Hydraulic Engineering
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Lecture – 51
Viscous Fluid Flow (Contd.)

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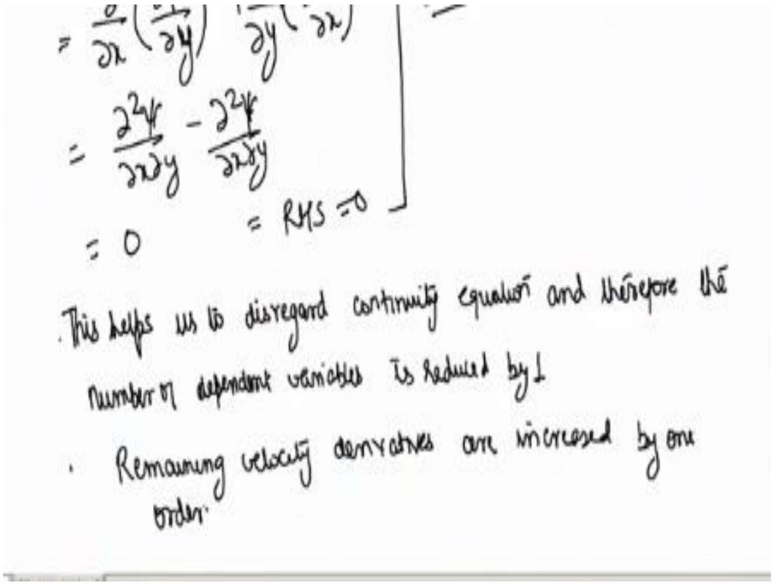

Welcome back student so, in the last lecture we have seen that the stream function satisfies identically the continuity equation, so now we are going to proceed forward so, this was the place where we left off and we are going to start now, okay. So, why, what is the importance of a stream function? This helps us to disregard continuity equation and therefore, this means the stream function.

And therefore, the number of dependent variables is reduced by 1, secondly there is a disadvantage to it as well, the disadvantage is that remaining velocity derivatives are increased by 1 order because as you see this is del square term, so therefore this is one of the disadvantages but on the other hand, the advantage is that the number of dependent variables were reduced by 1. Now, what is the physical significance of stream function?

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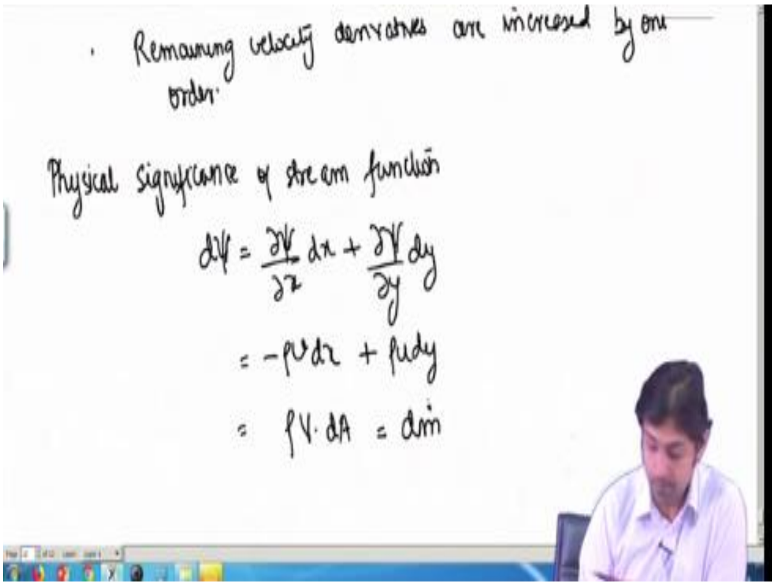

So, to look at it; if you see ψ can be written as

$$d\Psi = \frac{\partial \Psi}{\partial x} dx + \frac{\partial \Psi}{\partial y} dy$$

alright, this is how dψ can be written and you see del ψ by del x we have defined, minus rho v, equation number 8, we use equation number 8, so it becomes - rho v dx + del ψ by del y was rho u dy alright, so this is nothing but this is rho v; velocity will remain v, we are writing as vector into the area vector, alright.

