Hydraulic Engineering
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Lecture # 57
Computational fluid dynamics (Contd.)

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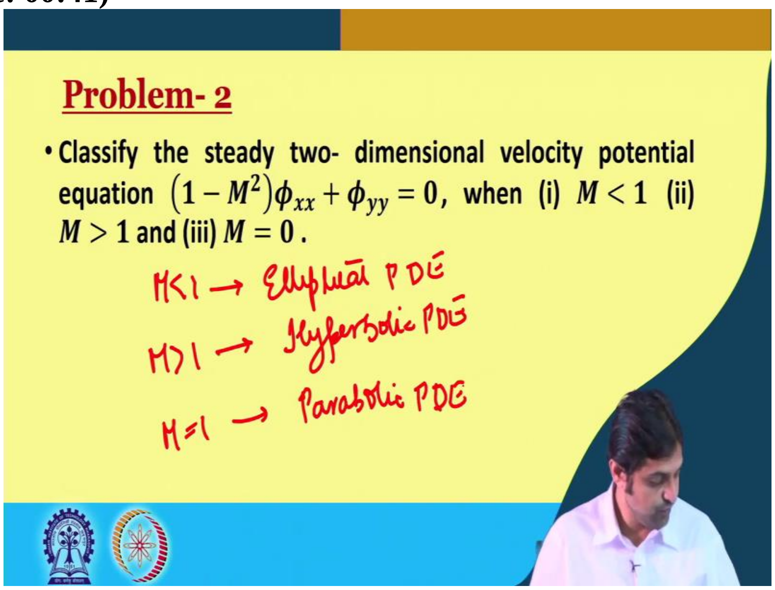

Welcome back students. So, in the last lecture we started with a problem and then I went back and showed you what different values of A B and C are. So, solve this problem we will use the whiteboard.

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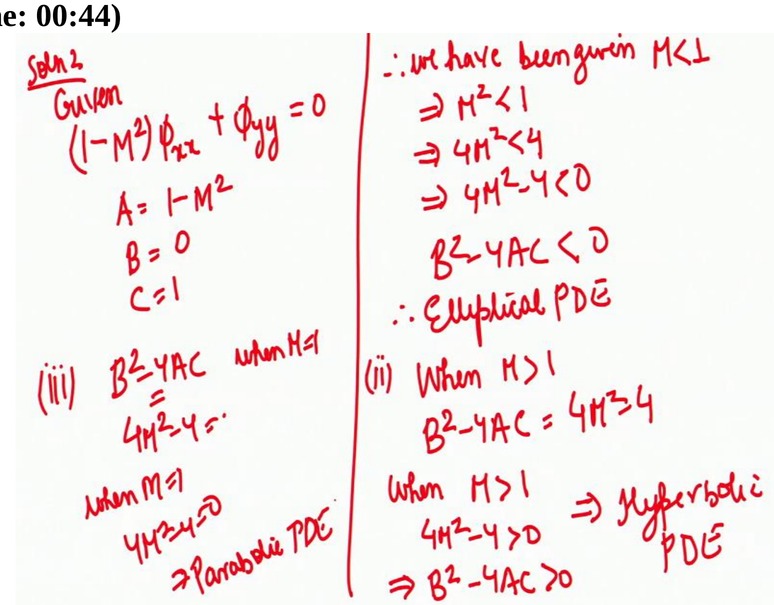

So, the best way is always to solution to given first we write what the things are given. So equation given 1 - M squared phi xx + phi yy. So drawing the analogy from the general equation, w write A = 1 - M squared, second, B is 0 and C is 1. So the first part, we say when m is less than 1, we have to calculate the value of b square – 4AC, so B is 0, so 0 - 4 is 1 - M squared into 1. So - 4 + 4 m squared are 4 M squared – 4.

We have been given M is less than 1 implies M squared will also be less than 1 which implies 4 M squared is less than 4 so if you bring 4 on this side, it will become 4 M squared - 4 is less than 0. Therefore, this value 4 M squared - 4 are B squared – 4AC will be less than 0 which means it is and elliptical PDE so in the second part, when m is greater than 1.

We have found out does not matter what the value but b squared - 4 AC came out to be 4M Squared - 4, right. So when M is greater than 1, this means 4 M squared - 4 is going to be greater than 0, which implies B squared - 4 AC is greater than 0, this implies it is hyperbolic PDE. Now the hard part is I will rub this one here B squared - 4 AC when M = 1. So B square - 4 AC came out to be 4 M squared - 4 right. So when M = 1 4 M squared - 4 = 0. This means it is parabolic partial differential equation.

So, this is how we have solved the problem. So I will just write the solution here. So, when M is greater than 1 it was elliptical PDE M greater than 1 it was hyperbolic And for M = 1 it was parabolic PDE. So, time to move onwards.

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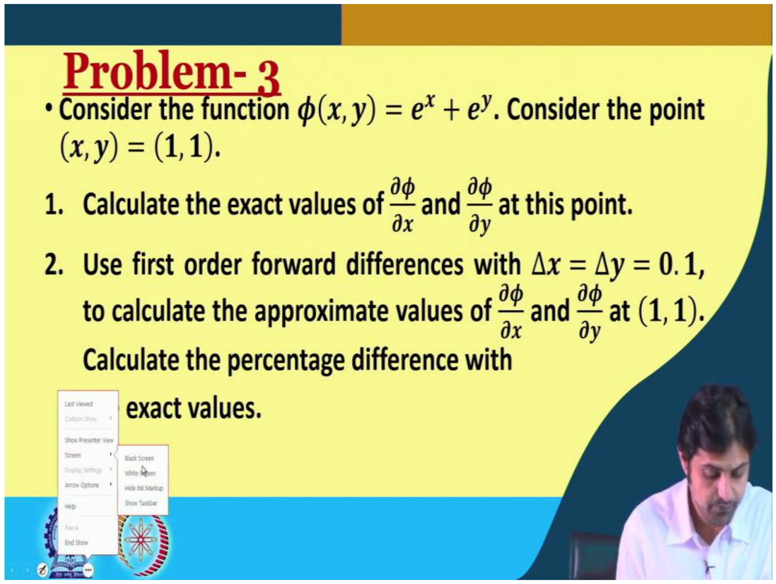

Will solve another problem. So, it says that considers the function phi of x y is e to the power x + e to the power y and we have to consider the point x, y as 1, 1 we have to calculate the exact values of del phi del x and del phi del y at this point at point 1, 1. Secondly, use the first order of finite difference with delta x = delta y = 0.1 to calculate the approximate values of delta phi by delta x and delta phi by delta y at 1, 1 and calculate the percentage difference with the exact value. So, this is the application of all the things that we had done in the last lecture and this will be a good way to understand.

