

Hydraulic Engineering
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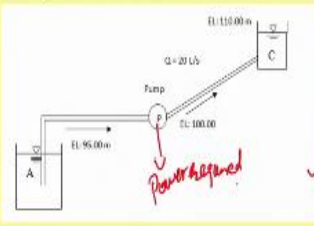
Lecture - 45
Pipe Networks(Contd.)

Welcome back.

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Class Problem

For the pumping set-up shown in Fig below estimate (a) power required and (b) the pressure at the suction side of the pump. Atmospheric head = 10.0 m. Assume both major and minor losses.



Pipe	Diameter	Length	f
AP	15 cm	20 m	0.02
PC	12 cm	300 m	0.02

Last class we solved this particular problem which is shown in the slide and we are going to continue in this lecture. I am going to solve yet another question. The question here is, for the pumping set-up shown in figure below, this figure, estimate the power required and the pressure at the suction side of the pump. We see that the atmospheric head here is 10 meters and we have to assume both the major and the minor losses.

You see there are two pipes and these things are given AP the diameter of the pipe is given, the length is given, the friction factor for each of these pipes is given. So how do we solve this? We have to estimate the power required by this pump and pressure required at this suction side of the pump.

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Soln 11)

Static Head = $110 - 95 = 15\text{m}$

$V_1 = \frac{Q}{A_1} = \frac{0.02}{\frac{\pi (0.15)^2}{4}} = 1.132\text{ m/s}$ (Suction pipe)

$V_2 = V_1 \frac{D_1^2}{D_2^2} = 1.132 \times \left(\frac{15}{12}\right)^2 = 1.769\text{ m/s}$ (delivery pipe)

$h_{f1} = \frac{f_1 L}{D_1} \frac{V_1^2}{2g} = \frac{0.02 \times 20}{0.15} \times \frac{(1.132)^2}{2 \times 9.81} = 0.174\text{m}$

$h_{L1} = 0.5 \times \frac{V_1^2}{2g} = 0.5 \times \frac{(1.132)^2}{2 \times 9.81} = 0.033\text{m}$



So we will continue with 11. So static head is $110 - 95$ you see, that is the static head. If you remember the figure and that is 15 meter, velocity in the pipe 1 would be Q/A_1 , the pipe 1 was before the pump, the pipe 2 was after the pump. Therefore, $V_1 Q$ is given as 20 liters per second, so it is 0.02 and area is $\pi/4$ and the diameter is also given 0.15, so it will come out to be 1.132 meters per second.

Similarly, for the delivery pipe, that is, velocity in the delivery pipe, this is a suction pipe and V_2 is the delivery pipe, we can, $A_1 V_1 = A_2 V_2$ and therefore V_1 can be written as $D_1^2 \text{ square}/D_2^2 \text{ square}$ and therefore 1 point or you can do also Q/A_2 , $15/12$, D_2 was 12 and therefore it will come to be 1.769 meters per second in the delivery pipe. So the major loss in pipe 1 will be $f_1 L/D_1$ into $V_1^2/2g$, f_1 is also given in pipe 1.

Length is 20 meters it is given, diameter we already know 0.15 meter into V_1 we found out, 1.132 whole square/2 into 9.81 and this comes to be 0.174 meter. So there will also be a inlet loss and that is 0.5 into, this is the minor loss, $V_1^2/2g$, so 0.5 into 1.132 whole square/2 into 9.81, so 0.033 meter. So this was the losses in pipe 1.

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$$h_{f2} = \frac{f L_2}{D_2} \frac{V_2^2}{2g} = \frac{0.02 \times 300}{0.12} \times \frac{(1.769)^2}{2 \times 9.81} = 7.975 \text{ m}$$

$$h_{e2} = \text{Loss at exit} = \frac{V_2^2}{2g} = \frac{(1.769)^2}{2 \times 9.81} = 0.160 \text{ m}$$

$$\text{Total head loss} = 0.174 + 0.033 + 7.975 + 0.160 = 8.342 \text{ m}$$

$$\text{Head delivered by the pump } H_f = \text{Static head} + \text{losses}$$

$$= 15 + 8.342 = 23.342 \text{ m}$$

$$\text{Power delivered by the pump} = \gamma Q H_f = 9.79 \times 0.02 \times 23.342 \text{ kW}$$

$$P = 4.57 \text{ kW} \quad \text{Answer}$$


We are going to see the h_{f2} , so major loss in pipe 2 will be fL_2 / D_2 into $V_2^2 / 2g$. So f_2 is also given 0.02, length is 300 meters that is given into D_2 was 0.12 into 1.769 whole square/2 into 9.81, that comes out to be 7.97. Similarly, there will be a loss at exit and at exit it is simply, $V_2^2 / 2g$, because k here is 1, so 1.769, it is 0.160. So total head loss is going to be $0.174 + 0.033 + 7.975 + 0.160$ and this is going to be 8.342 meter.

