Hydraulic Engineering
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Lecture - 40
Pipe Flow (Contd.,)

Welcome back student to yet another lecture of the pipe flow.

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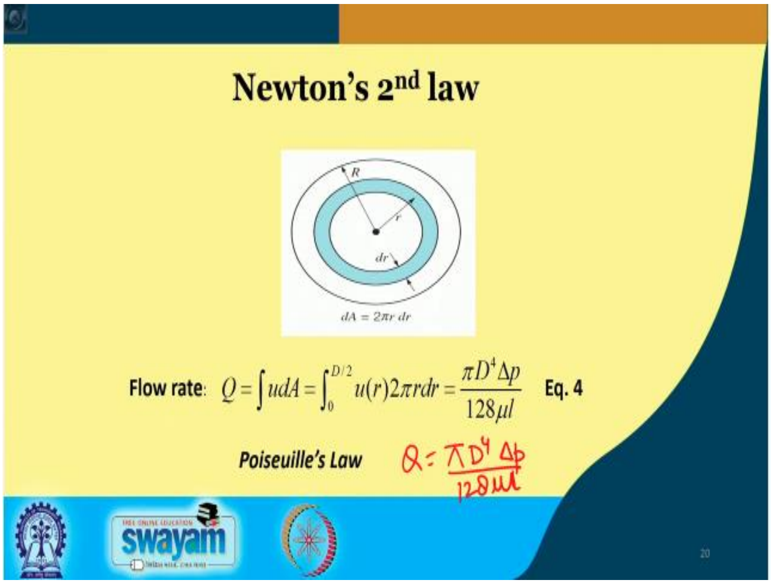

In the last lecture we derived the Poiseuille's law. The Poiseuille's law is $Q = \frac{\pi D^4 \Delta p}{128 \mu l}$ Q, where l is length of the pipe and delta p is the pressure drop. So, we have been able to relate the pressure drop with the discharge. So, proceeding forward,

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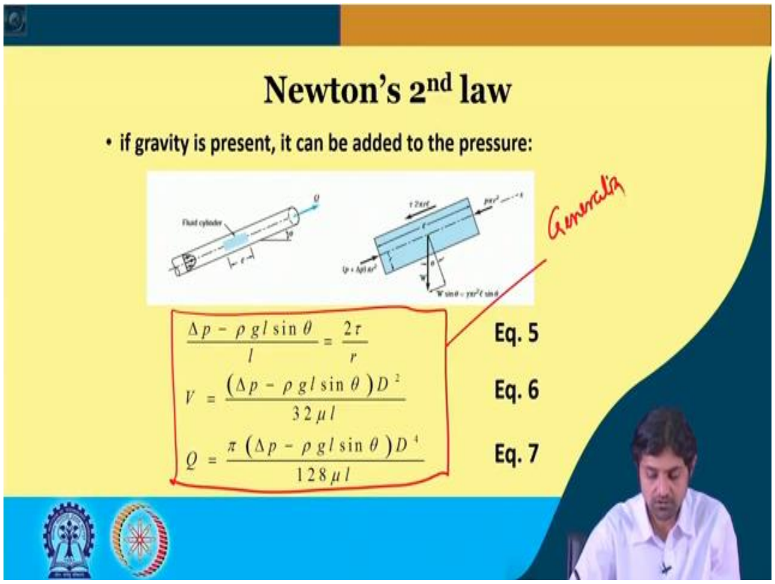

We will now see what if the gravity was also present and that if it can be added to the pressure. So, it is, earlier our pipe was horizontal, there was no gravity. But right now, the equations of motion will change. Because then there will be another, you know, this w sin theta, that is, pi r square l sin theta w sin theta, this is not in multiplied by ϒ, ϒ pi r square l sin theta will be another component that will be acting in this direction.

Because of which our analysis will change a bit but the what happens is, instead of delta p, we are going to get components like, minus ρ gl sin theta, in the equation number 1. When we are going to calculate the velocity, minus ρ gl sin theta here and the Poiseuille's law will change, where minus ρ gl sin theta. If, you want to verify, see, this is valid for all the angles. So, if you put theta =0, you see, this term will vanish, this term will vanish and this term, because sin 0 = 0.

