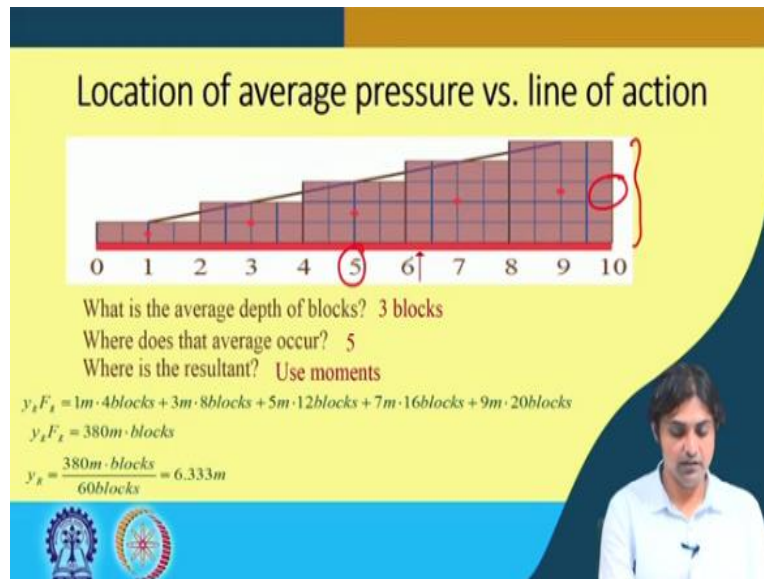


Hydraulic Engineering
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Lecture – 05
Basics of fluid mechanics-I(Cont.)

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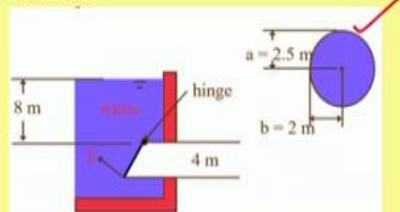


Okay. Welcome back. So, this is the again starting with the last slide from the last lecture and we figured out what was the average depth of the block and that came out to be, you know, the average depth was 3 blocks here, what where does that average occur? It was at block number 5 this average and we found out the resultant and the resultant came this. Okay.

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Example using Moments

An elliptical gate covers the end of a pipe 4 m in diameter. If the gate is hinged at the top, what normal force F applied at the bottom of the gate is required to open the gate when water is 8 m deep above the top of the pipe and the pipe is open to the atmosphere on the other side? Neglect the weight of the gate.



So, now we proceed with one of the real life examples of moment. So, the question is, an elliptical gate covers the end of a pipe 4 meters in diameter. So, this is the elliptical gate. I will delete this but I am trying to show where those different objects are. If the gate is hinged at the top here, okay, what normal force F applied at the bottom of the gate is required to open the gate when the water is 8 meters deep above the top of the pipe, okay, so this, and the pipe is open to atmosphere, okay, on the other side, here. Neglect the weight of the gate, okay. So, here we have seen, this is how this elliptical gate looks, this is the cross section, okay. So, once you have understood the question, we can proceed with these solutions now, okay.

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Magnitude of the Force

Pressure datum? atm Y axis?

$$F_R = p_c A$$

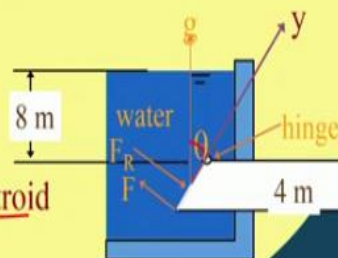
$$A = \pi ab$$

$$h_c = 10 \text{ m} \quad \text{Depth to the centroid}$$

$$F_R = \rho g h_c \pi ab$$

$$F_R = \left(1000 \frac{\text{kg}}{\text{m}^3} \right) \left(9.8 \frac{\text{m}}{\text{s}^2} \right) (10 \text{ m}) \pi (2.5 \text{ m}) (2 \text{ m})$$

$$F_R = 1.54 \text{ MN}$$



So, I have kept this figure on the right side. So, that you are able to follow, what we actually we are going to do. So, what is the pressure datum? This is the atmosphere, okay. So, the resultant force is going to be the pressure at the centroid into area that we have seen in the derivation in last lecture. What is the area? Area of the ellipse that we have seen is π into a b , okay. What is the h_c ? So, just taking, you know, for h_c first we need to understand, this is the y axis that we have assumed, okay. And this is the θ that we have assumed, okay. So, h_c is 10 meter actually. Why? 8 meter plus 2 because of the ellipse, depth to this centroid, very clear. So, F_R is going to be $\rho g h_c \pi a b$. So, ρ is 1000 kilogram per meter cube, g is 9.8 height of the centroid that we have seen is 10 meter, because 8 meter is this water depth and the depth location of the centroid is further 2 meters below this location so 10 meters. π is π , a is 2.5 and b is 2, as we have seen the values of a and b from the previous slide here is 2.5 and b is 2.

