Hydraulic Engineering
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Lecture # 55
Computational fluid dynamics (Contd.)

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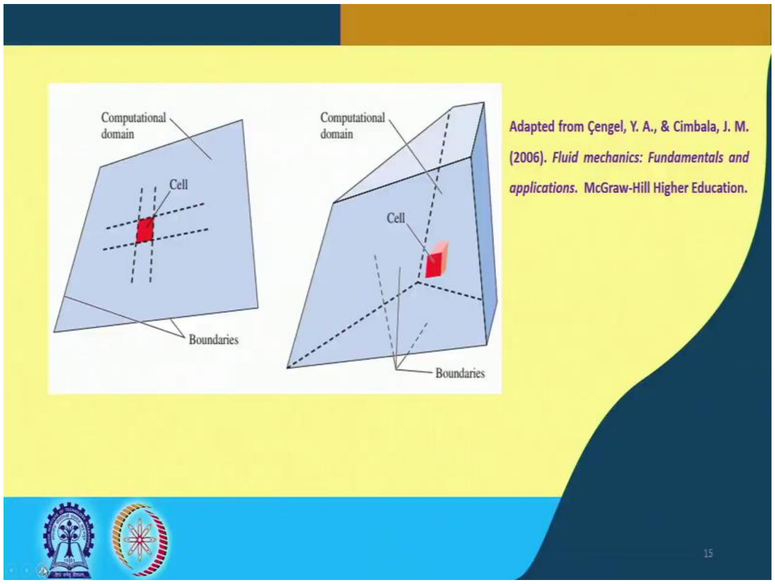

Welcome back students in this lecture, we will we are going to continue what we started in the last lecture with the grid generation. So, we finished that this slide. Therefore, we are going to proceed from this point onwards and see what further analysis can be done.

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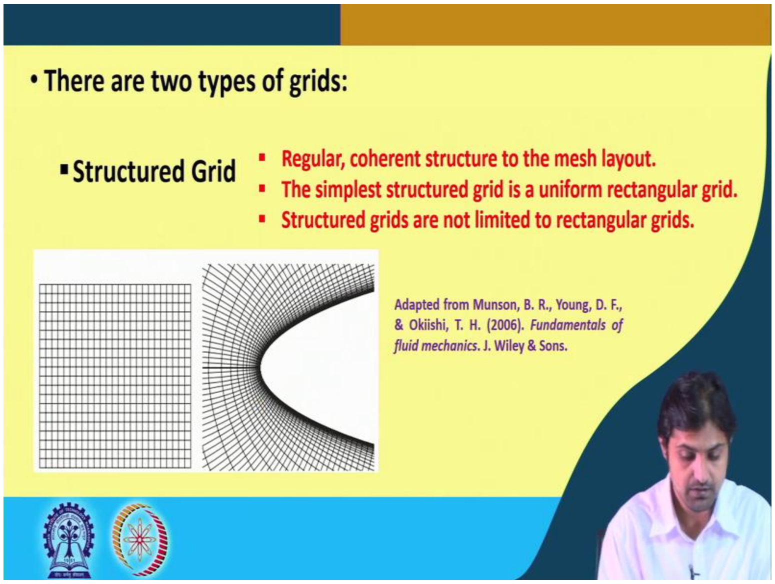

So, we talked about grids so, there are actually 2 types of grids. One is a structured grid and the structured grid means the grids are regular and coherent structure to the mesh layer. These are the simplest the structured grid and they are generally uniform rectangular grid those are called the structured grids. So, the structured grids look like this one here. So, you see they have a uniform rectangular grid. So these all the small shapes are rectangular.

Structured grids are not limited to rectangular grids only. So, these ones this could be off actually of any shape here it chose the rectangular grids only but you see these rectangular grids are decreasing in size.

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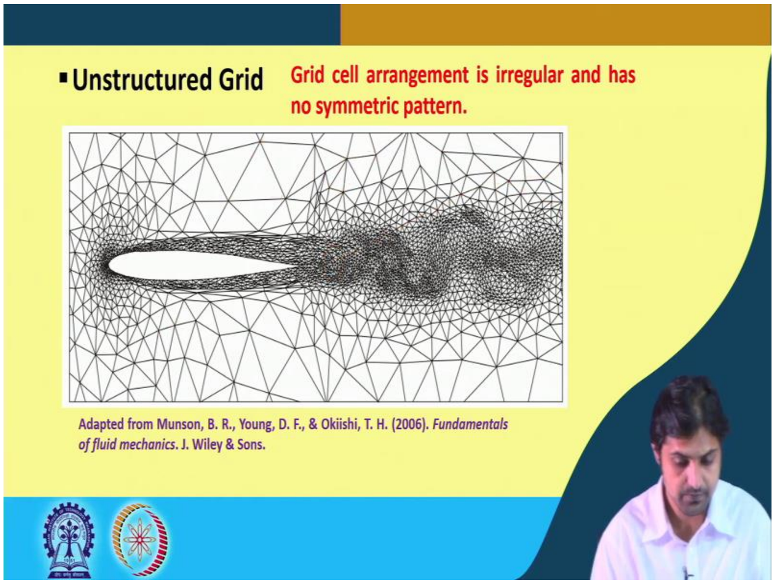

The second one are the unstructured grids. So, the grid cell arrangement is irregular and has no symmetry pattern if you see in the last one, there was a symmetry pattern here. If you consider this one you see symmetry cause this and here it is completely symmetry this one in the unstructured grids, the cell arrangement is irregular and has no symmetric pattern something like this, so, see the triangles, these are off, no specific, same type. You see this the cells here are very small, smallest smaller than these ones here largest.