Now the head delivered by the pump H_f will be static head, first it has to overcome this static head plus the losses. So it is going to be $15 + 8.342$, that is, 23.342 meter. Therefore, the power delivered by the pump is $\gamma Q H_f$, that is, the total head. So 9.79 into Q is 0.02 into 23.342 kilowatt. If you find 4.57 kilowatt. So the first part we have got this as the answer. Now we also have to find the pressure at the suction side of the pump.

So for that we are going to use the Bernoulli's equation and also taking the head loss into account.

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(b) By energy equation between reservoir A and pump B
 let P_s = Pressure at suction side of the pump

$$95 + 0 + 0 = 100 + \frac{P_s}{\gamma} + \frac{V_1^2}{2g} + \text{losses in suction pipe}$$

$$= 100 + \frac{P_s}{\gamma} + 0.065 + 0.174 + 0.033$$

Major Minor

$$\frac{P_s}{\gamma} = -5.272 \text{ m (gauge)}$$

$$P_s = 10 - 5.272 \text{ (abs)}$$

$P_s = 4.728 \times 9.79 = 46.29 \text{ kPa}$

→ Absolute

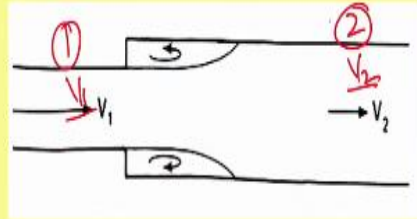
So part B will be solved by energy equation between reservoir A and pump B. So let P_s be the pressure at suction side of the pump, then we put the Bernoulli's equation $95 + 0 + 0$ will be equal to, the pump is at $z = 100$ and if the pressure $P_s/\gamma + V_1^2/2g + \text{losses in suction pipe}$. So this will be $100 + P_s/\gamma$ and $V_1^2/2g$ is going to be 0.065, because V_1 we have already found out, plus losses in the suction pipe is 0.174 major and the minor losses 0.033.

This is major loss, this is minor loss. So P_s/γ can be found out to be -5.272 meter. This is gauge. So the real pressure is going to be $10 - 5.272$, because we have already been told the absolute is 10, so it is going to be $P_s = 4.728$ multiplied by 9.79 to get it into form of kilopascal, so 46.29 kilopascal absolute. This is the value that we have got. So both the answers we got.

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Class Problem

A pipe enlarges suddenly from $D_1=240\text{mm}$ to $D_2=480\text{mm}$. the H.G.L rises by 10 cm calculate the flow in the pipe



Now we proceed next and we solve one more problem. A very simple problem, we say a pipe enlarges suddenly from $D_1 = 240$ millimeters to $D_2 = 480$ millimeters and the HGL rises by 10 centimeters, calculate the flow in the pipe. So this is V_1 and this is V_2 , so this is 1 and this is 2.

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Solution (2.)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_L = \left(\frac{p_2}{\rho g} + z_2 \right) - \left(\frac{p_1}{\rho g} + z_1 \right)$$

$$V_1 A_1 = V_2 A_2 \Rightarrow V_1 = 4V_2$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{(4V_2 - V_1)^2}{2g} = 0.1$$

