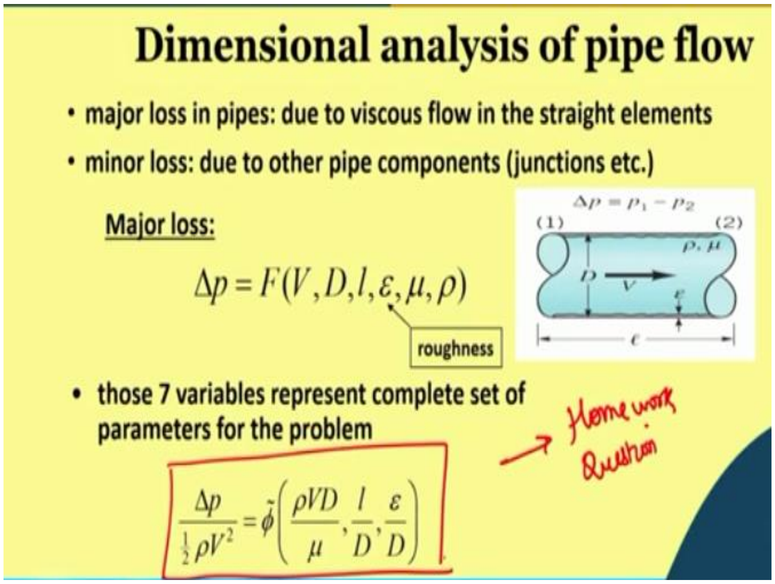
Hydraulic Engineering
Prof. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology – Kharagpur

Lecture 42
Pipe flow (Contd.)

Welcome back. So, last time, we solved a question, using the laminar and the turbulent shear stresses. We calculated the laminar viscous sublayer thickness, then we calculated the centerline velocity, using those equations of turbulent velocity profile and then in the end, we calculated the ratio of turbulent shear stress to laminar shear stress and found out that the ratio is more than 1000, it was almost 1217. Indicating that, in a turbulent flow, the ratio of turbulent shear stress to laminar shear stress is very, very high.

(Refer Slide Time: 01:05)



So, in turbulent flow, the dominating shear stress is due to the turbulence. Now, we are going to start a topic, dimensional analysis of pipe flow. So, pipes are generally considered rough and they have roughness, that is, so they have this roughness and because of this roughness, there is loss of energy due to viscous flow in the straight element and this loss that happens due to the viscous flow is called the major loss in pipes.

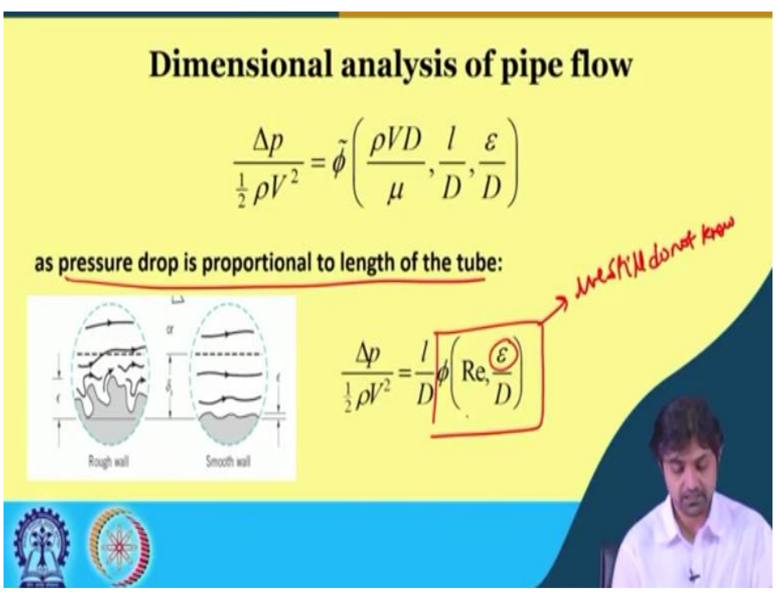
So, there are two types of losses in pipe, which is, one is major loss and the other is minor losses. The minor losses happen due to the pipe components. Suppose, for example, there are junctions at pipes or there is a bend or there is a contraction or an expansion, then also there will be some loss in the energy contained in the turbulent flow and those losses are called minor losses.

