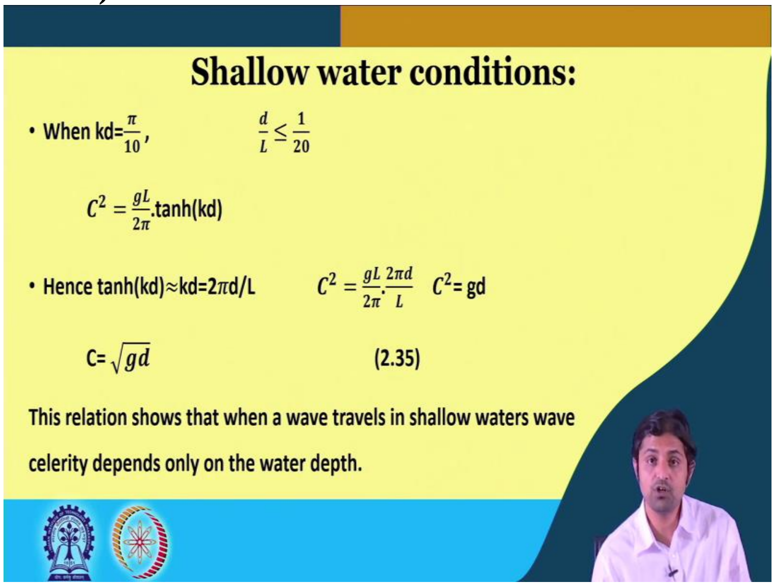
(Refer Slide Time: 05:38)



That is when d by L is greater than half tan hkd approaches unity and the wave characteristic are independent of water depth. One important thing to note is that here the term d does not appear in this equation because the term that contained to d was tan h and this the tan h term goes to work in case of Deepwater. Therefore, all the way of characteristic are independent of the water depth, this is an important result.

So, a lot is in FPS so, this is a different unit within SI unit which is the one that we are going to use will be 1.56 times T squared, this is the meter kilograms second unit SI unit. So, if equation 2.33 and 2.34 are used to compute wave celerity for shallow water condition. So, if we say that we are going to use the same equation for shallow waters, we are going to get some errors of the order of 20 to 50%., which is not good because we need to get the correct value of the velocities in shallow water. Shallow water occurs near the coast for example.

(Refer Slide Time: 06:50)

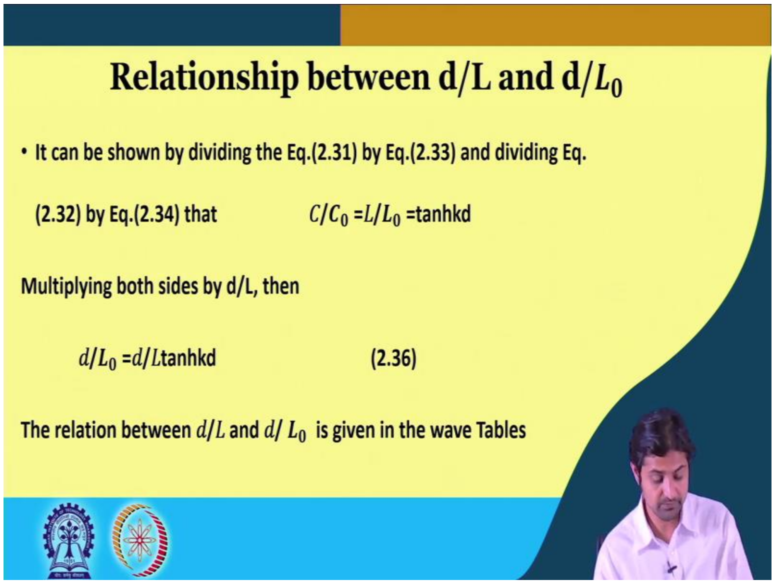


Now, what are the shallow water conditions kd = pi by 10., that is what we said 2 pi d by L that is what we found out from the table. Therefore, c square can be written as gL by 2 pi tan hkd then if k dan hkd can be written as kd = 2 pi d by L. So, this will become C squared will become gL by 2 pi into 2 by d by L right. So, 2 pi 2 pi gets canceled and c squared = gd same as what we have written here So, C will is under route gd.

So, in shallow waters the waves speed will depend on nothing but only the water depth in deep water it was not dependent on the water depth button shallow water, the only thing on which the wave speed depends on the is the water depth if you recall this is so, same equation as we got in open channel flow module remember we got C = under route gd by applying the principles of continuity momentum and energy balance.

So, in shallow waters wave speed is under route gd where gd the depth of the water. So, as I said this relationship shows that when a wave travels in shallow water waves celerity depends only on the water.

