Hydraulic Engineering

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Lecture- 04
Basics of Fluid Mechanics- 1 (Contnd.)

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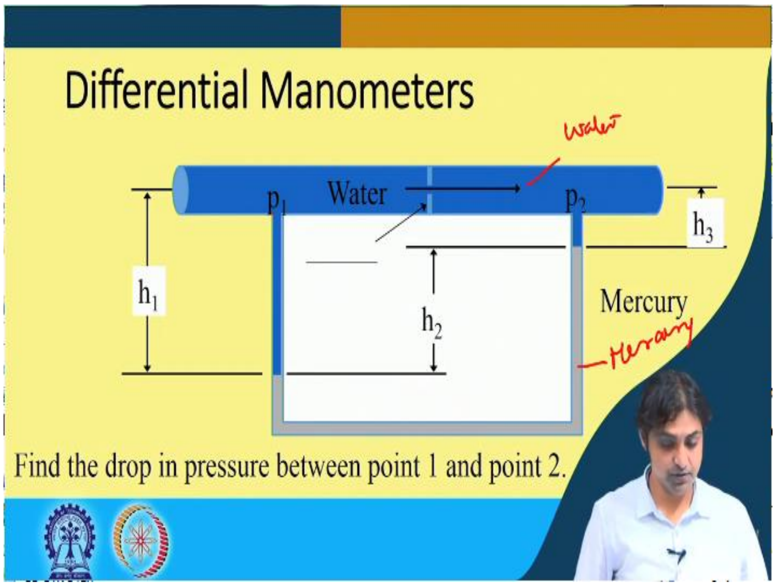

Welcome back to the lecture number 4 of this week. Last week we stopped sorry last lecture we stopped at differential manometers this was the slide that we were going to talk about, we saw some devices that can be used to measure pressures one of them was manometers in which a standard manometer and a differential manometer, so we are actually going to write down an equation or you can call it as a numerical as well, so this is the set up you know the blue one is this liquid here is water. And this is mercury.

So we need to find out the drop in pressure between points 1 and 2. So how are we going to approach this problem, so where if you see this is an orifice.

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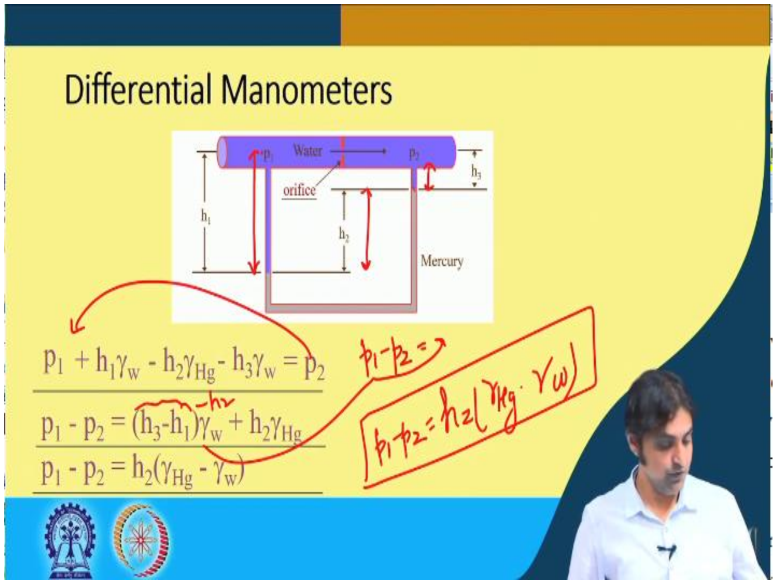

So now we are actually going to keep this figure in a small and just write down start writing down the equations variation with pressure if we go down we are going to add the pressure if we are going up we are going to subtract that pressure. So firstly P 1 this is P 1 here plus h 1 gamma w so what we are we are traversing around this direction okay h 1 gamma w because this is water minus h 2, so if we travels around this direction and reach this point, so, minus h 2 into gamma Hg again we are traversing across the water, so minus h 3 gamma w should be equal to P 2 very very simple equation. Correct? So we can write we can $p_1 - p_2 = h_2(\gamma_{Hg} - g_w)$. Right?

So this can be written as minus h 2 this term here. So it will be h 2 can be taken out and we can write P 1 - P 2 is equal to h 2 gamma Hg - gamma-water and this is the pressure difference. Very very simple question on differential manometers, and this is how it works. Okay? Nice. So, this concludes my first part where the pressure variation with location was their pressure at a point.

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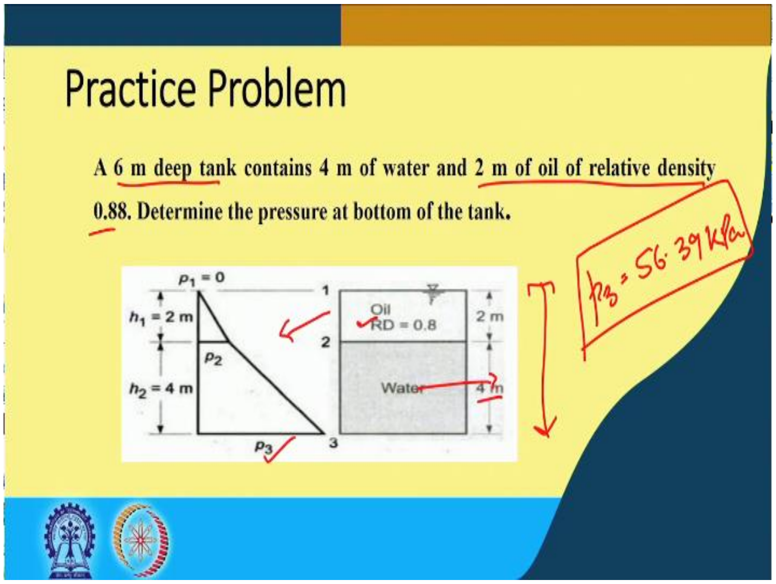

So we are going to solve some questions also, and this is quite important. So, what we say we have a 6 meter deep tank, so this is 6 meter, right? And contains 4 meters of water, so this is as it is written here very clear 4 meter and 2 meter of oil of relative density 0.88. So we have an oil the relative density is 0.88, we have to determine the pressure at the bottom of the tank. Okay? So, for your convenience we have drawn the pressure variation here on this side and this.Okay?

