Hydraulic Engineering

Prof. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology - Kharagpur

Lecture – 50
Viscous Fluid Flow (Contd.)

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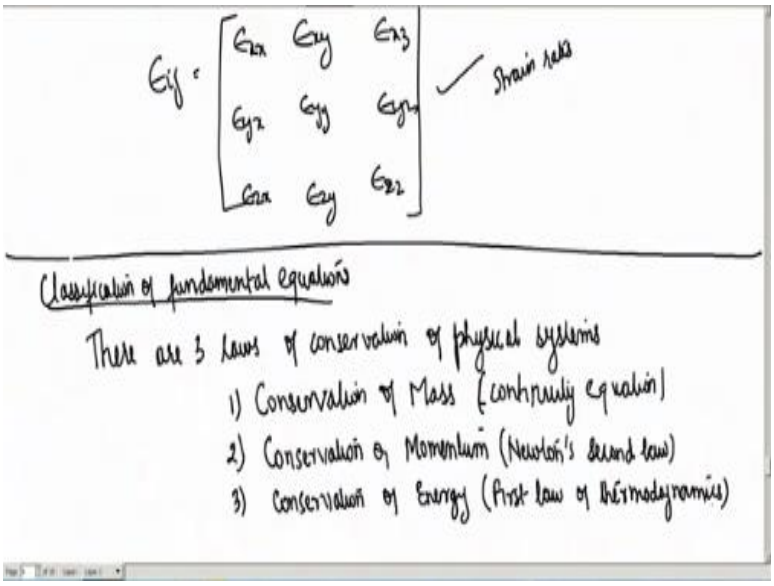

Welcome back students, to this week's and module's lecture. So, in the last 2 lectures we have gone through looked at the basic fluid properties, then we derived the rate of shear strain and this is an important property, which we are going to use in our further analysis. So, we are going to, so this is the last point where we concluded, written the extensional and the shear strain rates, extensional is epsilon xx, epsilon yy and epsilon zz.

Whereas, the shear strain rates are epsilon xy, epsilon xz and epsilon zy, so this is the matrix of the strain rates and the values of epsilon xx, epsilon yy and epsilon zz, we had already derived that epsilon xx was del u by del x and similarly, epsilon yy was del v del y and epsilon zz was del w del z. So, after that we are going to start the classification of fundamental equations.

So, actually there are 3 laws of conservation of physical systems. The first one is conservation of mass, this you have already seen that is continuity equation. The second one is, the second conservation equation is conservation of momentum and this is analogous to

Newton's second law. The third is conservation of energy and this has something to do with first law of thermodynamics.

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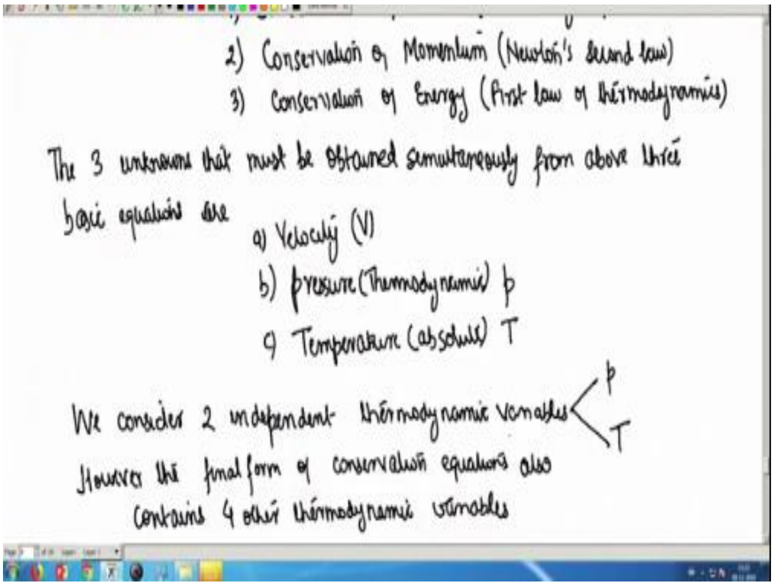

So, the 3 unknowns that must be obtained simultaneously from above 3 basic equations are velocity, second is pressure, so writing V, pressure which is thermodynamic by p and T as temperature, absolute temperature, given by T. So, we actually consider 2 independent, so when it comes to thermodynamic variables, we consider 2 independent thermodynamic variables, which are actually pressure and temperature.