(Refer Slide Time: 08:39)



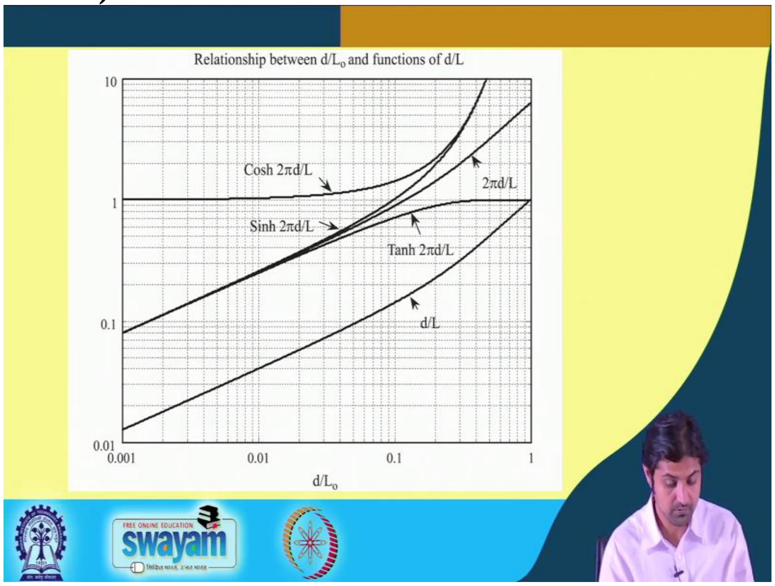
Now, relationship between d by L and D by L not if you divide the equation 2.31 by equation 2.33 and equation 2.32 by equation 2.34 we can simply get so, 2.31 and 2.32 are the equations which are general form 33 and 34 was doubt of deep water. So, if we divide those we can get C by C not = L by L not = 10 hkd. You can try this on your own. Now, if we must multiply both sides by d by L, we can get d by L lot = d by L tan hkd.

Now, the relationship between D by L not and d by L and D by a not is given in wave tables also. So, this is something if you want to calculate there is something called wave tables, which I would not show you in this course. But if you need to solve you can always use trial and error method for solution of the desperation relationship, but most of the question that you will be getting will be in your assignments will be based either on deep water or very simple that you can use the equations like there is present relationship or the velocity potential.

Where a lot of iterations and intermediate water determination would be required, I would not be giving those type of problems for you to solve, because the idea here is that you understand the basics of wave mechanics and therefore, if I give you more questions about intermediate waters, it will only complicated calculations. Therefore, the use of wave tables will not be this is not encouraged.

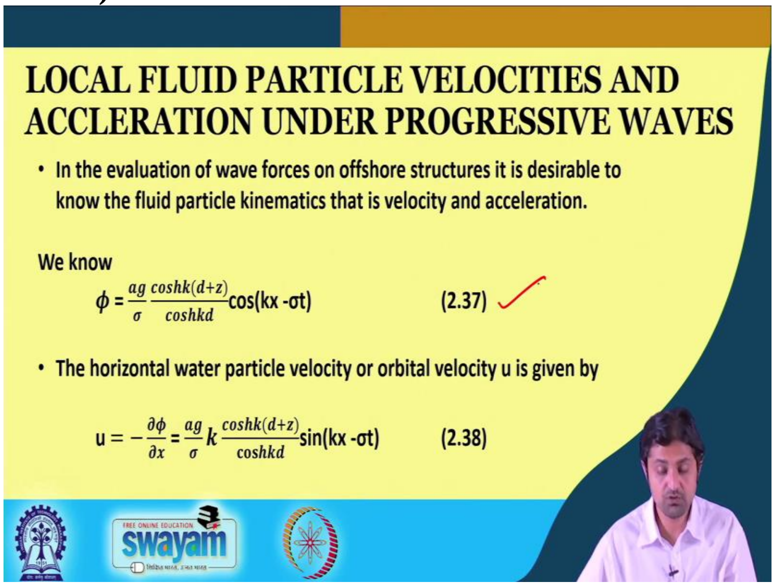
However, I can try to give you questions in your assignment 1 or 2 which will show exactly how to use the iterative formula, iterative procedure. And I can also post some couple of questions in the forum. Where you are going to use those iteration method for solving the wave amplitude or the intermediate water depth properties.

(Refer Slide Time: 11:14)



So, this is the curve which is which denotes the relationship between d by L and D by L not. So, the right hand side then this shows d by L not.

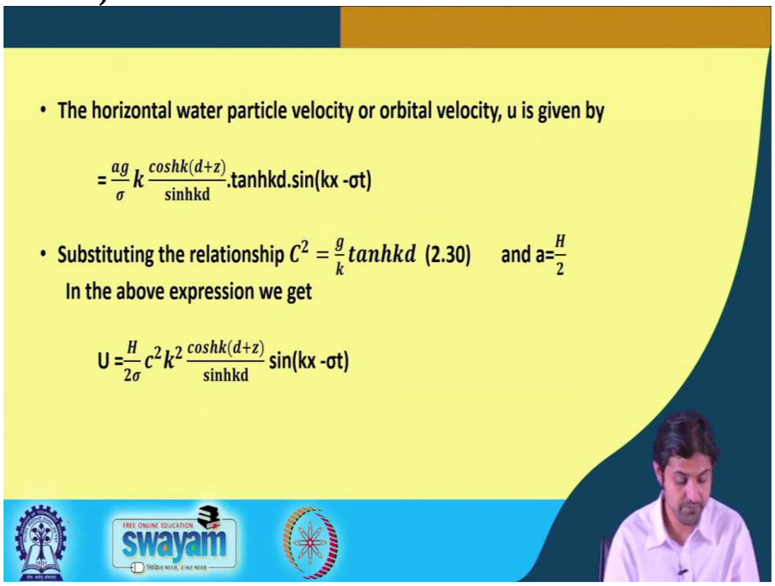
(Refer Slide Time: 11:37)



So, see proceeding forward now, we have found out the velocity potential we have found out the dispersion relationship now it is time to form find out the local fluid particle velocities and acceleration under progressive waves these So, in the evaluation of I mean first more importantly, why do we need to find out these velocities and acceleration because in evaluation of waveform on offshore structures, it is desirable to know the fluid particle kinematics that is velocity and acceleration.

