Hydraulic Engineering
Prof. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture – 06
Basics of Fluid Mechanics – II

Welcome back to this course of hydraulic engineering. This week, we are going to cover basics of fluid mechanics, this is the second week of this. So, we call this basic of fluid mechanics II the first topic in this series is going to be fluid kinematics.

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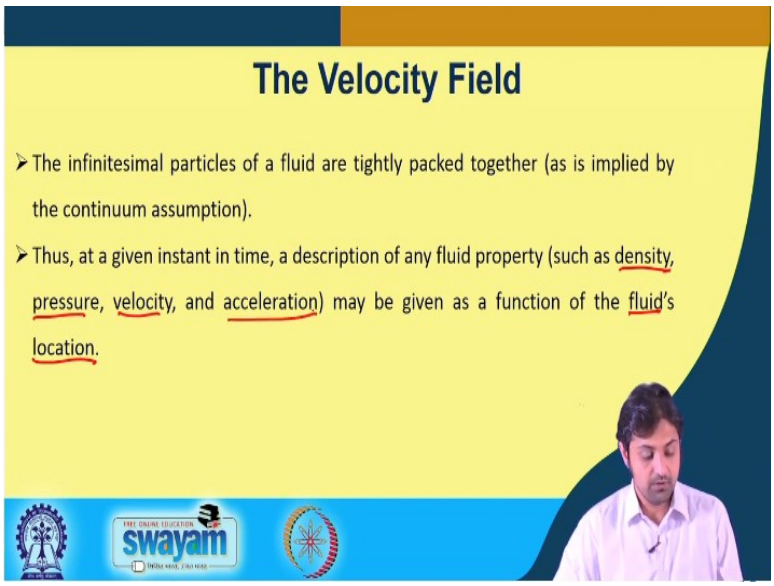

And yes, we can proceed. So, first, we will talk about the velocity field, what actually is this velocity field. You know, it from beforehand but too thorough to brush up your concepts will go through it once again. So, it is the actually the fluids can be assumed to be made up of infinitesimal small particles and they are tightly packed together. And this is implied by the continuum assumption. So, infinitesimal particles of fluids are tightly packed together.

Thus, at any instant in time, a description of any fluid property, such as density, pressure, velocity and acceleration, may be given as a function of the fluids location. So, that is the basic assumption that is the basic thing that we are going to start with that these properties such as density, pressure, velocity and acceleration can be described as a function of fluids location. So, this is quite important to start with, the fluid location and properties such as density, pressure, velocity and acceleration.

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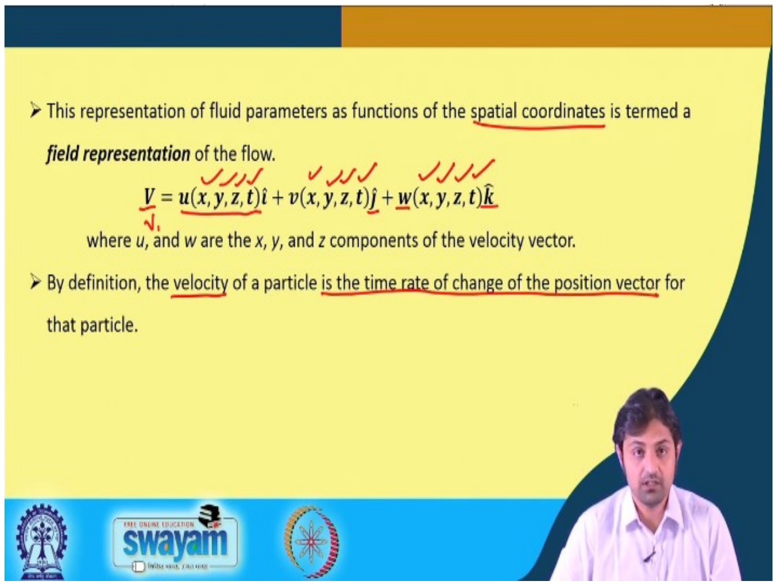

So, this representation of fluid parameters as function of a spatial coordinates is termed as field representation of the flow, spatial coordinate. So, that is, with respect to the space. So, in terms of x, y and z. For example, the velocity here can be written as u, u is a fluid velocity or the fluid speed in $\hat{i}$ direction, $\hat{i}$ is x axis plus v in $\hat{j}$ direction, $\hat{j}$ is y axis and w in $\hat{k}$ direction and each of u, v and w can be function of x, y, z and t, each of those, this one, this one, and this one here.

So, as I told you, u and w are u, v, and w are x, y and z components, u, v and w of the velocity vector. By definition, the velocity of the particle is the time rate of change of the position vector for that particle. So, the velocity is the time rate of change of the position vector for that particular particle, this you already know.

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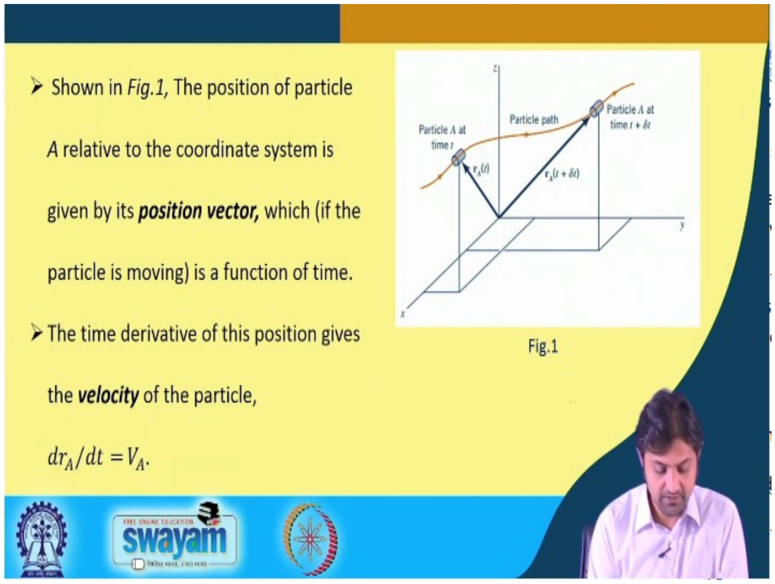

