Hydraulic Engineering
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Lecture # 58
Computational Fluid Dynamics (Contd.,)

Welcome back to the last lecture of this module computational fluid dynamics and the last lecture we ended at a point where we saw the Reynolds shear stress equation. We discussed about the different terms in it you do not need to worry about the mathematical equation of those terms, but some common terms like Reynolds shear stress, those things are recommended that you remember them, but not the complex terms. So, the up in proceeding in the direction we are going to study the closure problem. So, the effect of the Reynolds shear stress rho in tau i j as you can see on the slide now the effect of rho in tau i j on the mean flow is like that of a stress term.

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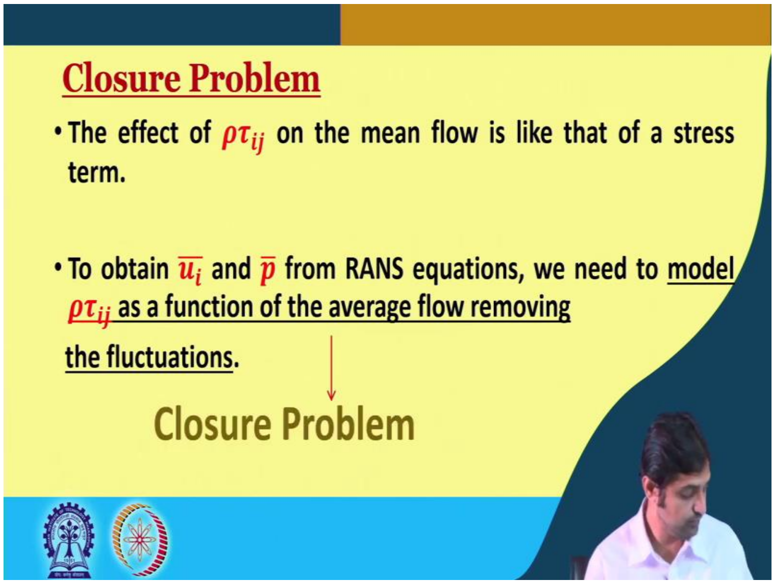

So I am going to write here rho in tau i j is actually Reynolds shear stress. So it is like shear stress to obtain u i and pressure. So, u i is the average flow velocities and pressure from RANS equation, we need to model this shear stress rho in tau i j is a function of the average flow so if you look at the equations. So take you back to this.

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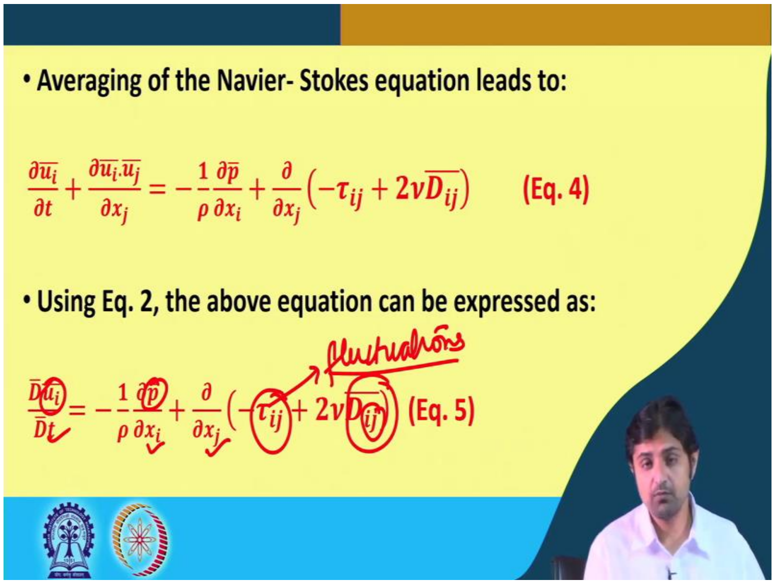

Where we have written the Reynolds average here you see this particular equation this is average quantity that we need to find out this we need to find out these are the independent variables. D i j we also know because this is the average value and it comprises of only this is the only term that has fluctuations and this increases our complexity. So, we need to be able to equate tau ij to something that is known known or some unknown which we are actually calculating.

So, the best ways to put it in some form of an average value and to do that, that particular problem is called a closure problem. So, we will go back to the closure problem so as I said, to obtain u i and pressure from Reynolds equation, we need to model this Reynolds's shear stress given by rho into tau i j as a function of average flow, this will remove the fluctuations and this process is called the closure problem.

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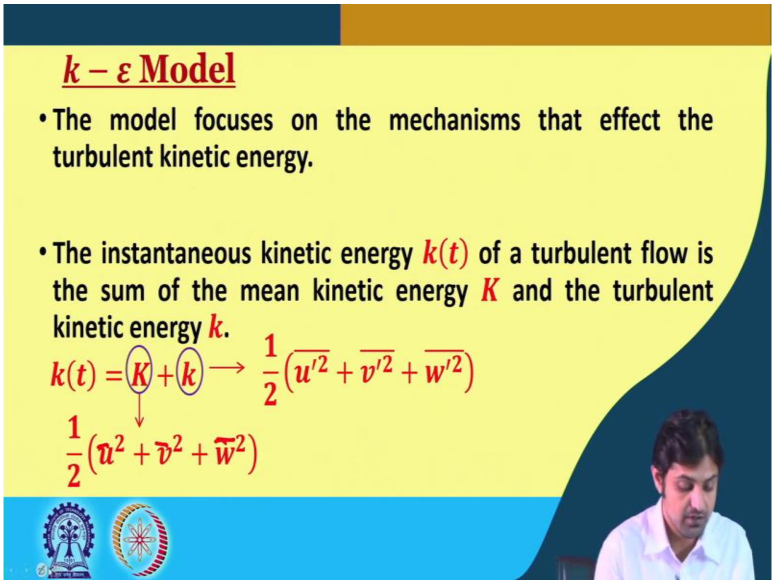

There are elegant ways the first of the methods is called the k epsilon model. So, this in this technique the model focuses on the mechanism that effects the turbulent kinetic energy, k stands for kinetic energy. So, the instantaneous kinetic energy k as a function of time k t of a turbulent flow is the sum of the mean kinetic energy and the turbulent kinetic energy k. So, k of t can be written as capital K + small k.