So, I am just going back. So, I mean, here. So, F_R finally comes out to be 1.54 Mega Newton, very simple to calculate, okay. Mega Newton is 10^6 , actually. So, fine?

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Location of Resultant Force

$$y_R = \underline{y_c} + \frac{I_{xc}}{y_c A}$$

$$\frac{I_{xc}}{A} = \frac{a^2}{4}$$

$$y_c = \frac{h_c}{\sin \theta}$$

$$y_R = y_c + \frac{a^2}{4h_c} \sin \theta$$

$$y_R = y_c + \frac{a^2}{5h_c} = \underline{0.125 \text{ m}} \quad x_R = \underline{0}$$

Diagram showing a vertical gate submerged in water. The water depth is 8 m. The gate is 4 m wide at the bottom. A hinge is at the top. The resultant force F_R acts at a depth of 0.125 m from the centroid. The angle θ is indicated.

$\sin \theta = 4/5$

So, we proceed to the other part now. So, what is the location of the resultant? So, we need to find out y_R and x_R , y_R is given by y_c , correct. So, I_{xc} / A from the set of the areas that we had seen in the shown in the last lecture is given as a square / 4. If you are confused, you can go and see it in the last lecture notes. y_c can be written as $h_c / \sin \theta$, this we have already seen

in the derivation, okay, where $\sin \theta$ is $4/5$ for m , because minor axis and major axis is 2.5 and 2 . So, y_R can be written as $y_c + a^2 / 4 h_c \sin \theta$. Right?

If you just put the value of I_{xc} and a , you will get this result this value. So, y_R is $y_c + a^2 / 5 h_c$, if you have substitute the value of $\sin \theta$ $4/5$ you will get $y_c + a^2 / 5 h_c$, and this comes out to be 0.125 meter. As ellipse we discussed these the it is symmetric about the centroid, so, we will not have any x_R and x_R was going to be 0 . So, very simple calculations here, correct.

(Refer Slide Time: 05:28)

Force Required to Open Gate

How do we find the required force?

Moments about the hinge

$$\sum M_{hinge} = 0 = F l_{tot} - F_R l_{cp}$$

$$F = \frac{F_R l_{cp}}{l_{tot}} \quad F = \frac{(1.54 \times 10^6 \text{ N})(2.625 \text{ m})}{(5 \text{ m})}$$

$$F = 809 \text{ kN}$$

The diagram shows a semi-circular gate of radius 2.5 m and width $b = 2 \text{ m}$. The center of pressure is at a distance $l_{cp} = 2.625 \text{ m}$ from the hinge. The total length of the gate is $l_{tot} = 5 \text{ m}$. A resultant force F_R acts at the center of pressure, and a force F is applied at the hinge to open the gate.

Now, the most important part is force that is required to open the gate. So, what all do we know? See, y_R is 0.125 , right. So, this will become $2.5 + 0.125$ that will be 2.625 , correct. And this is 2.5 meters, this is 2 and this is total length l_{total} . So, how do we find the required force here? So, we have to take moments about the hinge, hinge was here. So, moment about hinge will be 0 and this means force into length of total will be the resultant force into length of it, we already know length of the, you know, center of pressure and the resultant force, we already know.

We need to find out this F , we already know l_{total} . So, F_{total} will be $\frac{F_R l_{cp}}{l_{tot}}$ and on substituting the value 1.54 into 10 to the power 6 for $F_R l_{cp}$ is 2.625 as can be seen here, and l_{total} is very simple. It is 5 meters, right. So, yeah. So, this value will come out to be 809 kilo Newton. Okay,

so, this makes sense. Correct. So, this makes our problem complete. What I am going to do again, I will erase all the ink on this slide. And now we can just proceed to the other part.