It will be same as what we have derived before. So, you can also try to remember these equations, at least Poiseuille's law you remember, that is quite important, with or without the, I mean, the change in angle, you know, if the gravity is there or not with the sloping pipe. So, these are the generalized equation.

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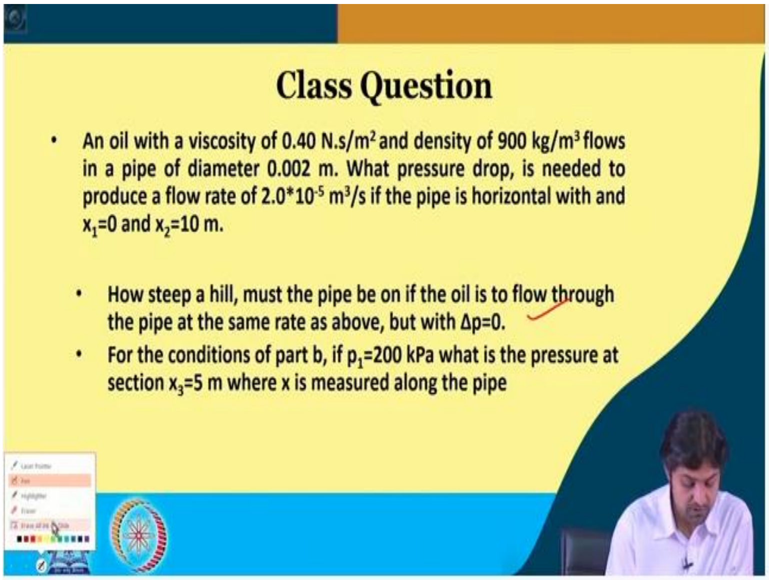

So, now, we are going to solve a question, which says that an oil with viscosity of 0.4 Newton second per meter square and density of 900 kilogram per meter cube, flows in a pipe of diameter 0.02 meter. What pressure drop, is needed to produce a flow rate of 2 into 10 to the power -5 meter cube per second if the pipe is horizontal with x1 = 0 and x2 = 10 meter? That is the first question.

Another question is, how steep a hill must the pipe be on if the oil is to flow through the pipe at the same rate as above with delta p = 0. And the third, for the conditions of part b, this one, if p1 = 200 kilopascal what is the pressure at section x3 = 5 meter, where x is measured along the pipe. So, this is the question that involves the use of Poiseuille’s law and also involves the last equations that we have seen, what happens if the gravity was present. So, to solve this, we will solve this in a white screen.

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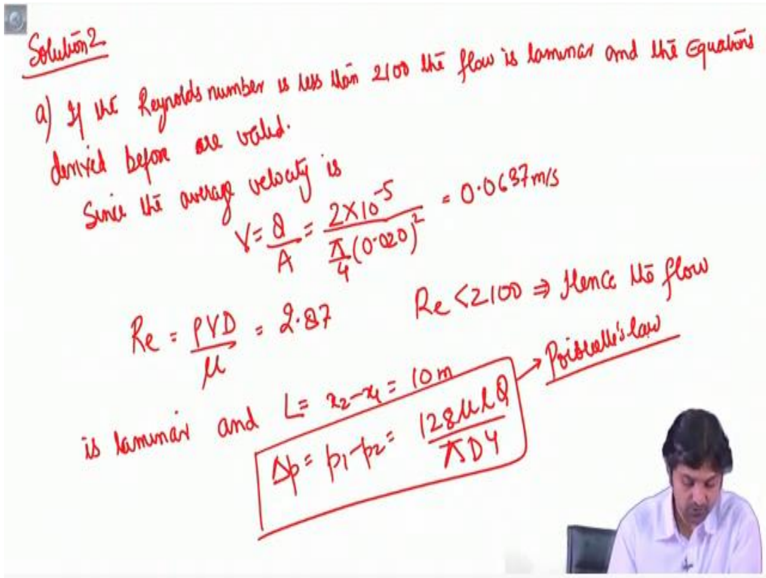

The solution 2, the first one. So, if the Reynolds number is less than 2100 the flow is laminar and the equations derived before are valid. Since, the average velocity is V = Q/A. So, Q is 2, into 10 to the power -5 and area is pi/4, into 0.020 whole square and this comes to be 0.0637 meters per second. And how do we write the Reynolds number? P VD/Mu and on putting this you will get Reynolds number, this velocity.