So, this is small k and capital K is written is simply half u square + v squared + w squared actually u bar v bar and w bar and small k is written is half of u dash squared + v dash squared + w dash squared.

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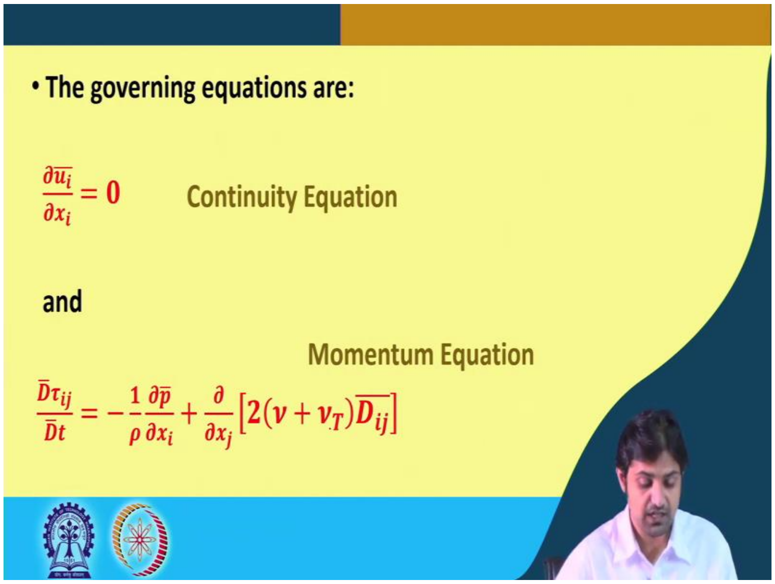

The governing equations for this are one we have continuity equation and the other is so, the way we write is So, D D t of tau i j that is what we are modeling is -1 by rho del p del x i + del x j into 2 nu + nu t into del i j. How does equations have been derived we are not going to discuss but these are the 2 equations that we use for momentum. See, it looks like a momentum equation itself so modeling of the Reynolds shear stress in as the momentum equation.

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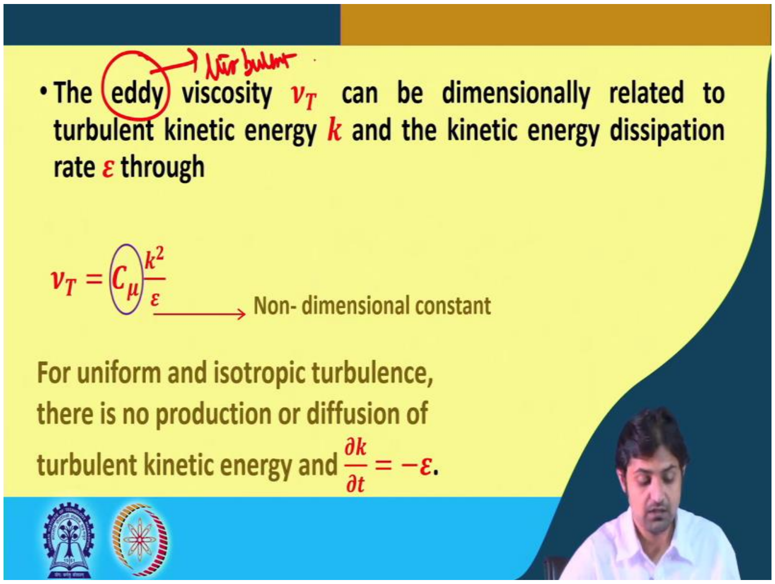

So, if you see there is a term called nu T. So, this nu t is the eddy viscosity. So, this actually is not should better be called as turbulent eddy viscosity, nu T, it can be actually dimensionally related to kinetic energy k kinetic energy dissipation rate epsilon through nu T is written as c mu into k square by epsilon. So, nu T we got also able to find on infinite terms of k and epsilon and we substitute this into this equation and C mu is a non-dimensional constant. Which values we know from experiments for uniform and isotropic turbulence there is no production or diffusion of turbulent kinetic energy, therefore, del k delta t will be equal to - epsilon E.

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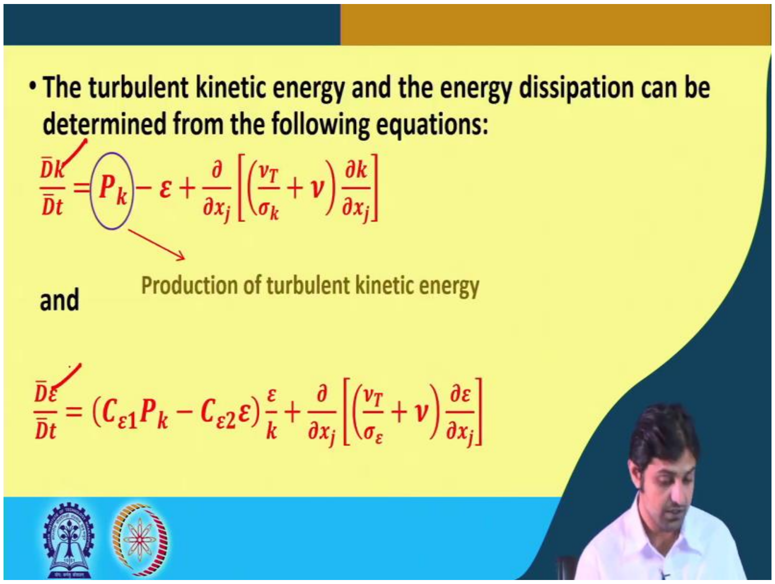

The turbulent kinetic energy and the energy dissipation can be determined from the following equation. So, now, we said that this is the equation which we are going to use for determining tau i j and that we can use in our average equation Reynolds average Navier stokes equation, we saw that there are some terms one is nu t that we do not know again others are again like del p bar, D i j bar. Those are known means do quantity that are in terms of average quantities.

So nu T is something that we yet do not know. And we write it nu to in terms of C mu into k square by epsilon. So, again, we have 2 things to again find out k and epsilon we still we do not know so, the next step is finding out this turbulent kinetic energy k and epsilon that that is how this equation gets in the model gets its name. And to be able to do that we have 2 another equation 1 is in terms of kinetic energy 1 is in terms of epsilon. And this is production of turbulent kinetic energy this term here. So, we use these 2 equation to solve for k and epsilon.