We know that phi = ag by sigma this we have derived this equation. The horizontal particle velocity or the orbital velocity u is given by del – del phi del x, definition of velocity potential, so, we get ag by sigma k into cos hkd + z by cos hkd into sine k x - sigma t.

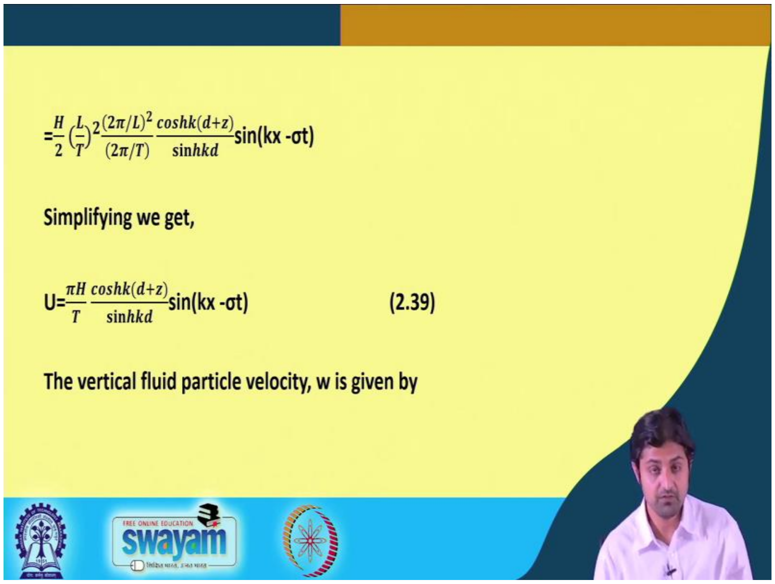
(Refer Slide Time: 12:37)



Whereas, so if we substitute the relationship c squared = g by k tan hkd and small a as H by 2 because this is way of height and this is wave celerity We get u = H by 2 sigma C squared K squared divided by cos hkd represent by sine hkd into sine k x - sigma t u it is enough if you remember the velocity potential equation because that you can differentiate and try to find out the velocities.

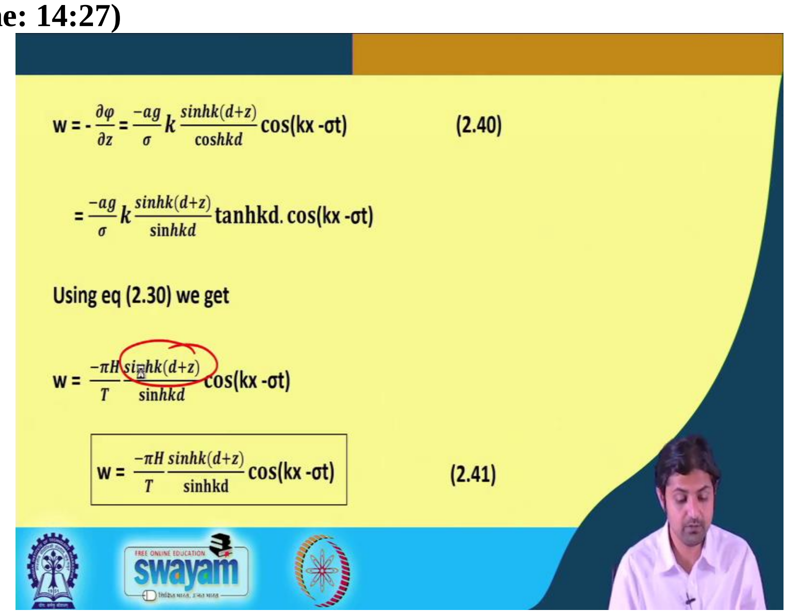
Maximum you will be given, find out the velocity that is that = 0 or at the water depth where is that = - d because though that is pretty simple to calculate or maximum velocity when you talk about maximum velocity, this will be assumed to be one for the maximum velocity case.