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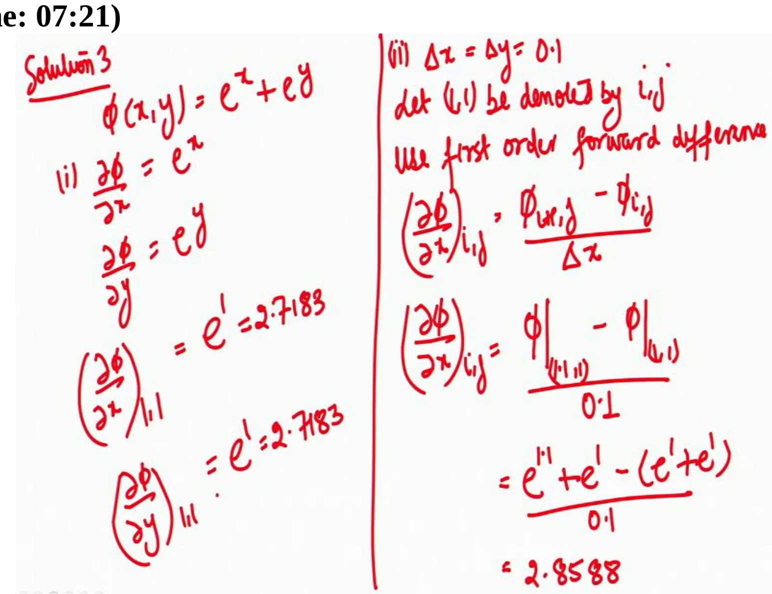

So, solution number 3. So, what we have been, we know that the function phi x, y has been given as e to the power x + e to the power y. So, for the first part, we should be able to calculate del phi del x will be e to the power x correct and del phi del y is going to be e to the power y simple partial differential so we have to calculate del phi del x at 1, 1, it is going to be e to the power 1 are 2.7183. Similarly, del phi by del y at 1, 1 e to the power y that is e to the power 1 2.7183.

So, that was a first question that we have solved the values of del phi del x at 1, 1 del phi del y at 1, 1. In the second part we have been given that delta x = delta y = 0.1 this is what we have to assume let 1, 1 be denoted by i, j. And if we use first order forward difference del phi del x at i, j can be written as phi of i, j + 1, j – phi i, j divided by delta x del phi del x i, j can be written as phi of phi at what .1 because delta x is .1, so 1 + delta x so that is 1.1, 1 – phi at 1, 1 divided by 0.1.

This will be e to the power 1.1 + e to the power 1 - e to the power 1 + e to the power 1 we just putting in the values and if you use your calculator, it is going to be 2.8588. That is del phi del x at 1, 1. So, we are going to it is everything here and proceed.

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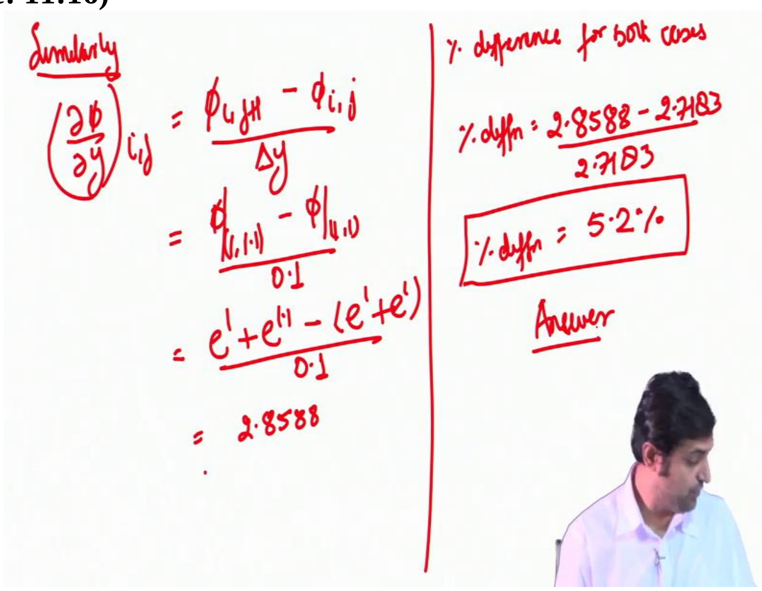

So, similarly del phi by del y i, j will be phi of i, j + 1 – phi of i, j divided by delta y are the value of the function at 1, 1 .1 – phi at 1, 1 divided by delta y 0.1 are e to the power 1 + e to the power 1 + 1 - e to the power 1 + e to the power 1 divided by 0.1. This is 2.8588. So, percentage difference for both cases is 2.8588 - 2.7183 divided by 2.7183 and this percentage difference come out to be if this 5.2%.

So, this is the answer. So, this was the simple most question and the solution for demonstrating the percentage difference and the concepts that we have learned in the last lecture. So, we proceed from this point now.

