Hydraulic Engineering
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Lecture - 41
Pipe flow (Contd)

Welcome back student to this 4th lecture of the pipe flow.

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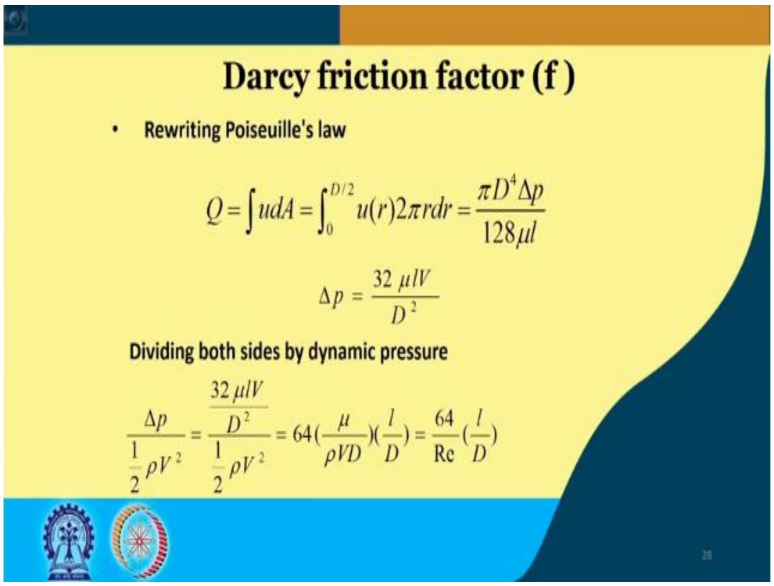

Last time we saw the Darcy friction factor and we saw that for the fully developed laminar flow the Darcy friction was 64/Re. Whereas, in terms of wall shear stress, it was 8 tau w/rho V square.

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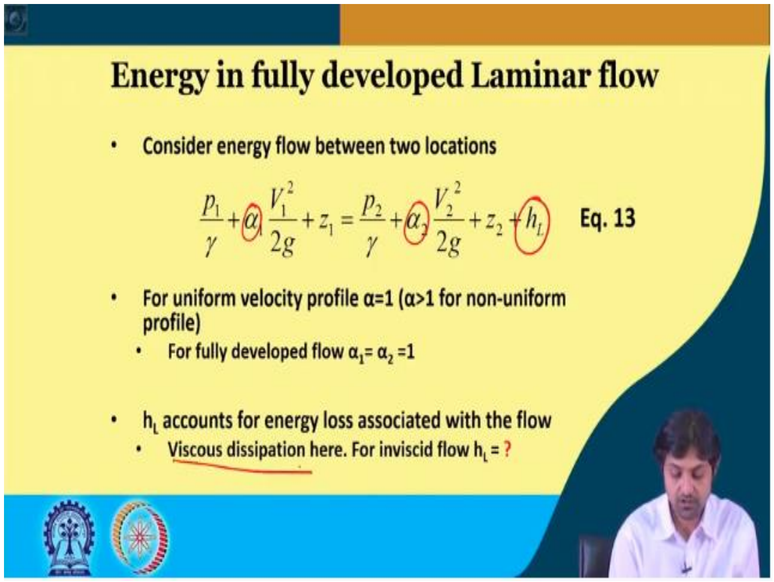

Now, we are going to see about the energy in a fully developed laminar flow. So, consider the flow between 2 locations and if we consider flow between 2 locations, we can write, Bernoullis equation. So, the pressure head, the velocity head and the datum head is equal plus the energy loss on the other side, if there is an energy loss, in terms of head loss hL. This alpha and alpha 2, we take 1 for the uniform velocity profile and alpha>1 for non-uniform velocity profile.

We are assuming, alpha = 1 because our velocity profile is uniform. So, for fully developed flow, alpha 1 = alpha 2 = 1. And here the head loss accounts for energy loss, associated with the flow, that I have told you before. So, we have written, Bernoullis equation between 0.1 and 0.2, accounting for the head loss or the energy loss and this is due to the viscous dissipation. For inviscid flow, there will be no head loss, hL was going to be 0.