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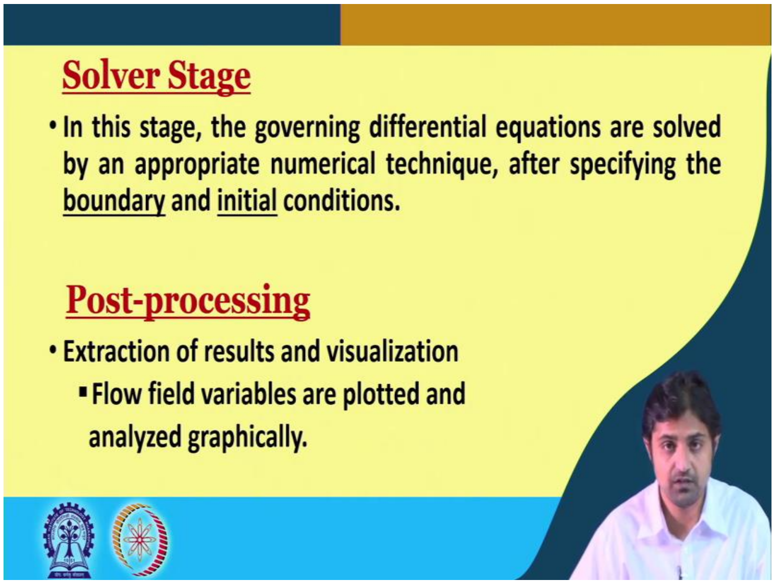

So, now after this regeneration we comes to the solver stage. So, in this stage that the governing differential equations are solved by an approximate numerical technique after specifying the boundary and the initial conditions. So, the actual real solution of those differential equations is done at this stage called the solver stage and in post processing, the extraction of results and visualization how the results appear is done. So, when it comes to extraction of results and visualization flow field variables are plotted and analyze graphically. So, this is the most common thing to do in the post processing.

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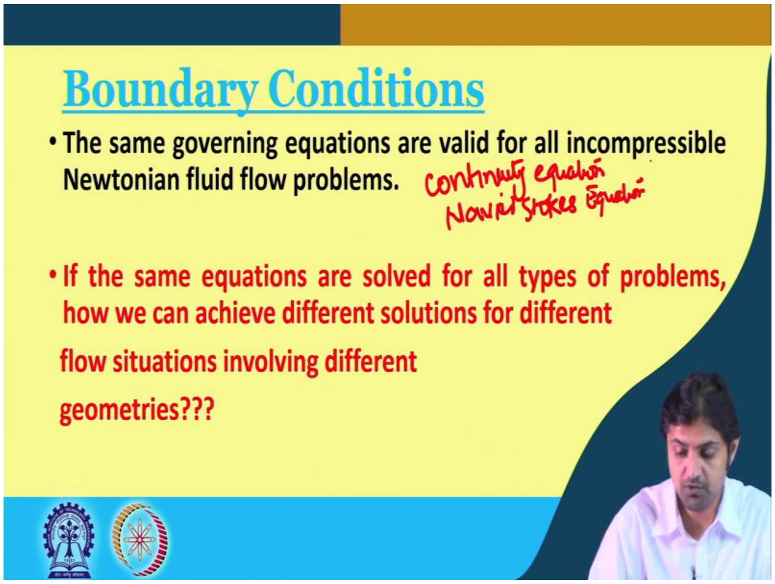

So, you use you know we mentioned about boundary and initial conditions and that is what we are going to discuss next. Boundary condition the same governing equations are valid for all compressible Newtonian fluid flow problems. So, if the same equations are solved for all types of problems, and how can we achieve different solutions for different flow situations involving different geometries that is the question because which governing equations are we going to talk we are talking about the continuity equation, Navier stokes equation.

So, this is common for all the fluid flow problems? Then how do we know and how do we achieve different solutions for different flow situations involving different geometries and this happens because of something called boundaries conditions.

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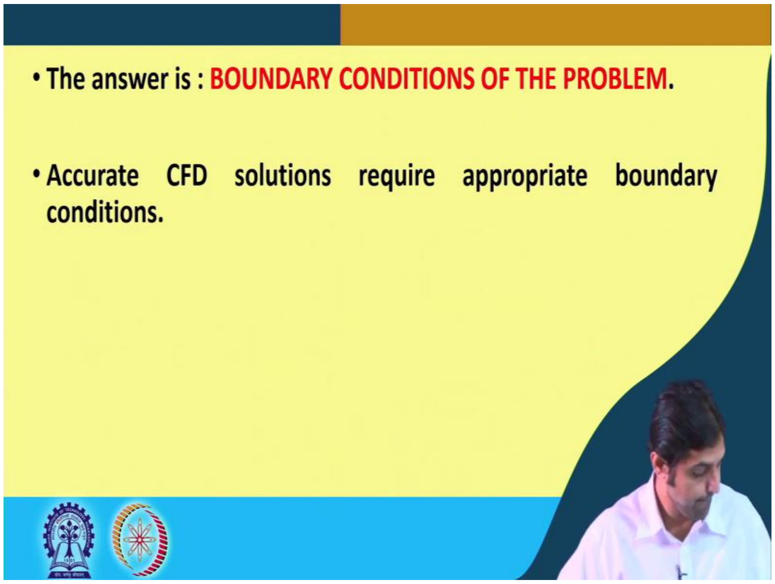

So the answer is boundary conditions of the problem. So, boundary conditions, I will tell you what that is for example, if the flow is occurring in a tank itself the velocity is 1 meters per second, this is one continuously occurring. So, this is 1 boundary condition, there could be other where you know the velocities could be 10 meters per second for example, so, these are different boundary conditions that is why we get different results.

Suppose, 1 of the boundary conditions could be this is closed at this end and the other one that it could be open. So, this is another boundary condition so, because of this difference in boundary conditions we have different solutions of the problem.

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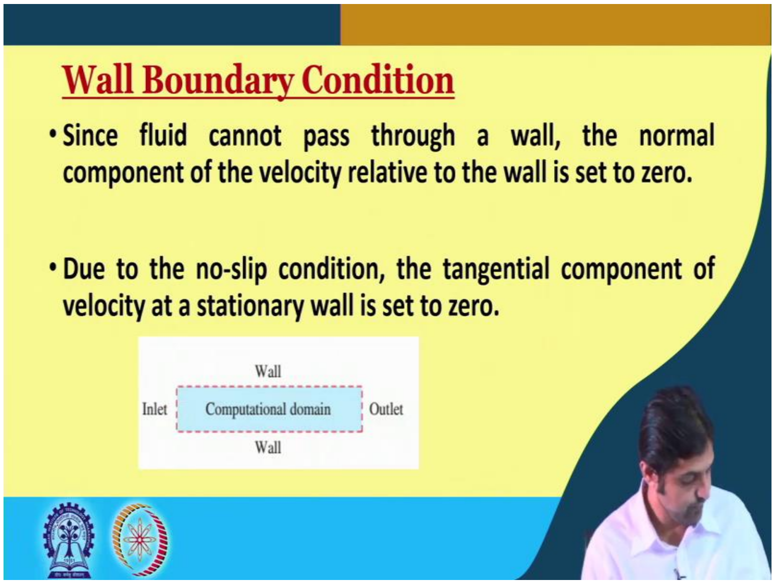

Now, as I said I mentioned about being closed and you know being open there is a wall at the boundary these wall and other concepts you have read in your previous lectures of hydraulic engineering. So, we are going to talk about the wall boundary condition, since fluid cannot pass through a wall the normal component of the velocity relative to the wall is set to 0 and this is what is this called this is called no slip condition. Therefore, due to the no slip condition the tangential component of the velocity at a stationary wall is set to 0.