So, there is one figure here, we as you look into the figure, x axis is in this direction, y axis is in this direction and z axis is in this direction. The position of the particle a, so a is our particle here, this one and this one. So, these particles have the same particle, but at two different times, at time t and at time $t+\delta t$ and this red line here represents the particle path. Let, me just see if I am able to do this, yes. So, this one here, is the particle path.

So, this, the position of this particle is given by the position vector, this which are of $r_A(t)$, this is the position vector. And here at time $t+\delta t$, this is $r_A(t+\delta t)$. So, I will go back here. So, so this is the position vector at time t and this is the position vector different position vector at time $t+\delta t$. So, considering the particle is moving so, particle has moved from this point to this point via this.

So, time derivative of this position, let me take out the ink first yeah. So, the time derivative of this position gives the velocity of the particle. So, time derivative of dr_A is given by $dr_A/dt = V_A$. So, velocity at point A or is the same thing what I have written and which is given on the slide as well.

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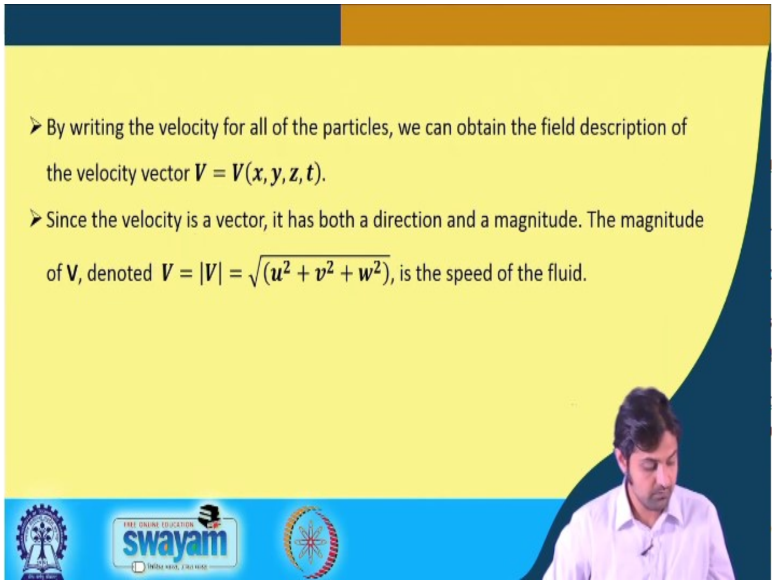

So, by writing the velocity of all the particles, we can obtain the field description of the velocity. So, if we write the velocity of all the particles in the system, we can obtain the field description of the velocity vector v as this V = V (x, y, z, t) using the equation described in the previous slide. Since, the velocity is a vector, it has both a direction and the magnitude, this is the basic concept that you know from before. The magnitude of V is given by $|V| = \sqrt{(u^2 + v^2 + w^2)}$ this is simply nothing but the speed of the fluid.

So, assuming a particle has a velocity here, this is the let us assume 2 direction, this is x and this is y. So, there will be a component and this is cos θ. So, this speed will be |V|. So, if this is let us say, u_1 is the x component of the velocity and u and v_1 is the y component of the velocity, so |V| in 2 directions will be $\sqrt{(u_1^2 + v_1^2)}$ this you already know.

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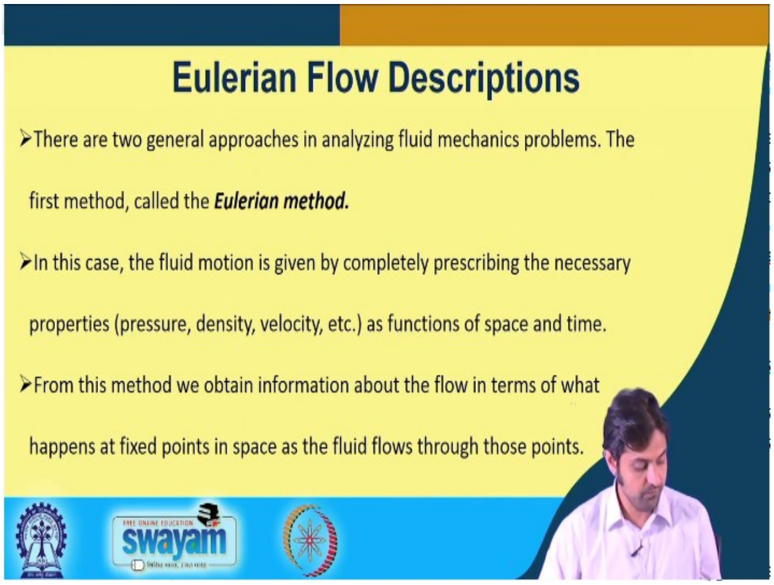

So, now there are certain descriptions of the flow, the two most famous are Eulerian and Lagrangian. So, we will revise that as well. What is the Eulerian flow description and what is the Lagrangian flow description? So, as I told there are two general approaches in analyzing fluid mechanics problems, the first method is called Eulerian method. In this case, the fluid motion is given by completely prescribing the necessary properties as a function of space and time.