We are going to get, Reynolds number of 2.87. This Reynolds number is less than 2100 which implies, hence, the flow is laminar and L = x2 - x1 = 10 meter. So, first we found out the velocity. Checked that using the Reynold number, if it is laminar or not. Yes, it was laminar and the pressure drop was p1 - p2 = 128, using the Poiseuille’s law. So, we are now going to substitute that.

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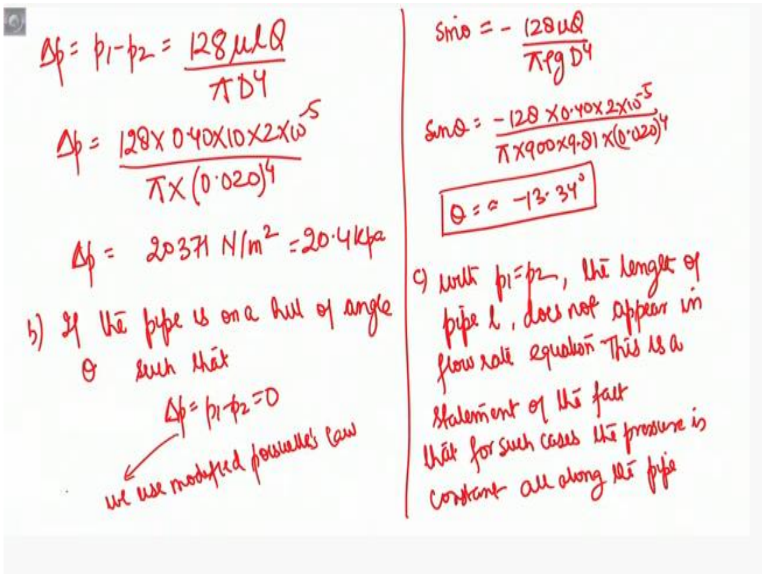

So, delta p, I will rewrite the equation, is equal to p1 - p2 = 128 Mu lQ / pi D to the power 4. So, delta p is going to be 128 Mu is 0.40, into 10 into 2 into 10 to the power - 5/pi into 10 to the power 4. So, pressure drop is going to be 20371 Newton per meter square or 20.4 kilopascal. This is the pressure drop that we found out using the Poiseuille's law. The second part says, if the pipe is on a hill of angle theta, such that, the pressure drop p1 - p2 = 0.

So, for this, we use modified Poiseuille's law and we can write, because delta p = 0, so we can use sin theta will be -128 Mu Q/pi ρ g into D to the power 4. So, sin theta is going to be -128 this is 0.40, into 2 into 10 to the power -5 into pi into 900 into 9.81 into 0.020 to the power 4 and theta is going to be approximately -13.34 degrees. So, in part a, we use simple Poiseuille's law. In the second one, we use the modified Poiseuille's law.

Now, the third part says that with p1 = p2, the length of pipe l does not appear in the flow rate equation. So, this is a statement of the fact that for such cases, the pressure is all along the pipe. See, for part, b we said there was no pressure drop, that is correct. So, that means the pressure is constant all along the pipe.