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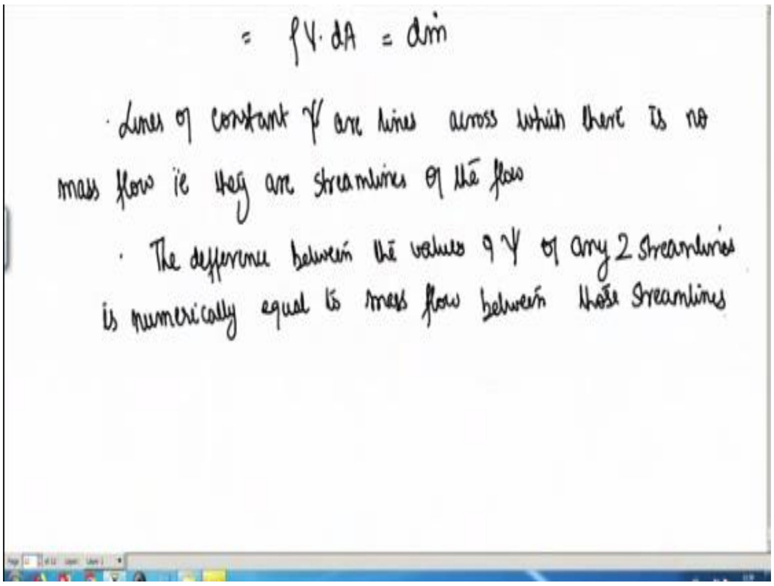

And this whole is mass flow rate, so this implies actually that lines of constant ψ are lines across which there is no mass flow that is they are streamlines of the flow. So, you are noticing in this particular module the way of approaching the streamlines and the continuity equation has been very different, we have started from scratch and tried to show what we have learnt in our fluid mechanics class.

Secondly, the difference between the values of ψ of any 2 streamlines is numerically equal to mass flow between those stream lines, alright. So, this is are the 2 physical significance of stream function that the long lines of constant ψ are lines across which there is no mass flow and also the difference between the values of ψ across any 2 streamline is numerically equal to the mass flow between those 2 stream lines, alright.

So, this concludes our continuity equation now, we are going to the conservation of mass. So, I think it is the right time to start the new page, alright.

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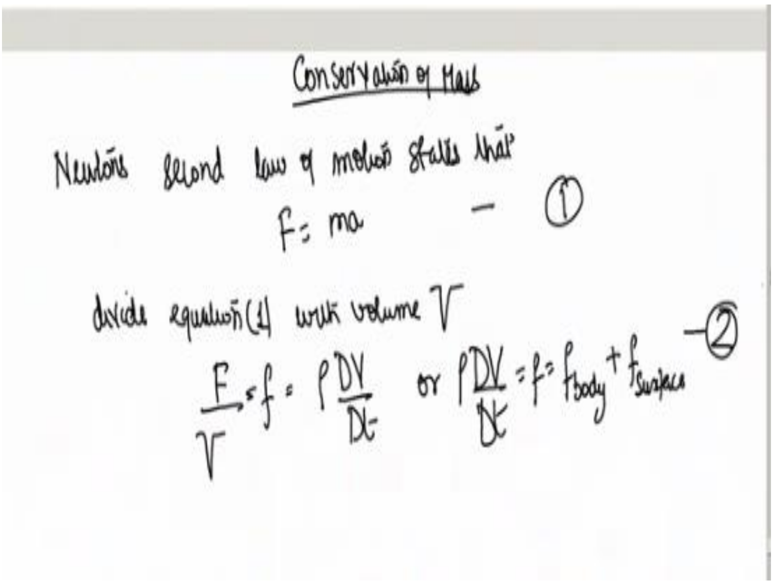

So, we write conservation of mass okay, Newton's second law of motion states that F is equal to mass into acceleration, so let us divide so, I am again writing equation number 1 because we have started a new topic, we call this conservation of mass so, this is equation number 1 in conservation of mass. So, if we divide equation number 1 with volume, so volume we are all; in this module we are denoting by V.