So, pressure, density, velocity, whatever properties you have, even acceleration are prescribed by describing them as a function of space and time. From this method, we obtain informations about the flow in terms of what happens at fixed point in space as the fluid flows through those points. So, this is very important. So, this is the main difference between the two descriptions. In Eulerian flow description; we describe the flow properties in terms of what happens at fixed points in space.

The observation is also taken from the fixed point in space suppose; we define a coordinate system from beforehand and all the things that are observed from the observer. So, this is x, this is y and this is z. So, if we if we put our observer here, for example, anywhere and it does not necessarily need to be at origin, and if we keep on observing suppose a particle is moving like this, if we keep observing from here, and we see what is happening at this particular point at a fixed point here, the flow description properties are Eulerian and in nature. Good.

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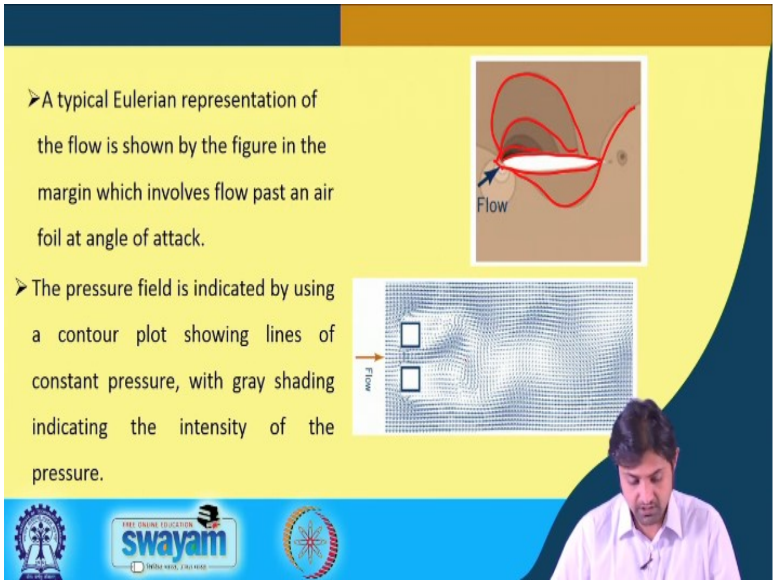

So, a typical Eulerian representation of the flow is shown by the figure, this one which involves the flow past an airfoil at an angle of attack. This is just to show Eulerian representation this is flow past an airfoil. You see, this is a foil, this is an object and this is the flow properties or the contour that are there. So, the way we have described it as one of the observer at a fixed point in space, this is Eulerian in nature. The pressure field is indicated by a contour plot showing the lines of constant pressure, with gray shading indicating the intensity of the pressure.

So, this is the flow and this is the pressure contour, you know. So, just want to make it very clear that all these results have been observed and obtained from previous experiments. So, this is just to show how we describe these Eulerian velocities for example or the pressure in this case, here.

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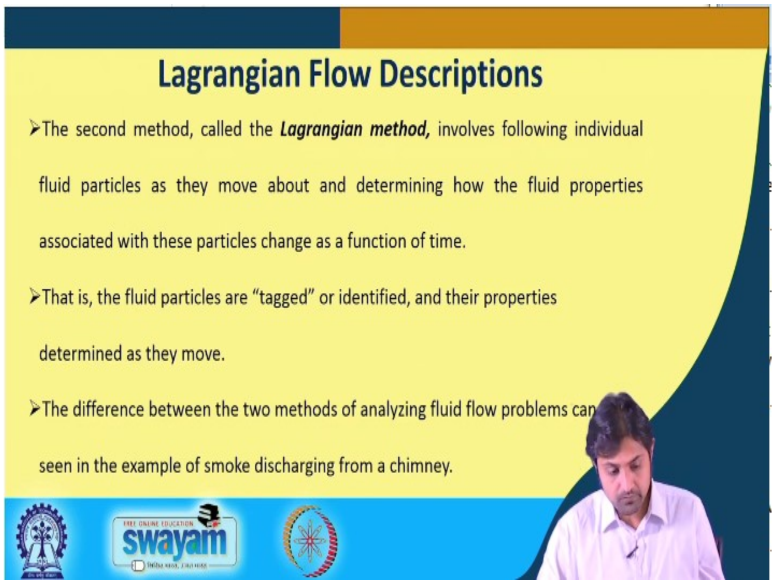

Now, what is Lagrangian flow description? So, the second method called the Lagrangian method here, involves following individual fluid particle as they move about and determining how the fluid properties associated with these particle change as a function of time. So, the key component is, is following the individual fluid particles. So, our frame of reference is fixed at one of the particle that is moving.

So, that means, it is a non it could be a non-Newtonian frame of reference. Earlier, we had a fixed in Eulerian we had a fixed point in space from where we were making the observation about a fixed point. Here, we are following an individual fluid particle, so, our frame of reference x, y, z is that the particle itself. So, whatever motion we are observing is relative to that particular particle. And this flow description is called the Lagrangian method.

And this is observed as a of this particle as a function of time. That is the fluid particles are tagged or tagged means just putting a marker on them or identified and their properties are determined as they move. So, just to summarize again, the difference between Eulerian and Lagrangian flow descriptions is that, Eulerian is description is a description taken from a fixed-point space. However, in Lagrangian flow description, the frame of reference is fixed to a moving particle. So, the difference between as I said the two methods of analyzing fluid flow problems can be seen an example of smoke discharging from a chimney.