However, the final form of conservation equations also contains 4 other thermodynamic variables. And what are those variables?

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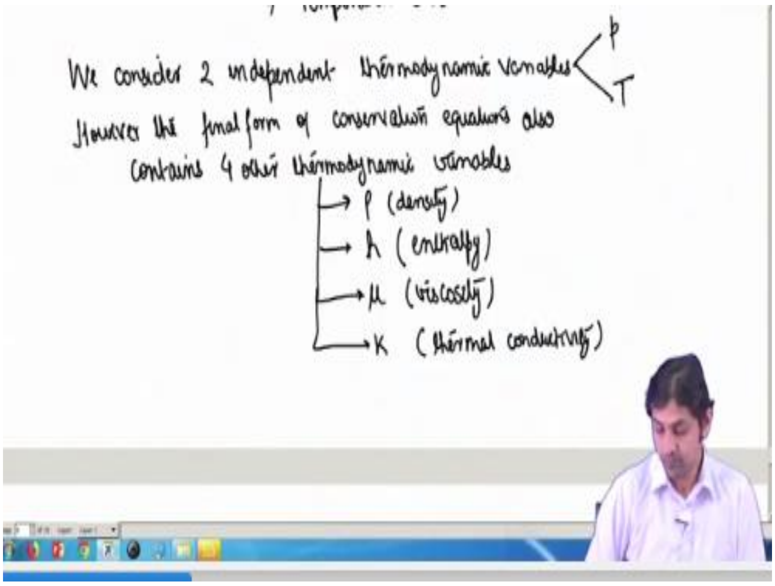

We are going to write it down, one is Rho also called density, second is H, that is, enthalpy, there is mu viscosity and K thermal conductivity.

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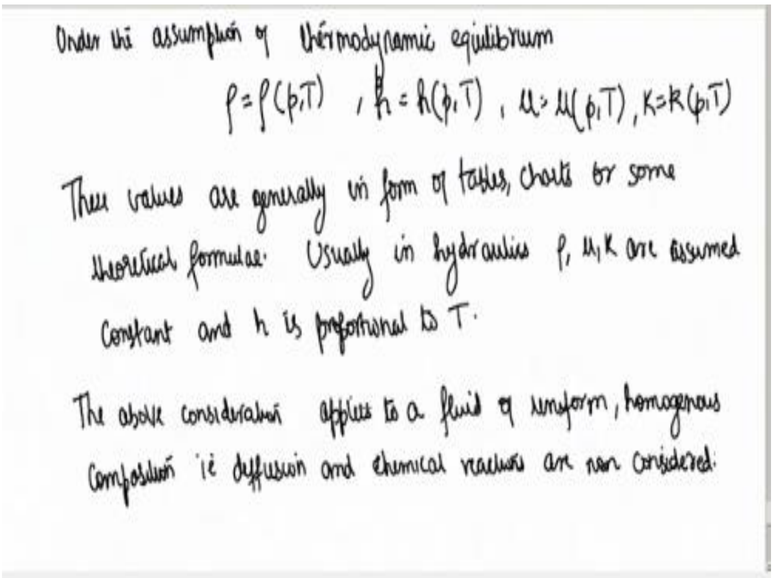

So, we go to the next page, so we assume something and that assumption is that we assume that there is a thermodynamic equilibrium. So, under the assumption of thermodynamic equilibrium rho is; rho as a function of pressure and temperature, h is also a function of pressure and temperature, so mu is equal to; mu is also function of pressure and temperature and K that is the thermal conductivity, K is also the; so basically, all these independent, I mean, dependent variable are a function of pressure and temperature under thermodynamic equilibrium.