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

$$\frac{6V_2^2}{2g} = 0.1$$

$$\Rightarrow V_2 = 0.57 \text{ m/s}$$

$$\Rightarrow Q = A_2 V_2 = 0.103 \text{ m}^3/\text{s}$$

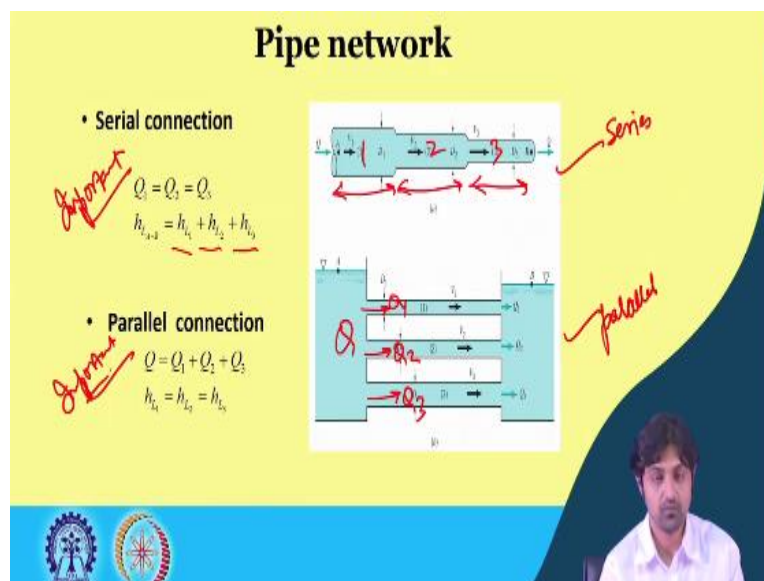
So we are going to solve this as well. So we can write Bernoulli's equation $\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1$ will be $= \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{whatever the head loss is}$. So we take V_2 on this side, so we can write $\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_L = \frac{p_2}{\rho g} + z_2 - \frac{p_1}{\rho g} - z_1$ and. So to find the relation between A_1 and V_1 sorry V_1 and V_2 we can say $V_1 A_1 = V_2 A_2$ which will give us, so the areas are double, so we are going to get $V_1 = 4V_2$.

And if we put this in these equations here and we also know that this is 0.1 that is what it says

that HGL rises by 0.1. Therefore, this will become $16 V_2^2$ square, this equation here, by $2g - V_2^2 \text{ square}/2g = 4 V_2^2 - V_1^2 \text{ square}/2g$ sorry this is minus because there was an entrance, you know, head loss is, what is this h_L , h_L is $V_1^2 - V_2^2 \text{ whole square}/2g$, if you remember the minor loss formula, is equal to 0.1 and if you solve this equation, you are going to get $6 V_2^2 \text{ square}/2g = 0.1$, which will give V_2 as 0.57 meters per second, implies Q is $A_2 V_2$, which will give 0.103 meter cube per second.

Very simple formula, the application we have seen using Bernoulli's equation, equation of continuity and because there is going to be a minor loss which is equivalent to $V_1^2 - V_2^2 \text{ whole square}/2g$. As we have said we have read the minor loss, the topic on the minor losses in the pipes. So we proceed forward from this question.

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So we talk about pipe networks. Pipe could be connected in series or parallel or combination of both. So this is a serial connection. So in the serial connection the important properties are that discharge Q_1 in this section, this section 2, this section 3 will be the same. However, the total head loss will be the sum of the head losses of individual sections, whereas in case of parallel connection the discharge will be the sum of all three.

So if there is a discharge Q , Q_1 , Q_2 , Q_3 , but the head losses will be the same in each of the pipe. So this is thumb rule, important thing to remember when you are dealing with pipe networks.

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Pipe Network

- A water distribution system consists of complex interconnected pipes, service reservoirs and/or pumps, which deliver water from the treatment plant to the consumer.
- Water demand is highly variable, whereas supply is normally constant. Thus, the distribution system must include storage elements, and must be capable of flexible operation.
- Pipe network analysis involves the determination of the pipe flow rates and pressure heads at the outflows points of the network. The flow rate and pressure heads must satisfy the continuity and energy equations.

So a water distribution system consists of complex interconnected pipes, service reservoir and or pumps which deliver water from the treatment plant to the consumer. The water demand is highly variable, whereas supply is normally constant. Thus, the distribution system must include storage element and must be capable of flexible operation. So pipe network analysis involves determination of pipe flow rates.

And pressure heads at the outflow points of the network. The flow rate and pressure head must satisfy the continuity and the energy equation. This is very true, as I told you in the last slide.