So, this becomes F by V is equal to small s is ρDV/Dt, correct so, that is force per unit volume or we can also write rho DV/Dt is equal to force small f and that can be written as; there are 2 type of forces; one is body force plus one is surface force and this equation we write as equation number 2, alright.

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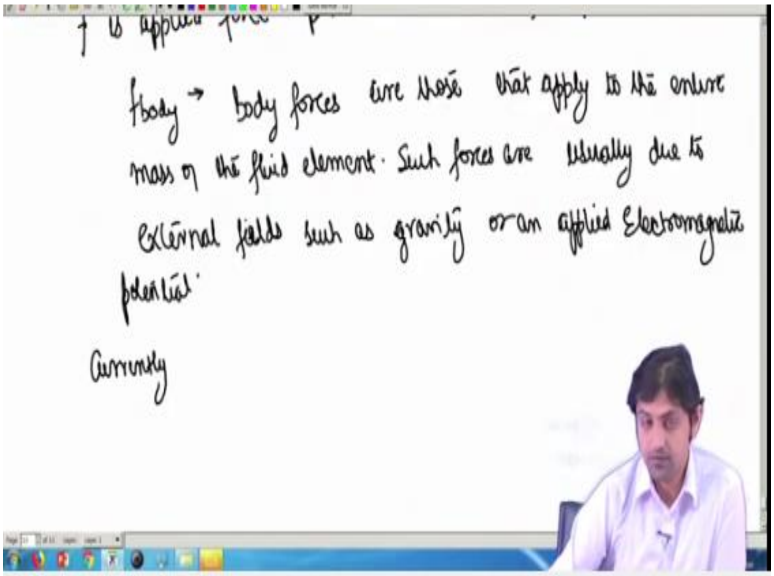

So, this f is applied force per unit volume on fluid particle, alright so, what is f body, we have to clarify that a little bit so, f body, so basically this is body force so, body forces are those that apply to the entire mass of the fluid element, okay. Such forces are usually due to external fields such as gravity or an applied electromagnetic potential, okay.

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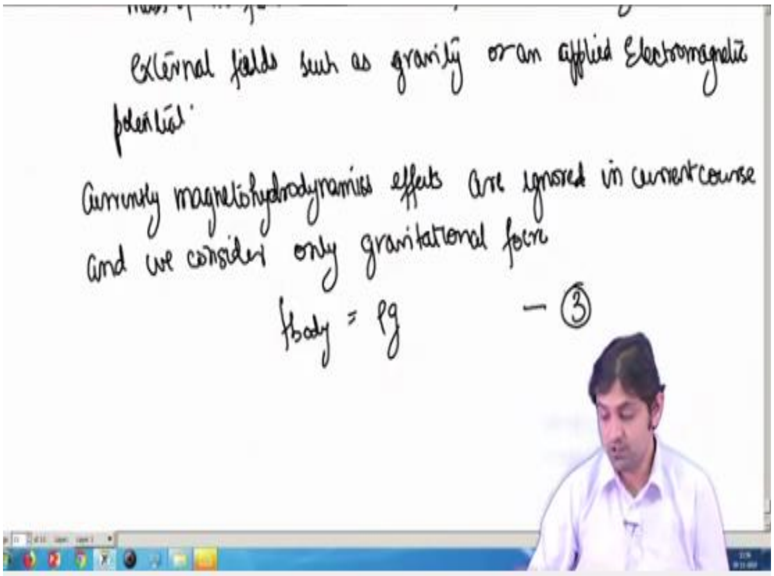

So, currently magneto hydrodynamics effects; so magneto hydrodynamics; hydrodynamics is dynamics, fluid dynamics for example, right and which is caused by the magnetic force, so these type of forces; magneto hydrodynamic effects are ignored in current course okay, current module basically and we consider only the gravitational body forces, okay. So, f body is written as rho g, alright and this we called as equation number 3, alright.