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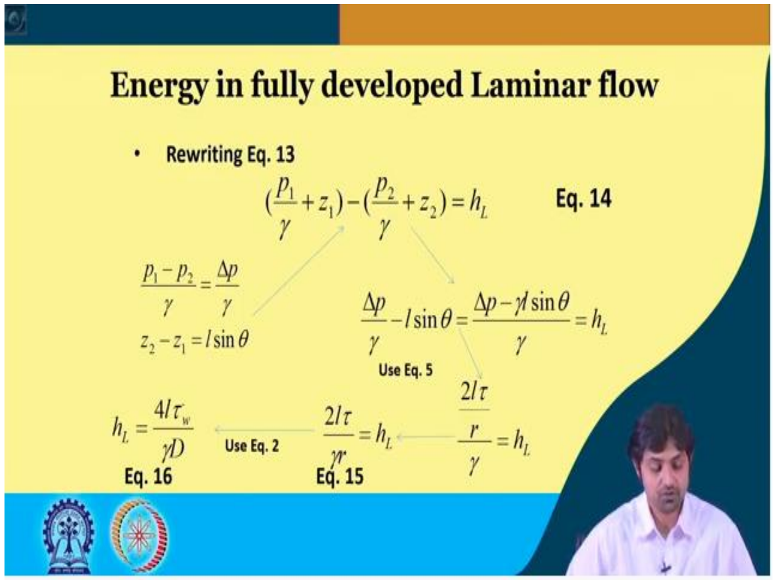

So, with this background, we will rewrite equation number 13. So, we will get, p1/gamma + z1 - p2/gamma + z2 = hL, for uniform velocity profiles. So, what we are going to get, p1 - p2/gamma = delta p by, because this is if we do p, so here is p1/gamma minus, so we can write, p1 - p2/gamma and this becomes delta p/gamma, z2-z1 we can always write l sin theta, the datum, if, the bed, the slope of the bed is theta.

And if we put that here, we are going to get, delta p/l – l sin theta = head loss or delta p – gamma l sin theta/gamma is head loss, with this simple manipulation. Now, if we use equation 5. You remember what the equation 5 was? Then, so p – gamma l sin theta using equation number 5, can be written as, 2 tau l/r. This will be the head loss. And then had loss can be simply written as, 2 tau l/gamma r. This is equation number 15.

Now, if we use equation number 2, 2 was using the wall shear stress and using equation number 2, if we substitute tau, in terms of wall shear stress and r, we will see, we will get head loss as, 4l tau w/gamma D. So, this is a quick formula for obtaining the head loss, in terms of wall shear stress, length of the pipe and diameter of the pipe. So, you should remember this one. And this is equation number 16.

It is just a matter of substituting one equation into the other and obtaining what we require. So, the beauty of this equation is that, you know, 4l tau w/gamma D, it depends only on the wall shear stress.

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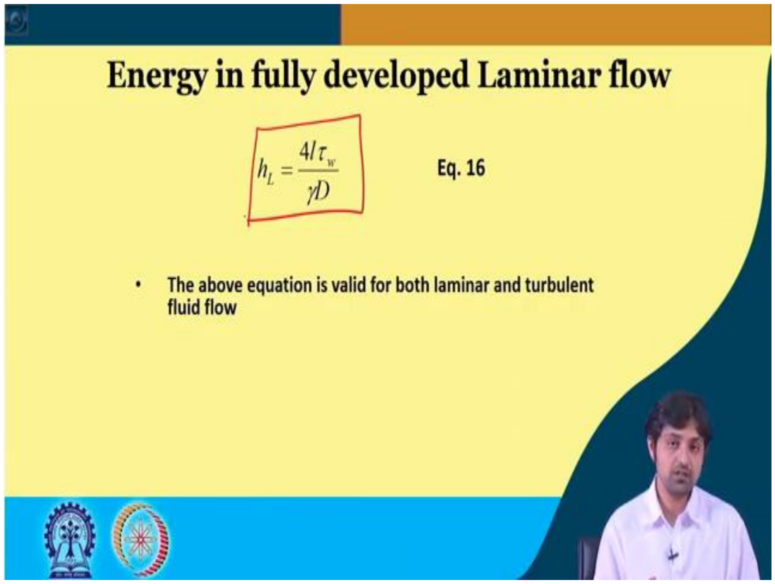

Now, we have written head loss as 4l into tau w/gamma D. This is equation number 16 and this equation is valid both for laminar and turbulent flow. So, we have not yet used any approximation, specific for only laminar flow. This particular equation, because the substitution that we have done, we have taken tau w and tau relationship, that was, valid for both laminar and turbulent flow.