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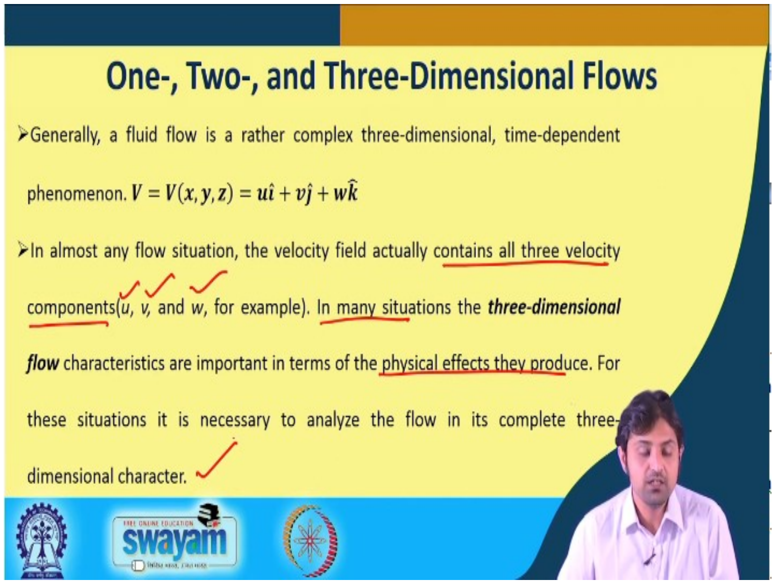

So, talking about one, two and, three-dimensional flows, generally, a fluid flow is rather complex three dimensional and a time dependent phenomenon. Any fluid flow in real life is all the three, it is complex three dimensional and time dependent phenomenon that means, the velocity will be a function of time, it will have a velocity component. So, this is time dependent means as a function of time. Three-dimensional means it will have x, y and z all the three components would be there as indicated by this equation V = V (x, y, z, t) which can be written as u, u is the velocity component in x direction, v is the velocity component in y direction and w is the velocity component in z direction. So, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors in x y and z direction, from your maths class you would already be knowing that | $\hat{i}$ | is 1, | $\hat{j}$ | is 1 and | $\hat{k}$ | is also 1 if we write the vectors here. So, this is just a basic revision. So, $\hat{i}$, $\hat{j}$ and $\hat{k}$ by multiplying $\hat{i}$, $\hat{j}$ and $\hat{k}$ we do not change the magnitude but just try to show the direction. Let, me rub this.

So, actually in almost any flow situations, the velocity field contains all the three velocity components, u, v and w. In many situations, the three-dimensional flow characteristics are important in terms of physical effects they produce. For these situations, it is necessary to analyze the flow in its complete three-dimensional character. So, for many flows in nature, three dimensional flows actually are very important and therefore, we need to analyze in all the three direction that is why u, v, and w all are important.

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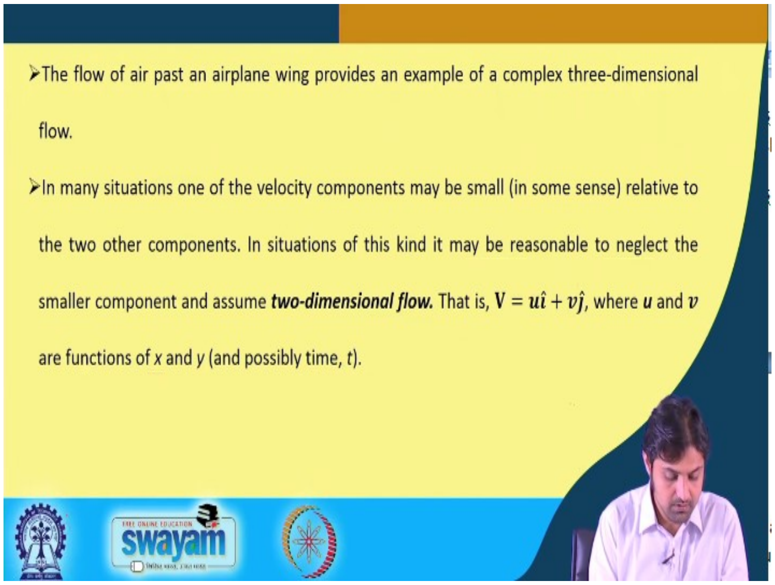

The flow of air past an airplane wing provides an example of a complex three-dimensional flow, how? Because airplane is moving, so, the velocity will be changing also in time and also because the plane is also moving the velocity will also be a function of x, y, and z. At different points the velocity could be really different. In many situations, one of the velocity components may be small. This can happen anywhere.

For example, suppose, there is a large tank and there is a flow in x direction and the tank is infinite in the y direction. And in that case, in one of the directions, if we do not vary the give the speed in y direction that component is going to be very small. So, but just going back to the example, the main part here is that in one of the velocity components may be small relative to the other component. In such situations of this kind, it may be reasonable to neglect the smaller component and assume two-dimensional flow.

So, as I would repeat again, we have talked that all the flows in nature are three dimensional, but some of the flows can be assumed to be two-dimensional, because one of the small one of the components would be very small in comparison to the other two components. And therefore, we can say that this is a two-dimensional flow. In this case here, where we have written $V = u\hat{\imath} + v\hat{\jmath}$ and we have neglected w because here it might have happened that the w << u, v. And here, u and v both here are functions of x, y and of course, it could be a function of time as well.