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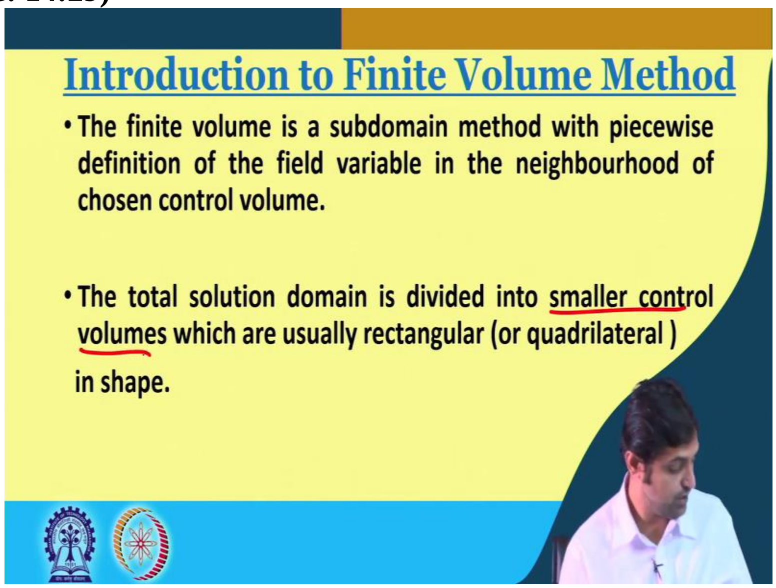

To we mentioned about other schemes of may be machine that is the introduction to finite volume method, what is finite volume method. So, the finite volume is the sub domain method with piecewise definition of field variable in the neighborhood of chosen control volume, so, the total solution domain in the Finite volume is divided into smaller control volume. So, the total solution to the total domain is divided into smaller control volumes. Normally they are rectangular in shape or quadrilateral inch.

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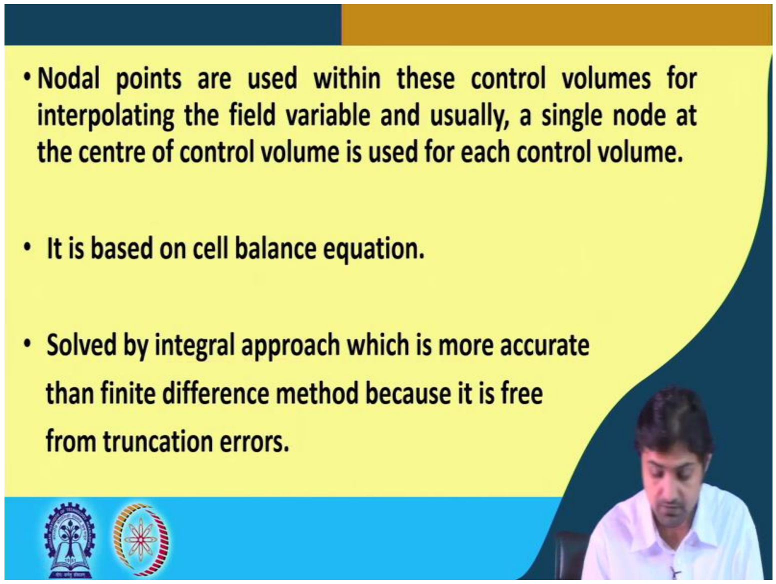

So, nodal points are used with these control volumes for interpolating the field variable and usually a single node at the center of the control volume is used for each control volume. So, this is the 1 single point in that control volume that is used and this is based on the cell balance equation. It is solved by the integral approach which is more accurate than the finite difference method because it is free from truncation error. So, one thing about the integral approach is that it is free from the truncation errors.

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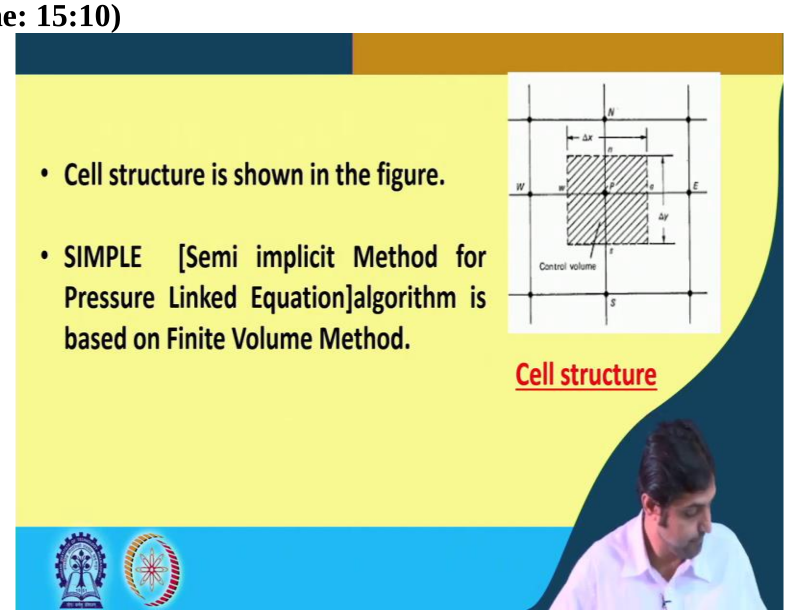

Cell structure is shown in this figure here, you see, this is the control volume the dashed no zone and this at the center point you know, we calculate the things simple is one method which is called semi implicit method for pressure linked equations. So, simple algorithm is based on finite volume method this is just one example of finite this was just something about the finite volume method.