So how are we going to this is a question that generally people do it for before in you know engineering starts or you must also have done it in a fluid mechanics class. So now we are going to approach this and as always what I am going to do is I am going to use a white screen.

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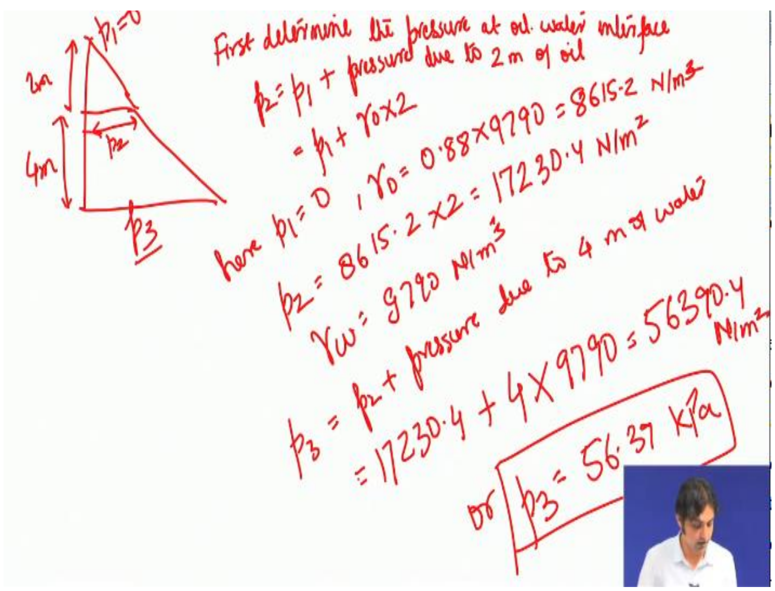

So we have already seen this equation so I will just draw the pressure prism I cannot should not call it pressure prism but yes so this is p 2, right? The entire thing here this is 2 meters, okay, so that, you remember pressure at atmosphere is 0 this is 4 meters, okay, and this is p3 we need to find this correct. So these are all the information that we have. So, first see steps are very important so you must be noting it down.

First, determine the pressure while water interface, that is, p2, so p2 is written as p1 plus pressure due to 2 meter of oil, very nice. So or p1 plus what is the pressure due to 2 meters of oil, gamma 0 into 2. Correct? Here p1 is equal to 0, right? Whereas gamma not which is the pressure of the oil it is 0.88 specific gravity into 9790 that is 8615.2 Newton per meter square and a Newton per meter cube sorry because this here.

So p2 can be written as 8615.2 into 2 that gives us 17230.4 Newton per meter square. So, you do not have to be very specific about 9790 you can also assume 9.8 so 9800 it is just what I have assumed, okay. So, that is the first step now we must also determine P 3, right. gamma w is we have assumed 9790 Newton per meter cube, okay. So p3 will be p2 + pressure due to 4 meters of water, right.

So p2 we already know, correct? So, p2 value, p2 is what we have found out was 17230.4 + pressure due to 4 meters of water is going to be 4 into 9790, okay, that gives us 56390.4 Newton per meter square, or p3 can be written as 56.39 kilo Pascal, okay. So, this was what the question had asked to calculate p2 and p3. If we just go back, you know, so determine the pressure at the bottom of the tank so bottom of the tank was p3 and that is exactly what we have found out. So, the answer was P 3 is equal to 56.39 kilo Pascal, okay. So maybe, so I have one another problem for you.

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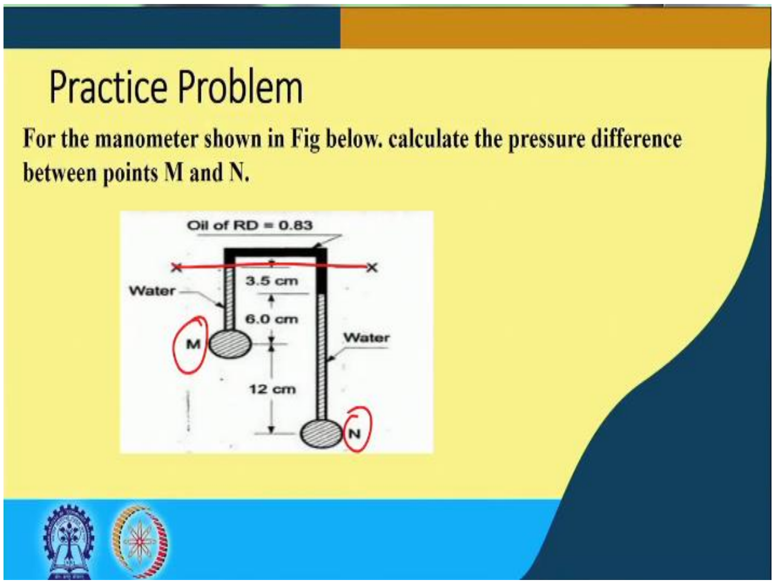

