Hydraulic Engineering
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Lecture – 36
Non-Uniform Flow and Hydraulic Jump (Contd.)

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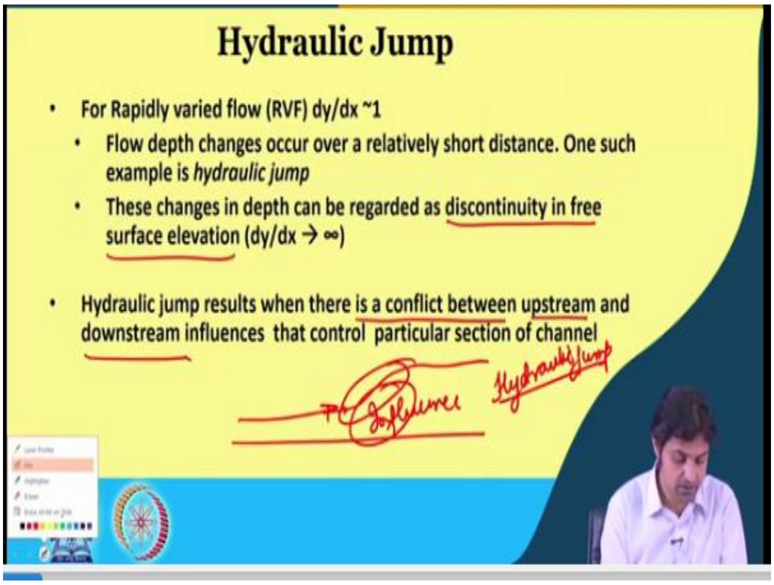

So, welcome back. We just started the topic hydraulic jump, at the end of our last lecture. So, we are going to continue with that hydraulic jump. Hydraulic jump is a type of rapidly varied flow and for rapidly varied flow, the definition is dy by dx is approximately equal to 1. This means, that the rate of change of depth with the stream wise direction is not very small.

So, the flow depth changes occur over a relatively short distance, that is the meaning of dy by dx approximately equal to 1. One such example, as I said before is hydraulic jump. So, these changes in depth can be regarded as discontinuity in free surface elevation. So, dy by dx can go to infinity as well, so if the depth over a small distance if it changes rapidly, it can be said that these are sort of a discontinuity in free surface elevation.

So, hydraulic jump results when there is a conflict between the upstream and the downstream influences that control particular section of the channel, so there is a conflict between the upstream and the downstream influences. So, suppose this is a channel, the flow is coming like this and there is some influence here, because of which there is a discontinuity, for example, in the free surface.

And because the water depth changes very rapidly, this is called hydraulic jump. This hydraulic jump can occur due to many reasons. One of them is sudden elevation bump that can occur or due to several other phenomenon that we are going to see in this lecture.

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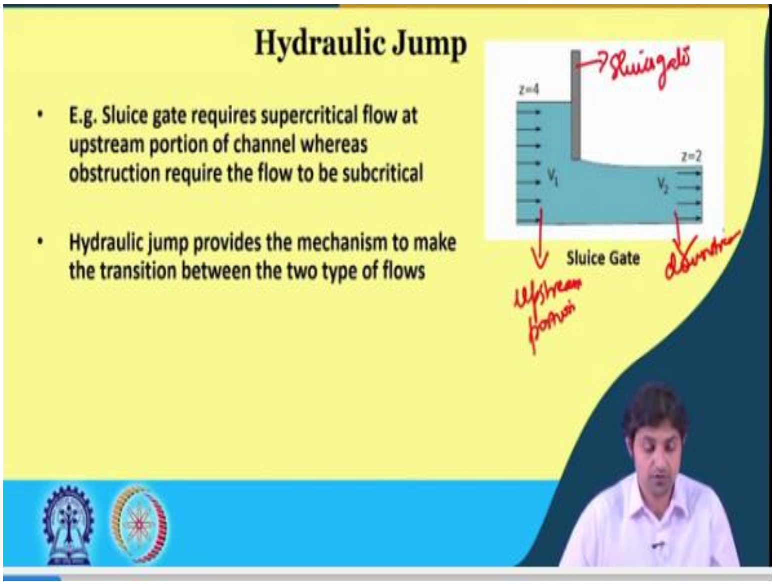

So, one of the examples is a sluice gate, so sluice gate is a gate like this, this requires that the supercritical flow at the upstream portion, downstream portion, so sluice gate requires supercritical flow at the upstream portion of the channel, whereas the obstruction require the flow to be subcritical. Though this phenomenon called hydraulic jump provides the mechanism to make this transition between the 2 types of flows.