(Refer Slide Time: 13:41)



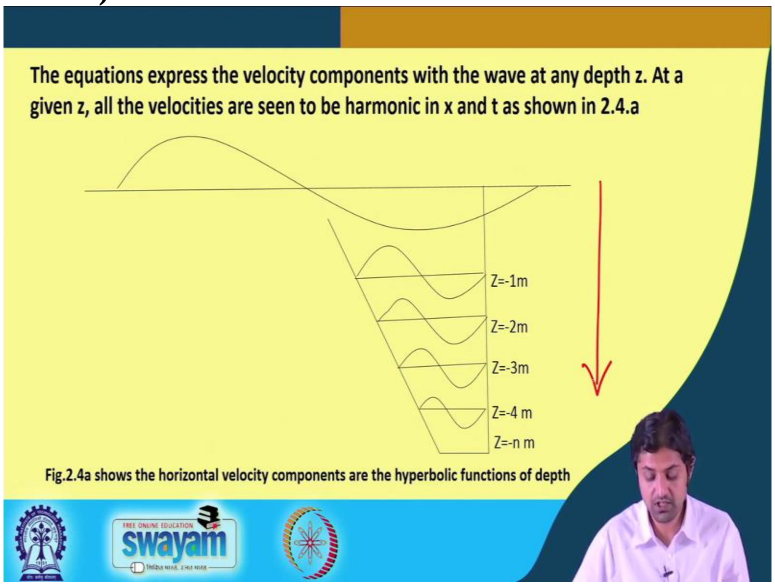
So, just doing manipulation writing in terms of l and t this equation you can remember but not mandatory simplifying you will definitely remember the velocity potential equation and therefore, you can know derive this U even for your assignments and exams, so it is pi H by T cos hkd + z divided by sin hkd into sine k x - sigma t, this is the periodic term periodic term in x and d sine k x - sigma t. And similarly we can find the vertical fluid velocity is given by – del phi del z.

(Refer Slide Time: 14:27)



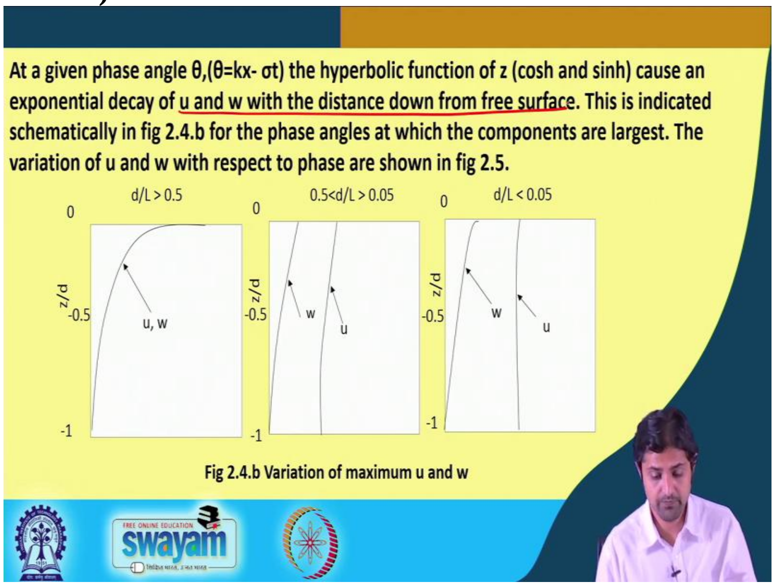
And therefore, we can you know similar substitutions will give us - pi H by T sin hkd + z divided by sine hkd. Again, you do not have to remember this but if you do it is no harm because more importantly it is the velocity potential that you must be remembering. So, w can return as - pi H by T sin hk into d + z divided by sine hkd into cos k x – sigma t.

(Refer Slide Time: 15:10)



So, the equation expresses the velocity components with the wave at any depth z at a given z all the velocities are seen to be harmonic in x and t as shown in figure the lets you know if we plot those its harmonic but with different said you know this is the variation which we are getting if we plot those velocities. So, figure 2.48 this figure it shows the horizontal velocity component or the hyperbolic functions of the depth you see sine hkd + z hyperbolic functions of the pleasant and same with you also you see one is U is in cos h k and this is in sin hkd we want w.

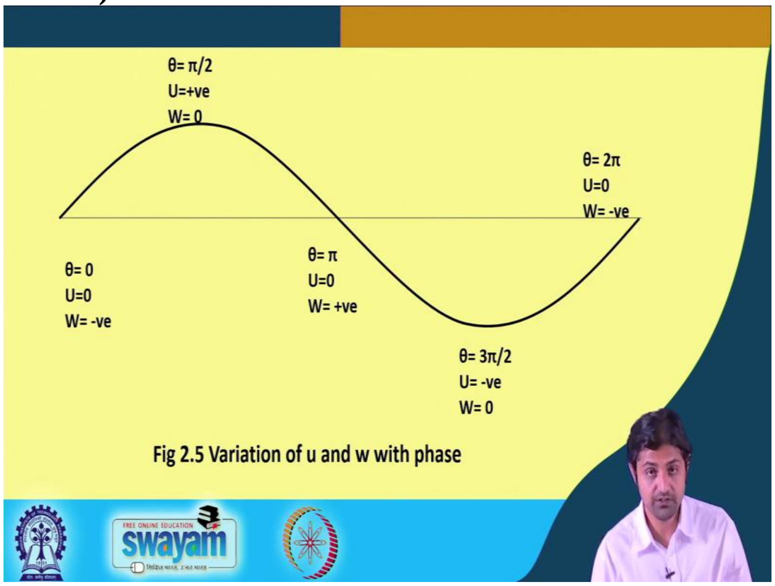
(Refer Slide Time: 16:06)



We say at any phase angle theta phase angle is k x - sigma t the hyperbolic functions of z that is cos h and sin h cause an exponential decay of u and w with the distance down from the sea surface, this is indicated schematically in this figure. These are just the findings, you might you should not put a lot of effort and concentration and finding these values but, I mean this is pretty true right that the there is an exponential decay of u and w with the distance down from the surface.