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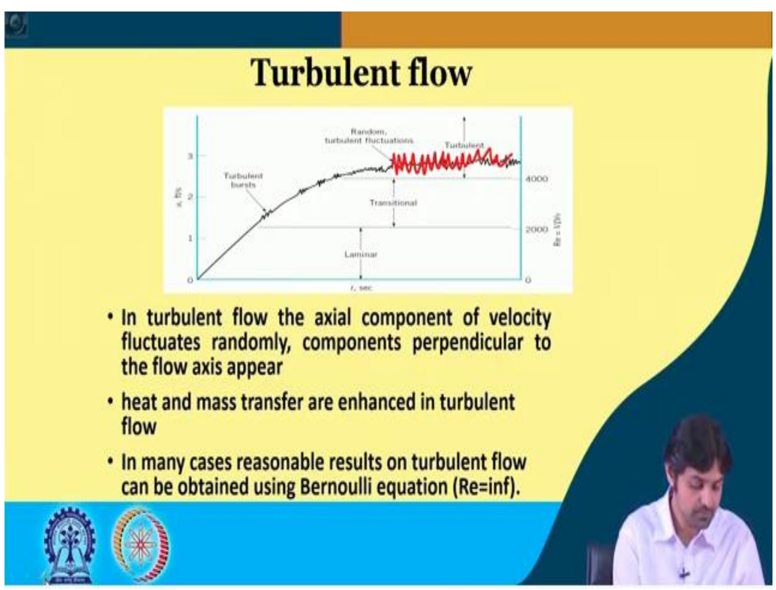

So, with this we conclude this energy in fully developed laminar flow and the also this one, since it is valid for turbulent flow and so also energy in the turbulent flow.

Now, we proceed to a broader topic called turbulent flow. Much of it we have already seen in a chapter called laminar and turbulent flow. But for specifically for pipe flow, we will go through it once again. The one that we have said was for a general case of flows, in the week number 3 that we went.

So, here there is a figure that shows, time on this axis and velocity on this axis. You see. as the velocity is increasing until some point, the velocity is less so this is the laminar region. The idea is if you keep on increasing the velocity, Reynold number will increase and the flow will become from laminar to transitional to turbulent. So, as you keep on increasing the velocity until this point, it is still transitional until Reynolds number of 4000.

And after that it becomes fully turbulent. So, in turbulent flow the axial component of velocity fluctuates randomly. The components perpendicular to the flow axis appear, you see here. In turbulent flow, heat and mass transfer are enhanced. That is another important feature that we know. In many cases, reasonable result on turbulent flow can be obtained using the Bernoulli's equation, many cases. For because the, I mean, by putting a Reynolds number as infinity.

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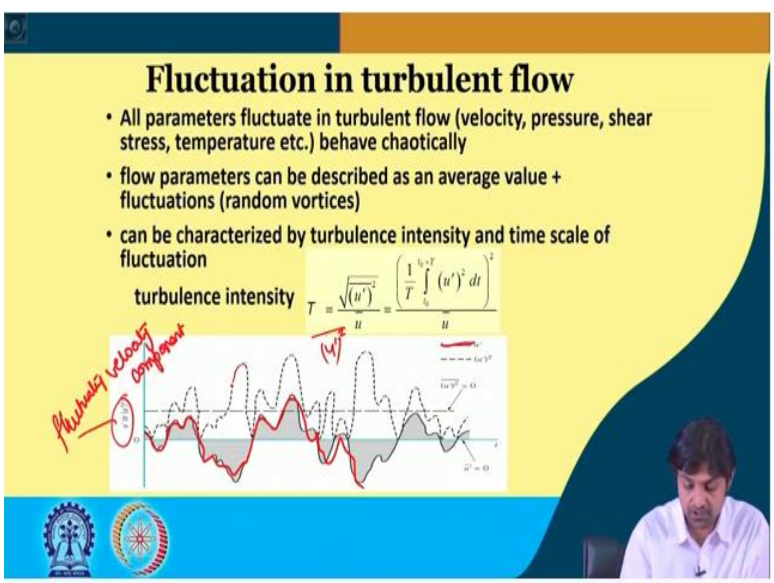

So, talking about the fluctuation in turbulent flow, all parameters fluctuate in turbulent flow. Whether it is velocity, pressure, shear stress, temperature, everything fluctuates and they behave chaotically, chaos is there. The flow parameters can be described as an average value plus fluctuation. This we have already seen many times, that u can be return as, u bar + u prime, v can be written as, v bar + v prime, w can be written as, w bar + w prime, same with pressure.

So, this is the average component, things with the bar and this is the fluctuating component. And these you see, there is bar and prime. So, they can be characterized by turbulent intensity and timescale of fluctuation. Turbulence intensity is nothing but under root of u prime whole square and then average taken over that to the average value. So, you see, here it is the fluctuation by average value.

So, you can assume, if the fluctuation is too high, the turbulence intensity will be high because it is fluctuation by average. So, in terms of wave averaged or, you know, time average, this can be written also like this, so something like this. So, you see, we have plotted again it as a function of time, the fluctuations. So, here it is not the average velocity, it is the fluctuating velocity component, as a function of time.

So, this line represents the fluctuation, this solid line. The dash, dash, dash represent fluctuation to the whole square and u and this line is the thin line, here, you see, so this is the fluctuation, the tick line. Whereas, this line, represents u dash whole square and this, the average of this quantity is represented by thin line like this, this one, this one, this one, this one, this one.