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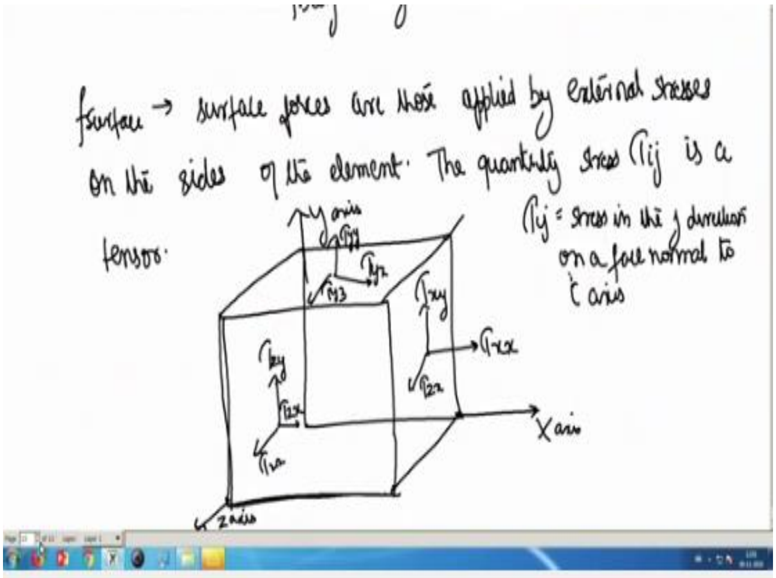

Now, the other fourth is f surface; surface force so, first the definition; surface forces are those applied by external stresses on the sides of the element. This quantity that is stress, τ ij is a tensor, so surface forces are those that are applied by external stresses on the side of the element and this is called τ ij, will write it in a much, alright and this τ ij is a tensor, okay. So, let us draw a figure, I am not very good at drawing but I will try, alright.

So, what we assume this dimension as x, alright, this has been assumed y and this side it is z axis, alright. So, these stresses on the side needs to be represented alright, so this is x side, so the normal stress here is going to be τ xx alright, so the shear stress on the this face but going in y direction is called τ xy, right. So, this particular face and the one that is acting here is τ zx.

So, if we similarly, write on the y surface alright, so this is the y surface where you know this is the; alright so, the up side is going to be τ yy alright, this is going to be τ yx and this side it is going to be τ yz. If we write on this surface so, on this surface, it is going to be τ zy because this is z surface τ zz and this is τ zx because it is in x direction, alright. So, τ ij is the stress in the j direction on a face normal to i axis, okay.

So, this has been our definition; τ ij stress in the j direction on a face that is normal to i axis alright, so we said that a stress; this is a tensor correct.

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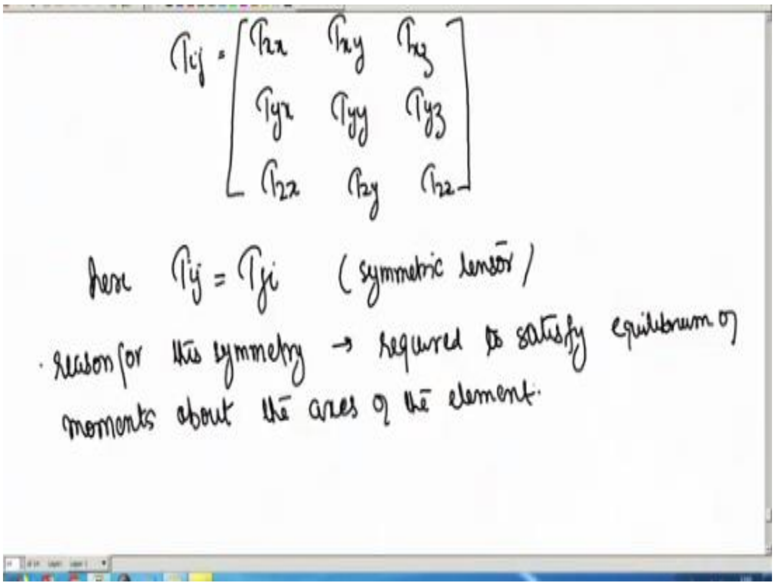