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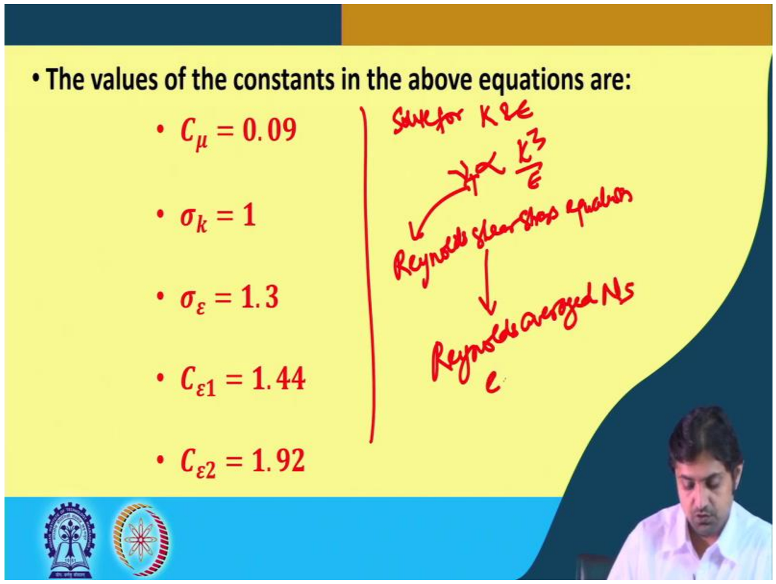

And the values of the constant in above equation are C mu = 0.09, sigma k = 1 if you see, C mu was there first, and then there is sigma k here then there is sigma E here and also there are some terms there is sigma there is the C epsilon 1 there is C epsilon 2. So, those things are already known these are the values that we use so, what we do is we solve the k and epsilon equations use those equations to find out the turbulent kinetic turbulent eddy viscosity nu T put it into their an Reynolds shear stress equation.

And use it in the average equation of Reynolds Navier Stokes equation. So, this for just writing it down solve for k and epsilon then nu T is related to k square by epsilon. Nu T, put this in Reynolds shear stress equation and use that in Reynolds average Navier stokes equations. It is a quite a complex way, but it gives good results.

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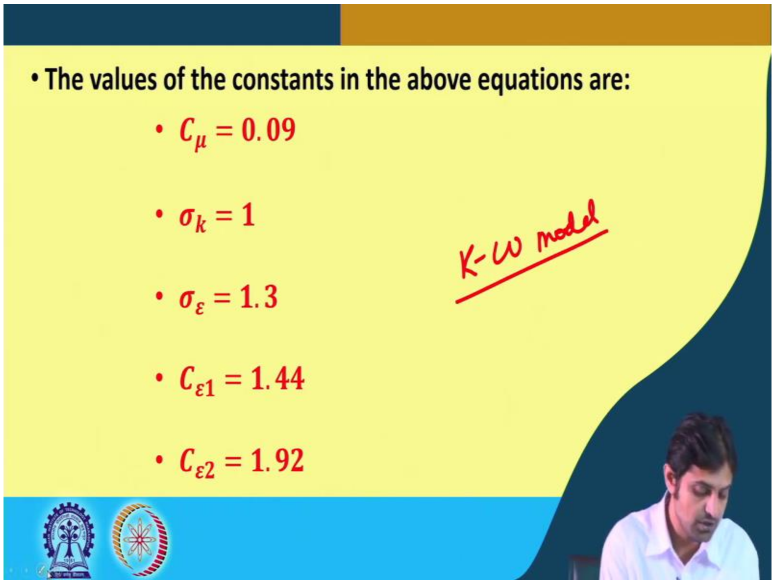

Most of the turbulence model in the world follow k and epsilon, there is one other turbulence model for k omega. So, k is the same kinetic energy omega is somehow related to dissipation epsilon, but that is also the outside the scope, but it is better to remember the name. So, the other model is k omega model like epsilon, these are the two world's most widely used turbulence models, and some of each of them have their advantages and disadvantages.

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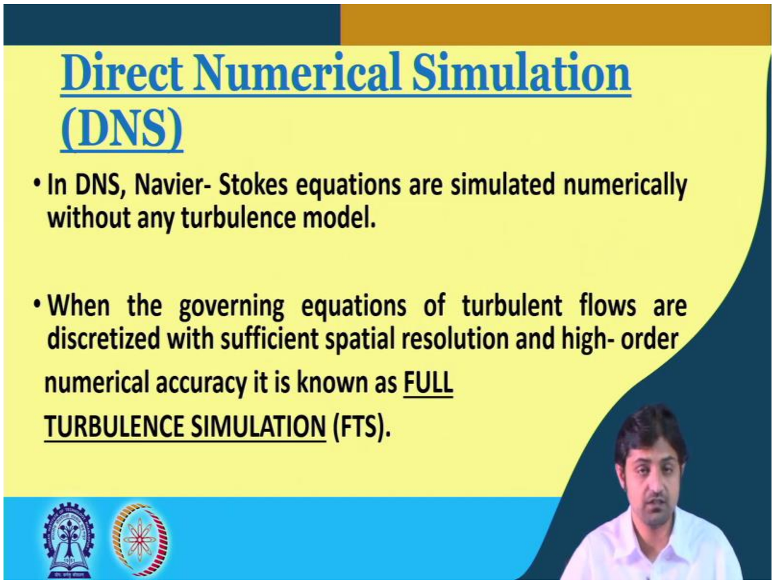

So, another method apart from Reynolds average navier stokes equation for solving the turbulence, for solving the computational fluid dynamics navier stokes equation is called direct numerical simulation. So, in direct numerical simulation DNS navier stokes equation or simulated numerically without any turbulence model. So, they solve for the exact solution when the governing equations of turbulent flows are discretized with sufficient spatial resolution and high order numerical accuracy, it is known as full turbulence simulation FTS.