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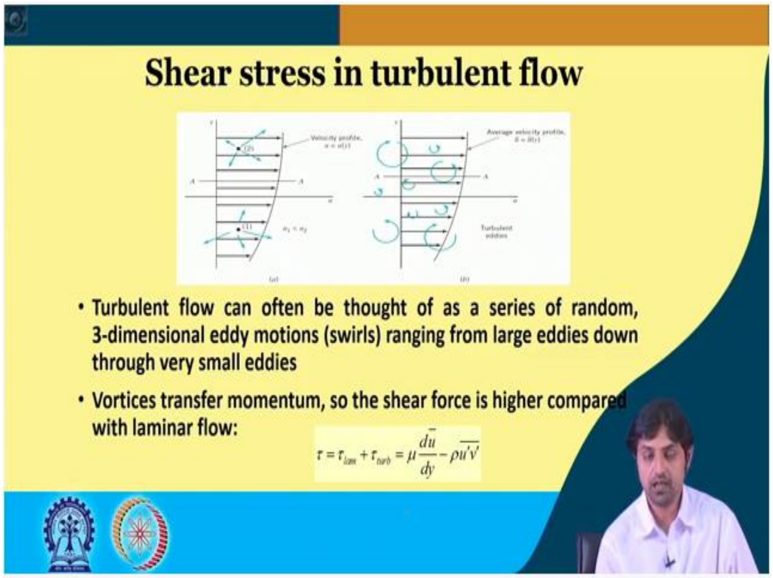

Now, talking about shear stress in turbulent flow. So, they represent 2 sub figures a and b. You see, this represents a particle having random velocities here and at a point 2 and this represents a velocity profile, a normal velocity profile. And this represents an average velocity profile, as you can see, we have plotted the turbulent eddies, here, here, see. So, eddies has been represented.

So, the turbulent flow can be often thought as a series of random 3 dimensional eddy motion, ranging from large eddies down to very small eddies. So, this is large eddie and these are small eddies. Vortices, these vortices they transfer momentum so the shear force is higher compared with the laminar flow. So, in turbulent flow, the shear force is much higher than with the, can compared with the laminar flow.

So, the total shear stress in a turbulent flow can be written as combination of 2 things, shear stress due to laminar and plus shear stress due to turbulence and this is for the laminar shear stress and this is turbulent shear stress. We shall go into more detail, when we study the modules of viscous fluid flow and computational fluid dynamics, about rho u dash v dash, they are called Reynold shear stress. So, therefore, the shear stress in a turbulent motion is much, much larger than the laminar flow.

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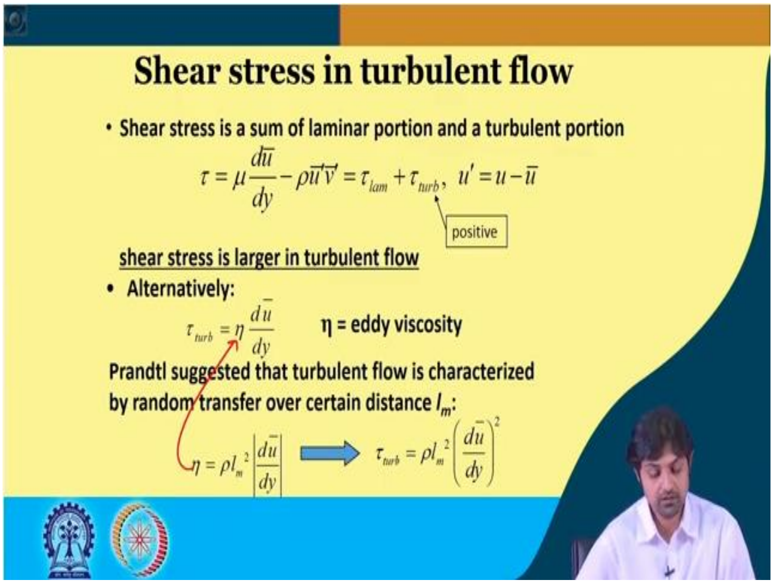

So, here, as I said the shear stress is the sum of laminar portion and a turbulent portion. As shown before, tau = Mu du dy – rho u dash v dash whole bar. So, tau laminar + tau turbulence, where this is fluctuation and this shear stress turbulence is positive and therefore it is sum of 2 shear stresses. Shear stress is larger in turbulent flow. Alternatively, we can also write, so instead of writing, we were writing tau = - rho u dash v dash whole bar.

But we can also write, tau turbulence as, tau turbulence can be written as, another viscosity into du dy and this eta is not equal to Mu, but much higher and this is called turbulent eddy viscosity. This is our approximation, where eta is eddy viscosity or turbulent eddy viscosity.