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Pipe Network

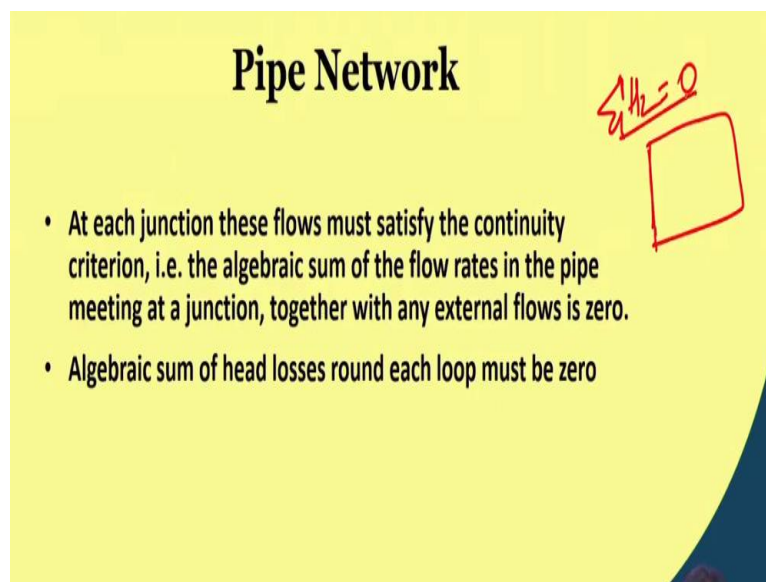
- The earliest systematic method of network analysis (Hardy-Cross Method) is known as the head balance or closed loop method.
- This method is applicable to system in which pipes form closed loops. The outflows from the system are generally assumed to occur at the nodes junction.
- For a given pipe system with known outflows, the Hardy-Cross method is an iterative procedure based on initially iterated flows in the pipes.

The earliest systematic method of network analysis is called the Hardy Cross Method and is known as the head balance or the close loop method. So pipe network is a topic where we are

going to study this famous method of Hardy Cross Method. It is known as the head balance or the close loop method. It is a very, very systematic way which can be used for solving the pipe networks.

So this particular method of Hardy Cross is applicable to a system in which pipes form closed loops. So the outflow from the system are generally assumed to occur at the junction nodes. For a given pipe system with known outflows, the Hardy Cross Method is an iterative procedure based on initially iterated flows in pipes. So Hardy Cross Method has a set rule and procedure, but this is for the solution, it is an iterative procedure.

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Pipe Network

- At each junction these flows must satisfy the continuity criterion, i.e. the algebraic sum of the flow rates in the pipe meeting at a junction, together with any external flows is zero.
- Algebraic sum of head losses round each loop must be zero

Handwritten notes: $\sum H_L = 0$ and a square loop diagram.

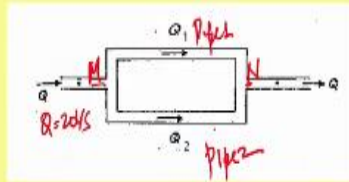
So what are the important points to remember in solving pipe networks using Hardy Cross Method? That at each junction the flow must satisfy the continuity criterion. What are these continuity criterion? The continuity criterion is that the algebraic sum of the flow rates in the pipe meeting at a junction together with any external flows is 0. Suppose, this is a node there is Q_1, Q_2, Q_3 . So $Q_1 = Q_2 + Q_3$,

So the net outflow at any junction should be 0 is the continuity criterion. Secondly, the algebraic sum of head losses round each loop must be 0. So sigma of head losses in one loop, like this, should be 0.

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Class Question

- A pipe 6-cm in diameter, 1000 m long and with $\lambda = 0.018$ is connected in parallel between two points M and N with another pipe 8-cm in diameter, 800-m long and having $\lambda = 0.020$. A total discharge of 20 L/s enters the parallel pipe through division at A and rejoins at B. Estimate the discharge in each of the pipe.



So with a simple continuity equation we are going to solve one class question. The equation is a pipe 6 centimeter in diameter here and 1,000 meter long and with $\lambda = 0.018$ is connected in parallel, so this is pipe 1 and this is pipe 2, between two points M and N and this other pipe is 8 centimeter in diameter and 800 meter long. So this was 1,000 meter long, this is 800 and the diameter was 6 and diameter was 8.

And this has a λ of 0.020, a total discharge of 20 liters per second enters. So from here something 20 liters per second enters the parallel pipe through division at A and rejoins at B. So it here and it rejoins here. Estimate the discharge in each of the pipe, very simple application of the concepts we have learned till now. So we are going to solve this question.

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Solution 13

$Q = Q_1 + Q_2$
 $0.02 = \frac{\pi (0.06)^2 V_1}{4} + \frac{\pi (0.08)^2 V_2}{4}$
 $V_1 + 1.778 V_2 = 7.674 \quad (i)$

For pipes in parallel
 $h_{f1} = h_{f2}$
 $\frac{16 Q_1 V_1^2}{2 g D_1^5} = \frac{16 Q_2 V_2^2}{2 g D_2^5}$
 $\frac{0.018 \times 1000 \times V_1^2}{0.06^5} = \frac{0.020 \times 800 \times V_2^2}{0.08^5}$
 $V_1 = 0.8165 V_2 \quad (ii)$