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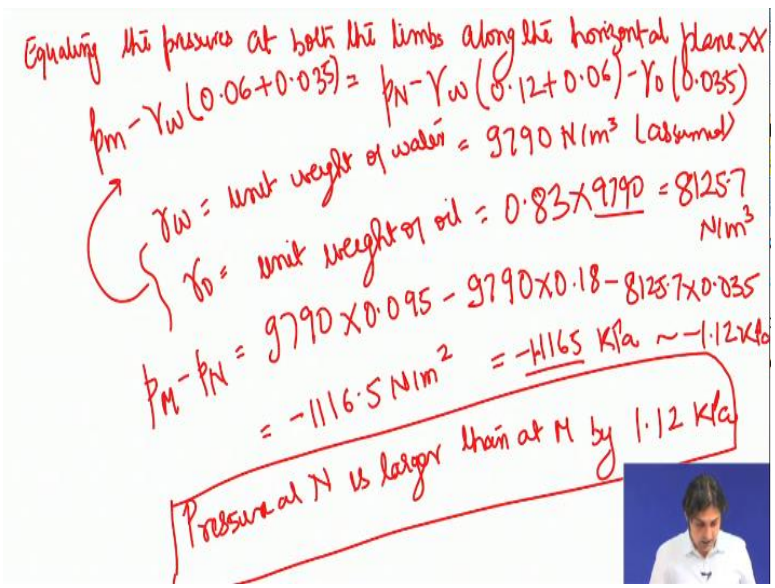

Before we go to the next concept, and that is, we have shown a manometer here in this figure. We have to calculate the pressure difference between points, M and N, this is point M and this is point N. The best way as I told you before, if you want to calculate the pressure difference or pressure at that point you have to start at one point where the pressure is known and traverse to the other point where you have to calculate the pressure.

If you go up you have to subtract that pressure if you go down you have to add that pressure. So, that is the one of the ways. Here, what we are going to do, we are going to equate the pressure at this level. okay. So, I think you must be drawing this figure, because I will not be redrawing it now. So but what I am going to do is, I am going to use that another white sheet to be able to solve this problem. Good. So, as I already told you we have to equate.

So, equating the pressures at both the limps as I told you, while discussing the figure along the horizontal plane, and what is that plan, xx. So, pm - gamma w 0.06 + 0.035 will give us pN – gamma w 0.12 + 0.06 - gamma not into 0.035. So here, gamma w is unit weight of water, which is equal to 9790 Newton per meter cube. And this value has been assumed, okay. This is a standard value. What is gamma not, is unit weight of oil. What is it going to be any guesses? The specific density was 0.83 into 9790, specific gravity when we say it is always our density that is in ratio with water no other fluid, okay.

So, that is why this the density of I mean unit weight of water is what we multiply to the specific gravity to get the unit weight of that oil, so here it comes out to be 8125.7 Newton per meter cube. So if we use these 2 values in equation here what we are going to get pM - pN is equal to 9790 into 0.095 - 9790 into 0.18 – 8125.7 into 0.035 and this is going to give us - 1116.5 Newton per meter square or - 1.1165 kilo Pascal. I mean, we do not have to be very, you know, where we can simply write 1.12 also. - 1.12 kilo Pascal okay.

So, the answer is going to be because it is negative that means pressure at N is larger than at M by 1.12 kilo Pascal, simple. So, the calculation of, you know, the pressure things is not that difficult. If you follow the chain rule, keep writing the pressure while going up and down this is very, very, very, simple, great. So, now we go back and this we have solved this problem now.

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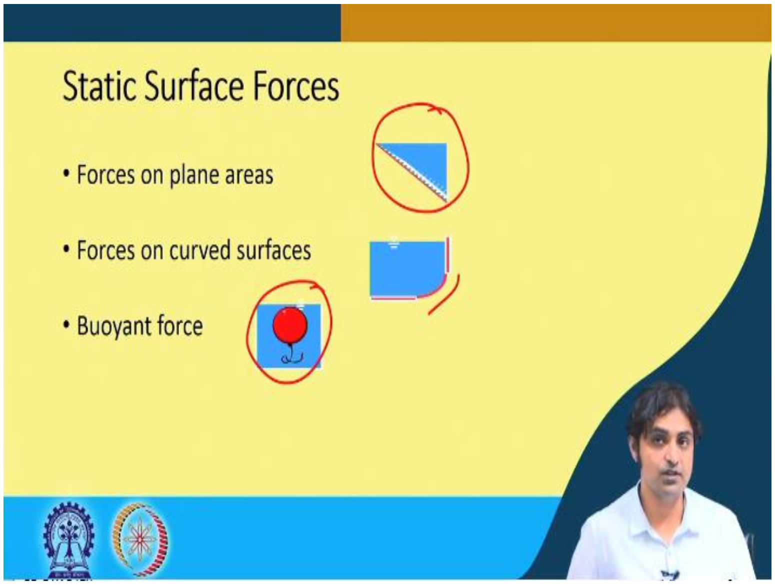

So, this is a fluid statics 2, I mean, I call it 2, because we are going to do the surface forces and body forces therefore we need to know, what are we going to study in fluid statics 2. So, in statics 2 its static surface forces, we are going to see force on plane areas, okay. We are going to see force on curved surfaces, like this. So, this is plane areas, this is plane, this is curved surface and we also will see the buoyant force, a very small detail of it, but I think this is very necessary. So, I mean, if we are teaching basics of fluid mechanics in this course in the beginning, so, buoyancy is an important concept that everybody must be aware of. Good.