So, you see there if there is an inlet here, there is a wall here there is an outlet. So, the velocity at this point does not matter what the velocity here is at this particular point anywhere around across this wall the velocity will be 0 due to no slip condition.

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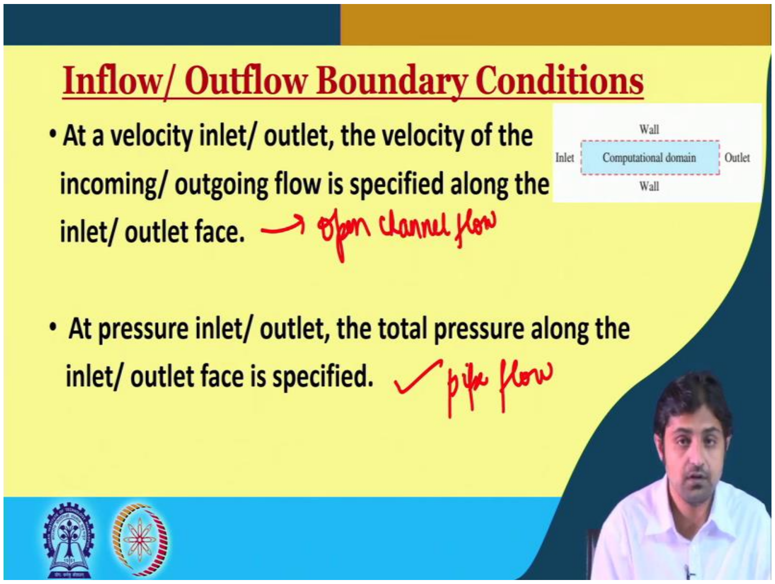

No not talking about the inflow and the outflow boundary conditions. So, you see this is inlet this is outlet. So, the here from here the inflow will be there, and from here out flow will be there. In means coming in and out means going out. So, at a velocity inlet or outlet the velocity of the incoming or outgoing flow specified along the inlet outlet phase at pressure inlet and outlet the total pressure along the inlet and outlet phase specified.

So, the in the inlet and outlet can be the specification can be in 2 forms, whether if whether we want to specify the inlet flow velocities at the inlet or outlet or the pressure. 1 typical example here is the for the pressure is the pipe flow. Here it is open channel flow.

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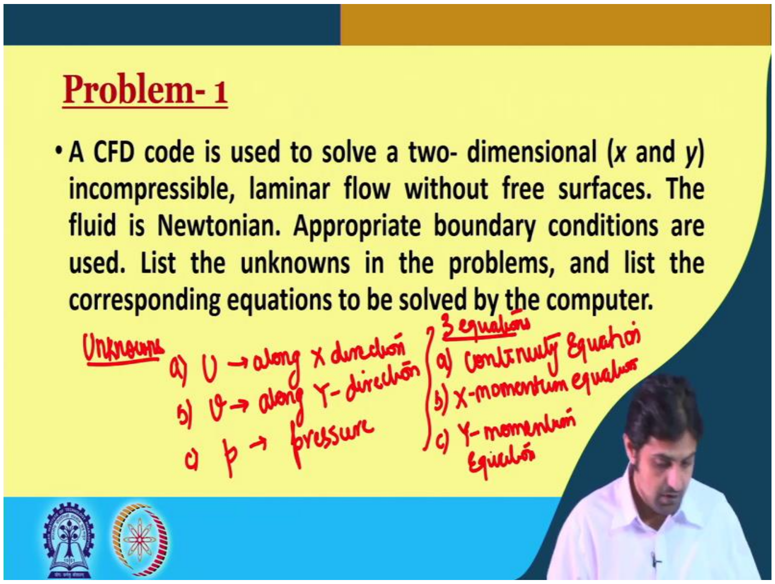

So, there is one question which I would want to know, solve solver discuss. So there is a CFD code which is used to solve a 2 dimensional in 2 dimension X and Y incompressible laminar flow without free surfaces. The fluid is Newtonian. So appropriate boundary conditions are used. So the question here is now lists the unknowns in the problem and list the corresponding equations to be solved by the computer.

So first important information that we have is that it is 2 dimensional in nature. So, the unknowns, you can start guessing the unknowns first is going to be velocity you which is along X direction. Secondly, the velocity V, which is along Y direction. The third one is going to be since there is no more third dimension there is not going to be a W direct W velocity, but there definitely will be p that is pressure and how are we going to solve these which equations are we going to solve for U V and P.

We are going to use 3 we see there are 3 equations. So, the first equation is very common conservation equation which is the continuity equation. The second one will be the X momentum equation so momentum equation in X direction. We will have a Y momentum equation. So, this is very quiet simple to you know imagine or guess that in 2 dimension only 2 dimensional velocities and the pressure is going to be the unknown and therefore, to solve we just need the three equations continuity, x momentum and y momentum equations.

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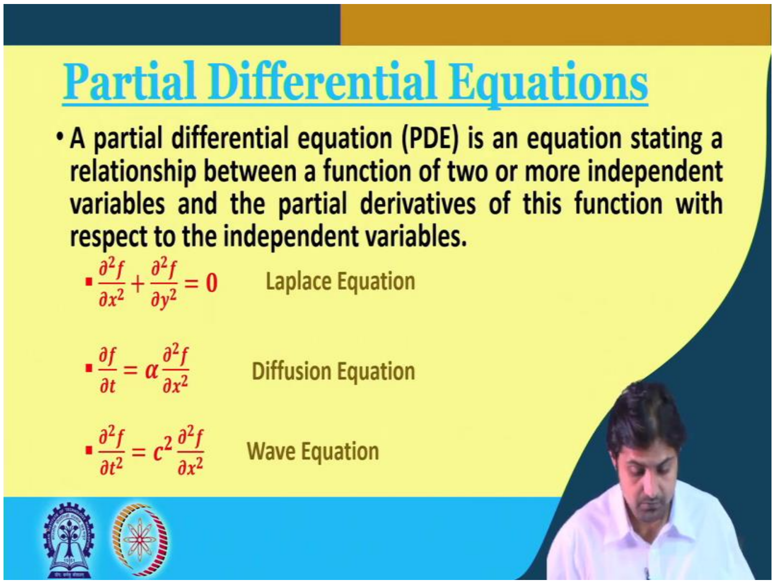

So, now, we come to the, in the solver stage we come to we have this way we have mentioned about the boundary condition and now we come to discuss about the partial differential equations. So, partial differential equation PDE is an equation stating a relationship between a function of two or more independent variables and the partial derivatives of this function with respect to the independent variable. So, this is the definition of partial differential equation.