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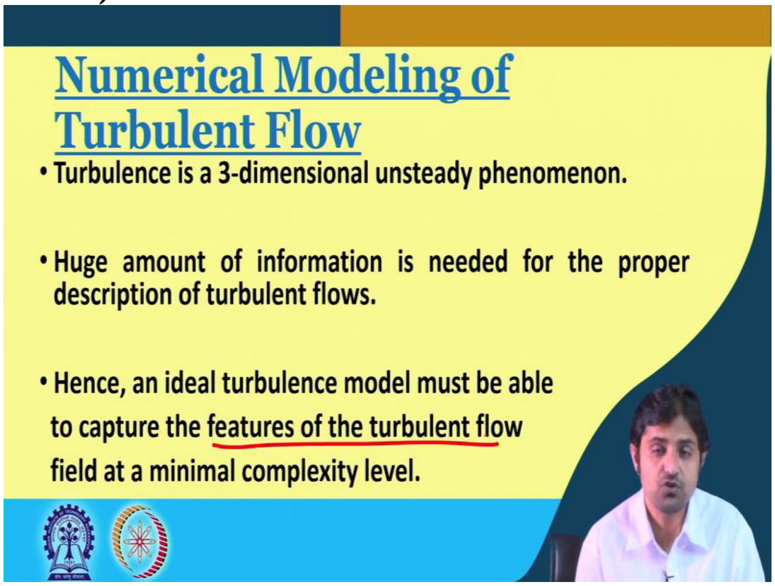

So, now going to the things that are very much relevant to our hydraulic engineering course that how do we model or do the numerical modeling of the turbulent flows because most of the flows in real nature at turbulent for turbulence is a 3 dimensional unsteady phenomenon. So, it is a function of time and it also function of a space. So, a huge amount of information is needed for the proper description of the turbulent flows that is a very well established fact.

Hence, an ideal turbulence method must be able to capture the features of the turbulent flow field at a minimum complexity level. So, ideally the best of the numerical model that can do this type of turbulent flow modeling should be able to capture the features of the turbulent flow with least complexity level. So, the model should be least complex as well.

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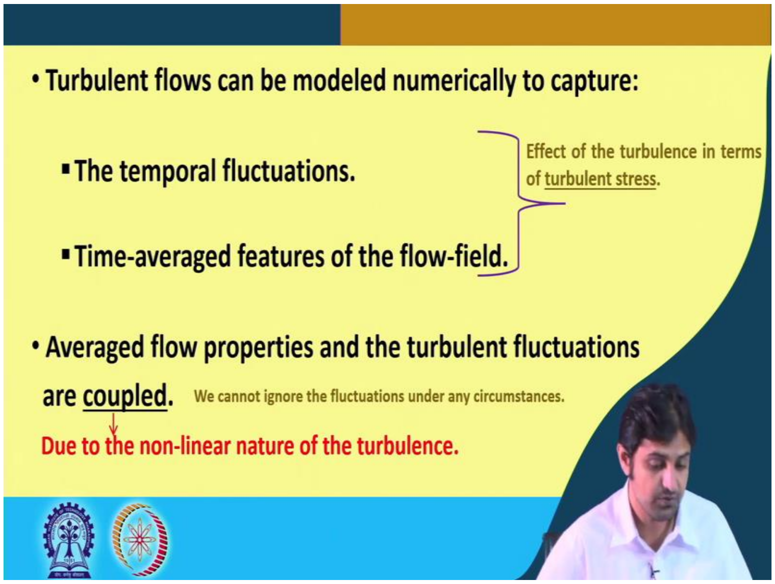

Turbulent flows can be modern numerically to capture the temporal fluctuations the time average features of the flow field. So, in the real velocity you know i mean the fluctuations mean the deviation from the average velocity and the time average features of the flow field. So, these because these 2 effect the turbulence in terms of turbulent stresses these 2 quantities, you remember we mentioned about you prime and you bar in our laminar and turbulent force flows.

So, average flow properties in the turbulent fluctuations are actually coupled and why are the coupled due to nonlinear nature of the turbulence, this is all the information that we know from before. And for this particular reason, we cannot ignore the fluctuations under any circumstances. So, an ideal turbulent model should be able to capture all these fluctuations too.

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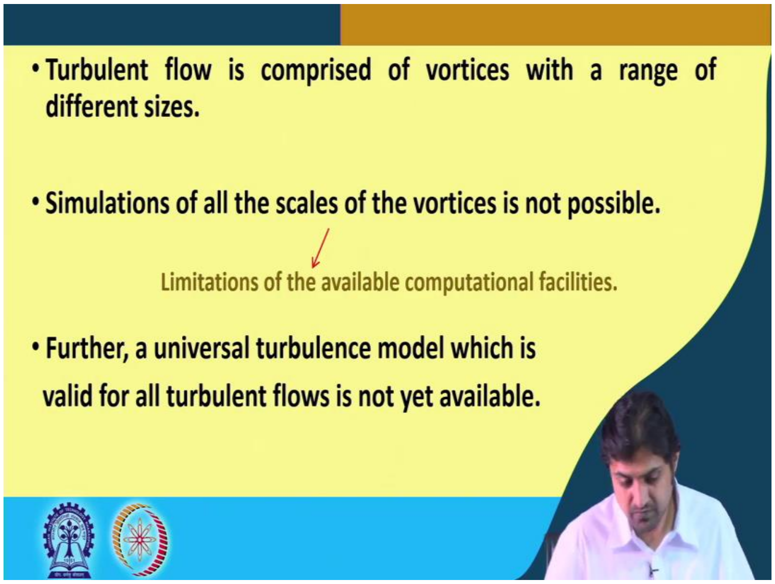

This turbulent flow is comprised of vortices with a range of different sizes. Simulations of all scales of the vortices is not possible, I mean, the vortices could be very, small as well. So, the modeling of the smallest of these vortices might not be possible, because the reason is we have limitations on the available computational facilities. The time and the quantum computing is the reality this might be an easier thing to do.

But for now, until that point, we still consider that our computational facilities where we are able to capture all the scales of what it says is not yet possible. Further a universal turbulence model which is valid for all the turbulence flows is not yet available as well.