So, we need to write down; we actually have written down τ ij in that figure that I will write it down, τ xx, you remember this similar type of tensor we had found in a strain, in the last one, I mean 2 lectures from now, I mean 2 lectures before okay, so here τ ij is equal to τ ji and what is this, what is the reason for symmetry? It is required to satisfy equilibrium of moments about the axes of the element.

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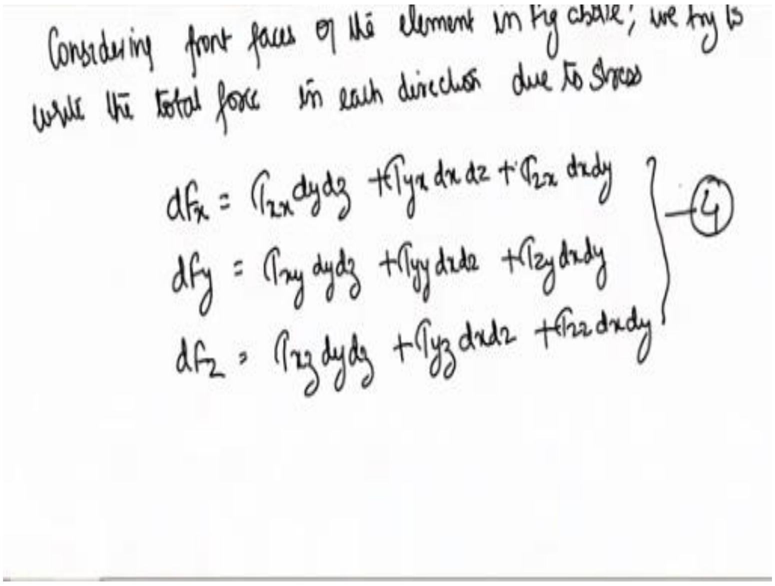

So, this symmetry is required to satisfy the equilibrium of moments about the axis of the elements. So, let us consider; so considering front faces of the element in figure above; actually, we wrote those stresses in the front you know, in the front faces itself, we try to write the total force in each direction due to stress so, dFx, in x direction so, τ xx is in this direction and is acting over the plane which is having an area of dy dz, I should actually show you once for this one.

So, if you see τ_{xx} is acting in this direction; x direction also, τ_{yx} is acting in x direction and τ_{zx} is also acting in this direction and multiplied by the corresponding so, τ_{xx} acts on area of dy dz, τ_{yx} acts on area of dx dz and τ_{zx} is acting on the area of dx dy alright. So, dF_x is τ_{yx} dx dz + τ_{zx} dx dy, so similarly, in y direction which write down the sum of the τ_{xy} into dy dz + τ_{yy} dx dz + τ_{zy} dx dz, τ_{zy} into dx dy.

And similarly, dF_z is written as τ_{xz} into dy dz + τ_{yz} dx dz + τ_{zz} into dx dy and this all 3 equation we call as equation number 4, alright.