So, for major losses during the dimensional analysis, we say that the pressure drop should be a function of the velocity in the pipe diameter D, length L of the pipe, mu the viscosity and the density of the liquid. Here, we have an additional element that is roughness height, epsilon. This is a new thing that we are going to take into account. Earlier, we said that the pipes were smooth, there were no roughness elements. So, pipe, there are roughness elements, like this.

In principle, there are roughness elements, like this. So, if we do the dimensional analysis, then we first write down K. How many? 1, 2, 3, 4, 5, 6 and then 7 and r is 3. So, how many dimensionless term? K – r, that we are going to get 4 dimensionless term. If, you do the Buckingham pi theorem analysis, we are going to get, so this is, as I said, was the roughness element, epsilon. This is the roughness element, as I showed you before as well.

We will get one dimensionless term, second dimensionless, third and fourth. As we said K – r, 7 – 3 = 4 dimensionless term and I ask you to do it as a homework question. If you have any problems, you can ask us about that in the forum. So, this is the general equation for pressure drop for a pipe flow with roughness element of epsilon E.

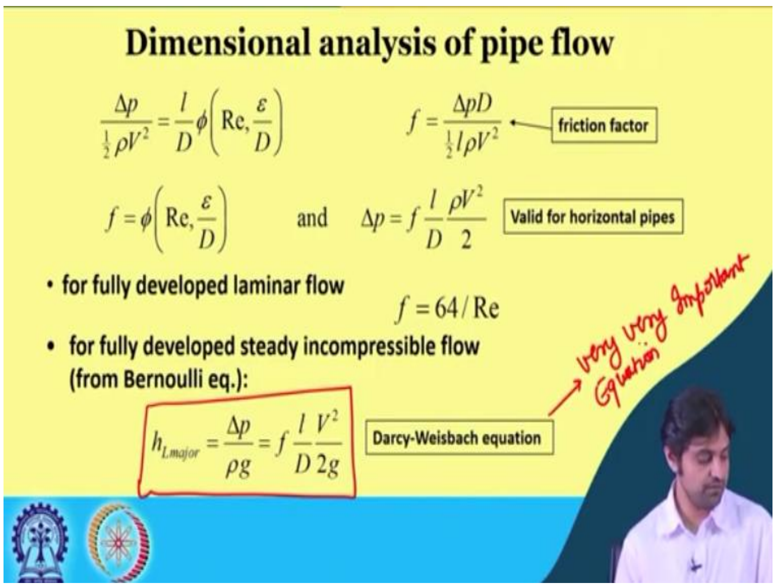
(Refer Slide Time: 05:03)



So, again we write down, delta P by 1/2 rho v square is a function of rho VD by mu l by D and epsilon by D. This is what? This is Reynolds number. This we know from before, rho VD by mu. Now, we already know from before, the pressure drop is proportional to the length of the tube. So, we can take l by D outside and we can write, delta P by 1/2 rho V square is equal to l by D, as a function of Reynolds number and epsilon by D.

How this depends, we still do not know. Until now, I mean, under the topics that we have covered, we still do not know, but other things we know, 1/2 Rho V square l by D, Reynolds number also we know. We do not know how to find epsilon and we also do not know how this function is dependent upon Reynolds number and epsilon by D. But that we will get to know in the upcoming slides.

(Refer Slide Time: 06:23)



So, again we write down the same equation, delta p by 1/2 rho V square is l by d and a function of Reynolds number and epsilon by D. So, we bring l by D, on the other side and define. So, if you take D on this side and l here and term is at f. So, f is nothing but delta p into D divided by 1/2 l rho V square and this we call friction factor f. So, f is a function of Reynolds number and epsilon by D.