So, it states it states the relationship between a function of two or more independent variables and the partial derivatives of this function with respect to the independent variable. For example, this equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

. So, these are independent variables x and y and f is an independent variable and this equation is called Laplace equation from your previous experience you should be knowing this another equation is

$$\frac{\partial f}{\partial t} = \alpha \frac{\partial^2 f}{\partial x^2}$$

So, this is a diffusion equation the third equation which we have written is

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}$$

and this equation is a wave equation. So, these are the 3 most common types of equation in partial differential equation that we know.

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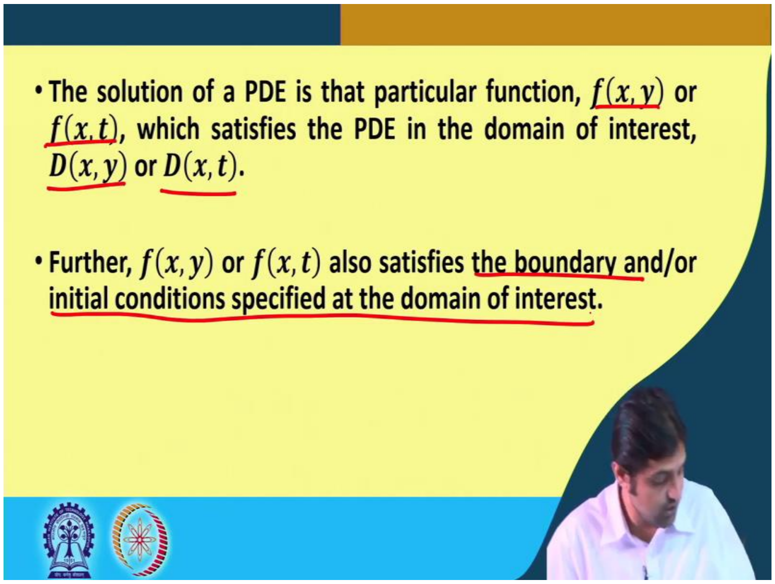

So, this is solution of the partial differential equation is that particular function f x y or f x, t, which satisfies the partial differential equation in the domain of interest domain is given by D(x y) or D (x, t), f(x, y) is when the function is based only on x and y, f(x, t) when if it is dependent on only x and t. Further f (x, y) or f (x, t). These are 2 different type of partial differential equations f x, y or f x, t they also satisfy the boundary and our initial conditions specified at the domain of interest. So, they must be satisfying they must be satisfied at all places in the domain or and at all times for which the calculation is being done.

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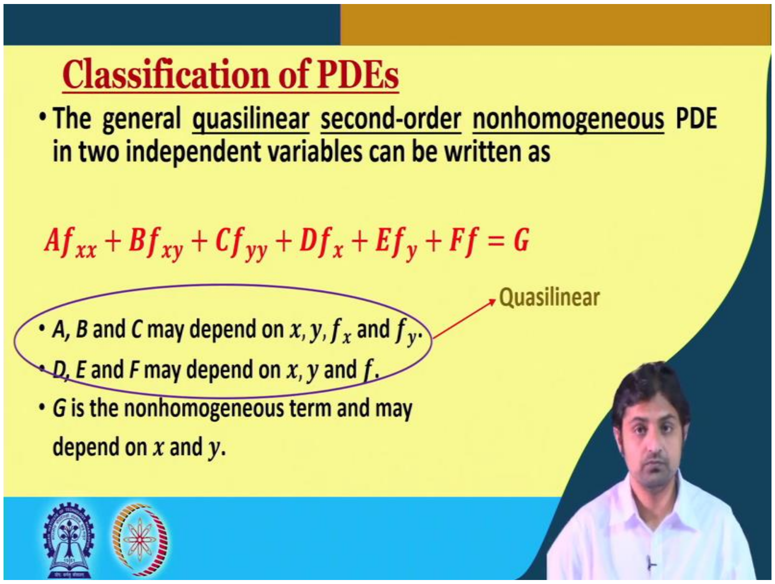

So, going to take a slight detour, because it is important that we try to remember what the classification of partial differential equations look like. So, they are general quasilinear second order non-homogeneous partial differential equation in 2 independent variables can be written as Af xx + Bf xy + Cf yy + Df x + Ef y + Ff = G, this is a general quasilinear second order we have x x, x y and y y that is second order non-homogeneous or partial differential equation.

So, A, B and C may depend on x, y, f x and f y, D, E and F may depend on x y and f. So, this is this means quasilinear. G is non-homogeneous term and may depend on x and y that is why we read non-homogeneous this is not equal to 0.

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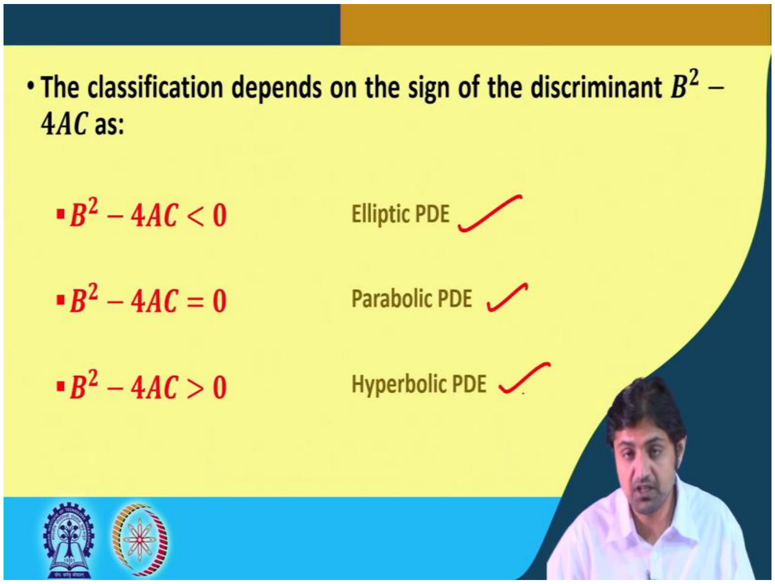

So, the classification depends on the sign of the discriminant B^2- 4AC as so, in this particular equation Af xx + Bf xy + Cf yy + Df x + Ef y + F f = G, the solution will depend on the value of B^2- 4AC. And how does it depend if B^2- 4AC is less than 0, then it is going to be elliptical partial differential equation. If B^2- 4AC = 0 is going to be a parabolic partial differential equation, if B^2- 4AC is greater than 0 then it is going to be a hyperbolic partial differential equation. You this you can recall from your mathematics class.