(Refer Slide Time: 07:14)

Forces on Plane Surfaces Review

- The average magnitude of the pressure force is the pressure at the centroid
- The horizontal location of the pressure force was at x_c (WHY?)
The gate was symmetrical about at least one of the centroidal axes.
- The vertical location of the pressure force is below the centroid. (WHY?)
Pressure increases with depth.

The slide features a yellow background with a blue header and footer. The footer contains two circular logos on the left and a video inset of a man in a white shirt on the right.

So, we also need to, you know, to wind up the, you know, the forces on the plane surfaces. Some of the important results, the average magnitude of the pressure force is the pressure at the centroid. This is what we have seen, correct. The horizontal location of the pressure force for that X_c , why? In this particular problem, the gate was symmetrical about at least one of the centroidal axis. In that case, it was $0 X_c$, the vertical location of the pressure forces below the centroid. This we have answered so many times that because the pressure increases with depth, correct.

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Forces on Curved Surfaces

- Horizontal component
- Vertical component

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So, now, this is sorry, yeah. Now we need to calculate the pressure on the curved surface. So, this surface here, you see this, okay. So, how? We need to know the horizontal component, we need to find out the vertical component because it is curved, right.

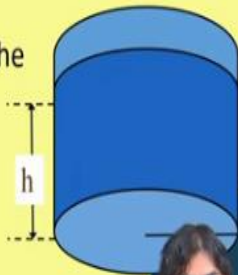
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Forces on Curved Surfaces: Vertical Component

- What is the magnitude of the vertical component of force on the cup?

$$F = pA$$

$$p = \rho gh$$

$$F = \rho gh \pi r^2 = W! \checkmark$$



So, this is one of the examples. My question to you is what is the magnitude of the vertical component of force on the cup? Tell me? Force is pA , correct. Pressure is ρgh , putting it ρgh , area is πr^2 , and this actually is the weight of the liquid supporting it. So that will be the force, vertical force.

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Forces on Curved Surfaces: Vertical Component

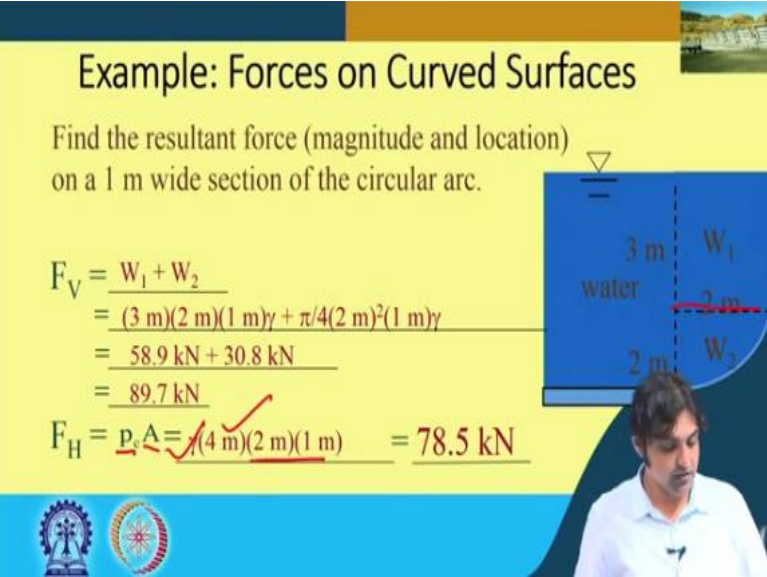
The vertical component of pressure force on a curved surface is equal to the weight of liquid vertically above the curved surface and extending up to the surface where the pressure is equal to the reference pressure. $\checkmark$



So, to sum the vertical component of the pressure force on a curved surface is equal to the weight of the liquid vertically above the curved surface and extending up to the surface where the pressure is equal to the reference pressure. This is the thumb rule. So, the vertical component of the pressure force on a curved surface is equal to the weight of the liquid vertically above the curved surface, okay. And extending up to the surface where pressure is equal to the reference surface.

So, that means up to the free surface, in our case. So, we will explain, we will understand this particular part using a solved example. This is pretty simple to understand.