So, there was a scientist called Prandtl. He suggested that the turbulent flow is characterized by random transfer over a certain distance lm, as we have already talked about this in our laminar and turbulent flow module.

So, eta can be approximated as, rho lm square into du/dy, where u this is average velocity. See, you have to appreciate that if we are able to write something, in terms of average velocity, it makes our life easier. And therefore, tau turbulence, if you substitute here, tau turbulence can be written as, rho lm square into du/dy whole square, indicating that tau turbulence is positive.

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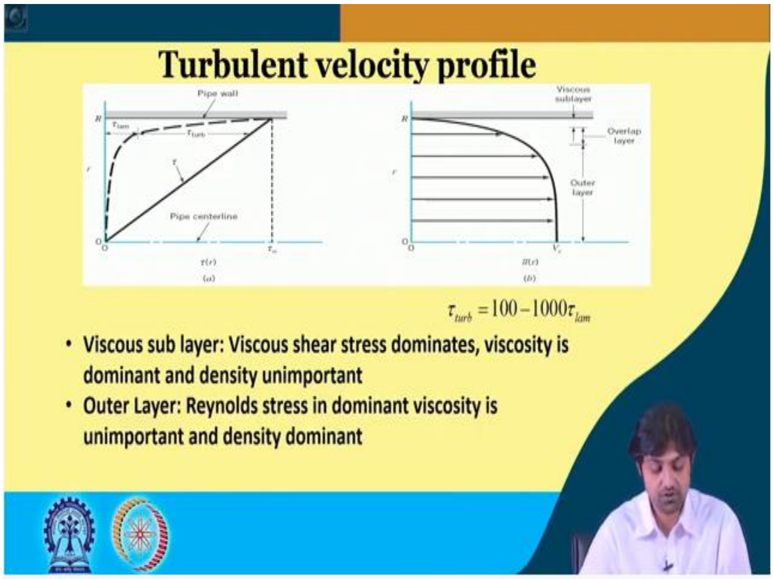

So, this is figure, showing different areas in a pipe flow, which is the layer so the area near the wall is called viscous sub layer, then there is something called the turbulent layer, then there is an overlap layer and then there is an outer layer until the centre line. So, you see, we have represented tau, as a function of r, this is the pipe center line and you see, if this is the total shear stress, at any point, you can see the contribution.

So, near the pipe the contribution of laminar flow is laminar is very high. But near the, say for example, at this point, so laminar is pretty high, but as soon as you move away from the wall, the laminar contribution compared to the turbulent contribution becomes less and less. So, actually we can write that shear stress due to turbulence is at least 100 to 1000 times of shear stress due to laminar.

So, laminar contribution is at least 100 to 1000 times less than the shear stress due to turbulence. Now, we said there are several layers in the, we have gone into lot of much detail in our laminar and turbulent. But for repetition, I would like to go once again very briefly, with this and try to solve some problem, as well. So, what is viscous sublayer? So, here, in this viscous sub layer, which is very near to the wall, the viscous shear stress dominates.

Therefore, viscosity is dominant and density is unimportant, as we have seen before as well. Outer layer, here Reynolds stress is dominant and viscosity is unimportant and therefore density is dominant.

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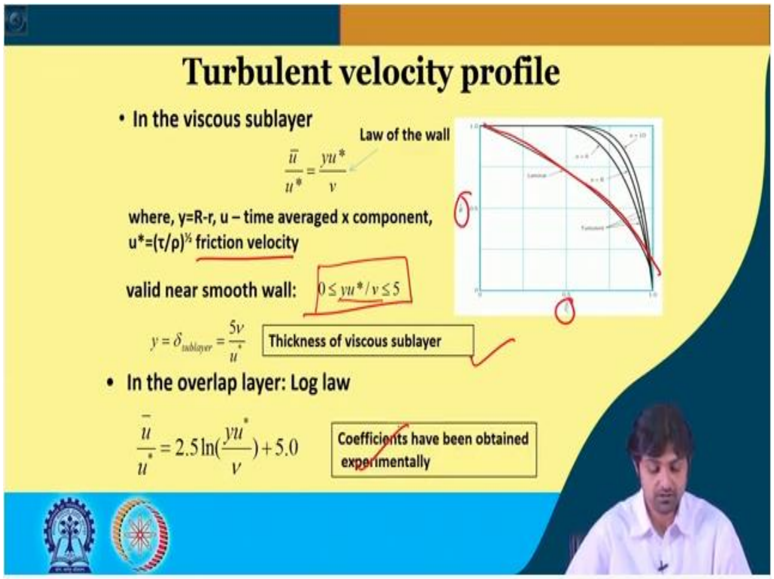

So, in the viscous sub layer, we can write, u bar/u star = yu star/nu. This is called the law of the wall, where y = R - small r, u is time average x component and u star is shear stress/rho to the power half also called as friction velocity. So, there is a curve figure, here and using this figure, this is u bar by centerline velocity and r/R is given the different, for different, you know, this laminar flow curve is given here and there is n = 8, n= 6.