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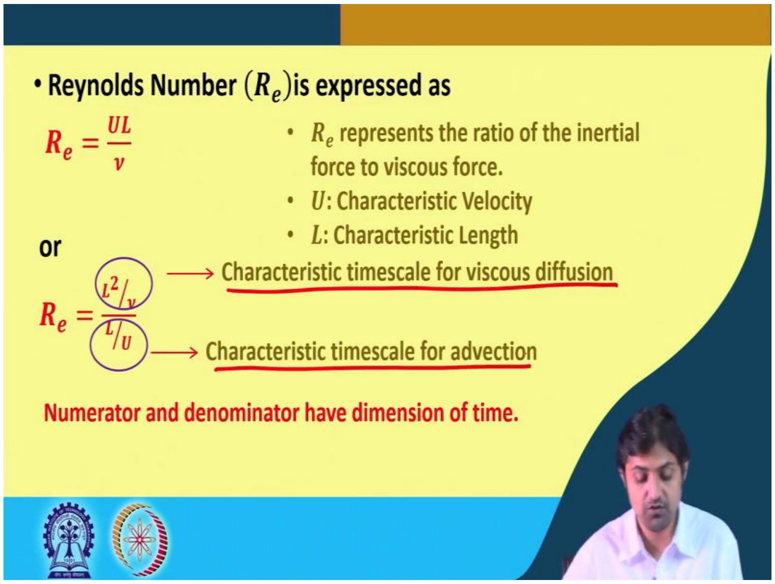

So, in this modeling Reynolds number is expressed as UL by nu, where R e represents the ratio of the inertial forces to viscous forces or Reynolds number, use the characteristic velocity L is the characteristic length or Reynolds number can also be written as square by nu divided by L by U. Numerator and denominators both have dimensions of time so, this L square by nu is the characteristic timescale for viscous diffusion whereas L by U is the characteristics timescale for advection.

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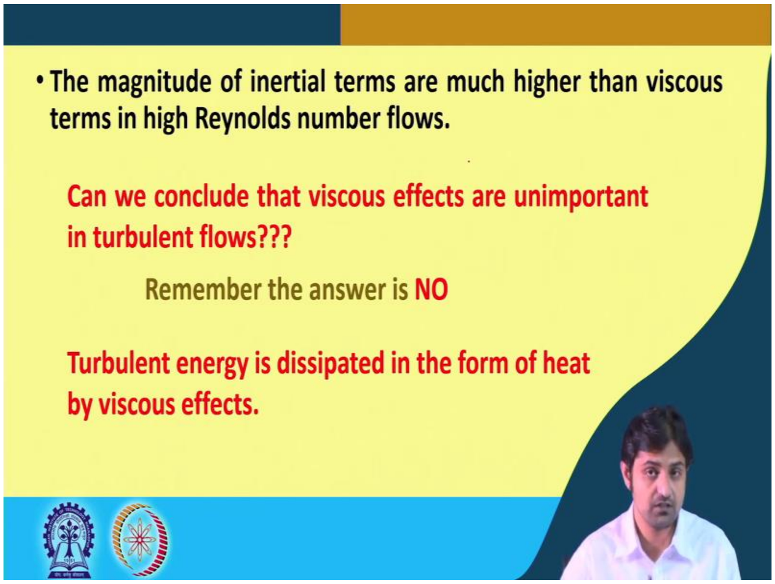

The magnitude of inertial terms are much higher than viscous term in high Reynolds number flows that is true, because it is the ratio of viscous diffusion because the ratio of the inertial forces to viscous forces. So, can we conclude that viscous effects are unimportant in turbulent flows no it is very important because turbulent energy is dissipated in form of heat by viscous effects? Therefore, the viscous effects are very important in high energy turbulent flows.

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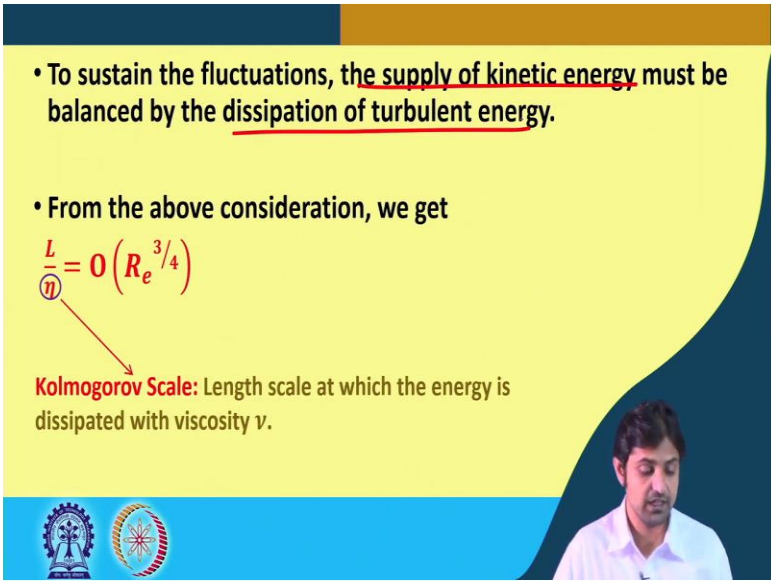

To sustain the fluctuations the supply of kinetic energy must be balanced by these dissipation of turbulent energy this is important thing supply of kinetic energy must be balanced by the dissipation of turbulent energy. So, from the above consideration what we get is all definitely outside the scope the derivation of this that L by eta is of the order of Reynolds number to the power 3 by 4 this this has been obtained and this eta is a Kolmogorov length scale. So, it is the length scale at which the energy is dissipated with viscosity nu you should remember the order and everything but the derivation is not required.