(Refer Slide Time: 09:49)



Example: Forces on Curved Surfaces

Find the resultant force (magnitude and location) on a 1 m wide section of the circular arc.

$$F_V = W_1 + W_2$$

$$= (3 \text{ m})(2 \text{ m})(1 \text{ m})\gamma + \pi/4(2 \text{ m})^2(1 \text{ m})\gamma$$

$$= 58.9 \text{ kN} + 30.8 \text{ kN}$$

$$= 89.7 \text{ kN}$$

$$F_H = p_c A = \gamma(4 \text{ m})(2 \text{ m})(1 \text{ m}) = 78.5 \text{ kN}$$

So, this is a gate, the all the components are indicated W_1 , the lengths are indicated 2 meters, W_1 , W_2 , 2 meters, 3 meters, water this is water, okay. So, let me erase this. We have to find the resultant force both the magnitude and location on a 1 meter wide section of the circular arc. So, actually this is 1 meter wide in that direction in the other direction not, shown here. So, what is going to be the vertical force? It is going to be what we according to the definition, it should be the sum of the liquid above this curved surface, extending from here up to the free surface.

So, basically what is that? $W_1 + W_2$, W_1 is very simple, it is 3 meters into 2 meters into 1 meter, because that is the dimension that is told, 1 meter wide section into γ of water. This one it is $\pi / 4 r^2$. So, $\pi / 4 r^2$ is a total area which is one fourth. So, $\pi / 4 r^2$ into 1 meter into

Y, very simple calculation. This comes to be 58.9 kilo Newton + 30.8 kilo Newton and in total it comes to about 89.7 kilo Newton, okay, very simple.

Now, we also need to, want to know what the horizontal component is. So, the horizontal component is the pressure at the centroid multiplied by area. But what area that is? So, area is 2 into 1. So, pressure at the centroid is, this is centroid 1 meter below, so, 3 + 1 = 4 Y H is the pressure and this is the area 2 meter into 1 meter in that. So, this the horizontal force is going to be 78.5 kilo Newton.

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Example: Forces on Curved Surfaces

The vertical component line of action goes through the centroid **Expectation???** of the volume of water above the surface.

Take moments about a vertical axis through A.

$$x_c F_v = (1 \text{ m}) W_1 + \frac{4(2 \text{ m})}{3\pi} W_2 \quad \frac{4R}{3\pi}$$

$$x_c = \frac{(1 \text{ m})(58.9 \text{ kN}) + \frac{4(2 \text{ m})}{3\pi} (30.8 \text{ kN})}{(89.7 \text{ kN})}$$

= 0.948 m (measured from A) with magnitude of 89.7 kN



The vertical component line of action goes through the centroid of the volume of water above the surface and that is very understood, you know. So, this is A, now we have to take moments about the vertical axis through A here. So, if we want to find out the line of action, that is, x_c , x_c we know already, right? Sorry x_c into F_v will be equal to the weight is acting, 1 meter from there, W_1 . However, W_2 I have told, if you remember, I told you to remember this equation $\frac{4R}{3\pi}$ in the lecture that we are going to use this and actually we are using this. This is 4 into 2 meter by the 4 into 2 / 3 pi and this, okay.

So, let me erase this, yeah. So, this is the $\frac{4R}{3\pi}$ and this will give us the location x_c . So, it will be 1 meter weight was 50, w_1 was 58.9 this we already know W_2 was 30.8 and the total vertical force we found over 89.7 in the previous slide. So, this is actually going to give us 0.948 meter and it

is measured from A with magnitude of 89.7 kilo Newton. So, this is the magnitude and this is the location.

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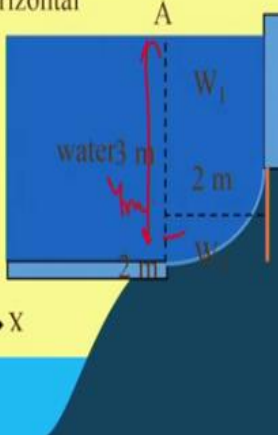
Example: Forces on Curved Surfaces