Variation of maximum u and w because cause and cause the you know if we take a phase angle fixed phi the whatever that is, that does not really matter because it will be the same for different z the periodic will be the same because cos theta or sin theta will remain the same.

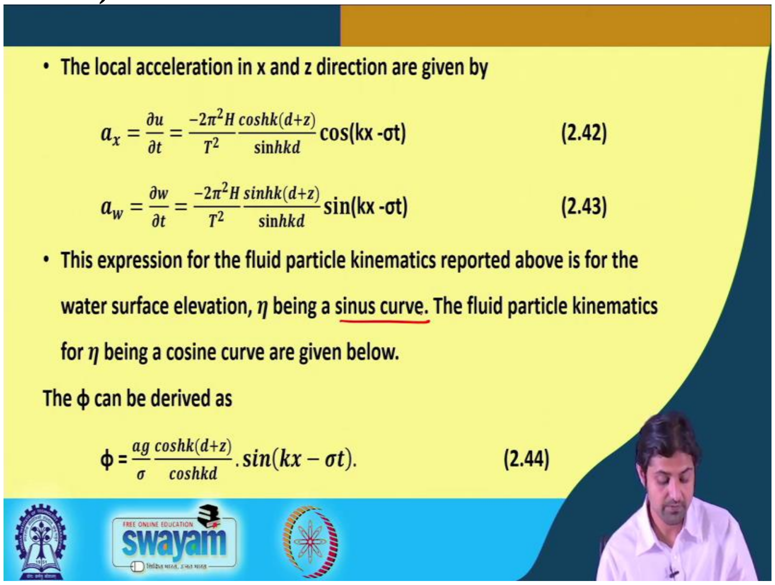
(Refer Slide Time: 17:12)



So here the phase angle is deemed to be 0 theta = 0, u = 0 and w is negative here theta is pi by 2 and u is assume u is positive and w is 0. At the top here theta = pi u is 0, but w is positive. At 3 pi by 2 u is negative and w = 0. And if you put in the velocity potential and u and w equation, different theta, different values you are going to get and the sign will be determined from there so this is just a summary of those results obtained from this equation more importantly, this is theta = 0 starting from here.

This is theta = pi by 2 this is theta= pi this theta will 3 pi by 2 and this is theta = 2 pi. So, phase is changing by pi by 2 from this point to this point and to this point finally. So, this is a variation of u and w with phase.

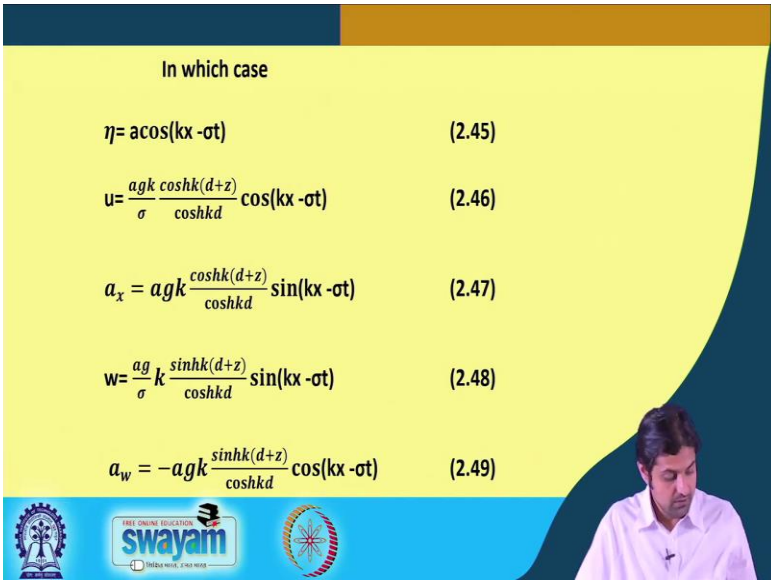
(Refer Slide Time: 18:20)



Now, we have talked about the velocity similar way we should talk about acceleration as well. So, a x is del u del t because u we have already obtained in terms of hd and cos h. So, we simply differentiate 1 small sign becomes cost here with a negative sin. And similarly a w will become pass will become sin with a negative sin, here it was signed, so it becomes cos and there was already a negative sign.