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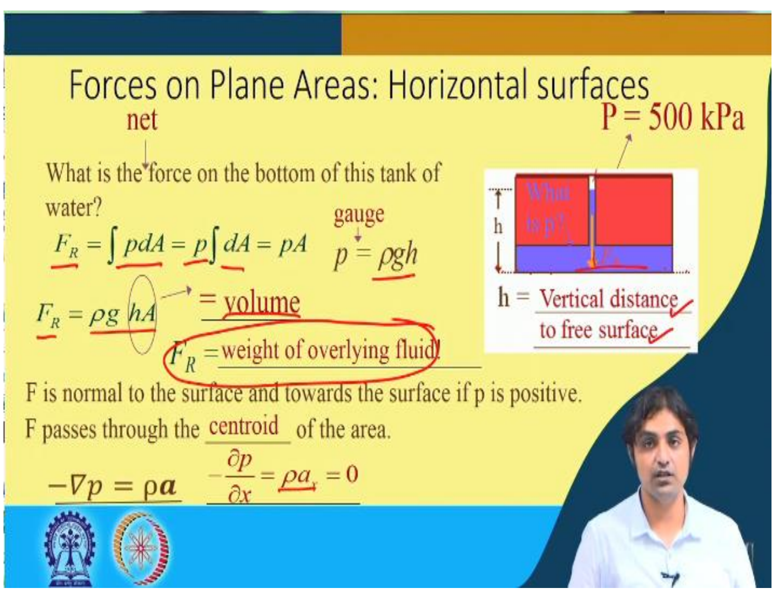

So, we have to see what are the forces on plane areas, that is horizontal surface. So, if you see, this is a figure that shows, you know, a horizontal surface a depth h, okay. So, this h is the vertical distance to free surface and this we what is the P here, okay. And what is the resultant force at the bottom, okay, and P we are assuming 500 kilo Pascal's, okay, that we are going to see. So, what is the force on the bottom of this tank of water actually, what is the net force on the bottom of this tank?

So, the force resultant force is going to be the integration of pressure into area, So, p is constant so it comes out and that becomes pressure into area pA, where p is rho gh, okay, this is the gauge pressure. So, $F_R = \int pdA = p\int dA = pA$

So, F R is actually nothing but the weight of the overlying fluid, okay. Also F is normal to the surface and towards the surface, if p is positive, okay.

F passes through the centroid of the area. This is an important information for you. And therefore, the change in pressure can be equated to rho into a, okay, or we can write in x direction $-\frac{\partial p}{\partial x} = \rho a_x = 0$

So, there is no acceleration in x direction here, and that is equal to 0. There is no pressure variation in x direction whatever there is, it is in z direction. Good.

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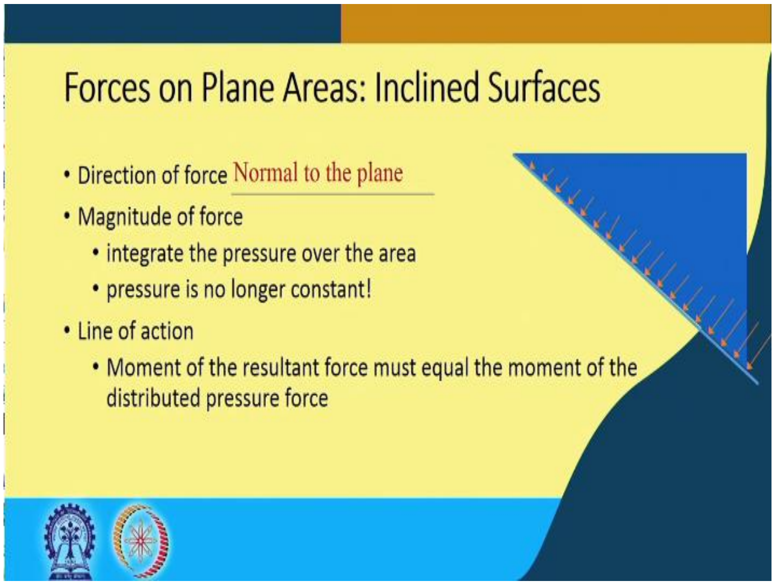

Another important thing is, we have to learn and revise again, what are the forces on the plane areas or the inclined surface. So, this has to be taken in a little bit of more detail. What will be the direction of the force? Always perpendicular, normal to the plane, right. So, the force will start acting like this, correct. What will be the magnitude of the force? We have to integrate the pressure over the entire area. Here, the pressure is no longer constant, because it is not it is not at one elevation it is varying see, the h is changing here, here it is different, here it is different, here it is different.

So, what is the line of the action? So far to find the line of action, we have to do the moment of the resultant force must be equal to the moment of the distributed pressure force. We have to do the moment balance to find the line of action we will see soon how are we going to tackle, that so I will just erase all ink on the slide. Good.

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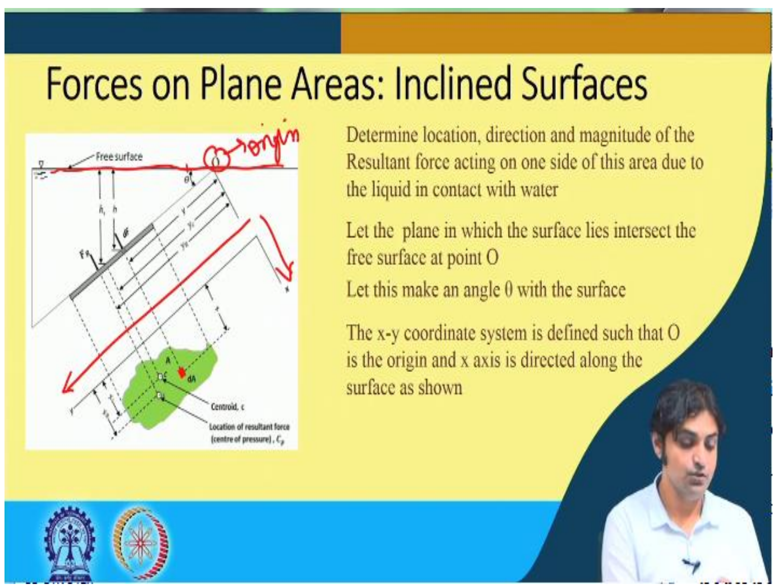

So, forces on plane areas, so this has been taken actually from Munson, Young and Okkiishi the derivation. So, but I think I will explain one by one what those things are, So, the question, the biggest question is you have to determine the location direction and magnitude of the resultant force acting on one side of this area due to the liquid in contact with water. If you see the body, okay. I will just erase because this was just to you know, okay, alright.