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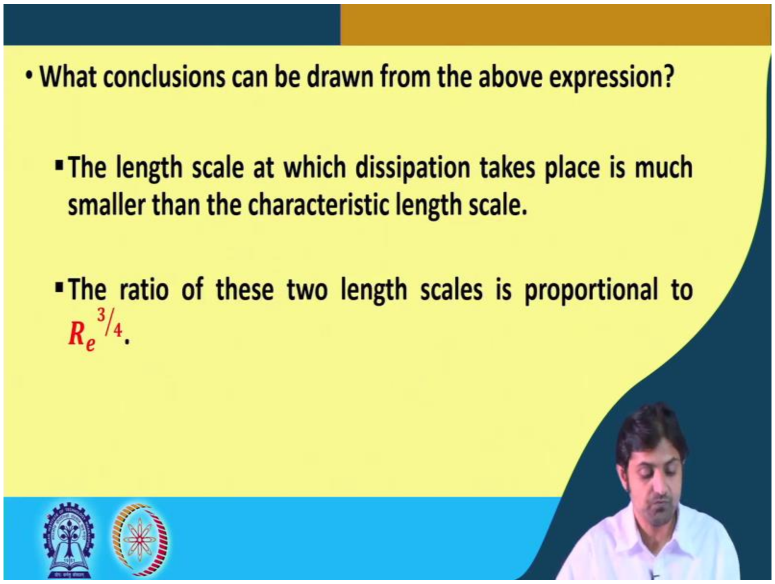

So, what can we convert conclusions can be drawn from the above expect expression this is important so, L by eta so length of the flow divided by the Kolmogorov length scale at which energy is dissipated is the function of Reynolds number to the power 3 by 4. So, if this is the equation what are the conclusions from it? This means that the length scale at which the dissipation takes place is much smaller than the characteristic length scale. L by eta is proportional to R e to the power so in turbulent flow and Reynolds number is very high.

So that means L by eta is very high. This means numerator is much, much larger than the denominator. That is implying the length scale at which dissipation takes place is much eta is much smaller than the characteristic length scale L. Valid conclusion from the equation. The ratio of these 2 length scale is proportional to R e to the power 3 by 4 as we have seen in the last slide.

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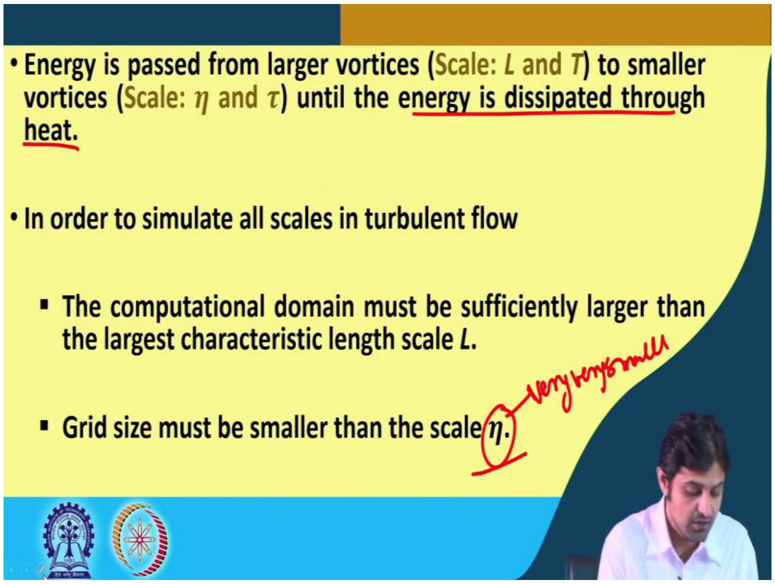

Second thing is the energy is passed from large vortices, that means, the flow is of the scale of length of L and T to smaller vortices that is, so, because see the dissipation happens at the Kolmogorov length scale. So, the eddies become transfer energy to each other they keep on losing the energy you know and until they become they come into range of this Kolmogorov length scale and at this point this energy is dissipated through the heat.

So, in order to simulate all the scales in turbulent flow So, now you see we have a length of scale from capital L to eta it is Kolmogorov length scale which is of the order of several order of magnitude less than millimeters the computational domain must be sufficiently large than the characteristic length scale L . So, to see there will be what is this which will be of the length L as well so, the computational domain must be sufficiently large than the character length L.

But also it is important that the grid size must be smaller than this may then this then this it is very small of the order of 10 to the power -5, 6 meters and length the scale of the typically of the order of let us say 1 or 2 meters or 10 to the power 1 meter, let us say so to be able to model all the type of energies energy in direct numerical simulation, the two things that are important is that the total domain means the study area should be larger than the length L which is of the order of meters several time 100 of meters but the grid size must be smaller than this Kolmogorov length scale which is very, small.

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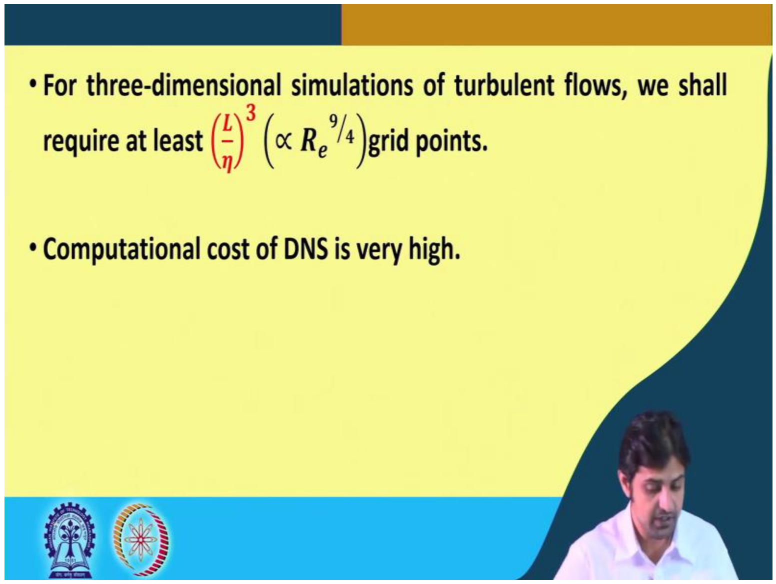

The implication of this is that for 3 dimensional simulation of turbulent flow we will require at least L by eta to the power 3 that means, Reynolds number 2 the power 9 by 4 grid points let us say R e is very, not very high Reynolds number let us see Reynolds number is 10 to the power 4. So, how many grids is needed? R e that is 10 to the power 4 raise to the power 9 by 4. So, we need 10 to the power nine grids almost so much high more than billions.

You know around billion grids, which currently looking at the capacity of our computational facilities it is not possible. So, these things like R e to the power 9 by 4, R e to the power 3 by 4 those are the term which you must the values which you must remember for the computational cost of DNS is very high.