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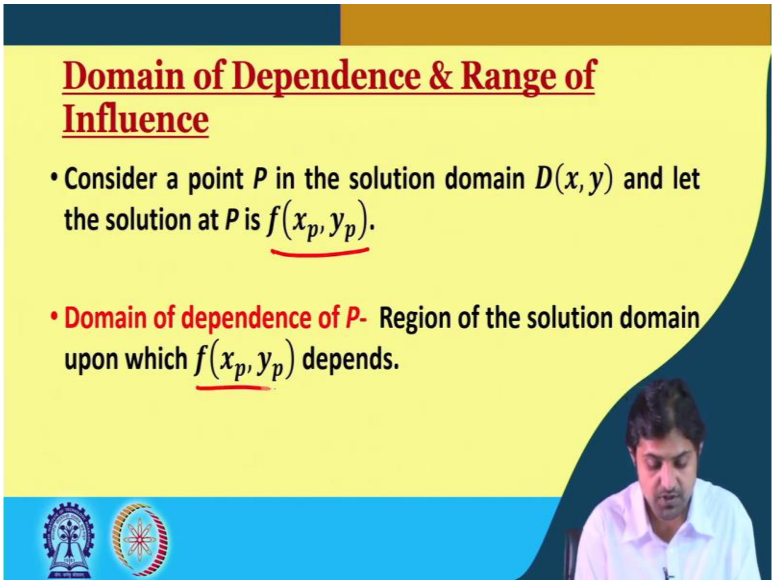

Now going to domain of dependence and range of influence. So, if you consider a point P in the solution domain, so, there is a point in the domain P and let the solution at P if we assume that the solution at that particular point P is f (x p, y p) we assume that then the domain of dependence of P is the region of solution domain upon which f(x p, y p) depends.

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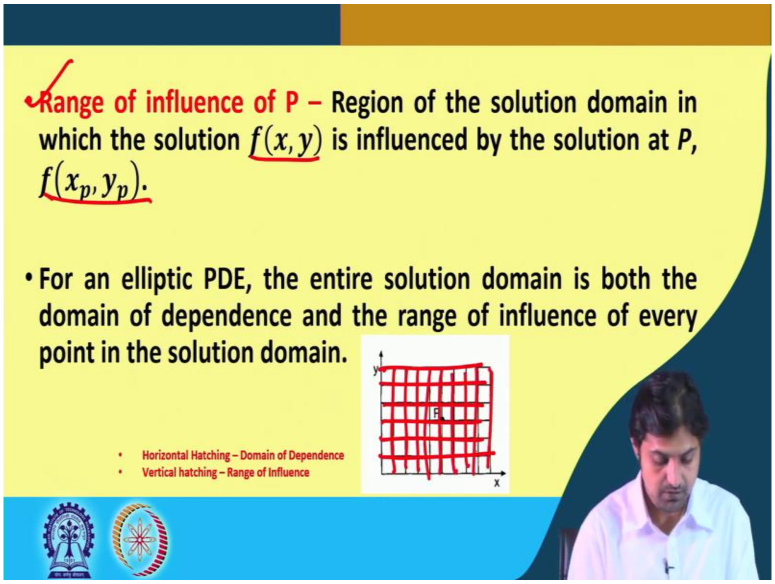

So, region of influence of P the region of solution domain in which the solution of x, y f of x, y is influenced by the solution at P which is f x p, y p. So, for an elliptical partial differential equation the entire solution domain is both the domain of dependence and range of influence of every point in the solution domain. So, there are 2 things domain of dependence of P and secondly, there is a range of influence of P which we have defined as the domain of dependence of phase the region of solution domain upon which x p f of x p, y p depends whereas, the range of influence of phase the range of solution domain in which the solution of except of x, y is influenced by the solution at p.

So, applying this to an elliptic partial differential equation the entire solution domain is both the domain of dependence and the range of influence of every point in the solution domain. So, the horizontal hatching here these ones shows the domain of dependence whereas the vertical hatching the shows the range of influence.

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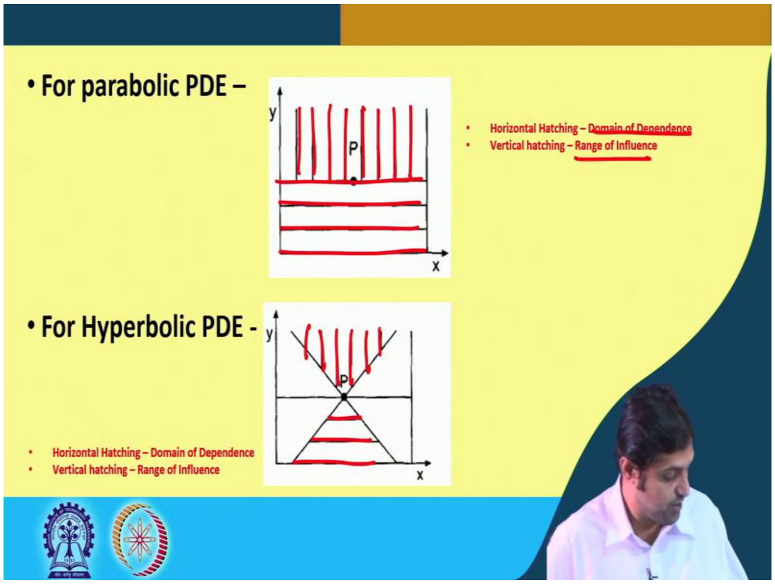

Now, for a parabolic PDE this is the domain of dependence so, horizontal hatching whereas the vertical hatching like these ones these are the range of influences. For a hyperbolic partial differential equation, the horizontal hatching shows the domain of dependence this one whereas the vertical hatching shows the range of influence. So, for 3 different type of solutions we said profiles, elliptic PDE, parabolic PDE and hyperbolic PDE.

For elliptic PDE we have seen that the domain of dependence is the entire solution and also the domain of dependence. Whereas, the parabolic and hyperbolic PDE the horizontal hatching as shown in the figures here shows the domain of dependence whereas the vertical hatching shows the range of influence.