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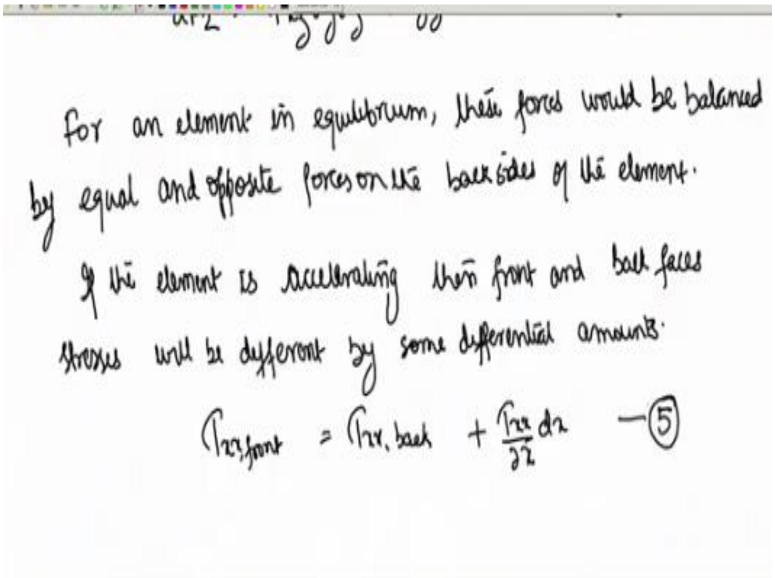

So, for an element in equilibrium, these forces would be balanced by equal and opposite on the backsides, right. So, if the element is in equilibrium, the forces that we have written dF_x, dF_y, dF_z these forces would should be balanced by equal and opposite I mean, forces on the backside, we have written the front faces alright, back sides of the element alright. Let us say just assume if the element is actually; if the element is accelerating let us say, it is accelerating, okay.

So, that means the forces are not balanced, accelerating, okay then, front and back faces forces or stresses will be different by some amount okay, some differential amount okay, so suppose τ_{xx} front, it is going to be τ_{xx} back, this is what it means, τ_{xx}, x sorry, you know this, so τ_{xx} x dx into del x into dx, we call this equation number 5, alright.

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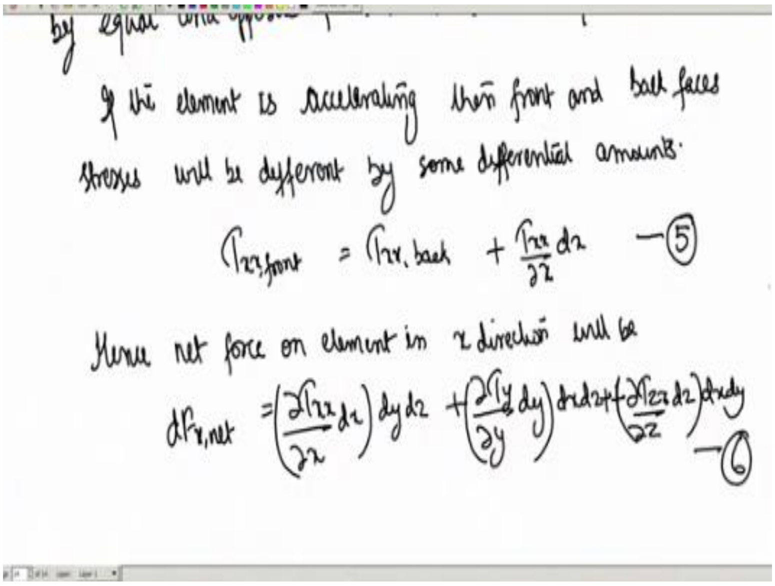

Hence, net force on element in x direction will be dFx net will be the differential right, so that is going to be now, del τ xx x dx into dx, this is the differential stress multiplied by dy dz, same here del τ y Y into multiplied by dy, alright into dx dz + del τ zx by del z into dz multiplied by dx dy alright, sorry and this is we call equation number 6. So, we go to another page now.