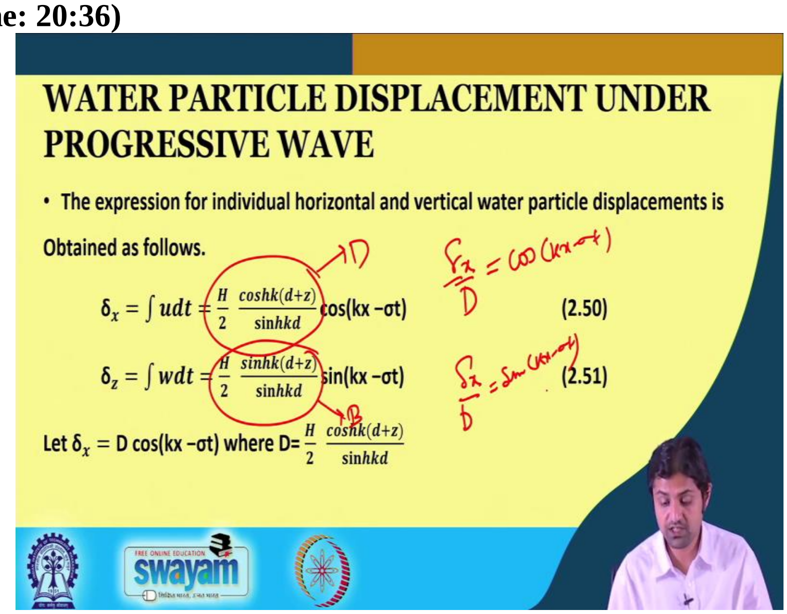
And in a w this course was then and it becomes sign after the differentiation with a negative sign so, this expression for fluid particle kinematic reported above is for water surface derivation being a sine curve. So, we started with a velocity potential where our time variation was sin k x – sigma t. So, similarly, if we assume a cosine curve, the velocities the pi can be derived as sine k x - sigma t correct.

(Refer Slide Time: 19:38)



So, in this case eta will be a cos k x - sigma t, the u will change u and a x and w and a w, so, if we assume a different velocity potential you remember we had to velocity potentials. So, we started doing all the calculation with the first velocity potential but instead of the first day we started with the second we will obtain this set of the wave kinematic parameters.

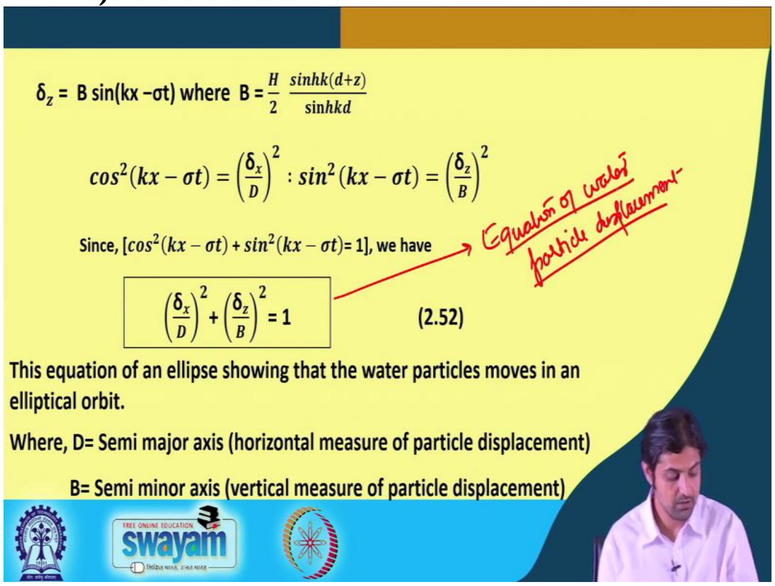
(Refer Slide Time: 20:36)



So we have studied the velocity potential, we found out the velocities under the progressive where we have found out the acceleration now importantly we have to find out water particle displacement is nothing but integral u times g T w times d T in extend that direction respectively. So, the expression of individual horizontal and vertical particle displacement is integral u dt u we already know in terms of h before.

So, the final results comes out to be h by 2 cos hkd + z phi by sin hkd into cos k x - sigma t. Similarly, the vertical displacement delta z is given by h by 2 sin hkd + z into sin hkd into sin k x - sigma t these are the periodic terms. So, once you derive the velocity potential everything can be found out. Now, if we say delta x = d cos k x sigma t k x - sigma t and we said D is h by 2 cos h k d + 0 divided by sin hkd and similarly.

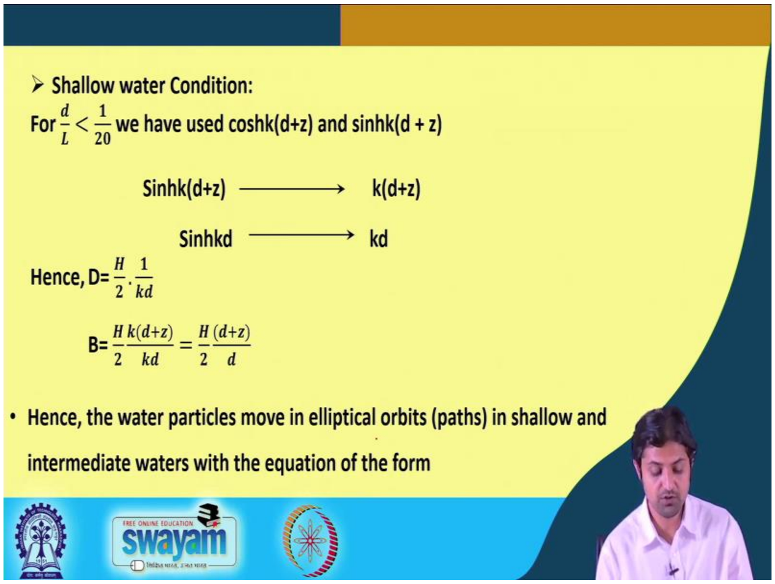
(Refer Slide Time: 22:00)



We assume delta z is B sine k x - sigma t where B is a different quantity called h by 2 sin hkd + z divided by sin h therefore, we can right cos squared k x - sigma t is delta x by d whole squared, let us go back. So, what we have said is this is D and this is B. So, if we take D down so, it will become delta x by d = cos k x - sigma t and if we take D down here it becomes delta z by D = sin k x - sigma t. So, sin squared theta + cos squared theta = 1 therefore, this is what we have used.