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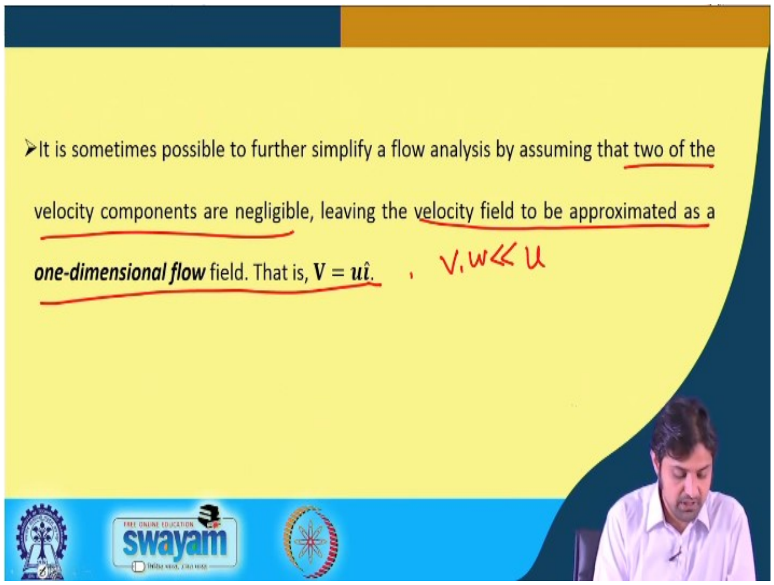

It is sometimes possible to further simplify a flow analysis by assuming that two of the velocity components are negligible. So, we saw that one component was negligible, but there can be situations where two of the flow velocity components are negligible, which would leave the velocity field to be approximated as one-dimensional flow field, that is $\overline{V = u\hat{\imath}}$, this means, that v, w << u. So, this is one dimensional flow in reality such flows can be there but it has to be very properly experimentally controlled, but in general as I said that the velocity is three dimensional in nature.

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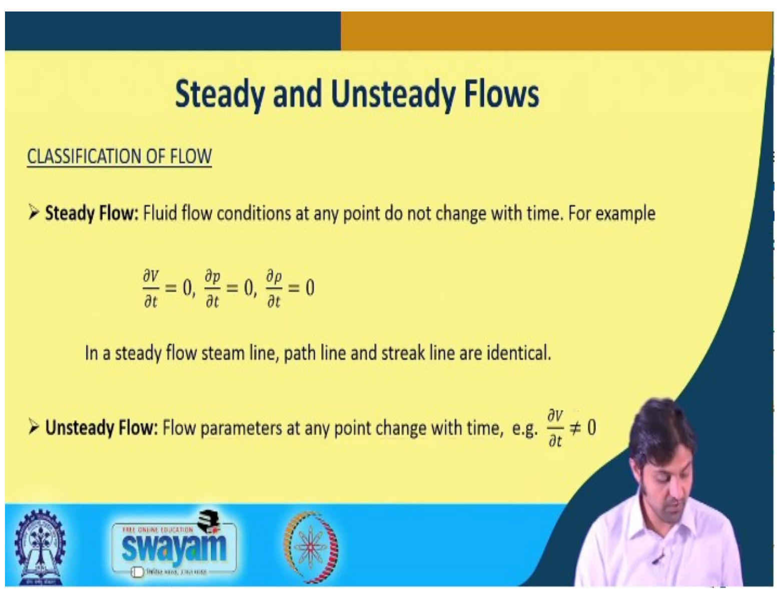

So, we have talked about some velocity descriptions, now, Eulerian, Lagrangian, and then we talked about three-dimensional flow, two-dimensional flow, one dimensional flow, now, it is time to talk about steady and unsteady flow. So, as you know from before probably from your fluid mechanics class is steady flow and unsteady flow, the main distinguishing component here should be time. How? That we are going to see. So, one classification of the flow is steady and unsteady.

In steady flow what happens is the fluid flow conditions at any point do not change with time, this is very important, do not change with time. For example, if the velocity component, velocity is v, pressure is given by p and density is given by ρ. So, the derivative with respect to time is going to be zero, there is no change in velocity, this means that, the change in, let me just go to the laser pointer, the change in velocity here with respect to time is zero. Same is the case with the pressure; the change in pressure with respect to the time here is equal to zero and the same fate is for the density. The change in density with respect to the time again here is zero. So, here and here.

So, in steady flow, stream line path line and streak lines are identical. We will, so, this is actually stream line not steam line. Sorry for the typo here. Path line and streak line are identical, this we will tell you in upcoming slides that what exactly are stream lines, what is path line and what is streak line. Good. An unsteady flow, unlike steady flow the flow parameters at any point changes with time that is any of the flow properties either it is velocity, so, $\frac{\partial V}{\partial t}$ will surely not be equal to zero that is the definition of or any property even $\frac{\partial a}{\partial t}$ is not equal to zero or del pressure as we saw del, sorry, density $\frac{\partial \rho}{\partial t}$ is not equal to zero and $\frac{\partial p}{\partial t}$ is also not equal to zero.