What that n is, we will come. So, very valid, I mean, this viscous sublayer is valid very near the smooth wall. So, this is valid. So, this viscous, this law of the wall is valid, which is very near to the wall. The value of which is given by yu star/nu <5. So, the thickness of the laminar sublayer can be written as, 5 nu/u star. This is important to remember. I think you must remember this value.

In the overlap layer, you see the overlap layer. You remember this overlap layer? This layer, this log law is valid by u bar/u star is equals to 2.5 ln yu star/nu + 5.0. I think we have gone through it in our other lecture. Here, the coefficients of 2.5 and 5.0 have been obtained experimentally.

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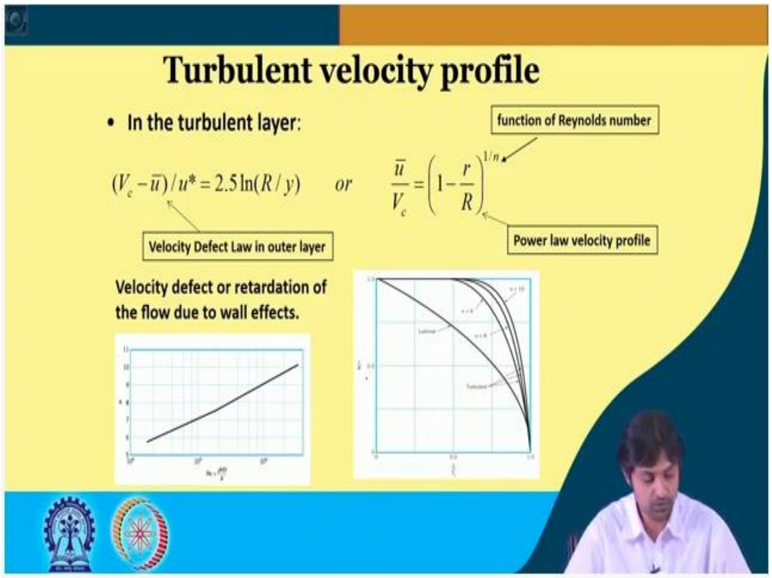

In the turbulent layer, we can also write it in form of centerline velocity, the graph of which was given here. Vc - u bar, this is called velocity defect law in outer layer is given as, 2.5 ln R/y or u bar/Vc is equal to, this was the equation, for which the curve was given. What is going to be the value of n, can be determined from this graph, with the corresponding values of, you know, u/Vc and r/R.

So, and this equation is called power law velocity profile and this n is a function of Reynolds number. So, velocity defect or retardation of flow due to wall effects, so this is the curve, where using which n can be determined. And if you determine n, from this graph then for corresponding n, you can use this table for finding, sorry, this graph for finding the relationship between u/Vc and r/R.

So, in our question, probably we will be using this thing, this particular concept. So, you, I mean, this is again a repetition, because we have gone through with this in the turbulent layer. But the problem that we are going to solve is completely new and will give you much more idea how to solve for this in the pipe flow.

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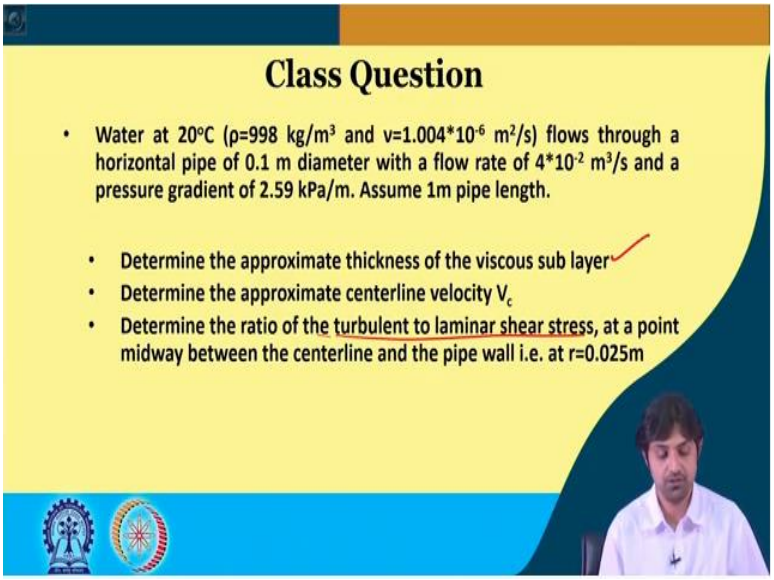

So, the class question is that, water at 20 degree centigrade, the nu and rho is given, flows through a horizontal pipe of 0.1 meter diameter with a flow rate of 4 into 10 to the power -2 meter cube per second and a pressure gradient of 2.59 kilopascals per meter. If we assume 1 meter pipe length, determine the approximate thickness of viscous sub layer, formula we know, determine the approximate centerline velocity Vc and determine the ratio of turbulent to laminar shear stress, at a point midway between the centerline and the pipe wall.