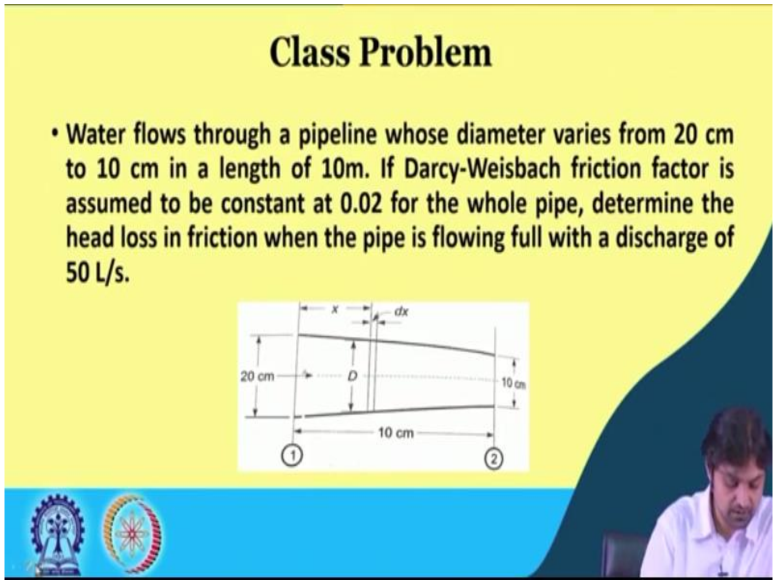
So, as I wrote, f is equal to a function of Reynolds number and epsilon by D. And if we are able to find this f, we can easily calculate delta p as f into, because see, if you use this equation again, we can write, delta p is equal to f into l by D into rho V square by 2. So, if we are able to calculate this f; we know l, we know D, we know rho, we know V, so everything will be calculated. So, the challenge is now, finding f and this is valid for horizontal pipes, this equation.

Now, for a fully developed laminar flow, we already found out that, what that f was, it was 64 by Re. From our previous lectures in pipe flow, we have derived that f is equal to 64 by Re for fully developed laminar flow. But for fully developed steady incompressible flow, we also know that the head loss is going to be, if we assume, delta p is the head loss and that will be transformed into energy loss, equivalent to rho gh. So, hL major can be written as, delta p by rho g.

Just concentrate on this. So, this is the head loss major and delta p by rho g can be, so if you use this equation and divide it by rho g, so what are you going to get, f will remain same f, l by D will remain same, rho and rho gets cancelled and we can get f into l by D into V square by 2g. So, the major head loss is going to be f l by D into V square by 2g. So, the major head loss, l we know, D we know, V we know and g is a constant.

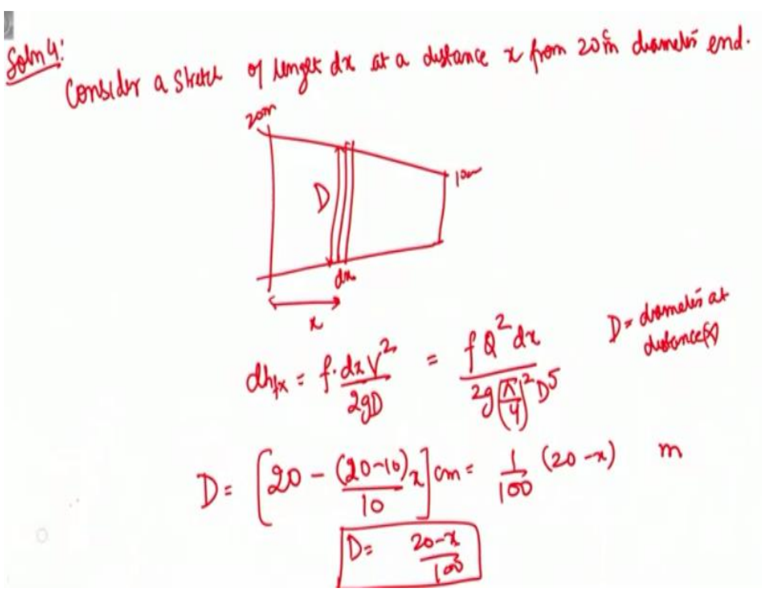
So, again the problem is, we still do not know f, but for f we know, it is a function of Reynolds number and epsilon by D and this particular equation is called the Darcy-Weisbach equation, a very, very important equation for the major head losses in the pipe. So, how do we estimate f? For f, if we would know Reynolds number and epsilon by D, everything would be, the recipe would be available to find out the friction factor f.