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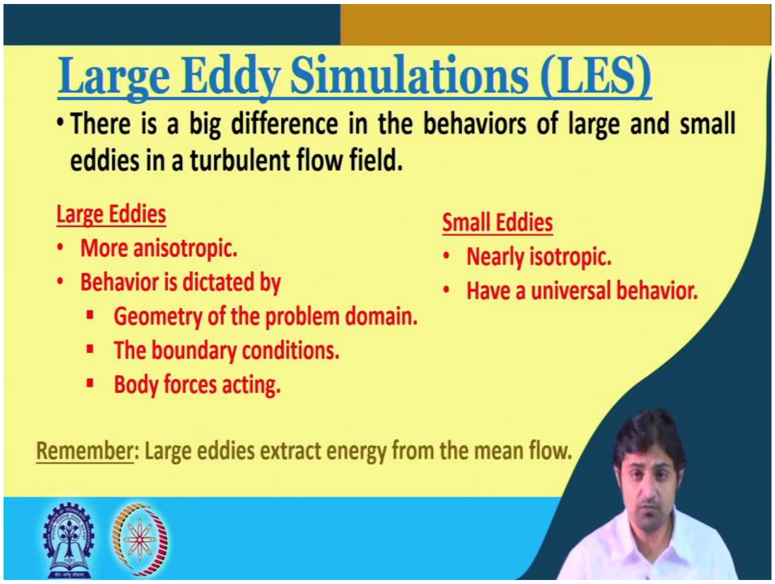

Another such technique is called Large Eddy simulation. See in the DNS one important thing to note was that we had the best accuracy but lot of computational time is required LES is sort of a tradeoff between the Reynolds average in Reynolds average we do many approximations so the results are not that accurate compare to DNS, but LES is something which is a tradeoff between DNS and Reynolds average navier stokes equation. So there is a big difference in the behaviors of large and small IDs in turbulent flow field.

We were talking in DNS about the length scales, we said that there will be vortices or a LES that are as big as the length of the flow, there will be LES at the time of dissipation if heat will be very, very small, let us say order of 10 to the power -5 -6 which also means that there is actually big difference in the behavior of these Eddies large Eddies will have a different behavior and smaller these will have a small bit, I mean, a different behavior.

So what are large Eddies they are large, Eddies are more, anisotropic. And their behavior is dictated by the geometry of the problem domain. And the boundary conditions the larger Eddies, they also must depend upon the they will also depend upon the body forces acting whereas small Eddies they are nearly isotropic they are very small they have not generally a universal behavior as demonstrated by Kolmogorov which we have not read in this course.

But just take it for granted that these have a universal behavior important thing to remember is that the large eddies extract energy from the mean flow, so, larger that is more than energy will be there, and they take energy from the mean flow that the flow real flow.

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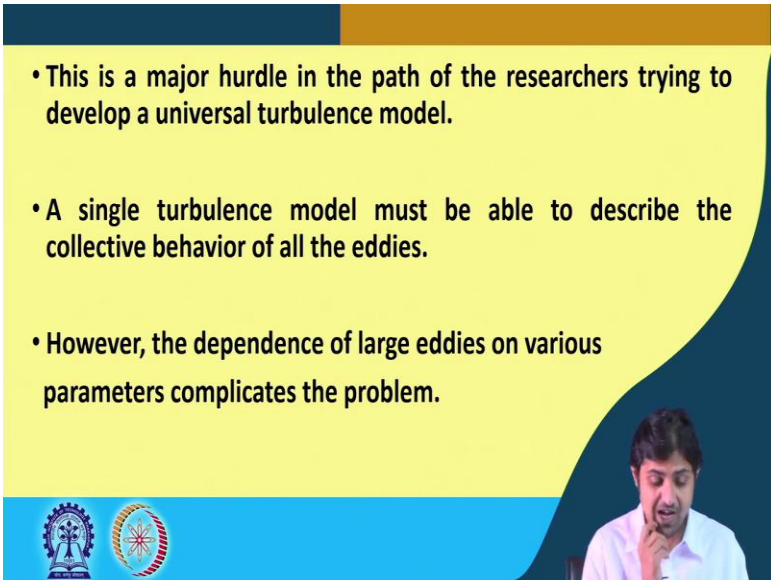

Whereas as small eddies take energy from the a little bit larger eddies which takes more energy from the larger Eddies than them. So energy is in form of a cascade. So, this this term is called Kolmogorov hypothesis. Again, I am telling it is outside the scope, but it is better to mention that this is actually a major hurdle in the path of research just trying to develop a universal turbulence model, which is what is the hurdle the variation of eddies from large scale to small scale?

And the fact that the largest scale eddies are not universal in nature only smaller eddies are a single turbulence model must be able to describe the collective behavior of all the eddies. However, in the dependence of large eddies on various parameter complicates the problem smaller eddies no problem because it is universal.

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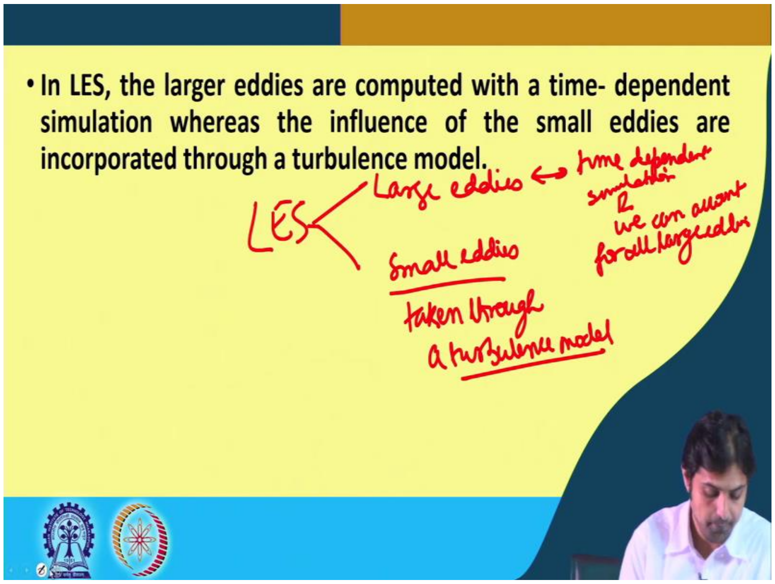

So, in LES the larger eddies are computed with a time dependent simulation where the influence of this small eddies are incorporated through turbulence model. So, we say in LES 2 parts large Eddies and there are small eddies. So they are solved through time dependent simulation and we account for everything for all large eddies but small eddies are so much the, the effects of the small eddies is taken through a turbulence model.