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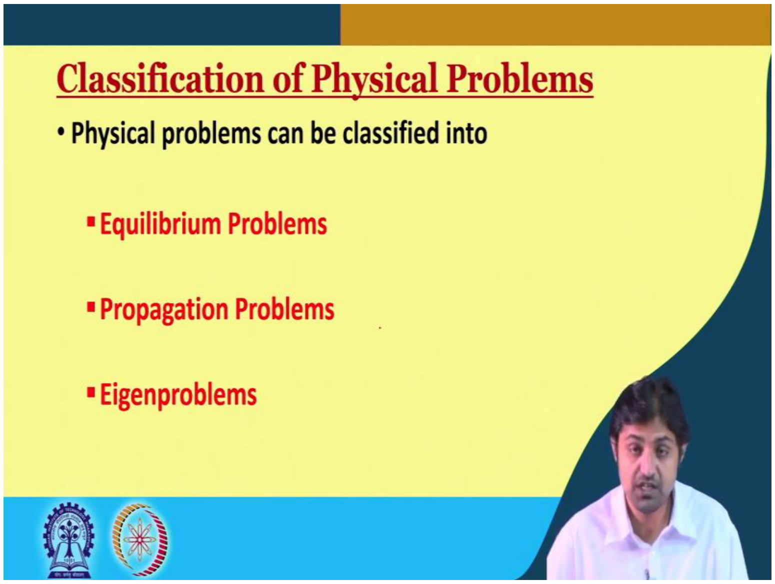

Now the classification of the physical problems can be classified into equilibrium problems or propagation problems, the third is Eigen problems. So, any physical problems can be classified into 3 different forms.

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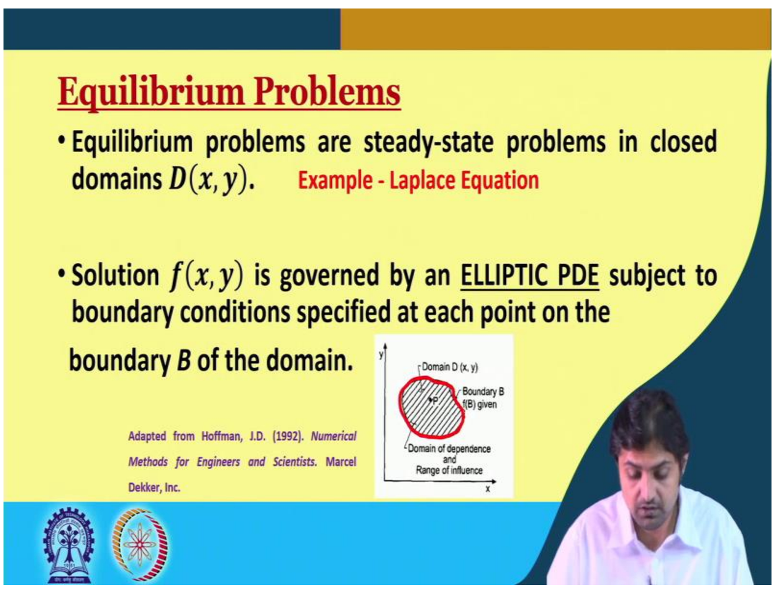

What are the equilibrium problems are the steady state problems in closed domain steady state means, there is no dependence on time there is an equilibrium. So, example is Laplace equation of such type of problems. So, here the solution f of x, y is governed by an electrical partial differential equation subject to boundary conditions specified at each point on the boundary B of the domain.

So, solution of Laplace equation is governed by an elliptic partial differential equation is an important thing to remember. And of course, the solution will depend upon the different type of boundary conditions that has been specified at each point on the boundary B of the domain. So, we have to specify suppose this is the you know, so, we have to specify boundary here also at all those points something like this.

So, if this is the figure you see you see, because this is an elliptical PDE the domain of independent dependence and range of influence are same all the domain and this is the boundary. So, we should be able to supply the boundary conditions at all the points, which is denoted by B here.

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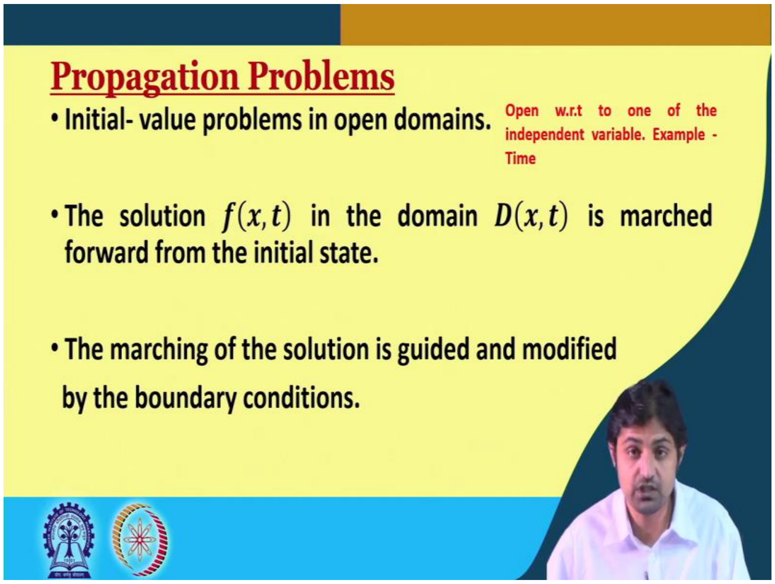

The second type of problems or propagation problems. So, an example is initial value problems in open domains. So, open with respect to one of the independent variables example time the solution f of x, t in the domain is marched forward from the initial stage. So, we know things that time t = 0, then we go from time t = 0 so let us say t = 2 seconds then time t = 2 seconds and time t = 3 seconds.

So, the marching of the solution is guided and modified by the boundary conditions. So, if we have a different boundary conditions, we still have to specify the boundary condition at time t = 0 initial at initial point, and we will also have to specify the initial problems at initial values at all the points in the domain. So boundary conditions and also the value at all the points in the domain initially to.