(Refer Slide Time: 10:28)



So, this before going to how to find out this epsilon by D, I have a small class problem, which we are going to solve. So, the question says that water flows through a pipe line whose diameter varies from 20 centimeters to 10 centimeter in a length of 10 meters. If the Darcy-Weisbach equation factor is assumed to be constant, so f is given constant at 0.02 for the whole pipe. Determine the head loss in friction, when the pipe is flowing full with a discharge of 50 liters per second. So, we are going to solve this question.

(Refer Slide Time: 11:14)

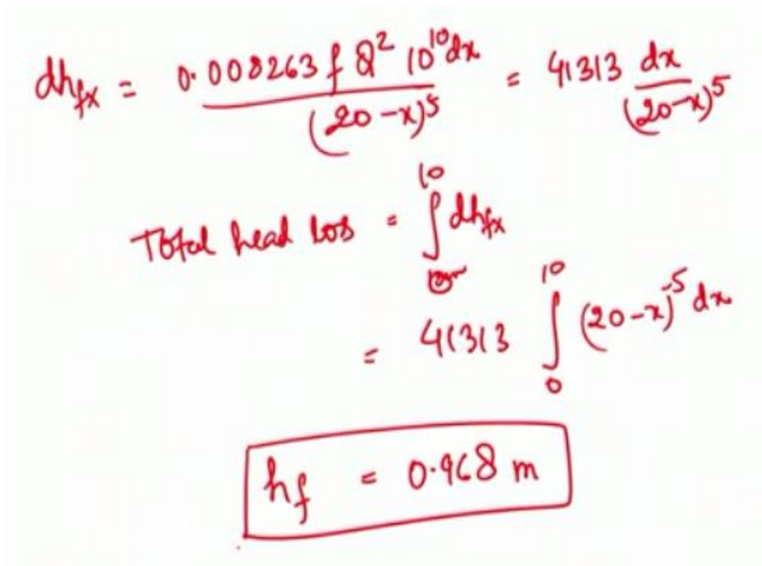


So, we say, consider a stretch of length dx at a distance x from 20 meter diameter end. This was 20 meter, sorry, 20 centimeter diameter end and this was 10 centimeter. So, we assume, at a distance x, strip of thickness dx here. So, dh fx will be f l, so l is here dx by, so V square by 2gd.

If we write in terms of Q, we can write f, friction factor f Q square dx divided by 2g pi by 4 whole square into D to the power 5 because Q square, V square will be Q square by a square and a is d to the power 4. So, we are going to get this equation.

Here, this is the diameter D, at distance x. So, we need to find what that D is and D can be written as, see, it is varying from 20, 20 centimeter. So, we can write to convert it to meter, one by 100 and this will come 20 - x in meters. This is the diameter. So, we proceed forward.