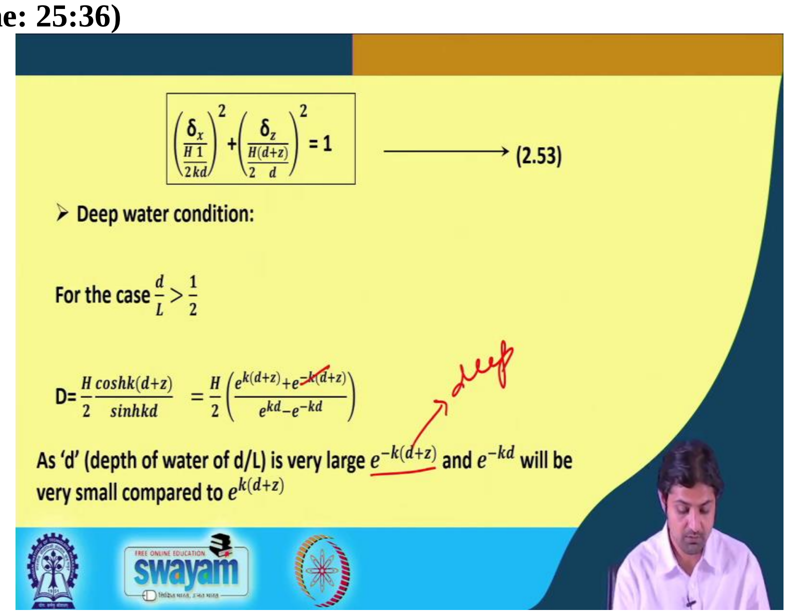
So, what we write is delta x by D whole squared + delta z by B whole squared = 1, this is in general what type of equation if D is not equal to be elliptical. So, this is an equation of analysts showing that water particle moves in in an elliptical orbit. So, we have proved particle moved in an elliptical orbit. Here D is the semi major axis at the horizontal measure of the particle displacement and B is these semi minor axis that is the vertical measure of the particle displacement. So the vertical the particle displacement this is an important equation.

(Refer Slide Time: 24:28)



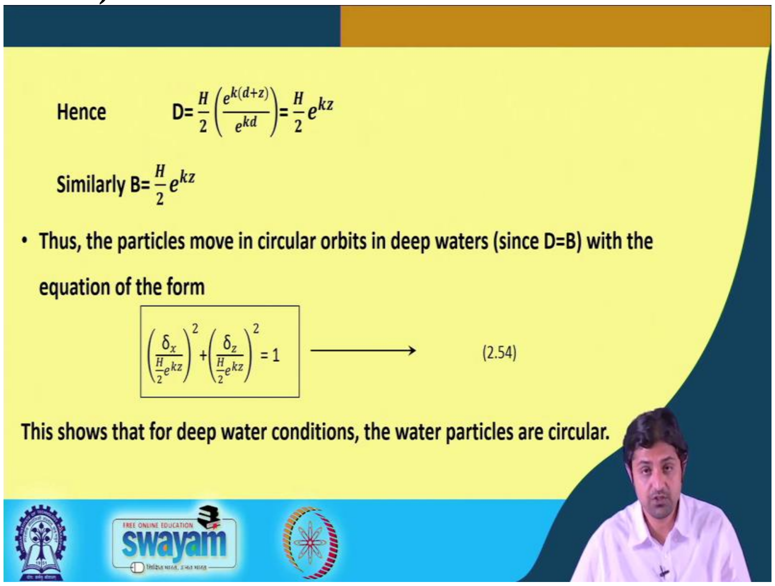
Now we analyze this displacement in shallow water. So what happens in shallow water for d by L less than 1 by 20 we have 0.05 we have used cos hkd + z and sin hkd + 2 sin hkd + z goes to k d + z and sin hkd goes to kd.. Hence, D will be what if we as you remember D was this equation D was h by 2 cos hkd + z and sin hkd in shallow water this becomes kd + z divided by kd. So this capital D becomes h by 2 into 1 by kd and B becomes h by 2 k d + z by kd hence the water particle in shallow water moved in analytical orbit in shallow and intermediate waters with the equation of the form.

(Refer Slide Time: 25:36)



If discuss just simply substituted in shallow and intermediate shallow and intermediate waters with this D and B but in deep water for the case d by L greater than half D becomes h by 2 e to the power k d + z + e to the power - kd + z divided by e to the power kd. - e to the power - kd as D is very large e to the power - k d + z and e to the power - kd will be very small correct because this deep now, so, if it - sin they will be very small compared to e to the power k d + z. So, this can be taken away.