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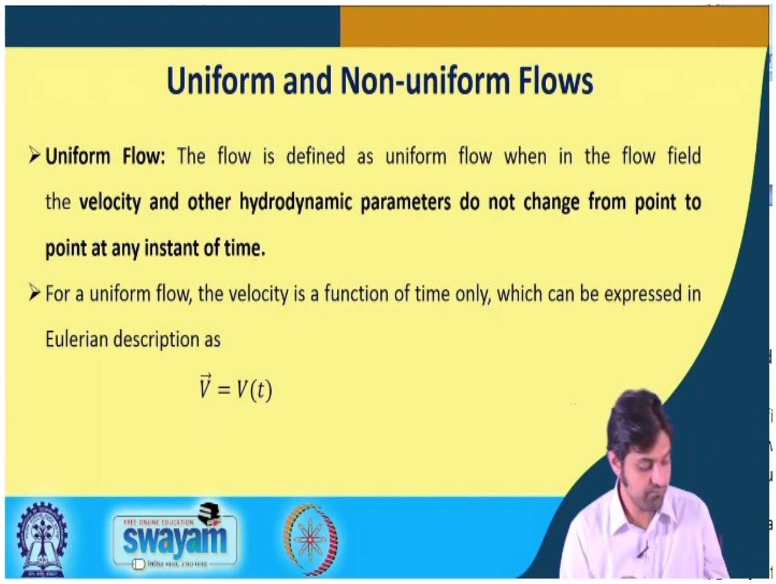

So, we have seen steady and unsteady flow. Now, we are going to see uniform and non uniform flows, the definition of the uniform and non uniform flow. Uniform flow, the flow is defined as uniform flow when the flow field that is the velocity and the other hydrodynamic parameters do not change from point to point. So, in a steady flow it was not changing with respect to time, here, these properties should not be changing with respect to the space at any instant of time.

So, uniform flow is the one, where the fluid properties do not change with respect to the space. For a uniform flow, the velocity is a function of time only that is it. Which can be expressed in Eulerian description as $\vec{V}$ = V (t). So, v $\neq$ v (x y z t), but only v = v (t). So, these x, y, z components vanish when it is a uniform flow.

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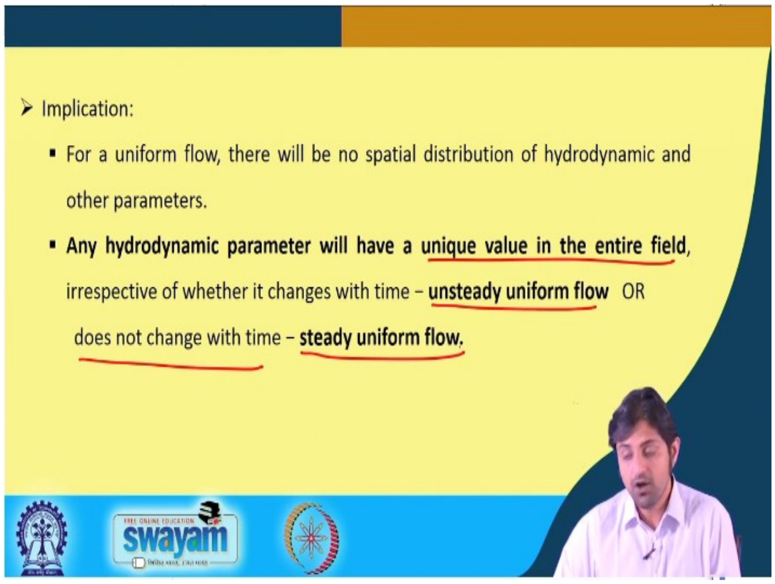

Implications for a uniform flow; there will be no spatial distribution of hydrodynamic and other parameters, as we told, because the parameters really does not depend upon the location. Any hydrodynamic parameter will have a unique value in the entire field, irrespective of whether it changes with time. Therefore, now, we have two parameters here is space and time, time was with if something depends upon time, it will be unsteady, if something does not depend upon time it will be steady.

So, combining the steady and unsteady with uniform and non uniform we can have either unsteady uniform flow, which means the velocity will be the only a function of time, or actually it might not even change with time as well, this type of flow can be called a steady uniform flow. So, the flow properties are neither going to change in space or with respect to time. Good.

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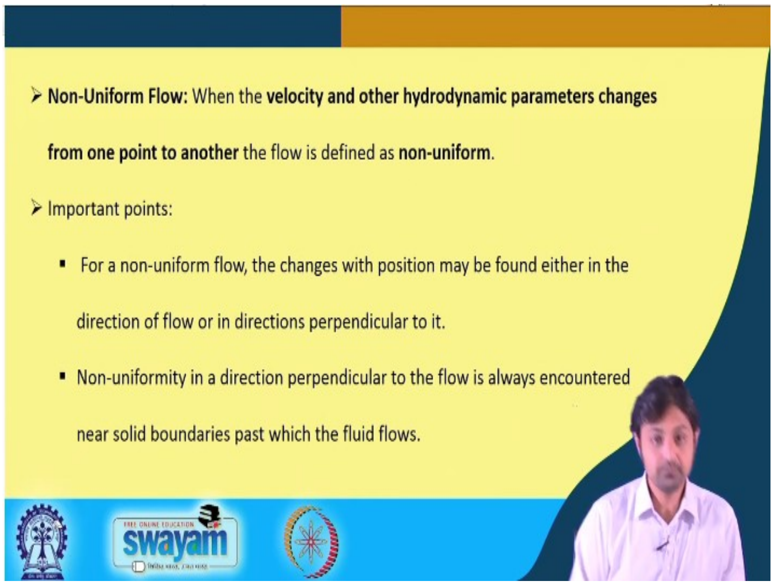

Now, non-uniform flow, when the velocity and other hydrodynamic parameters changes from one point to the other, which is exact opposite of uniform flow. So, the properties in uniform flow were not dependent on the position of the particle or from in these respect to the space, but non-uniform flow it will change from one point to the other and such flows are called non uniform flows. These are the some of the terms that you will be you will be encountering again when we start doing the open channel flow for example, that is why it is important to go through it once again.