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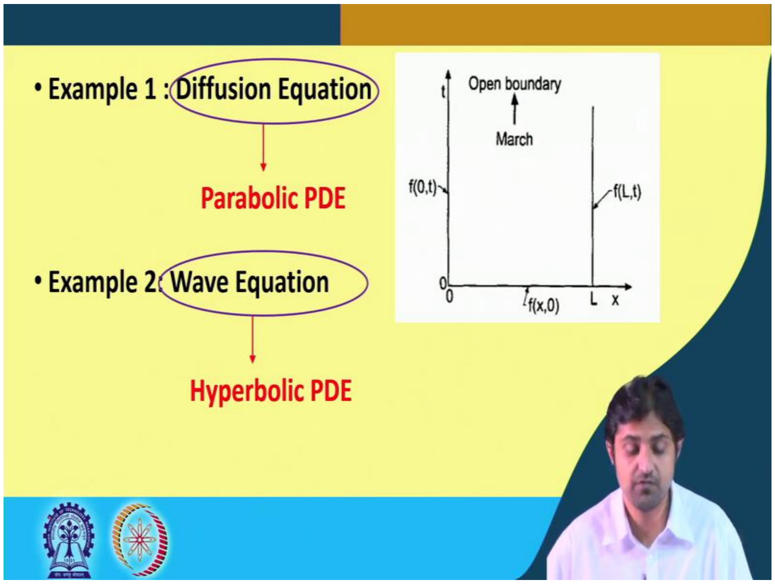

Example 1 here is the diffusion equation, you see this is an open boundary and we have to go from it is in x and this is in time t, this is x direction. So, this are solved by the parabolic partial differential equation parabolic PDE diffusion equation, second example is a wave equation. So, the wave equation is solved by the hyperbolic PDE hyperbolic partial differential equation. So, as we have seen 3 different type of equations have different partial differential equation profiles 1 was elliptical, Laplace equation the diffusion equation has parabolic partial differential equation and a wave is hyperbolic partial differential equations.

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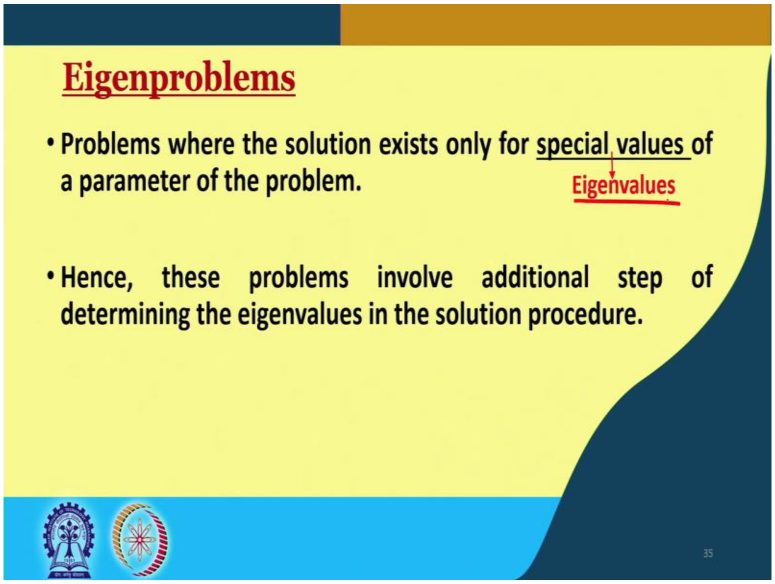

Now the last one in this set is the Eigen problems. So, problems where the solution exists only for special values of parameter of the problem. So, it the solution will not be there for all the values of the parameters. So, and these special values are called the Eigen values hence, these problems have involved additional step of determining the Eigen values is the solution procedure.

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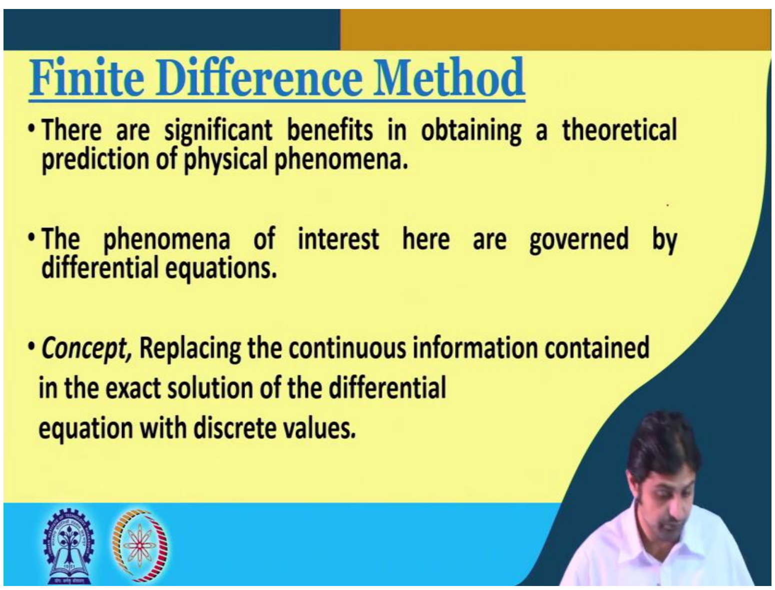

So, this is about the 4 type of problems that we were talking so, now, from this point onward we will proceed to the discretization technique that is the in we start with the finite difference method first. There are significant benefits in obtaining a theoretical prediction of a physical phenomenon. So, the phenomenon of interest here are governed by differential equations concept that is replacing the continuous information contained in the exact solution of the differential equation with discrete values.

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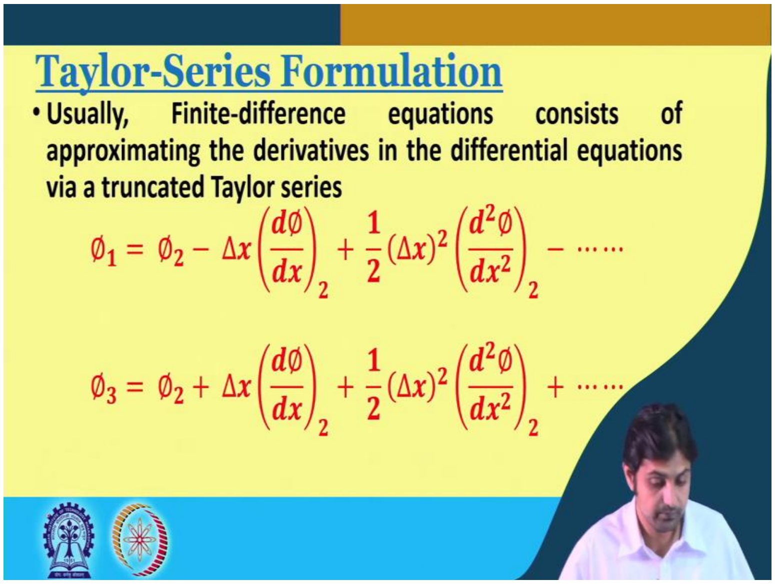

There is something called the Taylor Series Formulation which we generally use, usually finite difference equation consists of approximating the derivatives in the differential equations via a truncated Taylor series. So, how do we approximate the derivatives in the differential equation using a truncated Taylor series, which looks like this? I am pretty sure you have read that in your math class.