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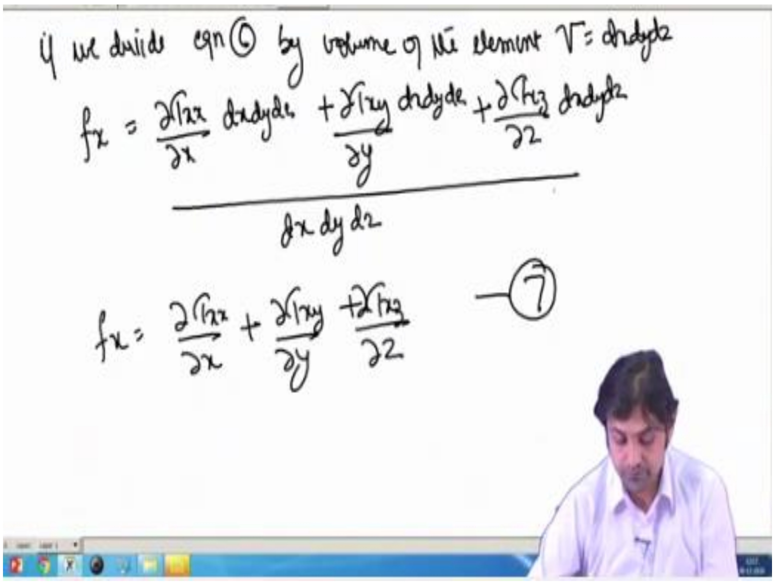

If we divide equation 6 by volume of the element, which is equal to dx, dy, dz, right we get, so differential force by unit volume we call small fx, correct is equal to; it will become del τ xx del x, so dx dy dz + del τ xy dx dy dz divided by dx dy dz, so dx dy dz will get cancelled from up and below both so, fx can be written as simply del τ xx by del x + del τ xy by del y + del τ xz by del z and this is equation number 7, alright.

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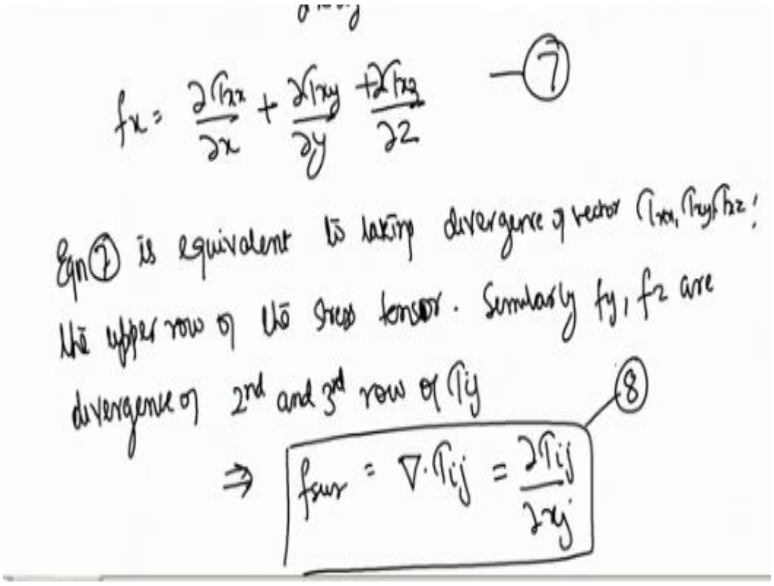

So, equation 7 is equivalent to taking divergence of vector τ xx, τ xy and τ xz that means, the upper row of the stress tensor; a stress tensor is what; the tensor that we have should written here, alright. So, this is equivalent to taking the divergence of these divergence of vector I have written velocity here, a stress tensor okay, so similarly fy and fz are divergence of second and third row of τ ij, okay.

This simply means that total f surface force will be summation of these forces so, in vector terms we can simply write τ ij divergence or del xj, alright, this is equation number 8 and it is a very nice point to finish our current lecture and then in the next lecture, we can proceed from this point and go ahead and finalize our you know, the Navier's stokes equation derivation and try to cover as much as we can in the next lecture. Thank you so much for listening.