(Refer Slide Time: 14:09)

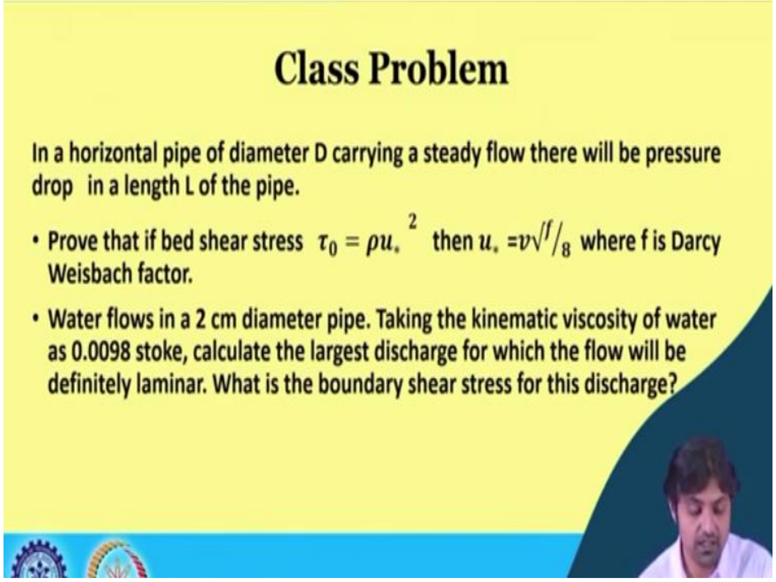


So, dh fx will be 0.008263 f into Q square into 10 to the power 10 dx divided by 20 - x to the power 5. This will be, if you calculate, it will be 41313 into dx by 20 - x to the power 5. So, total head loss will be integral of dh fx from 10 centimeter to, so I mean, from 0 to 10. So, total head loss is going to be 41313 integral 0 to 10, 20 - x to the power -5 dx and total head loss hf is going to be 0.968 meter. So, this is how you solve this question, a very simple application, because the Darcy-Weisbach equation friction was already given.

So, this is one question. So, until now, what we have done? We have obtained the pressure drop and as a function of another quantity, pressure drop was as a function of Darcy's friction f, where f is some function of Reynolds number and epsilon by D. In laminar flow, fully developed laminar flow, it was only a function of Reynolds number, as 64 by Re.

But before going in detail into that, we just solved, if we are given that friction factor f, Darcy's friction factor f, how are we able to apply these formulas of dh fx to find the head loss.

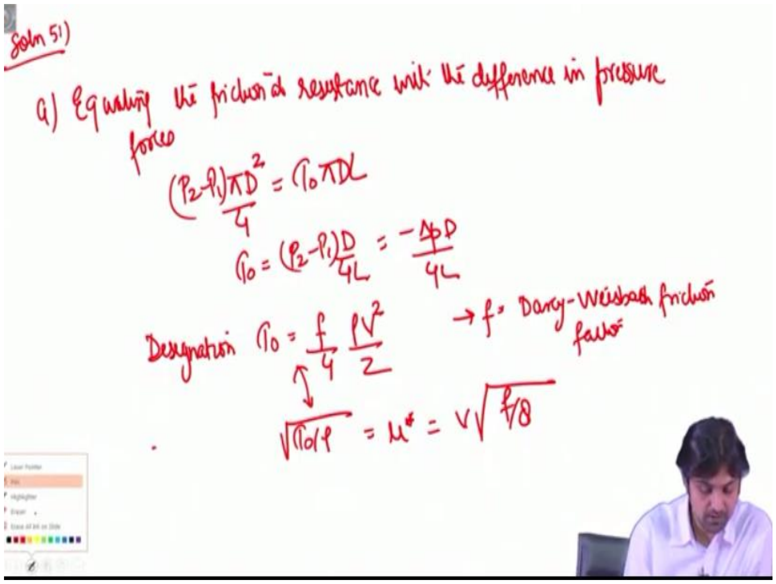
(Refer Slide Time: 16:53)



Now, there is another question. It says that in a horizontal pipe of diameter D, which is carrying a steady flow, there will be a pressure drop in a length L of the pipe. It says, prove that if bed shear stress is tau not is equal to rho u star square, then u star can be written as nu into under root f by 8, where f is Darcy Weisbach friction.