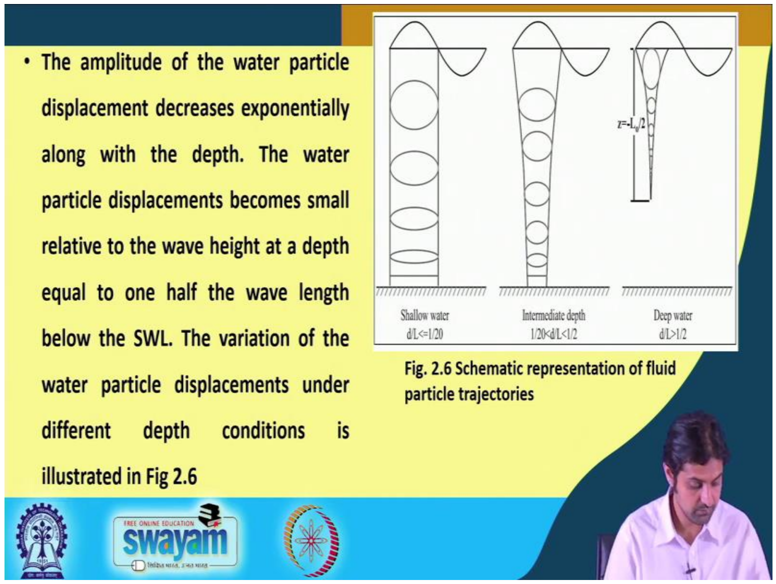
(Refer Slide Time: 26:25)



And therefore, we can right h by 2 e to the power kd + z divided by e to the power kd and simply h by 2 e to the power kz. This is important. Similarly, if we do the same process with B, we will again get h by 2 e to the power kz because deep water that is going to happen again. Therefore, you see, both B and D are same. So, in an ellipse if both the major and minor x is the same, it is nothing but a circle.

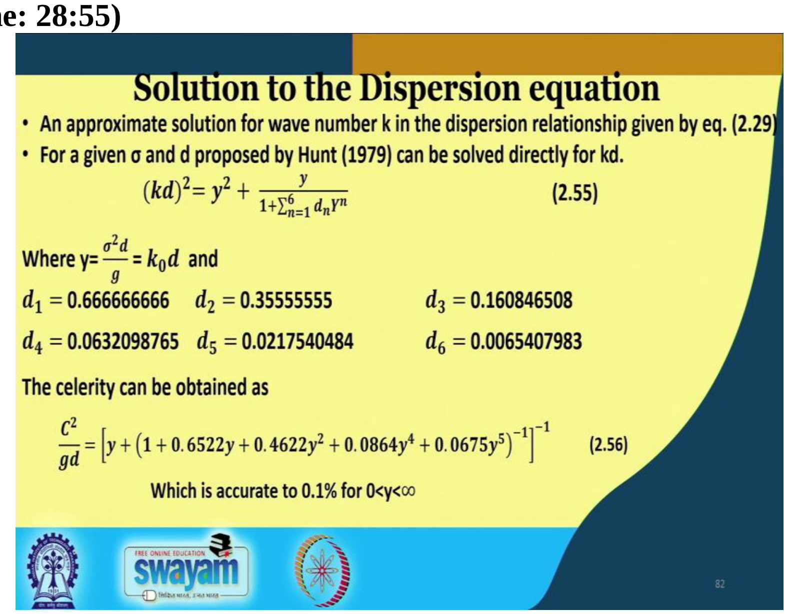
Therefore, the particles move in circular orbits in deep water since these equal to be with the equation of the form delta x divided by h by 2 e to the power kz + delta z h by 2 e to the power kz whole squared = 1. So, see these things are very important these are the type of questions which you can expect what type of orbit in deep water then you have to take mark the correct 1 as a circular orbit elliptical orbit. This shows that for deep water condition the water particle trajectories are circular.

(Refer Slide Time: 28:08)



So, you see, this is the representation the schematic representation of fluid particle trajectories the amplitude of the water particle displacement it first of all it decreases exponentially along the water depth the water particle displacement becomes small relative to the wave height at a depth equal to one half the wavelength below this still water level that is the deep water the variation of the water particle that displacement under the depth condition is illustrated in this.

(Refer Slide Time: 28:55)



So, there is 1 last thing before the end of this lecture, I have included in the slide but does not have any significance for this particular course. You see the dispersion relationship, I said there are 2 solutions. One is either using the wave tables and the second is trial and error. But some scientists like hunt he came up with the direct solution for kd. He said kd squared = y squared + y divided by 1 + n goes from 1 to 6 dn Yn.

And this final equation can be used in mat lab or Excel to determine for different wise so if you can put it on a computer code using mat lab or Excel, you the you There is no need for you know, iteration or any other method. So this is just for information purpose. You do not need do not need to remember maybe the name you know the if somebody is interested in doing further research or going for a master is program or you know, something related to research they can use this equation as such. So, with this actually this equation is accurate for 0.1% for y going from 0 to infinity.

(Refer Slide Time: 30:23)

So, I think this is a fine point to stop. In the next lecture, we will conclude this module of invested flow that is wave mechanics and we start with the pressure distribution and the progressive waves from the next lecture and finish this module. And this will also be the end of the course, but I will talk about it in the last lecture of this course called hydraulic engineering. Thank you so much for listening. See you in the next lecture.