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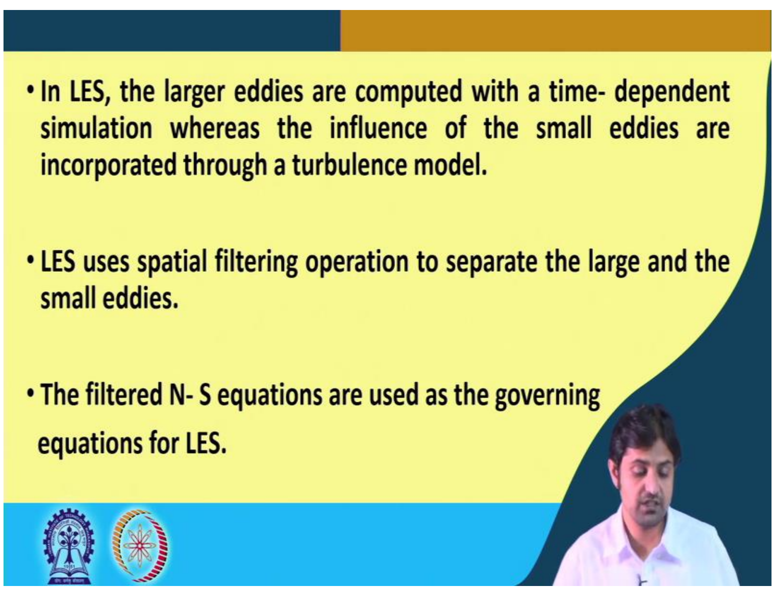

So our domain size does not is not mean we do not need to be, you know, accounting for the smaller eddies. We make our grid size that it captures the largest of the eddies. So, LES uses spatial filtering operation to separate the large and the small eddies as I have told you and the filtered navier stokes equation are used as the governing equation for large Eddy simulation.

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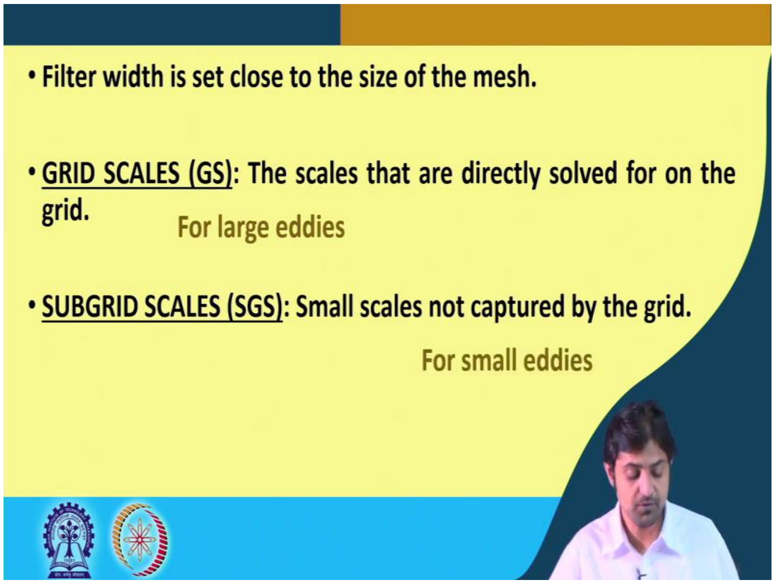

Now, what would be the filter means up till what time what lens should we cut it the filter width is set to be close to the size of the mesh. There are some terms that when we LES when we use the size of the mesh 1 is the Grid Scale GS. The scales that are directly solved for on the grid are called the grid scales for large eddies they are they are the grid scales are used for the large eddies and for the smaller one the Subgrid Scales SGS.

So, small scales are not captured by those grid correct because they are larger in size to capture those eddies. The grid should be at least more I mean smaller than those so sub grid scale modeling is used for smaller eddies.

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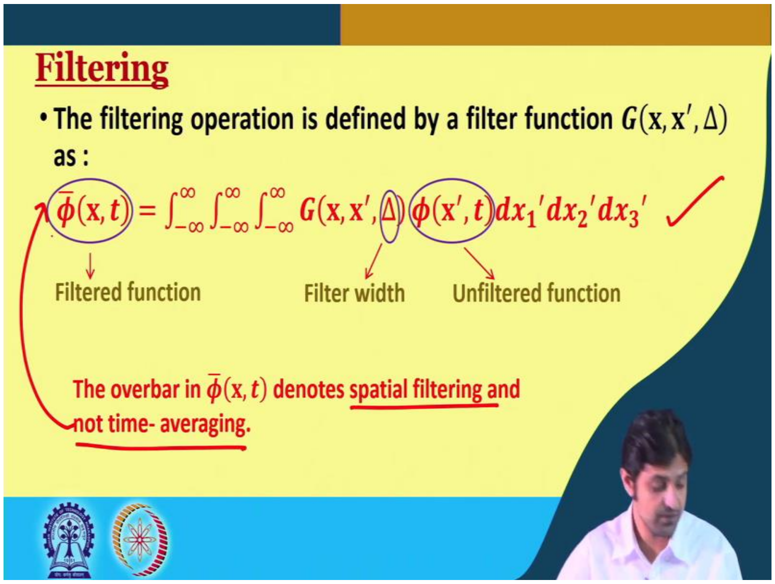

Now the filtering the filtering operation that we have talked about is defined by a filter function g of x, x dash, delta as it is a very complex but there is a filtering operation that you must know that in LES we use a filtering operation to cut off the smaller eddies and solve in reality for the larger eddies and then approximate this smaller eddies is using the turbulent model. So this is a filtered function delta is a filter with which is mostly said to the size of the mesh.

And this is the unfiltered function, phi x dash t. The over bar in phi bar x t denotes is spatial filtering and not time averaging. This is very important. So until now, when it there was bar we used to do average in time, but this 1 is spatial filtering spatial filtering. So, average in space.