See, if we the thickness of viscous sub layer, we will also find the centerline velocity and for finding centerline velocity, you see, we had this curves here. We can find n here first and then using n we can find the centerline velocity. And the best part is that we will be able to appreciate by finding out the ratio of the turbulent to laminar shear stress, at some point. So, we will solve this question in this lecture.

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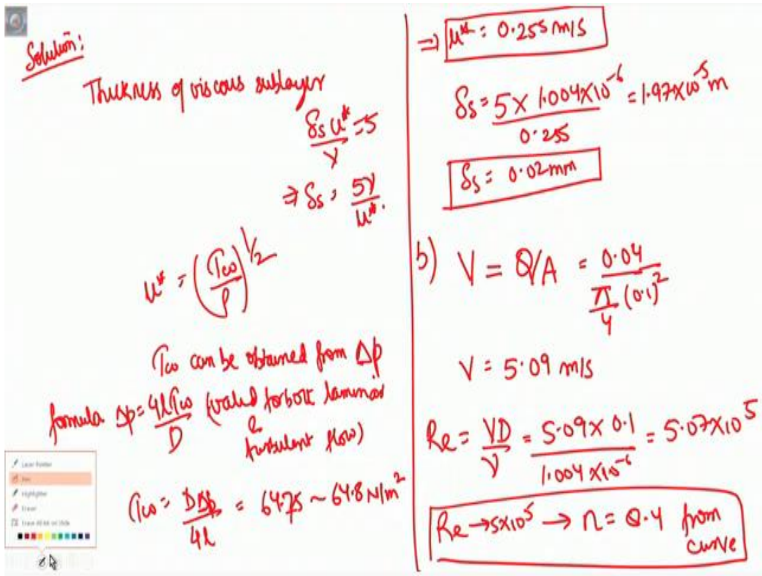

And we will finish this particular lecture after the solution. So, thickness of viscous sublayer is going to be delta s u star/nu = 5, so delta s 5 nu/u star. So, we do not know u star, but u star can be written as tau w/rho to the power half. And how do we obtain the wall shear stress tau w? So, tau w can be obtained from pressure drop formula of delta p is equal to, you remember, 4l tau w/D that we derived.

It is valid because it is valid for both laminar and turbulent flow. So, tau w would be D delta p/4l and after substituting the value, we are going to get 64.75 or 64.8 Newton per meter square. Therefore, we can get u star as 0.255 meters per second and therefore, we can get thickness of viscous sublayer as 5 into, nu was 1.004, into 10 to the power -6/0.255 and that gives us 1.97 into 10 to the power -5 meter or delta s is 0.02 millimeters.

So, you see how thin this viscous sub layer is. Now, second, we have to obtain the centerline velocity. So, we know that the centerline velocity can be obtained from the average velocity, using that power law velocity profile. So, first of all we find average velocity as V = Q/A, Q was 0.04 pi/4 into 0.1 square. So, velocity we are getting 5.09 meters per second.

Corresponding Reynolds number will be VD/nu.Values we already know, V is 5.09, diameter is 0.1 and it is 1.004 into 10 to the power -6. So, it comes to be 5.07 into 10 to the power 5. So, with this Reynold number, using Reynold number of this 5 into 10 to the power 5, corresponds to the value of n = 8.4 from the curve. This is important to note it down. So, we look at Reynold number and we look at the curve.

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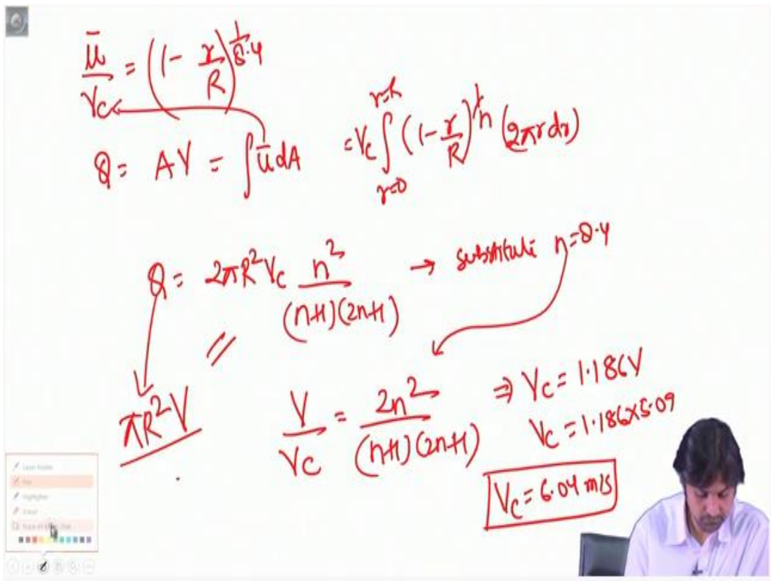