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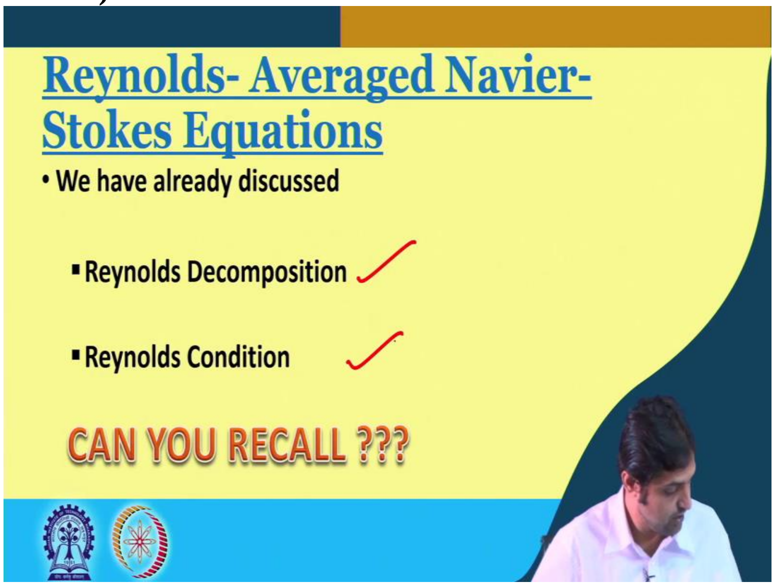

So, with this background, we come to one of the most important techniques to solve these navier stokes equation and this is called Reynolds average navier stokes equation as the name indicates, what we do is, we do the averaging of the Navier stokes equation Reynolds because Reynolds was the first scientist to do it. So, we have already discussed this term called Reynolds decomposition we have discuss this and laminar and turbulent flow module and the Reynolds condition. If you are able to recall just in just in a case where you are not I have included that again as a quick repetition.

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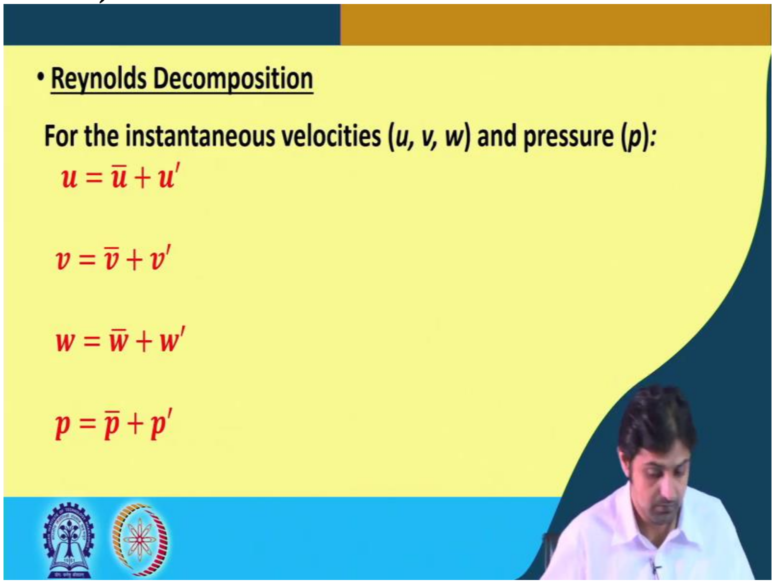

So, what is Reynolds decomposition for the instantaneous velocity is u v w and the pressure p u can be written as u bar + u prime. And v is written as v bar + v prime this is the bar whatever is in the bar denotes the average value and whatever is with the prime it indicates the fluctuations. Similarly, w is written as w bar + w crime and pressure is written as p bar + p prime.

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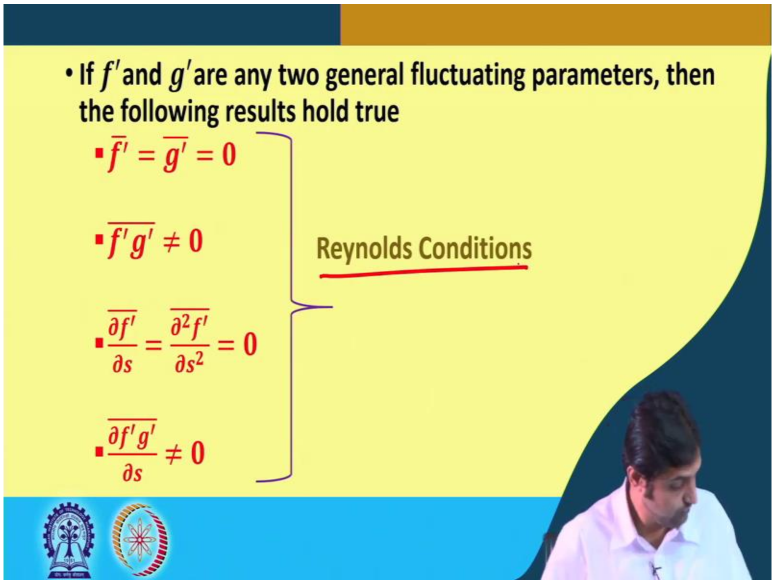

So, if we say that f dash and d dash or any to general fluctuating parameters, we also have dealt this in our previous slide, then the average of f bar and g bar is going to be 0 so, f prime whole bar and g prime whole bar is 0. Also we know that f prime multiplied by g prime and taking the average of that quantity is not equal to 0. If you recall we had Renault shear stress in u prime w prime. If this was 0 then there would be no shear stress but this is not equal to 0 there is a finite shear stress.

The third is the del f prime. So, the del s of the fluctuation the first differential and also the second differential and if we take the whole bar = 0 also if we multiply both the fluctuations f dash the dash take the differential and average it will not be equal to 0. So these are called the Reynolds condition. So, this we have already seen before.