These values; so which values; these values, rhos, h, mu and K, these are these values are generally in form of tables, charts or some theoretical or in theoretical, you know, formula. Usually in hydraulics for our purposes, you know, rho, mu, K are assumed constant and h is proportional to temperature. So, in this particular course, we are going to deal mainly with the rho and mu, which will be considered constant.

So, the discussions above; so, the above consideration applies to a fluid of uniform homogeneous composition, which means diffusion and chemical reactions are not considered. So, with this background in mind, we will get started with our first equation.

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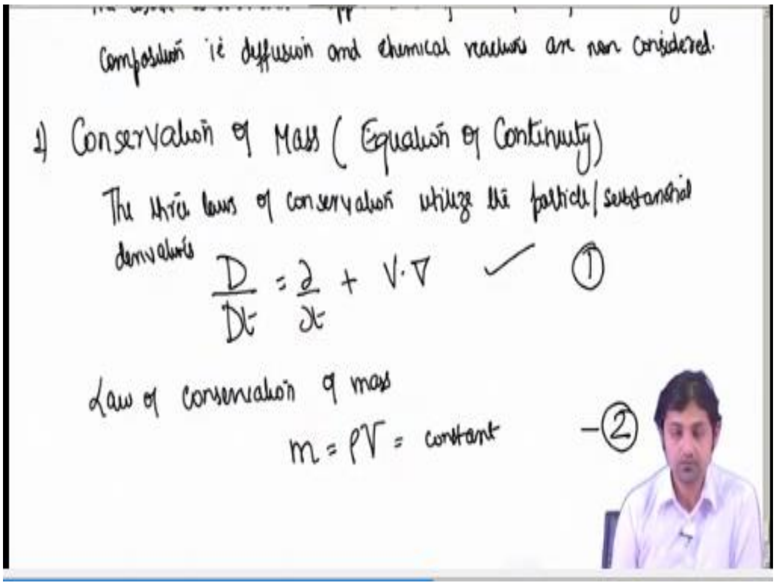

So, our first equation is conservation of mass or also called equation of continuity, so actually all the 3 laws for the derivation require the particle derivative or a substantial derivative or the material derivative, which we have already seen in our lecture, I mean, previous lectures. So, I will take you to that where we wrote the material derivative.

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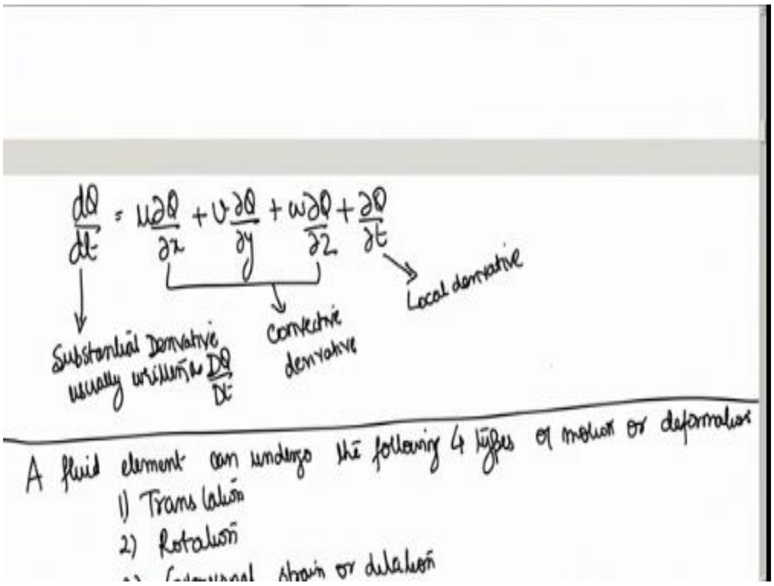

So, here, so a material derivative of any quantity is written as dQ dt as del Q del t + u del Q del x + v del Q del y + w del Q del z, so this is the material derivative. So, we are going back to; so the 3 laws including this one, the 3 laws of conservation utilized the particle substantial derivatives. So, that is, D Dt is equal to del del t plus, so this is what we utilize, actually in this particular course, we are going to deal with only the conservation of mass and conservation of momentum.