The location of the line of action of the horizontal component is given by

$$y_R = y_c + \frac{I_{xc}}{y_c A} \quad \text{Here, } h_c = y_c$$

$$y_R = y_c + \frac{a^2}{12h_c} \quad \frac{I_{xc}}{A} = \frac{a^2}{12}$$

$h_c = 4 \text{ m}$

$$y_R = y_c + 0.083$$


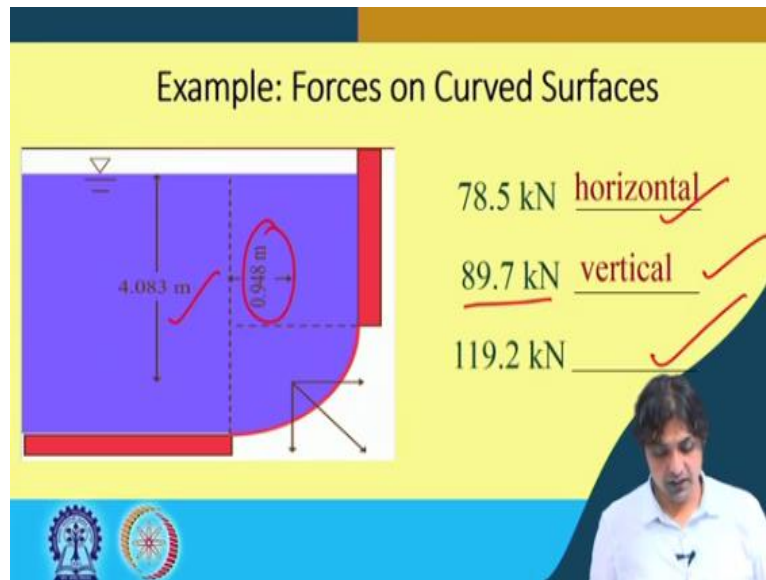
The location of the line of action of the horizontal component is given by

$y_R = y_c + \frac{I_{xc}}{y_c A}$. This is what we have seen before here, h_c is equal to y_c , the height of the centroid is the actually also the y because we have assumed an axis in the vertical direction.

So, y_R is going to be $y_c +$ if you look at this, so, $\frac{I_{xc}}{y_c A}$ will come out to be, you know, because of a square / 12.

If you assume, b and a as this I_{xc} / A is going to be a square / 12, as I have already indicated before, now, they have also the, you know, this one here. So, h_c is going to be in this case, what h_c is. So, 3 meter + 1 meter, so, that is 4 meter. So, y_R is $y_c + 0.083$ using this calculation, okay, because.

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So, now to sum up, this is the horizontal force 78.5, this is 89.7 the vertical force and 119.2 is the resultant force and, yeah. The distances are shown, .948 meter that we have calculated you know 4.083 meter also we had calculated. So, this solves our this problem of forces on curved surface. So, this will, you know, give an overall picture of what is happening in this, you know, forces on curved surface.

So, the trick is, that you use always the force balance and the moment balance. Wherever, the curved surface is, the as I showed the vertical force is, the weight of the liquid above the curved surface. until the, you know, where the atmospheric pressure is or until the free surface. And the pressure the horizontal force will be the pressure at the centroid into the area, projected area, projected area, in the in because surface is 3 dimensional, but in the plane of 2d the projected area that is very simple to do that and so, yeah.

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Buoyant Force



- The resultant force exerted on a body by a static fluid in which it is fully or partially submerged
 - The projection of the body on a vertical plane is always zero
(Two surfaces cancel, net horizontal force is zero.)
 - The vertical components of pressure on the top and bottom surfaces are different



So, we will be proceeding now to the topic, last topic that is buoyant force after which we will also solve some problems. So, I would ask you the resultant force exerted on a body by a static fluid. This is quite an important term here in which it is fully or partially submerged. It is going to be the projection of the body on a vertical plane is always you have to fill this is going to be 0. This means, that 2 surfaces cancel the net horizontal force is 0. That is very important. The vertical components of pressure on the top and bottom surfaces are different however. So, in the horizontal plane the forces cancel to surface cancel therefore, there is no horizontal force, but in the vertical components of pressure on the top and bottom surfaces are different. The reason is very true, because at the top the pressure will be different and at the bottom the pressure will be different.

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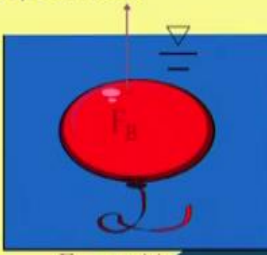
Buoyant Force: Thought Experiment

Place a thin wall balloon filled with water in a tank of water.

- What is the net force on the balloon? Zero
- Does the shape of the balloon matter? No
- What is the buoyant force on the balloon?