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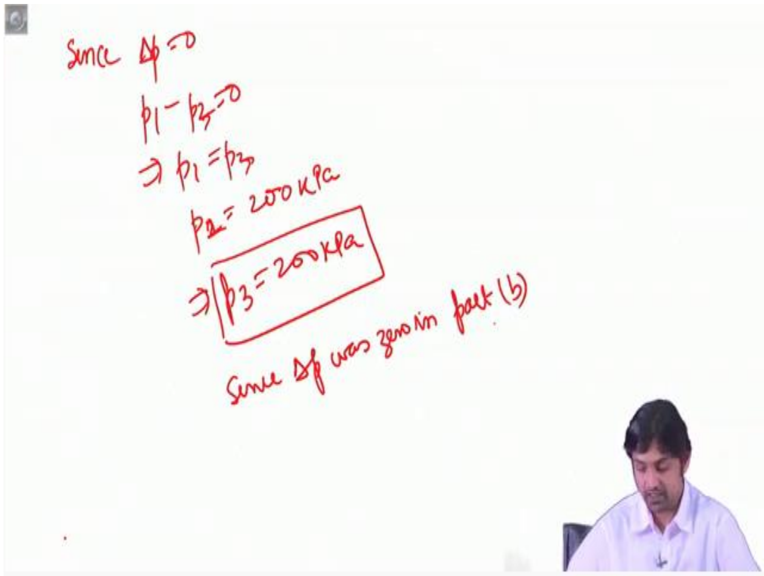

Since, delta p = 0, p1 - p2 = 0. So, p1 is equal to p2. And p2 is equal to how much? Sorry, p1 = 200 kilopascal, sorry, or yes, p or 3, sorry. Therefore, p3 is also going to be 200 kilopascal, since delta p was 0, in part b, very simple, because there was no pressure drop per unit length. So, the pressure would be the same as, what is at 0. So, we finish this one here. So, in the previous question, we have seen the application of Poiseuille's law and also the modified Poiseuille's law, in presence of the gravity.

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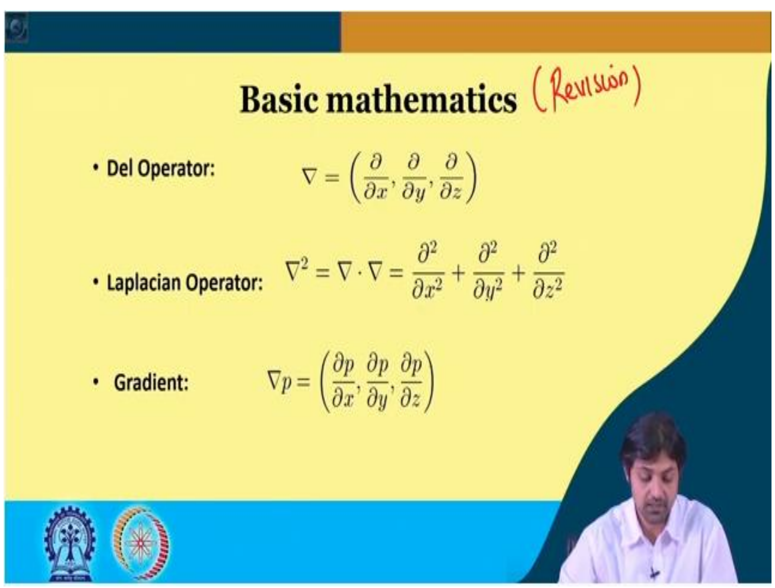

With this point, we move forward to our second way of finding the derived, for finding the laminar flow in the pipes, for fully developed flow. But before that we should revise some basic mathematics, for revision and these concerns our mathematical operator. For example, there is an operator called del operator, which every one of you know, it is