The third equation; the first law of thermodynamics, the conservation of energy is outside the scope, so we are not going to deal with that in this particular course, but for the people who study mechanical engineering that law is also very much applicable. So, writing now, law of conservation of mass. So, what does this law says? Law of conservation of mass, we said that this particular equation is will be used for the derivation of this equation of continuity.

So, the law of conservation of mass says that mass is equal to rho and this is V, I am writing a different V, because V we mostly say in hydraulics, v as velocity, so this is constant, so that is conservation of mass. So, this equation we call 1 for now and this equation we will call equation number 2.

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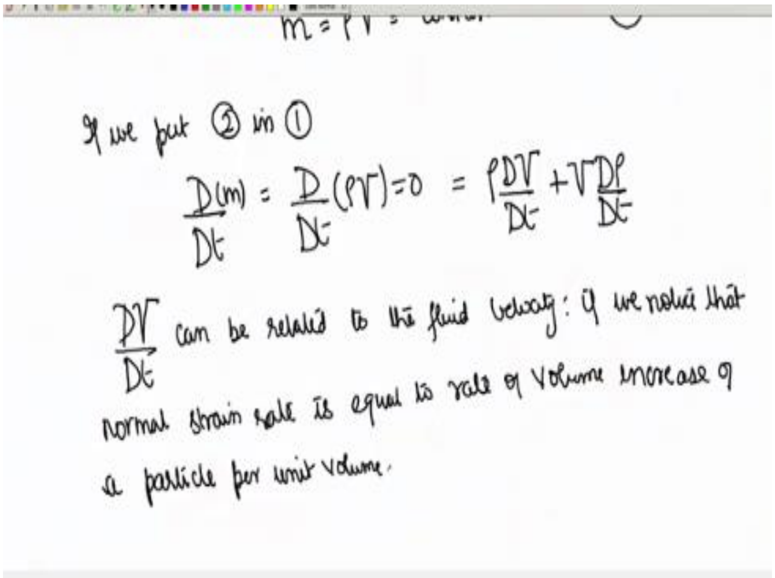

So, if we put equation number 2 in equation number 1, this becomes del D m/ D t is equal to 0, is equal to by chain rule rho. Now, this you see, there are 2 terms on the right hand side, rho DV Dt + VD rho Dt, where V is the volume. So, actually the rate of change of volume; so, the total change in volume, so DV Dt can be related to the fluid velocity. How? If we notice that the normal strain rate is equal to rate of volume increase of a particle per unit volume.

So, the normal strain rate actually is equal to the rate of volume increase of a particle per unit volume, so we go to another page.

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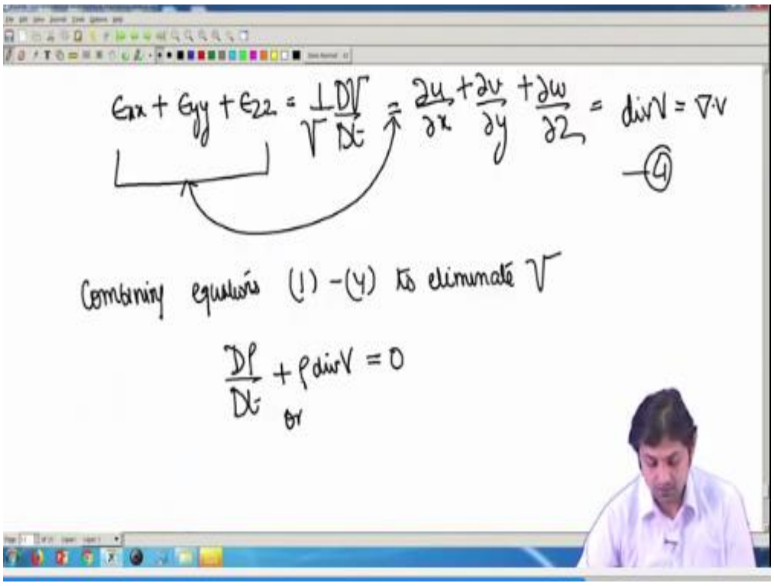

That means, epsilon xx + epsilon yy + epsilon zz is equal to; this is the definition 1 by V DV Dt, so we also know that this is equal to; from our previous lectures derivation, epsilon xx was del u del x, epsilon yy was del v del y and this was epsilon zz was del w del z, this was what we actually had derived and this actually is nothing but divergence of V or this; so this is the velocity, because it is u, v, w.