Weight of water displaced

weight of water that an object displaces is called buoyant force



$F_b = \rho g V$



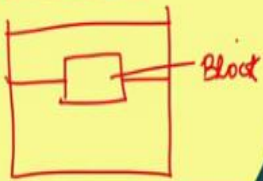
So, this is if you do try to do an experiment you place a thin water balloon filled with water in a tank of water, something like this. So, this is the balloon. And the, you know, this is filled with water and this is also filled this is placed in a tank of water. So, the buoyancy force, we call it $\rho g V$. I am asking what is the net force on the balloon? Should be 0. It is in if it is in equilibrium, net force should be 0. Here, does the shape of the balloon matters? No, because if you do that experiment, it does not really matter.

Now, the question is what is the buoyant force on the balloon? Actually, I have not derived it, but since you have already done it in your fluid mechanics class, the buoyant force on the balloon is actually the weight of the water displayed. So, this is the most important thing to note that the weight of water that an object displaces is called be buoyant force, this is very important. So, proceeding forward with these 2, you know, this particular concept, what is buoyant force? We are going to actually solve a problem.

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Problem on Buoyancy

- A block of wood of density ρ_1 floating completely submerged at the interface of two liquid ρ_2 and ρ_3 . If V_2 is the volume of the block in the upper liquid and V_1 is the total volume of the block, Show That

$$\frac{V_2}{V_1} = \frac{(\rho_3 - \rho_1)}{(\rho_3 - \rho_2)}$$


So, it says a block of wood of density ρ_1 is floating completely submerged. I think the spelling is wrong but I delete it at the interface of 2 liquid ρ_1 and ρ_2 , ρ_2 and ρ_3 yeah. So, it should be ρ_2 and this should be ρ_3 , I am very sorry for this typing error. If V_2 is volume of the block in the upper liquid and V_1 is the total volume of the block we have to show this equation. First, of all when you have any questions like this, the first and the most important step here to do is draw the figure.

So, I am not very good at this, but anyways we will try. So, let us place the block here first. This is block; let us put a liquid layer here and another liquid layer here. So, block, this density is ρ_1 , for example, this is liquid 3 because we have assuming density as ρ_3 and this is liquid 2 that is why this is ρ_2 . So, I think this is the correct representation and now we also have to say this is here.

So, this is the, you know, this is the complete diagram. Now, I think, the question would be more clear to you. So, as always I am going to use, sorry, I am going to use white screen.

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Let V_2 = volume of the block in the upper liquid (Liquid 2)

$V_1 = V_2 + V_3$ ✓

Since the block is in equilibrium, consider the weight of components of the block and hence the two displaced liquids to obtain

$\rho_1 V_1 g = \rho_2 V_2 g + \rho_3 V_3 g$

Thus $\rho_1 V_1 = \rho_2 V_2 + \rho_3 V_3$ ✓

or $V_2 = \frac{\rho_1 V_1 - \rho_3 V_3}{\rho_2}$

$\frac{V_3}{V_1} = \frac{V_3}{V_2 + V_3} = \frac{V_3}{\frac{\rho_1 V_1 - \rho_3 V_3}{\rho_2} + V_3}$



So, we say let v_2 is equal to volume of the block in the upper liquid that is liquid 2. So, V_1 will be equal to $V_2 + V_3$. Since, the block is in equilibrium, consider the weight of components of the block and hence the 2 displaced liquids to obtain. So, how do we get, that $\rho_1 V_1 g$ is equal to $\rho_2 V_2 g$ Plus $\rho_3 V_3 g$. Thus, $\rho_1 V_1$ is equal to $\rho_2 V_2 + \rho_3 V_3$. That is correct or we can simply V_2 is equal to using this equation here, $\rho_1 V_1 - \rho_3 V_3$ divided by ρ_2 or V_3 / V_1 on the other hand can be written as V_3 divided by $V_2 + V_3$ from here.

And this can be V_3 now $v_2 + v_3$, v_2 we can use from here. And this equation has actually come all the way from here. So, it will be $v_3 +$ and v_2 we are going to use so it will be a $\rho_1 V_1 - \rho_3 V_3$ divided by $\rho_2 + V_3$ fine. So, we will be expanding this in the next white screen. So, I am going screen on white, white screen.