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

. Now, the Laplacian operator is

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

. Gradient operator, it is

$$\nabla p = \left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right)$$

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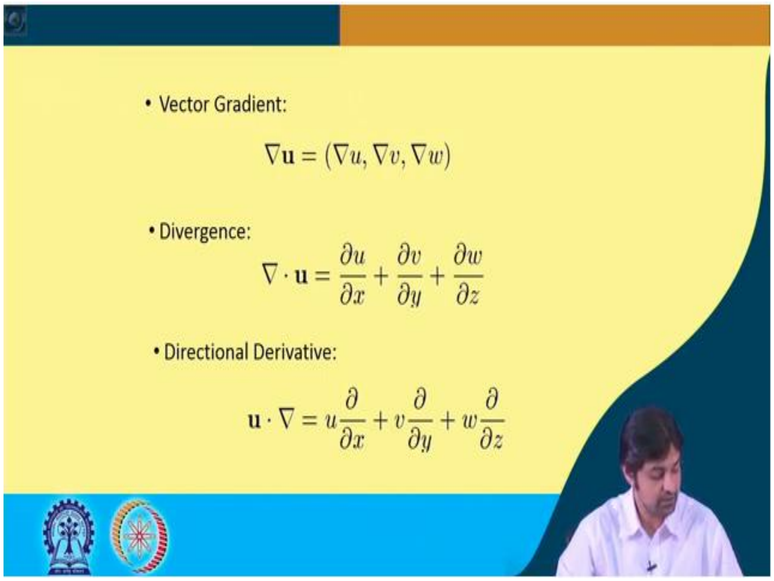

There is a vector gradient, that is,

$$\nabla \mathbf{u} = (\nabla u, \nabla v, \nabla w)$$

. There is a divergence operator, that is,

$$\nabla \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

. These are all basic mathematical equations that you know from before. There is one directional derivative that

$$\mathbf{u} \cdot \nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

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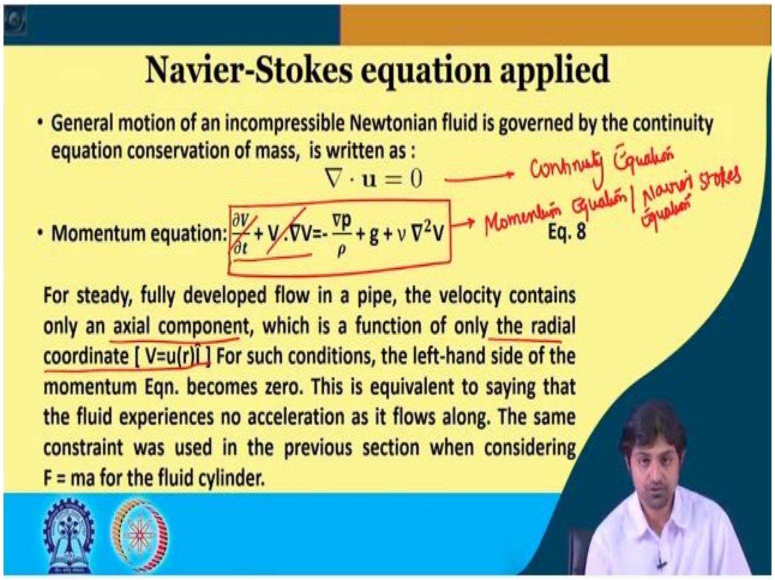

So, with this thing we are proceeding ahead for derivation of the laminar flow through pipes, using the Navier-Stokes equation. So, the general motion of an incompressible Newtonian fluid is governed by the continuity equation and the conservation of mass. This continuity equation is written as, del dot u = 0, this is continuity equation. Whereas, the momentum equation is written as, this or Navier-Stokes equation and this is equation number 8.

So, you should remember that for steady fully developed flow in the pipe the velocity contains only an axial component, which is the function of only the radial coordinates r, that we have talked in the beginning of this chapter itself. So, for such conditions, the left hand side of the momentum equation becomes 0, because it is steady. This is equivalent to saying that the fluid experiences no acceleration as it flows along.

So, convective acceleration and everything is 0. This is acceleration, local acceleration, there is convective acceleration. The same constraint was used in the previous section, when we considered F = ma, for fluid cylinders. So, left side of the Navier-Stokes equation becomes 0, because there is no acceleration in the flow.