And then a question is given that water flows in a 2 centimeter diameter pipe. Now, taking the kinematic viscosity of water as 0.0098 stoke, calculate the largest discharge for which the flow can be definitely laminar. So, largest discharge will be at largest velocity, that means, we will have to utilize some Reynolds number, you know. And then in the end, it says, what is the boundary shear stress for this discharge. So, we will go one by one for the solution.

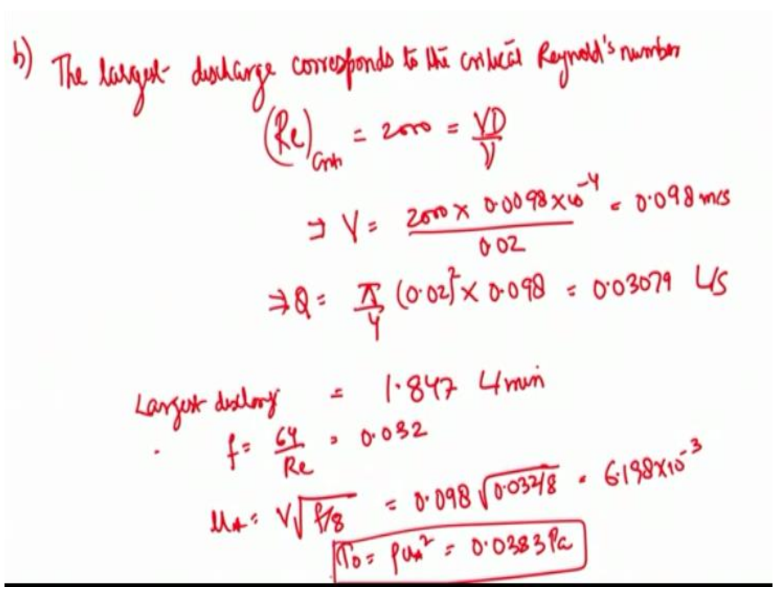
(Refer Slide Time: 17:57)



So, the first part is pretty simple. You have to equate the frictional resistance with the difference in pressure forces. So, instead of delta p, we write, P2 - P1 into pi D to the power 2 by 4 is equal to tau not into pi DL. So, tau not, using this can be written as P2 - P1 into D by 4L or minus delta P D by 4L, because P is P1 - P2. Now, designation tau not is f by 4 rho V square by 2. Here, f is Darcy-Weisbach friction.

So, under root tau not by rho is equal to mu star is equal to V under root f by 8. This is how we get from this equation. Now, starting with the second part, part B.

(Refer Slide Time: 20:18)