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$$\frac{V_3}{V_1} = \frac{\rho_2 V_3}{\rho_1 V_1 - \frac{(\rho_3 - \rho_2)}{\rho_2}} = \frac{1}{\frac{\rho_1 V_1}{\rho_2 V_3} - \left[\frac{\rho_3 - \rho_2}{\rho_2} \right]}$$

$$\Rightarrow \frac{\rho_1}{\rho_2} - \left(\frac{\rho_3 - \rho_2}{\rho_2} \right) \frac{V_3}{V_1} = 1$$

$$\text{or } \frac{V_3}{V_1} = \frac{\left(\frac{\rho_1}{\rho_2} - 1 \right)}{\left(\frac{\rho_3}{\rho_2} - 1 \right)} = \frac{\rho_1 - \rho_2}{\rho_3 - \rho_2} \quad \checkmark$$

So, V_3 / V_1 is going to be $\rho_2 V_3$, we are just simply expanding this, $\rho_1 V_1 - \rho_3 - \rho_2$ divided by ρ_2 or $1 / \rho_1 / \rho_2$, $V_1 / V_3 - \rho_3 - \rho_2$ divided by ρ_2 , or this will we take the denominator upside, so, it will be $\rho_1 / \rho_2 - \rho_3 - \rho_2$ divided by ρ_2 into V_3 / V_1 is equal to 1, or you simplify then V_3 / V_1 come out to be $\rho_1 / \rho_2 - 1$ are divided by ρ_3 minus $\rho_2 - 1$ or $\rho_1 - \rho_2$ divided by $\rho_3 - \rho_2$ and this was what was asked in the derivation.

So we have solved this problem, it is just nothing just simply forced balance in the vertical direction and some manipulation. I think, you should practice that more carefully in your home.

So, now we are going to, you know, do the last.

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Problem on Buoyancy

- A body weighs 20N and 10N when weighed under submerged conditions in liquids of relative densities 0.8 and 1.2 respectively. Determine its volume and weight in air.




The problem says, the body weighs 20 Newton and 10 Newton when weighed under submerged conditions in the liquid of relative densities 0.8 and 1.2 respectively. Determine its volume and weight in the air. So, yeah, what I am going to do again a screen it is very simple problem, but I think it will give you a thorough brush up. Let, I have not selected the pen. Now, I will select the screen, white screen.

(Refer Slide Time: 29:39)

all weight in air = W and volume = V
 weight in liquid of RD = S $W_s = W - \gamma S V$
 $W_{s1} = W - \gamma S_1 V$
 $W_{s2} = W - \gamma S_2 V$
 $W_{s2} - W_{s1} = \gamma V (S_2 - S_1)$
 $\Rightarrow V = \frac{W_{s2} - W_{s1}}{\gamma (S_2 - S_1)} = \frac{20 - 10}{9790 \times (1.2 - 0.8)}$
 $= 2.55 \times 10^{-3} \text{ m}^3$
 $W_s = 20 + 9790 \times 0.8 \times 2.5536 \times 10^{-3}$
 $= 40 \text{ N}$
 or $V = 2.5 \text{ L}$
 $W_s = W_{s1} + \gamma S_1 V$

So, let weight in air be W and volume is equal to V . Weight in liquid of relative density is equal to S be W_s is equal to $W - \gamma S V$. So, the W_{s1} is going to be $W - \gamma S_1 V$, W_{s2} is going to be $W - \gamma S_2 V$. So, what we do is we simply subtract first from the second. So, it will be $W_{s2} - W_{s1}$ is equal to $\gamma V S_2 - S_1$, simply, this – this. and Therefore, we can get V is equal to $W_{s2} - W_{s1}$ divided by $\gamma S_2 - S_1$ and we can simply substitute the values 20 - 10 in our case, γ we already know 9790 into $S_2 - S_1$ is 1.2 - 0.8 and this comes out to be 2.55 into 10 to the power - 3 meter cube or the volume in liters can be written as 2.55 liters and W_s can be written as W_{s1} into γ into S_1 into V . So, I will take this calculation here. So W_s can be written as 20 you know + 9790 into 0.8 into 2.5536 into 10 to the power - e actually, we are using this equation here. So, it should be W_s is equal to W W_s weight in air W_s . So, it will be W yeah, so 20 will add actually and this will come out to be 40 Newton. So, yeah, that W_{s1} we have already found out because it was 20.

So, we are using this equation, so this is not very true. So to calculate weight we are using this equation actually, fine. So this solves our question number last. So, with this we conclude the week one where we have studied the basics of fluid mechanics one this week. Next week we will be exploring fluid kinematics, fluid dynamics and the Bernoulli's equation and we will conclude the basics of fluid mechanics one that would be required for studying the other important chapters of hydraulic engineering. So, this is all for this week. Thank you so much. See you next week. Bye.