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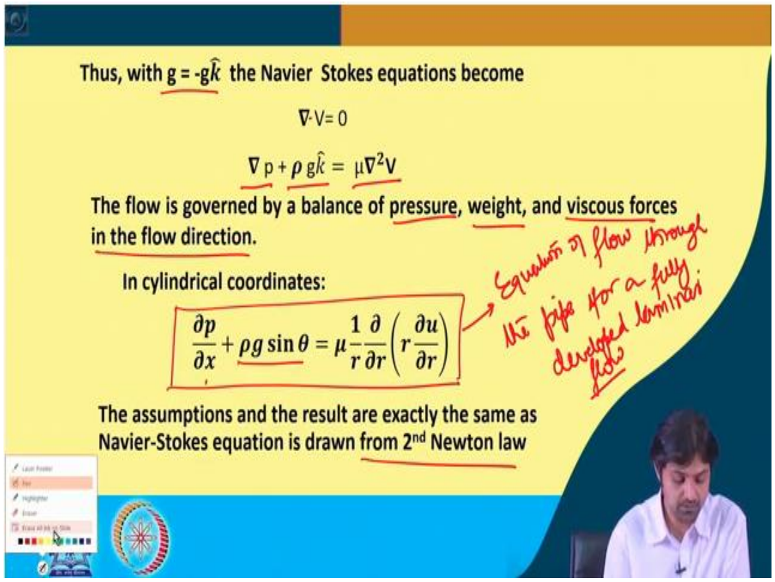

Therefore, if we write, with g as, minus g, in terms of vector, the Navier-Stokes equation, so the continuity equation will be del dot v = 0 and the right hand side can be written as delta p, delta p + ρ gk + Mu del square V. Now, the flow is governed by the balance of pressure, weight and viscous forces in flow direction. In cylindrical coordinates, we can simply write, so del p is del p del x + ρ g sin theta. Whereas, Mu in radial direction 1/r del del r into r del u/del r.

So, this is the using the Navier-Stokes equation, equation of flow through the pipe for a fully developed laminar flow. So, here are the assumptions and the results are exactly same as the Navier-Stokes equation, which is derived from the Newton second law, just written in a more differential form.

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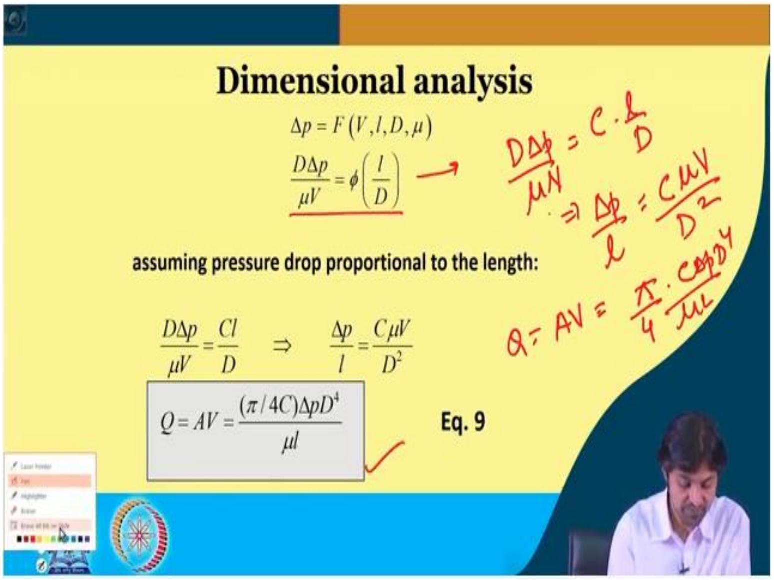

Now, the third way I said was dimensional analysis, from your dimensional analysis module. You remember that pressure is a function of V, l, D, and Mu. So, if we take delta p/l as a function of V, D, Mu or let us take, K is how much, 5, r is, so 1, 2, 3, 4, 5. So, K is 5 and r is 3. So, therefore number of dimensionless terms will be K–r, that is, 2 dimensionless term, so, pi 1 and pi 2. We get this using dimensional analysis.

So, using dimensional analysis we get this result. Now, assuming that the pressure drop is proportional to the length, so we say that the D delta p/Mu V is directly proportional to the length so it becomes C into l/D. So, from here, because you might not be, so D delta p/Mu V is some function of l/D, but we say proportional to the length.