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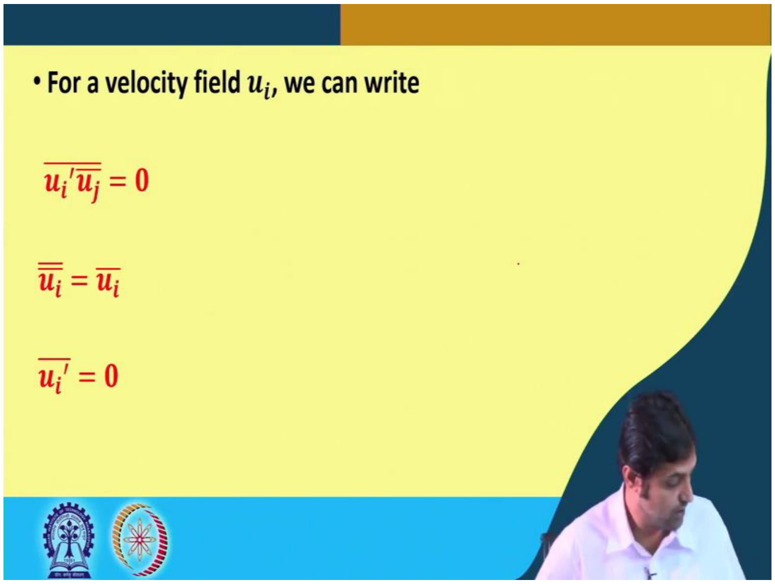

So, for a velocity field ui we can write ui prime into uj bar to the whole bar = 0. Also ui double bar = ui bar. If you take the average of the average it is the average itself and the fluctuation if it is average it is 0, this is Reynolds averaging.

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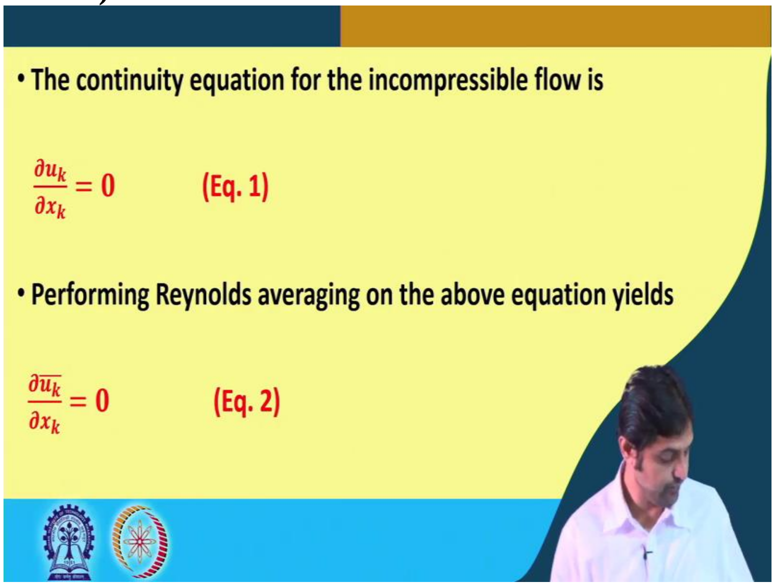

So the continuity equation for the incompressible flow is this Del uk by del x k = 0, right. And if we do the we perform the Reynolds averaging on the above equation, it is going to yield one equation that is del uk bar by del x k bar = 0, right? You do not have just uk you can write uk bar + uk Prime. And so Del, uk by del x k will be del uk bar by del x k + del x k uk bar and this will go to 0 So, this will can be represented as this.

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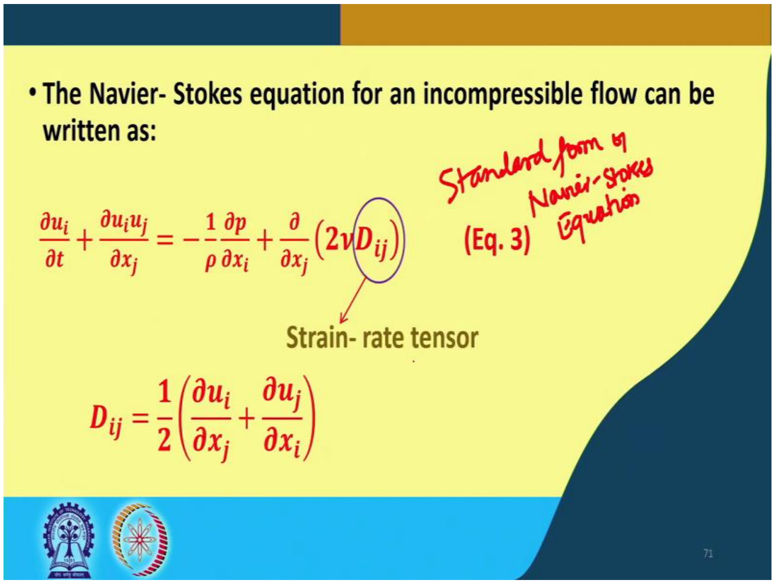

The Navier stokes equation for incompressible flow can be written as Del ui Del t + Del ui uj by Del x j = - 1 by rho Del p del xi + del xx j to nu Dij this is a standard form of. So, dij here is a strain rate tensor which we have seen in the last lecture in our viscous fluid flow that dij returns half of Del ui by Del x j + Del uj by Del xi. I think this is not a problem because we have derived it.