And this equation we call equation number 4, combining equations 1 to 4 to eliminate this volume. What we can get? We will get; so, we are going to get D Rho Dt + rho divergence of V, so this is velocity V not the volume V is equal to 0.

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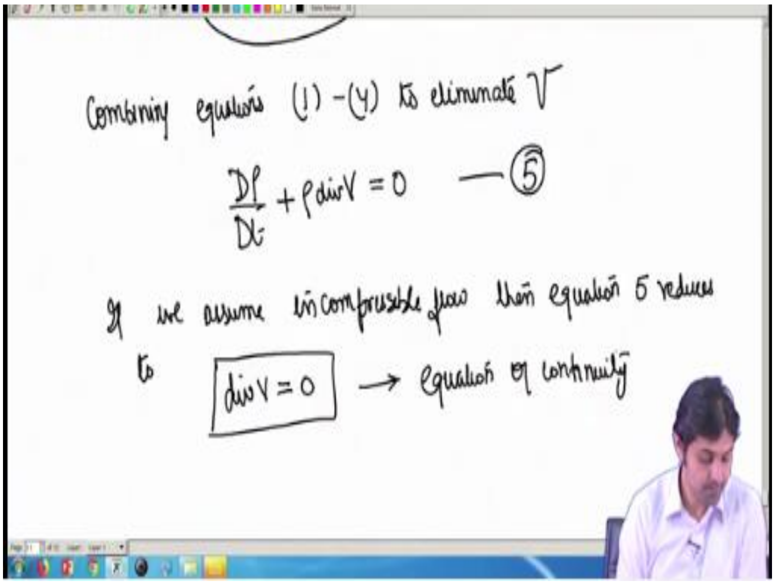

So, this is equation number 5. If we assume incompressible flow, then reduces to divergence of V is equal to 0 and this is nothing but equation of continuity. So, what we have done is; we have been able to derive the equation of continuity.

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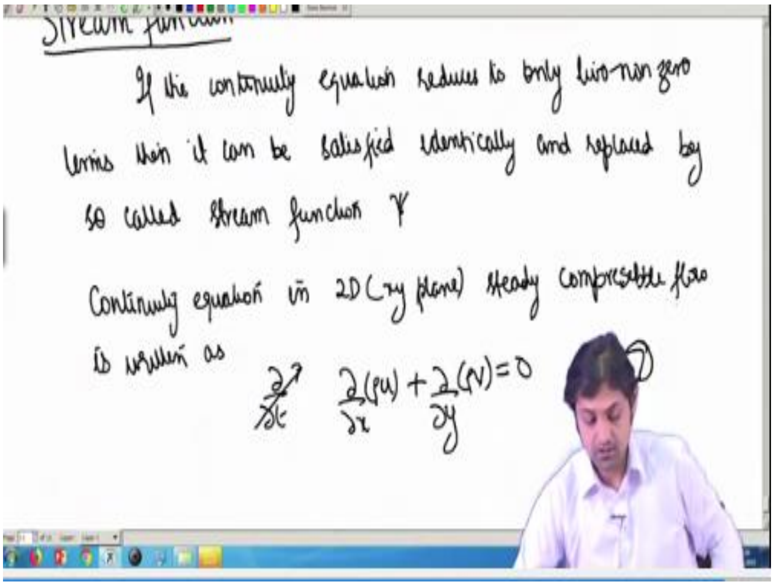

So, we; and this is equation number 6, so we look at a related concept called stream function now. What is stream function? If the continuity equation reduces to only 2 nonzero terms, then it can be satisfied identically and replaced by so called stream function, which is given by psi. So, continuity equation in 2D, let us call it xy plane, steady compressible flow is written as, so it is steady that means there is not going to be any del del t term.