Solving for V_2 by substituting (ii) in (i)
 $V_2 = 2.73 \text{ m/s}$
 $Q_2 = A_2 V_2 = \frac{\pi}{4} (0.08)^2 \times 2.73$
 $Q_2 = 0.0137 \text{ m}^3/\text{s}$
 $V_1 = 0.8165 V_2 = 2.23 \text{ m/s}$
 $Q_1 = 0.0063 \text{ m}^3/\text{s}$

Recheck the answer
 $Q_1 + Q_2 = Q$
 $0.0063 + 0.0137$
 $= 0.020 = Q$
 OK

So there was pipe, so Q will be $= Q_1 + Q_2$. So total discharge is 0.02 and we know the areas

$\pi/4$ into 0.06 let say velocity is V_1 here and in the pipe 2 it is V_2 0.08 V_2 . So this is the equation for the continuity equation for discharge. So for pipes in parallel we also know that the head losses should be equal in both pipes. So this actually can be written as $V_1 + 1.778$, $V_2 = 7.074$, if you solve this, it will come this.

This is the one equation that we have got from here and now $h_{f1} = h_{f2}$ which means $\lambda_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = \lambda_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$. So let us substitute the value, λ_1 was 0.018, length was 1,000 first pipe into $V_1^2/2g$, so $2g/D_1$ comes out to be 0.06, this will be 0.020 into 800, this comes to be 0.08 into V_2^2 square. So taking it here, we will get $V_1 = 0.8165$.

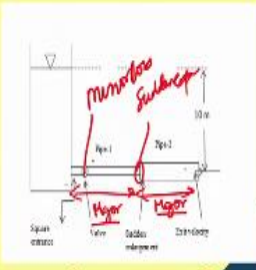
So we have two equations in V_1 and V_2 , so 1 and 2. Solving for V_2 by substituting V_1 by substituting 2 in 1, we will get V_2 as 2.73 meters per second. So Q_2 will be $A_2 V_2$, so $\pi/4$ 0.08 whole square into 2.73, so Q_2 will come to be 0.0137 meter cube per second. Now if we know V_2 , we can get V_1 from here, V_1 will be 0.8165 V_2 , this will give us V_1 as 2.23 meters per second.

And therefore, Q_1 will be $A_1 V_1$ will give us 0.0063 meter cube per second. So this is the answer, but one important step is it is better if we recheck the answer by $Q_1 + Q_2$, if it is equal to Q or not. So 0.0063 $Q_1 + 0.0137$, it is coming out to be 0.020 which is equal to Q . So our answer is okay. So this was one other question.

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Class Problem

- A horizontal pipeline, 50 m long, is connected to a reservoir at one end and discharges freely in to the atmosphere at the other end. For the first 25 m length from the reservoir the pipe has a diameter of 15 cm and it has a square entrance at the reservoir. The remaining 25 m length of pipe has a diameter of 30 cm. The junction of the two pipes is in the form of a sudden expansion. The 15 cm pipe has a gate valve ($k=0.2$) in fully open condition. If the height of the water surface in the tank is 10 m above the centreline of the pipe, estimate the discharge in the pipe by considering the Darcy Weisbach friction factor $f = 0.02$ for both the pipes. [Include all minor losses in the calculations].



Now the one more question is there is a horizontal pipeline which is 50 meters long and is

connected to reservoir at one end and discharges freely into the atmosphere at the other end. So for the first 25 meters, I have shown this diagram here. So the first 25 meters length from the reservoir, the pipe has a diameter of 15 centimeter and it has a square entrance at the reservoir. The remaining 25 meter length of the pipe has a diameter of 30 centimeter, so this is the long, long pipe.

And this is pipe 1, I mean the things that we were talking about, diameter is different here, diameter is different here. So junction of the two pipes is in the form of a sudden expansions. So we have been told sudden expansion that means there is going to be a energy loss, minor loss. It is also told that there is going to be a there is a gate valve in a 15 centimeter pipe. So there is going to be another minor loss here.

So if the height of the water surface in the tank is 10 meter above the centerline and the velocity, estimate the discharge in the pipe by considering the Darcy Weisbach friction factor of 0.02 for both the pipes. So this is given because there will be also major losses between this two sections, major losses. So we have to, this is a wholesome, a complete question that involves both the major and the minor losses.

So we are going to, actually I think this is a nice point to stop. In the next class we are going to solve this particular question in detail because then it will be fresh in your mind and then we will proceed to our last topic that is Hardy Cross Method. So thank you so much for listening. I will see you in the next lecture.