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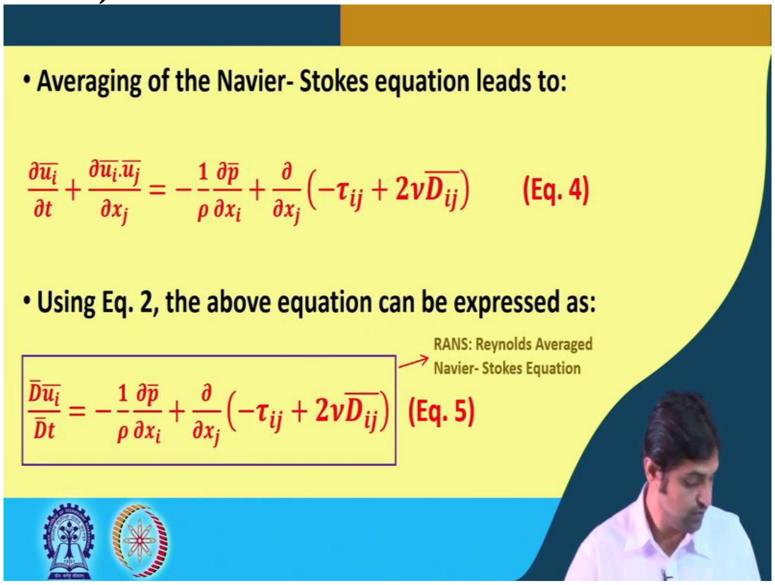

So, if we do the averaging of navier stokes equation, if we put u = u bar + u prime and take the whole average of the equation so, u = u bar + u prime and v = v bar + v prime and w = w bar + w prime and p = p bar + p prime and put it into Reynolds equation and do the whole average of the equation. We are going to get equation like del ui by del t whole bar, del ui bar del uj bar So, if you see this is average term, this is also average term and this is also average term right.

So, if we use equation to the above equation, you see you remember that equation to this one here Del uk bar by del x k = 0 using the continuity equation it can be written as Del ui bar by delta t = - 1 by rho del p bar by del xi + del x j of - tau i j + 2 nu dij. As if this is equation number 5. So, this actually is nothing but Reynolds average Navier stokes equation. There is some things that we need to find out what is tau ij dij we already know.

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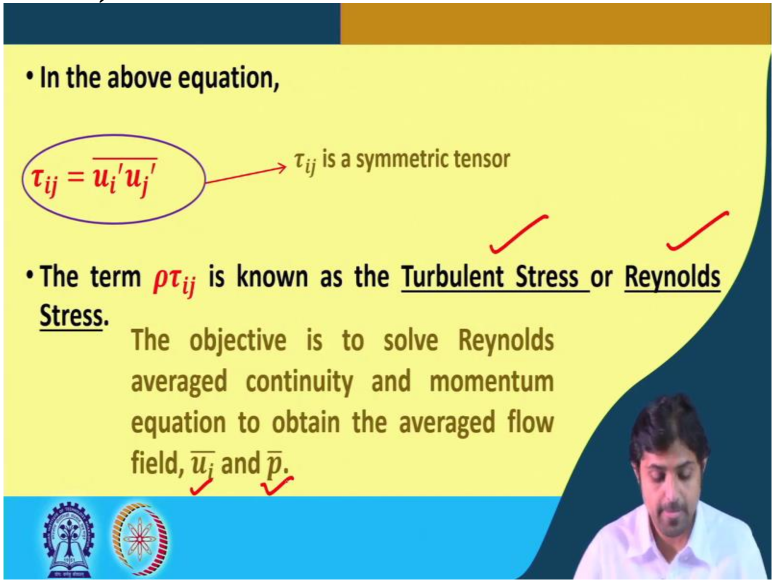

So, in the above equation tau ij is ui prime multiplied by uj prime this is generally called Reynolds shear stress and as we have read about it, is much more than the kinematic stress and tau ij here is a symmetric tensor symmetric means, ui prime uj prime = uj prime ui prime. So, the term rho into when rho multiplied by tau ij is known as turbulence stress or Reynolds stress. The objective is to solve the Reynolds average continuity and momentum equation to obtain the average flow feel ui bar and rho.

So the objective of this thing is to solve the flow for ui bar and p bar we want to find out the average quantities that is so that is why we do some turbulence modeling.

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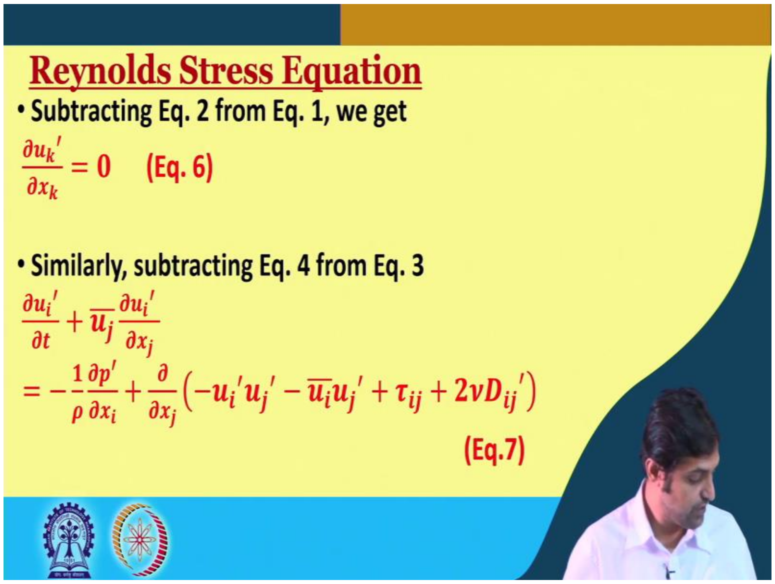

So how do we do it we solve the Reynolds stress equation if we subtract equation to the beginning equation, second equation from equation 1 we can get Del uk prime divided by del xk = 0. Similarly, if you subtract equation 4 from equation 3 you see question number 4, this 1 is 4 and this 1 is 3. So, what we are doing is we are subtracting the average to a general equation in both the cases.

We are going to get Del ui prime del t + uj prime original equation - averaged equation same is here original equation - average equation and why do we do this to obtain fluctuation equations. So, with that this is how we are we obtain the equations with fluctuation terms in terms of differential equations.