What we say, let the plane in which the surface lies, intersect the free surface. So, this is the free surface here, okay, and let the plane in which the surface lies the body intersect at point O, okay, right. Good. And let this make an angle theta with the surface, right. The xy coordinate system is defined such that O is the origin. So, this is also the origin, you know, and the x axis is directed along the surface as shown so this is x axis, okay. Sorry, this is x axis and this is y axis, good. So, this is just explaining you, and we are going to look at the individual terms when we describe those, you see many things here, centroid, location of resultant forces, you see this FR, you see dF, but we will come to it one by one, good.

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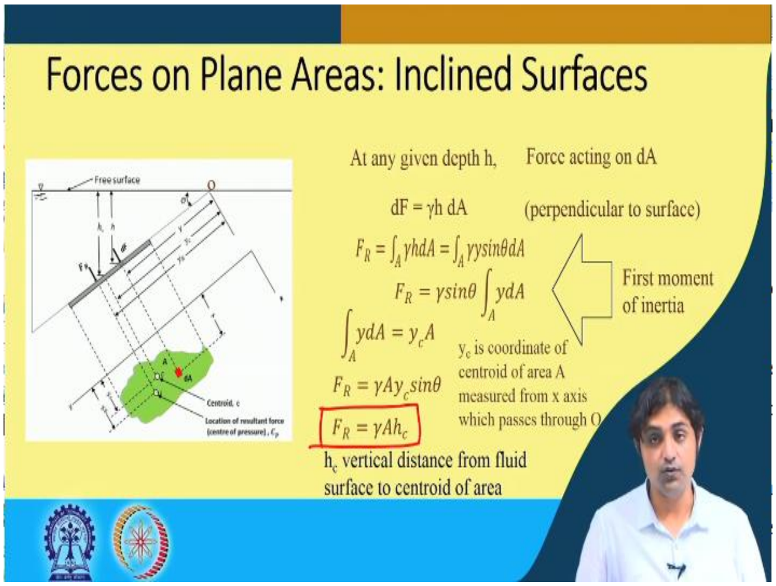

So, what I have done is, I have kept this image on the left side and how we are going to. So, for any depth h, okay. Let we say that the force acting on an area dA will be because it is at depth h, dF will be pressure into area right gamma h dA, this is the dF, this is perpendicular to the surface, that is very important. So, to find out the resultant force what should we do? We do gamma h dA integrated over the entire area, right.

h here, if we start if we try to write down in terms of Y this can be related to theta as h is y sin theta and gamma is gamma and dA is dA, okay. So, F R can be written as gamma sin theta can come out and this is y dA, okay, integral of y dA, very important, if you remember from your earlier classes, this is first moment of inertia. And this integral y dA can be written as y c into A, where y c is the location of the centroid of the object, right. And F R can be written as

$$F_R = \gamma A y_c sin\theta$$

. So now, you know gamma, you know A, you know sin theta and for a particular object you also know y c, the centroid location, right. Where y c is the coordinate of the centroid of the area A measured from x-axis which passes through O. So, orientation you have to be very careful about, okay. I will just erase all the ink now. So, or we can also say gamma A from here and y c sin theta. What is y c sin theta? This is y c, right? y c sin theta can be written as h c, h c is the height of the centroid from the free surface.

So, h c is the vertical distance from fluid surface to the centroid of the area. Now, we have actually simplified into very common, I mean, very simple equation. I will erase, so I will just write the important one, so this is the important one, F R is equal to gamma A h c this is one important result to note down at this point in time. Very good.

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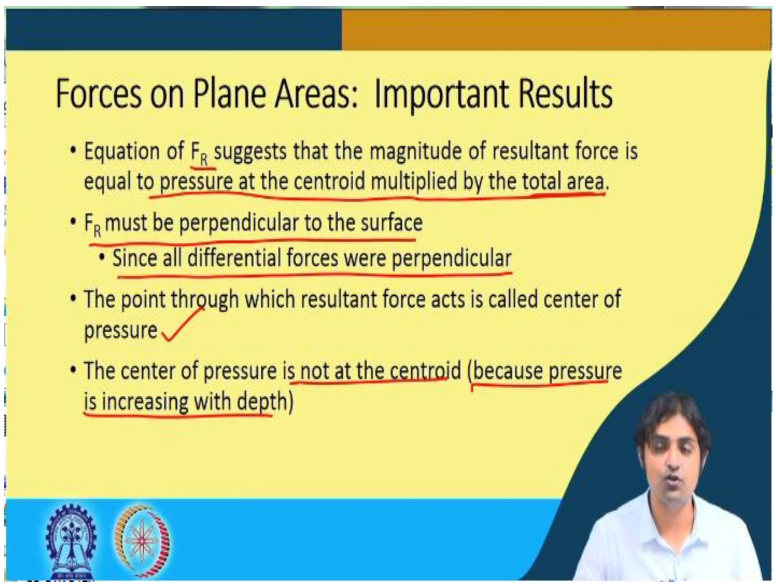

So, we proceed to the next slide, okay. So, this equation of F R suggests, that the magnitude of the resultant force is equal to the pressure at the centroid multiplied by total area. As you have seen here, see, pressure gamma A multiplied by the total area, right. F R also must be perpendicular to the surface that is very important. And the reason is since all the differential forces were perpendicular that we have counted.