Now, important points are for a non uniform flow, the changes with position may be found either in the direction of flow or in the direction perpendicular to it. So, we have said that, for a non uniform flow, the particle properties or the fluid part fluid particle properties changes with position. So, position or the direction can be infinite, but generally, it is a customary in fluid mechanics to calculate the changes in two directions. One direction is the flow direction.

Flow direction means for example, the river flows say in one direction x direction, then we will calculate the changes of the properties in the direction of the flow. In our case, it is x direction and the direction perpendicular to it. So, suppose, there is a river flowing like this and our coordinate system is x and y, for example, this is two-dimensional space. So, the best way to calculate the changes is in this direction and this direction perpendicular. Good.

So, non uniformity in a direction perpendicular to the flow is always encountered near solid boundaries past which the fluid flows. We will come across it later in our lectures, that how near solid boundaries this velocity component changes and that is one of the main reasons why near the solid boundaries we need to, you know, calculate the changes and thus they introduce non uniformity in the direction perpendicular to the flow.

So, if suppose, an example is there is a plate the fluid will be flowing like this. So, there will be velocity here, but this plate is at rest, we will come to it later as well, but since the fluid is flowing across this, then this is at rest and the particle adjacent to it will try to be at rest as well or the particle nearest to this plate will be at rest. Here, the velocity is suppose, V of the river I mean the stream velocity here, is zero. So, as you can see during this distance the velocity goes from V to zero. Good. Alright then. Nice.

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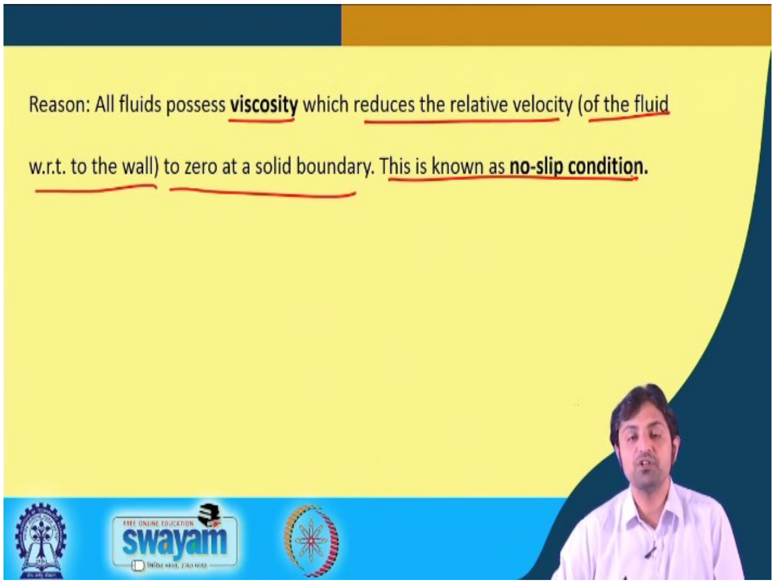

Reason, as I was telling you before for the, I mean, in the earlier point that we said, all fluids possess viscosity, sorry I will use the pen, all fluid possess viscosity. Viscosity in a layman's term or in a old term can be said as friction in fluids for example, and because of the friction, this reduces the relative velocity of the fluid, with respect, to the wall to zero at solid boundary as I was telling you and this condition is called as a no-slip condition and because of the no-slip condition, there is a non uniformity of the flow in the y direction.

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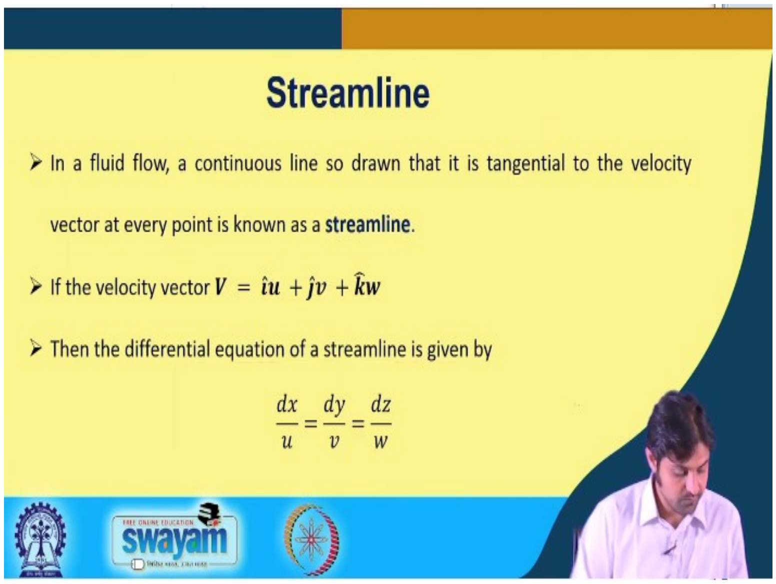

Now, what is a stream line? In a fluid flow, a continuous line so drawn that it is tangential to the velocity vector at every point is known as stream line. So, what is the streamline it is a continuous line such that this line is tangential to the velocity vector at every point. So, if you draw such a hypothetical line this is called a stream line. Suppose, if the velocity vector is given by in $\hat{i}$ direction as u, in $\hat{j}$ direction as v and in $\hat{k}$ direction as w, better to write it like this, $u\hat{i} + v\hat{j} + w\hat{k}$.

Then the differential equation of the stream line, this you have already done before but I will straight away go ahead and write the equation is $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$. So, this is the equation of the stream line. Alright.