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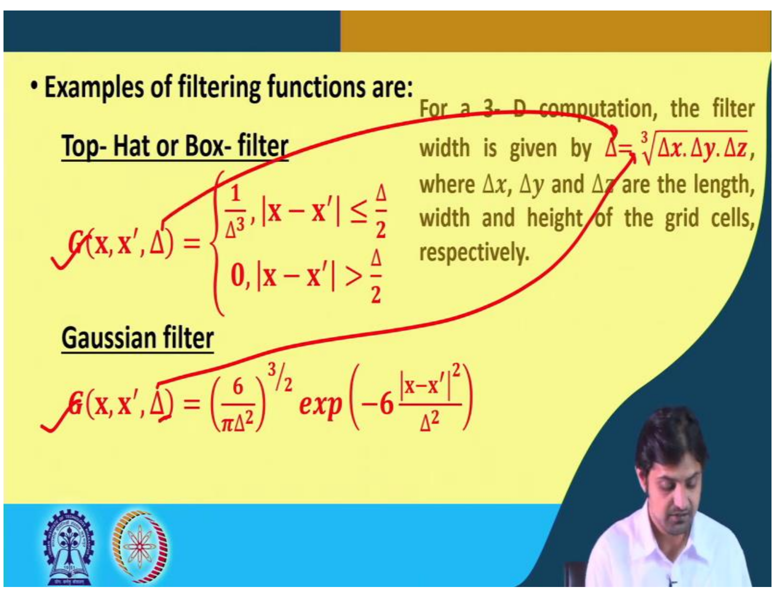

So examples of filtering functions are there is a top hat or box filter, where g x see, you see there is a G here, this is a filtering function and this therefore this is a filtered function. So g xx can be given by different box filter, it is a different study in its own is a Gaussian filter. So remembering the name of the filter is good enough you do not need to remember those equations. So for 3d computation, the filter with this is important.

The filter of it is given by this you must remember that delta is given by cube root off under root x, root y and root z where delta x, delta y and delta z or the length width and height of the grid cells respectively. So, you saw what G is and what delta is this delta is here.

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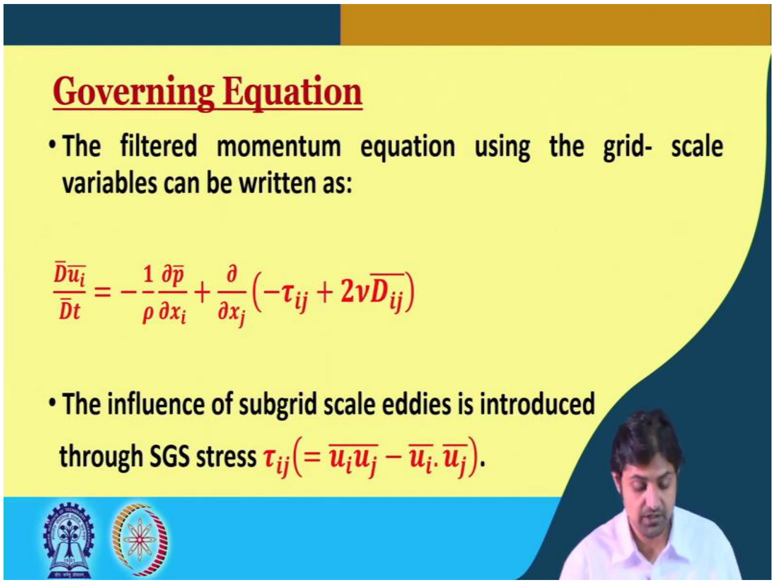

So, now coming to the governing equations of LES the filtered momentum equation using the grid scale variables can be written so, this is again it is a space averaging. The influence of sub grid scale eddies introduced through SGS stress, sub grid scale stress as SGS and that have written as tau i j you see here is a term tau i j just similar to the Navier stokes equation and that is written as i u j u i u j whole bar - u i bar into u j bar, these are against space average in LES.

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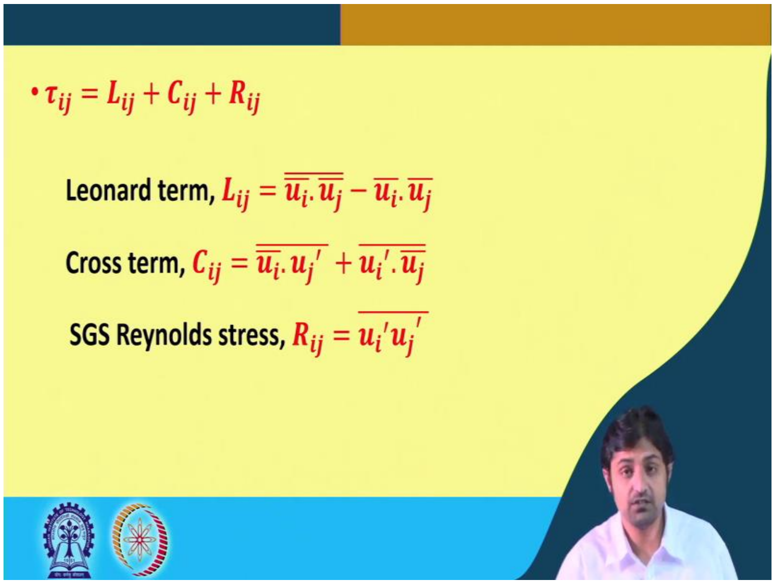

Where tau i j is L i j + C i j + R i j. The L i j is the Leonard term given by this there is a cross term called C i j the SGS Reynolds Stress R i j.

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So, this gives you an overall idea the differences between the Reynolds shear stress in a little bit more detail. The DNS the idea of DNS, why it is computationally so expensive has been reduced. We went through the term like R e to the power 3 by 4 number of grid points are equal to R e to the power 9 by 4. The basic idea of LES where we studied that we basically use the filtering function to model the larger these various sub grid scale modeling is used to model the smaller Eddies.

So, information like this what are the different terms Leonard term, Cross term and SGS Reynolds stresses and things like that have been covered in the turbulence modeling part. Many complex equations are not supposed to be remembered by you, but an overall idea about those equations or whatever required in this particular module. And this actually concludes our module on computational introduction to computational fluid dynamics.

I hope you enjoyed this particular session, this particular module of the course and as always, these are the references of then I mean the reference books that you can actually use. And thank you so much for listening and I will see you next week.