If you see, all the forces these were all perpendicular. So, this is let us say dF 1, dF 2, dF 3. So, if you add this the sum can change, but not the direction, right. The point through which the resultant force acts is called the center of pressure. This is you might have heard in your fluid mechanics class what calculate the center of pressure. So, this is what the center of pressure is a quick revision for you again. The center of pressure is not at the centroid. And what is the reason? Because the pressure is increasing with depth, that is important again, another important result to note down, okay. We have seen the resultant force value and we have discussed some of the properties of resultant force. Now, we must also be able to find out what corresponding x R

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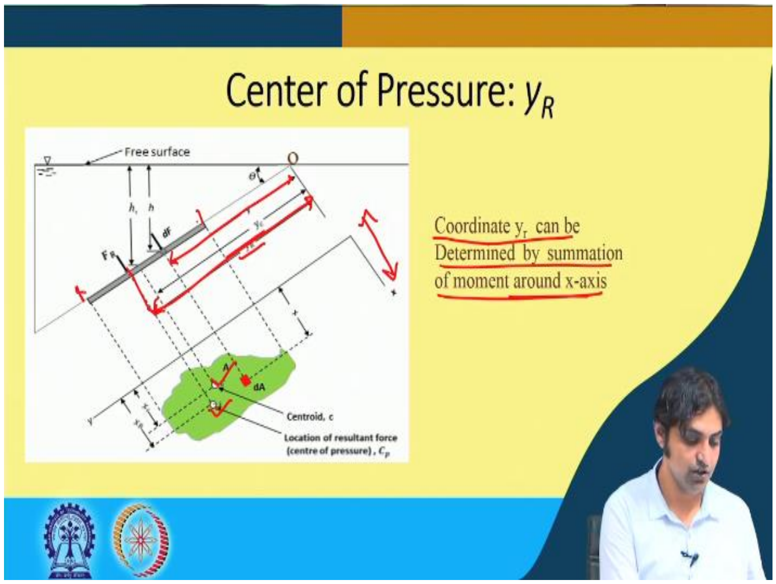

and y R is. So, let us say there is the one of the coordinates in y direction for the center of pressure centroid is here this is center of pressure right is y R, which is not equal to y c, okay. In that particular case what we are going to do is, coordinate y R can be determined by summation of moment around x axis. So, for finding y R the moment equilibrium should be done around x axis.

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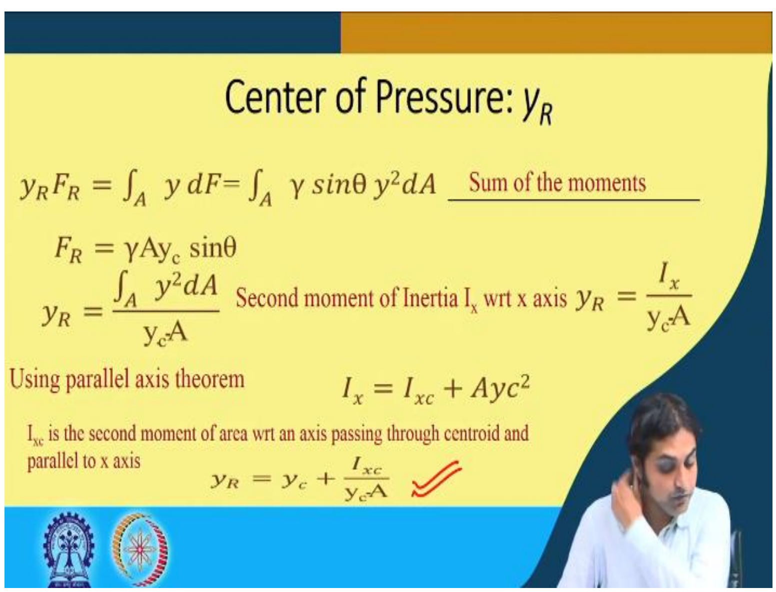

So, the way we do it y R, F R. So, c y R into F R, so this is y r into F R, this will be equal to integral y dF, right. So, what is y dF?. So this is dF and this is y, so all the summation beginning from here, until here, all these small small dF that is there. So, that will be y can be written as, sorry, dF can be written as gamma sin theta y dA, correct. This we have already seen in the previous slide this is the sum of the moment, where F R was gamma Ay c sin theta that we have seen before. So, y R now can be written as, integral y square dA divided by y c into A. I will erase the ink.

So, you can see integral y square dA, okay, divided by y c into A, because gamma sin theta gamma sin theta cancels, right. So, this is actually the second moment of inertia with respect to the x axis, very good. So, we are coming at some conclusions now, where y R can be written as I x second moment of inertia, correct. Now, if we use the parallel axis theorem. So, because see this is around an x axis which is not fully independent of the coordinate center, right system. But if we have a coordinate system passing through the centroid of the object, then we are able to you know, calculate very easily does not matter how do we orient our coordinate system.

So using parallel axis theorem I x can be written as $I_x = I_{xc} + Ayc^2$, very simple, that, this you have read before as well. So, I xc is the second moment of inertia with respect to an axis passing through the centroid and parallel to x axis, therefore, the equation of y R can be written as y c, is the centroid plus I xc divided by y c into A, I xc is known, y c is known and A is known, right.

So, this is an important let me, so, this is the important equation again. So, $y_R = y_c + \frac{I_{xc}}{y_c A}$

. If you remember this equation you will always be able to find out the center of pressure coordinate y R.