So, we will have only del del x, because it is compressible, del del y of rho v is equal to 0, this is equation number 7.

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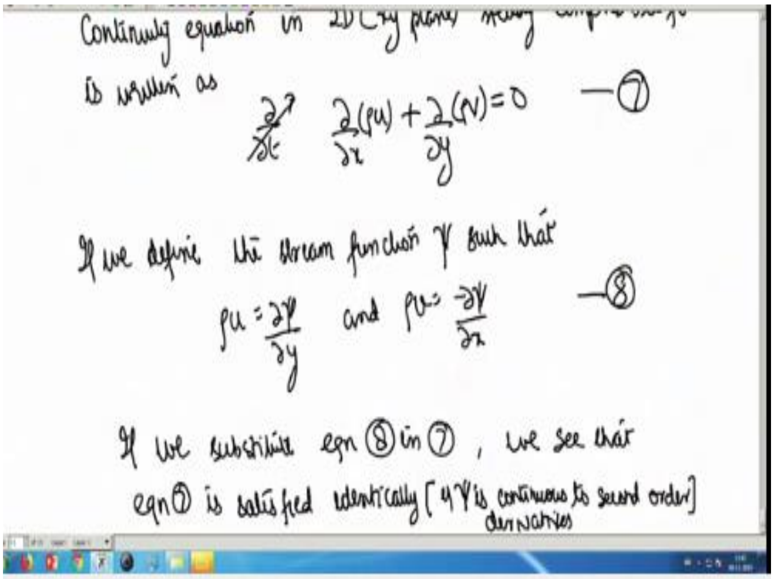

If we define the stream function psi such that rho u is del psi by del y and rho v is - del psi by del x, so first we have written the continuity equation in 2 dimension and if we are defining a function which we will call stream functions such that it satisfies rho u is equal to del psi by del y and rho v as del psi by; - del psi by del x. Then, if we substitute equation 8 in equation 7, we observe, we see that equation number 7 is satisfied identically.

Of course, the condition here is; psi is continuous to second order derivatives. Before we conclude I would like to you know just look at it. How?

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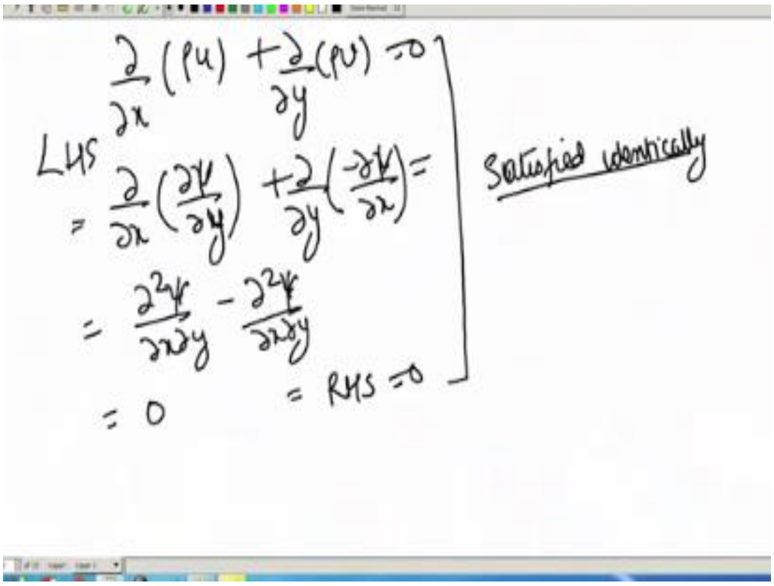

So, del del x rho u. How, I mean, by how is it satisfied identically? So, rho u is del psi del x, so it will be del del x of; sorry, plus del del y of rho v is minus del psi del x, so this means, so LHS and RHS was so; so I think this is right time to finish today's lecture and we start from where we are going to; we start in the next lecture from where we have just stopped. And thank you for listening. I will see you in the next lecture.