So, we can use this formula, u bar/Vc = 1 – r/R, the centerline velocity formula, divided by1.84 and we also know that Q = A into V or integral udA. So, we use u bar from here. So, Vc will come out and r = 0 to r = R 1-r/R to the power 1 – r and integral 2 pi rdr, we integrate, and after integration we will see, we will get 2piR square Vc into n square/n + 1 into 2n + 1. Now, we substitute, n = 8.4 and this Q is also equal to 2 pi R square. Sorry, Q is not 2 pi R square.

This will be pi R square into the velocity that we had obtained last time. That was 5.09 the average velocity. Because this is also equal to Q. So, now, if we equate both of those, what we are going to get? V/Vc = 2n square/n + 1 into 2n + 1 and we substitute the value here. Therefore, Vc will be 1.186 V and V already we know 5.09. So, Vc is going to be 6.04 meters per second. So, that is part b.

If, you remember, for laminar flow this Vc was 2 times the average velocity. But in turbulent flow Vc is not 2 times because V was 5.09 and Vc is not even 1.2 times also Vc. So, but the centerline velocity is lesser than what it should be in the laminar flow.

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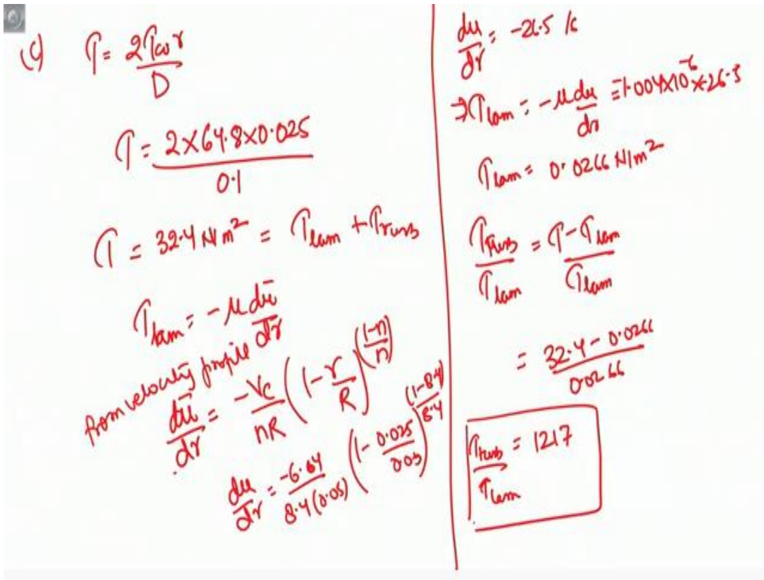

Now, the c part, we know, we are calculating shear stress, it is 2 tau w r/D. So, tau is 2 into, tau w we have already calculated as 64.8, r is, we have been given. We have to find this distance r, at what distance small r is given. So, this is coming out to be tau is 32.4 Newton per meter square. And this is sum of both laminar plus turbulent. We also know that tau laminar is given as, minus Mu du bar dr, from the velocity profile law.

And then, from the velocity profile law, if we try to find out du bar du/dr, it is going to be -Vc/nR into 1 – r/R, in velocity profile, we can write. So, we will calculate du dr, so minus it is, we already calculated and n was, we already found out from the curve, 0.05 into 1-0.025 already given, radius was 0.05 into 1 - 8.4/8.4. So, du/dr is coming to be -26.5 per second. Therefore, tau laminar will be -Mu du dr and that comes out to be -Mu is 1.004 into 10 to the power -6 into 26.5.

So, tau laminar is coming out to be 0.0266 Newton per meter square. So, ratio of tau turb to tau laminar will be tau - tau laminar/tau laminar. So, 32.4 - 0.0266/0.0266 and this ratio comes out to be 1217 almost. So, you see, the ratio of the turbulent shear stress to laminar shear stress is of the order of thousands, as they have claimed before.

So, I think, this is a fine place to, you know, stop this lecture and we will continue with our further topics in our next class. Thank you so much for listening.