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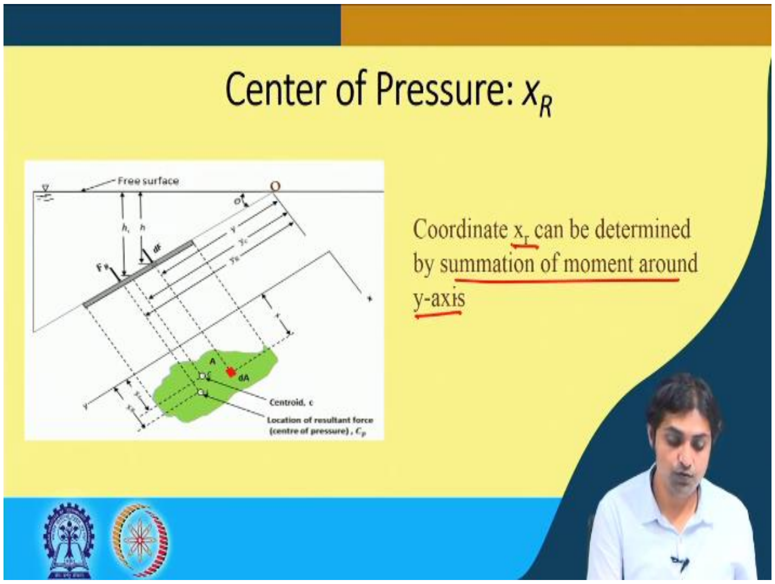

So, we also need to find out X R. What is the logical way? We will do the summation of moment around y axis now. So, for X R we need to do the moment calculation around y axis so around this axis.

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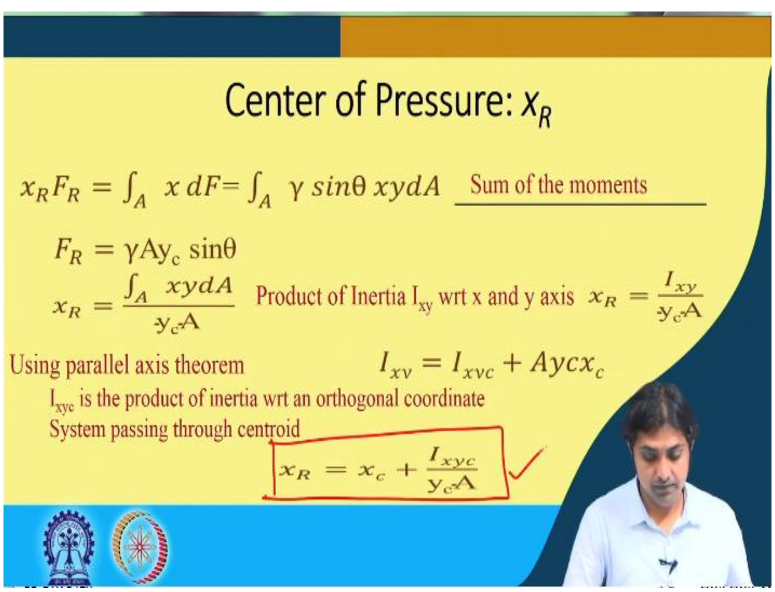

How? See, X R into F R. Where is X R? This is X R. So, X R into F R, okay. Because, yeah, is equal to integral x dF, so dF, we already know. So, it is gamma sin theta y dA, right, and this is some of the moment we know, F R is this one, gamma A yc sin theta. Therefore, x R will come out to be integral x y d A / y c A, and this is the product of inertia I xy with respect to the x and y axis.

Therefore, x R can be written as I x y / y c into A. Again the same thing happens that we need to translate this moment of inertia or product of inertia I x y with respect to the centroid, so that it becomes independent of that particular coordinate system. So, we can write, if this is I x y, okay, I x y c can be written as I x y is equal to I x y at centroid plus A y c x c. If we put this in this equation, here I x y c is the product of inertia with respect to an orthogonal coordinate system passing through the centroid.

And therefore $x_R = x_c + \frac{I_{xyc}}{y_c A}$

. So, this is an important result here, this is good. Another important result for we have obtained y R and x R.

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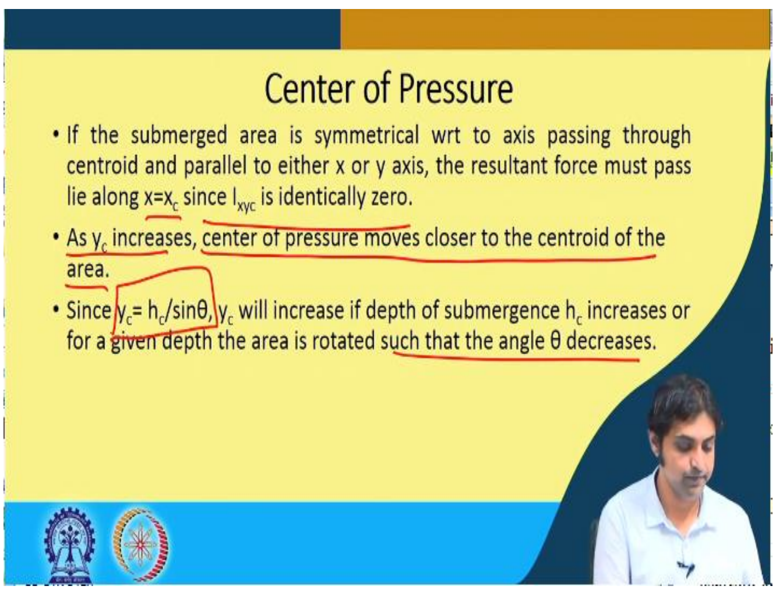

Now, the center of pressure sum. If the submerged area is symmetrical with respect to axis passing through centroid and parallel to either x or y axis the resultant force must pass a lie along the x is equal to x c since I x y is identically 0. See, if the submerged area is symmetrical with respect to axis passing through the centroid, then Ix y is identically 0. So, x r will be 0. Now, as the y c increases the center of the y coordinate of the centroid increases, center of pressure moves closer to the centroid of the area, very, very it is easy to see from the equation that y R was y c +.