So, we write, constant C into l/D, this equation. Therefore, we can write, delta p/l, l we bring this side, other parameters we take that side, it becomes C Mu V/D square or Q = area into velocity. So, area is pi/4 D square. Therefore, we will get, pi/4 into C delta p D to the power 4/Mu l, as this equation. Now, you see, there is something, some constant called C.

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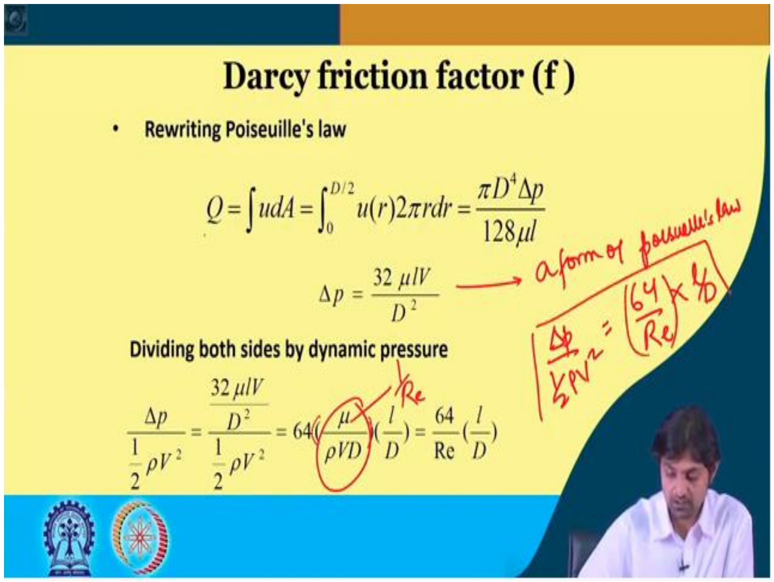

We are going to look into something here that is called Darcy's friction factor. If, we rewrite the Poiseuille's law, Poiseuille's law was pi D to the power 4 delta p/128 Mu l. Therefore, delta p could be written as, 32 Mu l V/D square, in terms of velocity. Now, if we divide both sides by this side, so this is in terms of Q and this is in terms of V. So, this is also a form of Poiseuille's law.

If, you divide both sides by dynamic pressure, like half ρ V square, so delta p/half ρ V square will be 32 Mu l V/D square/half ρ V square. So, this will be, this half becomes, goes up and become 64 and this we can take out, Mu/ρ VD into l/D and this Mu ρ VD/ Mu is Re. So, basically, we can write, delta p/half ρ V square = 64/Re into l/D. So, you see, we would see later, that this 64/Re is termed as Darcy friction factor f. Darcy means this is for the laminar flow.

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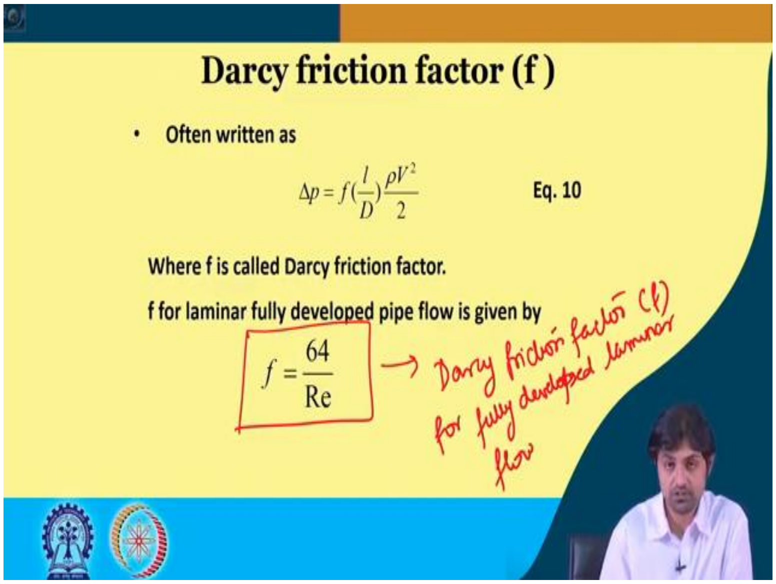

So, this is often written as, delta p = f into l/D ρ V square/2, with this equation number 10 analogy, we say that Darcy friction f for laminar fully developed pipe flow is given by 64/Re and this is Darcy friction factor f for fully developed laminar flow. And this is you must remember this.