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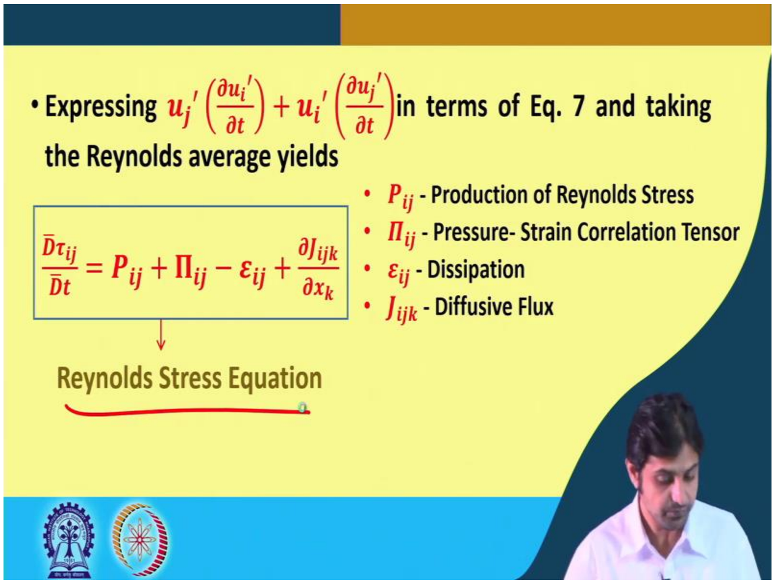

If expressing uj bar into Del ui bar Del u uj prime into del ui prime del t + ui in terms of equation 7. So, if we express that in terms of equation 7 here, you know and this is called the Reynolds stress equation where this dij is called the production of Reynolds stress, this pi ij is the pressure strain correlation tensor and epsilon ij is a dissipation where j ijk is diffusive flux you can try to do this at your own, but the derivation of this is outside the scope of the current course, I have shown you the methods how to get 1 equation from the other but the details is not in your course curriculum.

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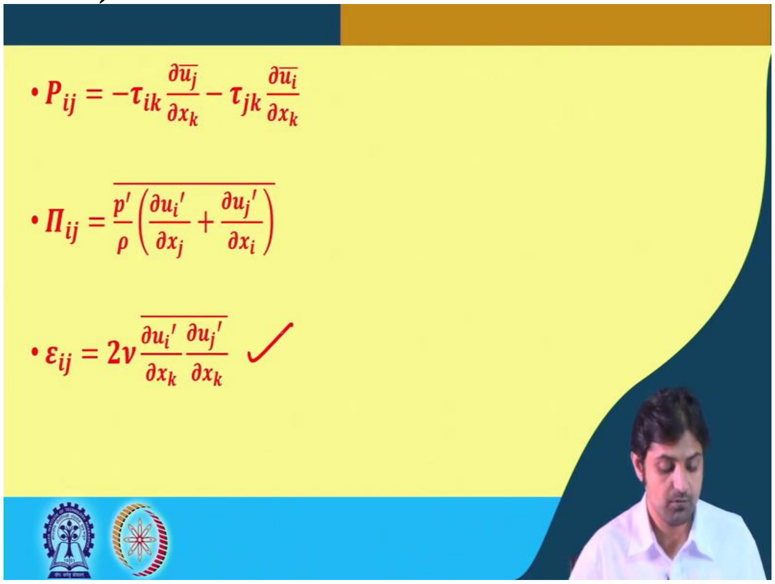

And this is Reynolds stress equation, equation number Pij is given as - tau ik del uj by del xk - tau jk del ui del xk this is the expression of the dissipation production and pressure strain correlation factor epsilon ij is the dissipation it is expressed like this to nu del ui prime del xk del uj prime del xk whole bar.

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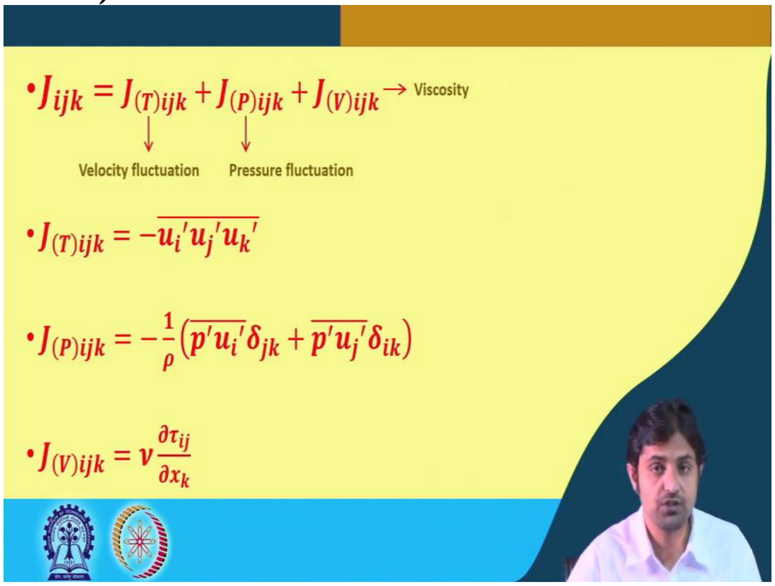

Where Jik is given in this for this is velocity fluctuation, this is pressure fluctuation and this is fluctuation due to the viscosity. Therefore, JT ijk can also be written as ui prime uj prime and uk Prime are you see, this velocity fluctuations is written like this the pressure fluctuation is written like this and viscosity fluctuations is written like this. So, we have seen what all these terms of we have seen that all what all these term means, for this Reynolds stress equation.

You see this one here, Pij pi ij a production of Reynolds stress dissipation diffusive flux and pressure strain correlation factor. So, you do not need to remember all the details here, but the way of doing it is quite an important and somebody who would be interested in doing further research or further studies in M.Tech this will be a complete course in itself. So, now, how to close this problem is the important question and that will be the topic of our next lecture.

And we will also proceed with techniques like direct numerical simulation and large at simulation for the modeling of computational fluid dynamics. So, this will be all for today. Thank you so much for listening and I will see you in the next lecture.