You know if yc is too large then does not matter what the other term is. Since, yc is equal to hc by sin theta, y will increase if depth of submergence h c increases or for a given depth the area is rotated such that an angle-theta decreases. This is very obvious from this particular equation.

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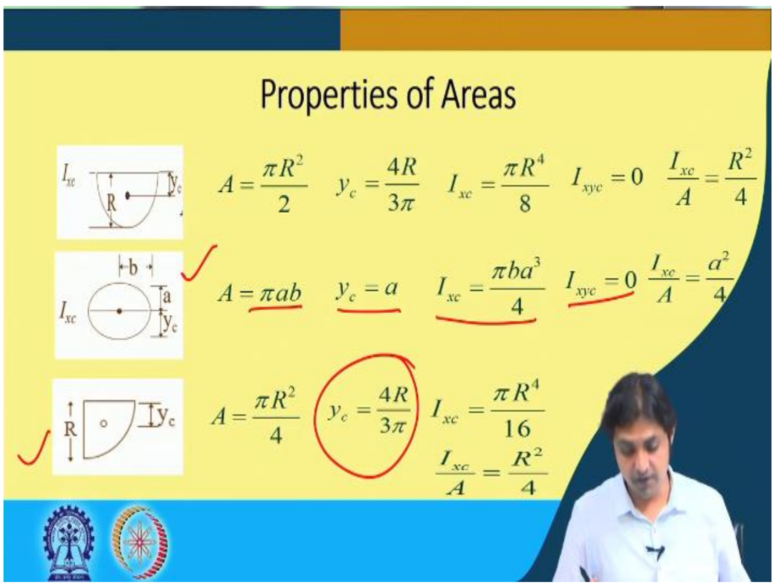

So now, there are some properties of area, which you will be supplied with. This is very easy to know, hand on, you do not need to remember some other with this slide if you had, so area in this particular of rectangle area is a b, depth of the centroid is a by 2, I x c b a cube by 12. So, this is symmetric about the centroid, right. So, I x y c is 0, where I x c by A is a square by 12, I mean you can just simply see. Similarly, for this triangle, isosceles triangle, this is for circle, I think, you can you know, note it down these figures are quite important.

The circle for example, even objects like this because in fluid mechanics in hydraulics you will encounter structures like this having this type of gate, ellipse is also important. So area is Pi ab, y c is a, I x c is Pi b a cube by 4, I x y c is 0, because it is also symmetric around the centroid. Similarly, this here, so this y c is 4R by 3 Pi, this value is important because, we are going to use it because most of the gates that are used in hydraulics have these type of openings we will come and we will see in this type of you know thing.

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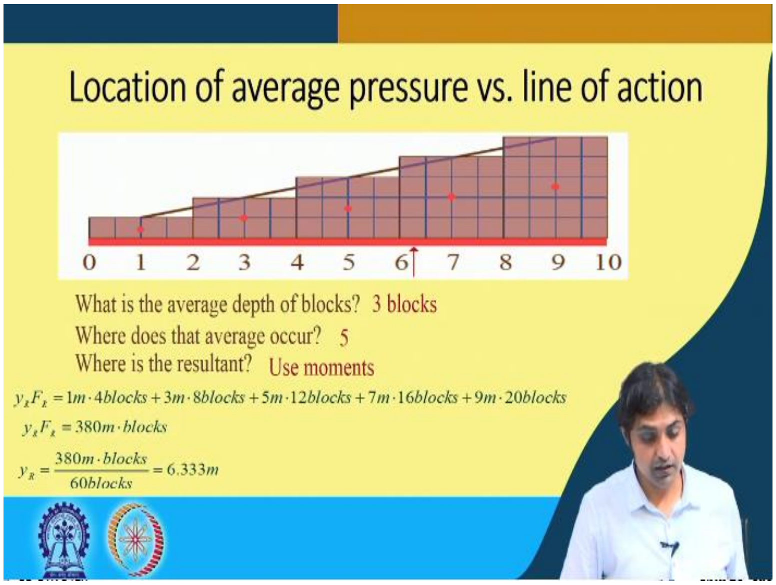

So, this is the probably the last slide of this lecture I would want to each and every one of you, this a very basic question, which I have taken from an English author, location of average pressure versus line of actions. We will see in practice, how this moment and those things are calculated with the help of these blocks. This is quite easy to understand. So, my question is what is the average depth of the blocks? Anyone tell me what is the average depth? So, this is 1, 2, 3, 4, 5.

So, what is the average depth 3, right? Where does this average occur tell me? Where is this average occurring the depth tell me? So, it is happening at, see, at 5. Are you able to get this, right? See, this is, correct, to see that most number of the block number 3 is at 5, you know, 1 2 3 4 5 6 6 8 9 10. So if you take the depth. Now the most important, where is the resultant actually? How are you able to find this resultant? We will use moment balance here.

So, correct? So, what we are going to do? We are going to do y R into F R, resultant force is equal to 1 meter. So, this is 1 meter. How many blocks are there, tell me? 4 blocks plus, see, 1, 2 3, 4, okay? Now, 8 blocks into 3 meters, right, 1 2 3 4 5 6 7 8 into 3 meters. Now we have 12 blocks, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 12 blocks that 1. Similarly, we have 16 blocks we have

20 blocks and if we do this summation so y R F R is equal to 380 meter blocks, okay. And therefore, 380 meter blocks in the total number of blocks is 60 blocks.

So, we divide and we see y R will be 6.333 meters. So, this is quite an intriguing example, where you can practice it on your own. And if you do not understand, we will explain that in the forum if you please, raise your questions there, that is good. So now, I will end this particular lecture and we will proceed to the last lecture of our this week which is continuation of fluid statics where we will see some more examples and how the forces on the surfaces is calculated more, great, see you.