So, phi 1 is written as phi 2 - delta x del phi by del x at 2 + half delta x whole squared del squared phi into del x whole squared at 2. So, at series like this goes on with alternate - and + signs. So, this is called the truncated Taylor series or phi 3 can be written as phi 2 + delta x del phi del x + half del x squared and it can go on like this.

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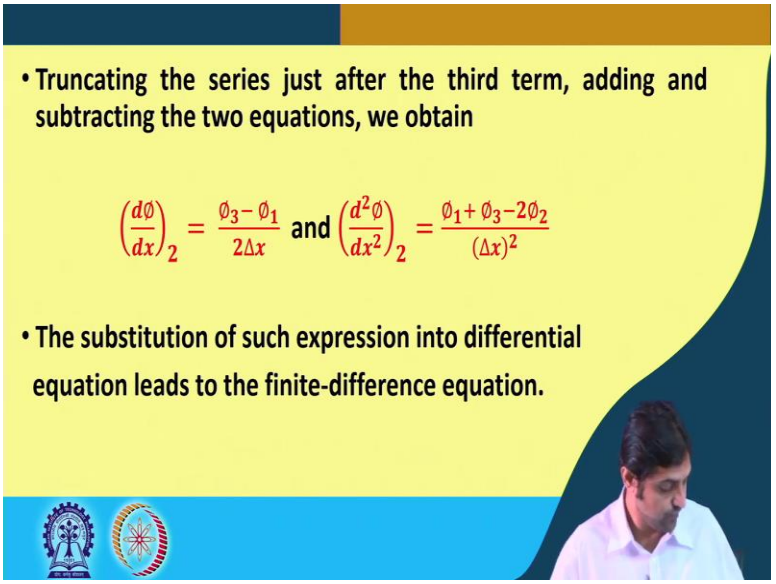

So, truncating the series just after the third term adding and subtracting the 2 equations so, you see there were 2 equations phi 1 and phi 3 both were return in terms of phi 2. If what we do if we just do you know if we first stopped the series just after the third term and add and subtract the 2 equation then we obtain del phi by del x at .2 will be phi 3 - phi 1 by 2 delta x and also del square phi by del x square can be written us phi 1 + phi 3 - 2phi 2 by delta x square.

So, what we have seen here, we have used the truncated Taylor series in general to write these values phi 1 and phi 3 and phi 2 and in once we add those and once we subtract those we get terms like these so, the substitution of such expression into differential equation leads to finite difference equation.

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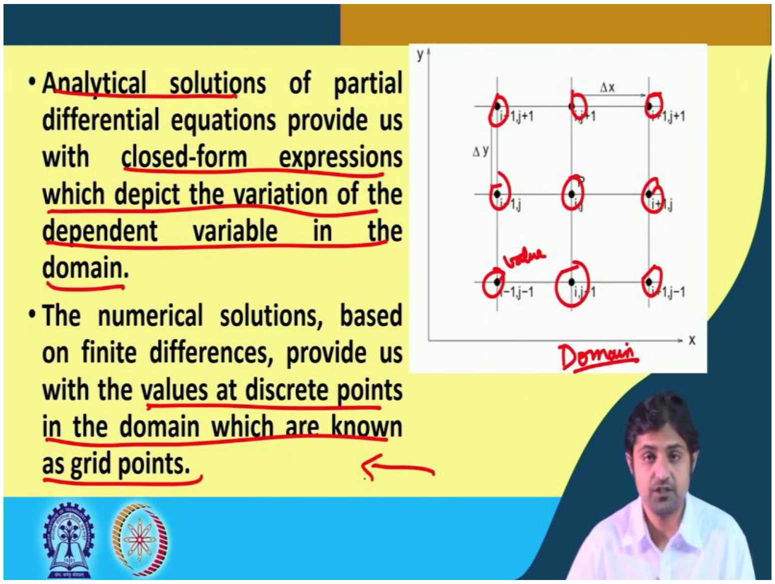

Say like this so, analytical solution of the partial differential equation provide us with closed form expressions which depicts the variation of the independent variable in the domain, so, this is the domain here and the numerical solution based on finite differences provide us with the values at discrete points in domain which are known at grid points. So, the difference between the analytical or the mathematical solution.

What does analytical solution of the partial differential equation provide us they provide us with closed form expressions we depict the variation of the dependent variable in the entire domain. Whereas, using the numerical simulation based on finite difference it provides us with the values at discrete points in the domain. So, it will provide us the value here. Here, whereas mathematical solution will give us closed form expression which is valid everywhere not only at some points.

So, of course, if we are able to obtain the analytical solution that is the best case scenario, but in majority most of the cases actually majority of the cases we are not able to do that therefore, we resort to the technique of finite difference method.

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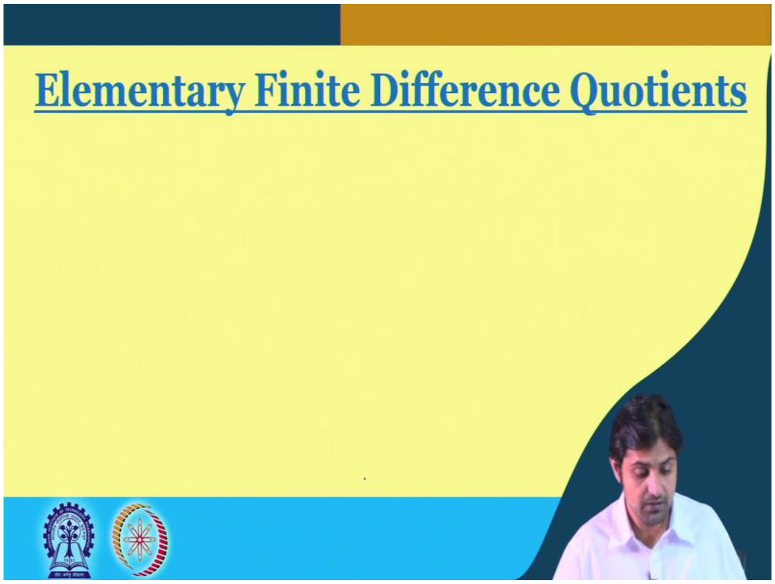

So, I think this is a nice point to stop and in the next lecture we will start with elementary finite difference quotients. So, thank you so much for listening and I will see you in the next lecture.