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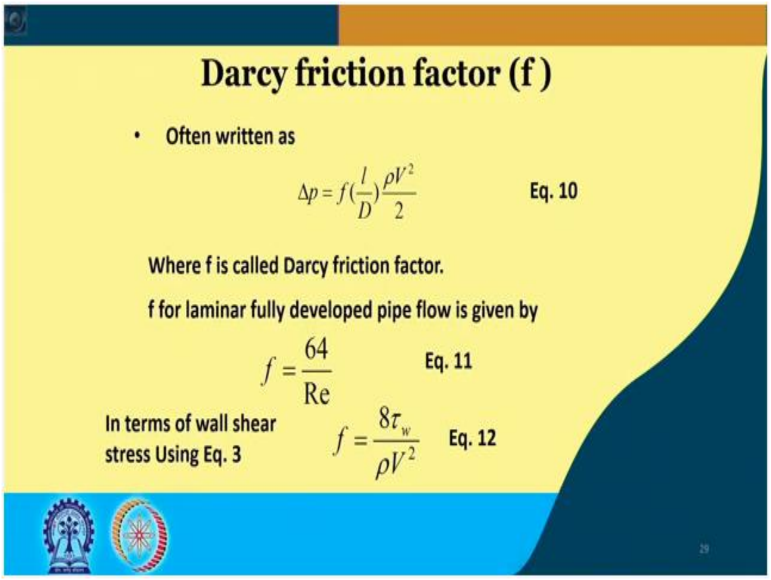

This is equation number 11. So, now in terms of wall shear stress, if we use equation number 3, f can be written as, so equation if we use this f l/D into ρ V square/2. If we if in terms of wall shear stress using equation 3 we will get f = 8 tau w/ρ V square. This is quite simple. Do it as your homework. If you have any problems, you can ask that to me in your, in the forum and this is equation number 12. It is just a matter of equating to obtain this, in terms of wall shear stress.

So, I think this is a nice point to stop this lecture and I will see you in the next lecture where we are going to continue the topic of energy in fully developed laminar flows.

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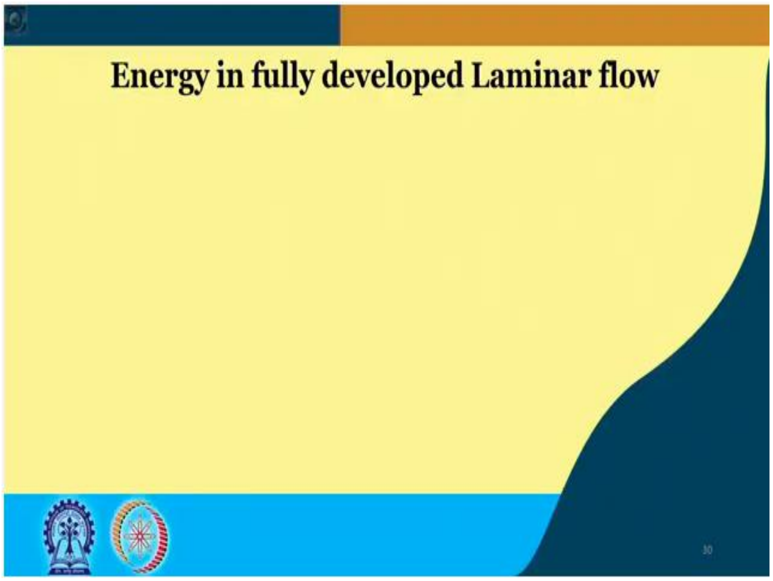

Thank you so much for listening and I will see you in the next class.