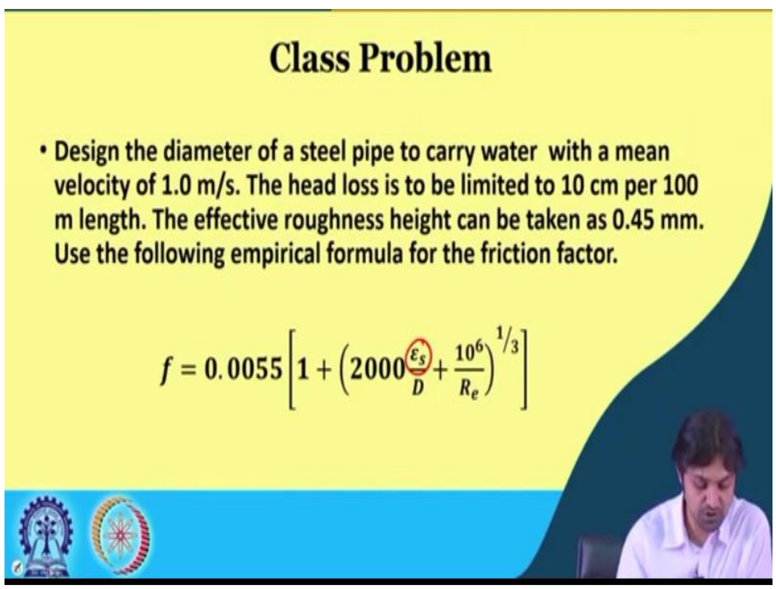


The largest discharge corresponds to the critical Reynolds number, which is for the largest it can be for laminar, it can be maximum 2000. Remember? Reynold critical will be 2000 is equal to VD by nu, which will give us velocity as 2000 into nu is 0.0098 into 10 to the power -4 divided by the diameter, 0.02 gives us 0.098 meters per second. Therefore, Q will be pi by 4 into diameter whole square into this velocity. This is going to be 0.03079 liters per second.

Instead of meters, we have multiplied 10 to the power 3 and made it in liters per second or if in terms of minutes, you can simply say, multiply it by 60 and say 1.847 liters per minute. So, this is the largest discharge, also f can be written as 64 by Re, which is comes 0.032. So, u star can be calculated at under root f by 8, f is 0.098, we have already found out. f was 0.032 divided by 8 eight or 6.198 into 10 to the power -3.

Similarly, tau not will be rho u star square, which we have calculated and it will come out to be 0.0383 Pascal. So, this is yet another simple question of the Darcy-Weisbach equation, very simple. So, we go back.

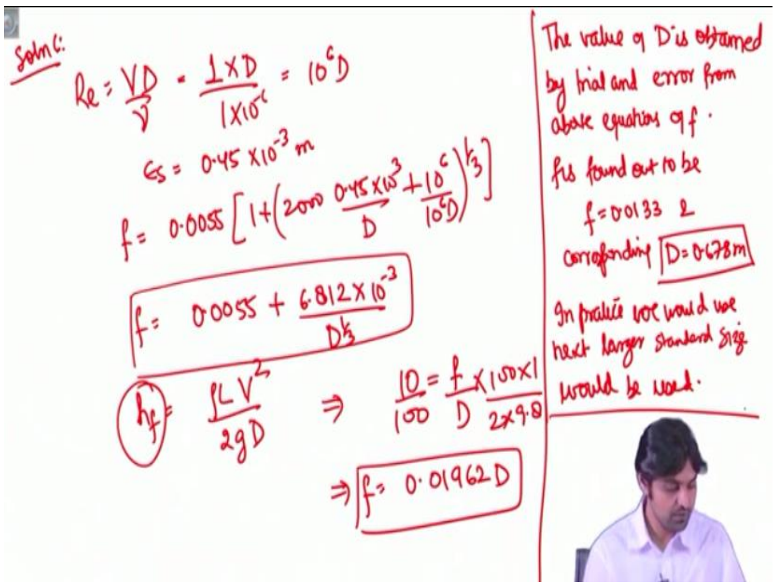
(Refer Slide Time: 23:20)



And there is one more problem. So, this is the last problem before the end of today's lecture that we are going to do. And it says is, design the diameter of a steel pipe to carry water with mean velocity of 1 meters per second. The head loss is to be limited to 10 centimeter per 100 meter length. The effective roughness height can be taken as 0.45. Now, use the following empirical formula for friction factor.

So, in this question, you see, we already have been given Es, this thing. So, f we can calculate. We have been given the functional dependence of f, how to find out f. So, it is a pretty simple problem, if we are given f. We just you need to follow the basic steps. How do we solve this problem?

(Refer Slide Time: 24:17)



First, we find the Reynolds number. So, VD by mu and V 1 meters per second, diameter D, let it be this and mu is 1 into 10 to the power -6, so it becomes 10 to the power 6D in terms of diameter. We have already been given Es, that is, 0.45 into 10 to the power -3. Now, if we substitute, this in this value, the f which we have given, because most of the things we know already, into 0.45 into 10 to the power 3 by D. Because D is we do not know, that is what we have to find out, into D.