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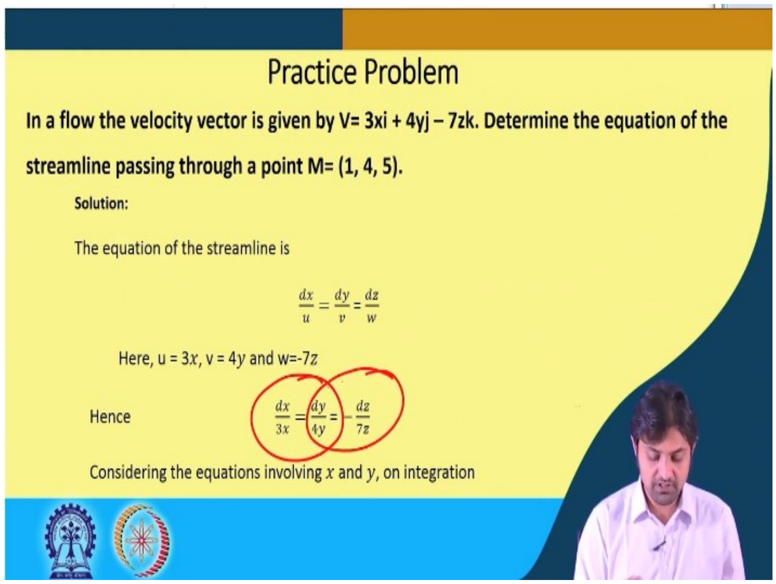

So, now, we will end this lecture with this practice problem, first, we will finish this practice problem and then finish it. The question is, in a flow the velocity vector is given by $3x\hat{\imath} + 4y\hat{\jmath} - 7z\hat{k}$. This means the u component is 3x, y component is 4y, and z component is minus, I will just do the eraser, - 7z. Now, the question is, determine the equation of this stream line passing through a point M and where M is given by (1, 4, 5), the procedure is very simple.

We have done this equation before, the equation was the equation of the streamline is given by $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$, this we have seen in the last slide. This is the most important equation and we already know u, for example. So, we can write as I wrote before, u is 3x, v is 4y, w is 7z, very it is very important to note that we must be writing these components whatever things we know from before when we start solving a problem we should be writing it down first.

For example, u = 3x we have written and so on. So, this equation can be written as $\frac{dx}{3x} = \frac{dy}{4y} = -\frac{dz}{7z}$ because Z was - 7z. Let, me take away this ink. Now, the procedure is we will solve this one at a time, $\frac{dx}{3x} = \frac{dy}{4y}$ and secondly we are going to solve this two.

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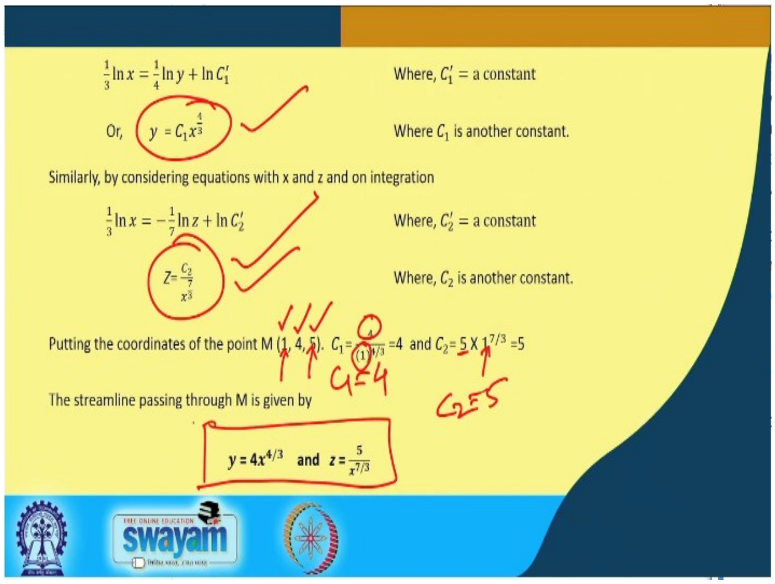

And when we integrate the first equation will can be written as $\frac{lnx}{3} = \frac{lny}{4} + lnC_1'$, where C_1' is a constant. And how do we obtain this C_1 that will talk about it next. So, in other word, we can simply write because all these are ln we can write, $y = C_1 x^{\frac{4}{3}}$. And on integrating the second equation as I told this one here, $\frac{dy}{4v} = -\frac{dz}{7z}$.

So, either we can do integrate this equation the second, or we can also use this equation these two equations together. So, in this practice problem we have taken the x and z. So, this gives $\frac{lnx}{3} = -\frac{lnz}{7} + lnC_2'$, where C_2' is again a constant. So, we can get simply $z = \frac{C_2}{x^{\frac{7}{3}}}$. So, these are the two equations that we got, correct. Now we need to obtain C_1 and C_2, what is the way we have said?

Find the equation of the stream line which passes through a point M, where the coordinate of M was given, x was 1, y was 4, and z was 5. This also means that this (1, 4, 5) should also be satisfying this equation and this equation, and on substituting in y in this equation, we can get C_1 because y here is 4, so, that is, 4 and x was 1, so we have put 1 here, in this way, we get C_1 is $C_1 = 4$.

Similarly, using 1, 4, 5, and putting it into z equation here again, C_2 can be written as $zx^{\frac{7}{3}}$. z is 5, right, so, this is 5 here, and x was 1 again, and this gives $C_2 = 5$. So, the stream line passing through M can be given by $y = 4x^{\frac{4}{3}}$, and $z = \frac{5}{x^{\frac{7}{3}}}$. So, this is the problem where we saw how to solve the streamline equation. And this is enough for today. So, we will resume in the next lecture. Thank you so much for watching.