So, f can be written as, in final, it can be 0.0055 + 6.812 into 10 to the power -3 divided by D to the power 1 by 3. And hf will be fL V square by 2gD. So, head loss has already been given, it says 10 in 100, that is, what it said. If it says the head loss is limited to 10 centimeter + 100 meters of length. So, head loss has been given. So, we can write, 10 by 100 is equal to f by D into 100 into 1 divided by 2.981.

This will give us f is equal to, in terms of this is 0.01962 D. So, we have two equations for D, sorry, two equations of f. So, the value of D is obtained by trial and error, from above equations of f and f is found out to be 0.0133 and corresponding D comes out to be 0.678 meters. So, if there is a practical problem in practice, we would use next larger standard size, because the pipe diameters does not come, it is very difficult to design 0.678 meter.

Maybe 0.7 meters is available, so we are not going to use 0.678, but the next higher size available. So, this concludes our this particular question. I am moving ahead.

(Refer Slide Time: 29:01)

Equivalent roughness for pipes

Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

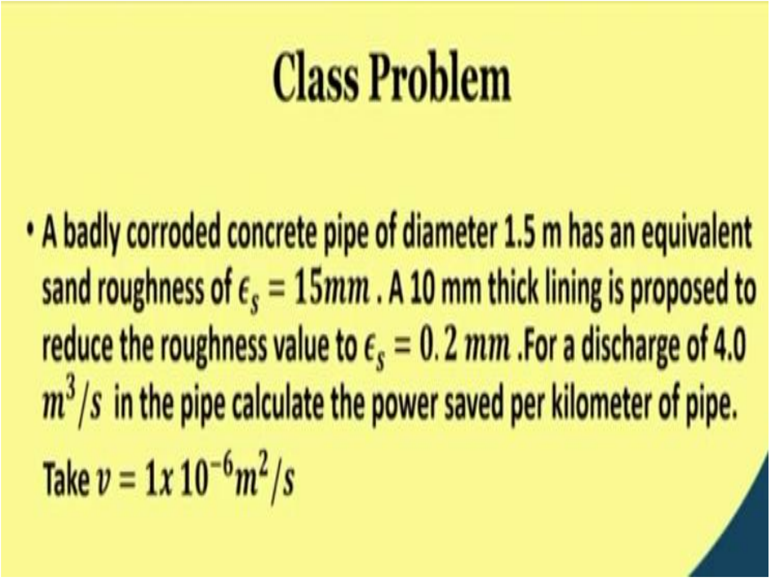
	Equivalent Roughness, ε	
Pipe	**Feet**	**Millimeters**
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

So, before we close this class, I would like to take this. What is the equivalent roughness for the pipes? Epsilon. Sorry. What is epsilon? So, epsilon is a material property and this epsilon can be obtained for practical senses, with tables like this. Here, it is given that for riveted steel epsilon is of varies from 0.9 to 9.0. Concrete it is going to be 0.3 and 3.0. And wood stave, for example, 0.18 to 0.9. So, for every type of pipe, a corresponding epsilon will be told to us from before.

For your purpose, most likely the epsilon will be given in the question because otherwise, you will have to, I mean, otherwise you will have to look it from the table, if we do not give you epsilon, will give you the table. So, you look which type of pipe is there among these for example, and you find out epsilon and then proceed with your problem. So, now you remember f, the Darcy friction was a function of Reynolds number, epsilon by D.

Reynolds number we know how to calculate, epsilon would be given and D would also be given. So, we know everything, but how it is dependent we do not know. This part we still do not know.

(Refer Slide Time: 30:33)



So, to answer that we have to proceed further, but before that there is a class problem. This is one question that we would like to do again. But I would like to close this lecture now and start our new lecture by solving this particular problem. Thank you so much for listening and I will see you in the next week.