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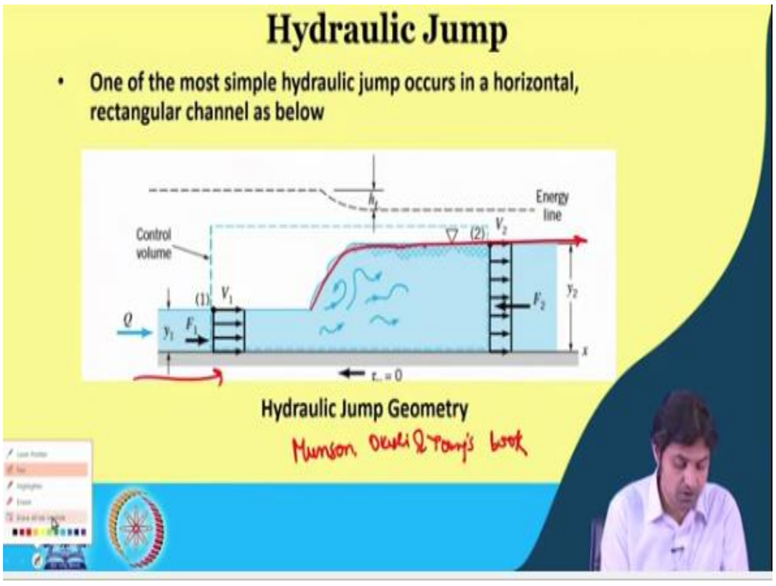

So, one of the most simple hydraulic jump occurs in a horizontal rectangular channel as below, we are going to see the one of the simplest occurrence of hydraulic jump. So, this figure has been taken from Munson, Okiishi and Young's book Fluid Mechanics. So, if there is a flow that is coming like this, in this direction and the water surface elevation due to the influence rises like this, then this is the simplest most case of hydraulic jump.

This we are going to derive the equation for hydraulic jump and we will take the help of this particular figure.

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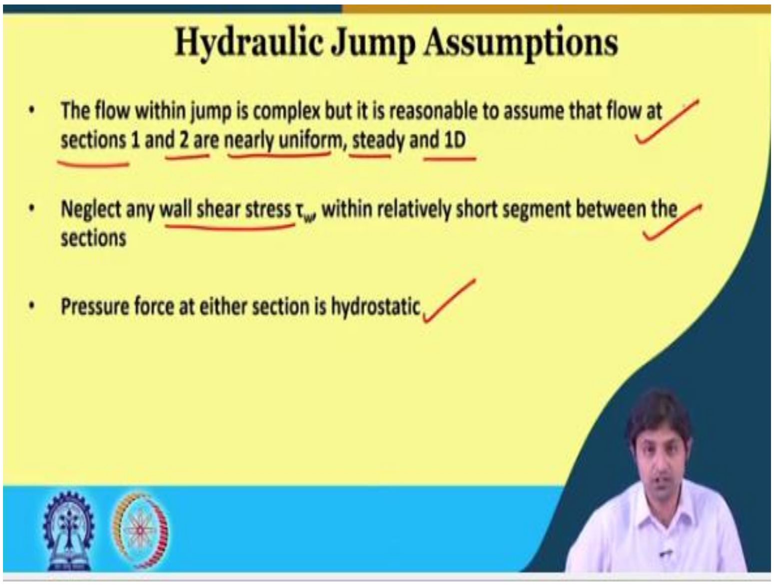

So, the flow within the jump. So what is the area within, what is this within the jump? So, this is the jump, this is the jump area, so the flow within the jump is complex but it is reasonable to assume that flow at section 1 and 2 are nearly uniform steady and 1D. So, one of the approximations is, however this is a jump and this area, this flow is quite complex in nature, but we can assume at sections 1 and 2, so this is 1 and 2 and the control surface that we have assumed starts at section 1 and goes like this and finishes at section 2.

So, this is the control surface that we have taken. So, at section 1 and 2, we are taking the flow to be nearly uniform, steady and one-dimensional in nature. We have to neglect any wall shear stress within relatively short segments between the sections. So, we neglect the wall shear stress tau w, for the derivation of the hydraulic jump.

Since, the at section 1, the flow is nearly uniform, steady and 1D, the pressure forces that either section will be hydrostatic. This is another assumption. This is one assumption and this is the, the first assumption is that at sections 1 and 2, the flow is considered to be uniform, steady and one-dimensional. Second, is we have to neglect any wall shear stress within relatively short segments between the sections. The third assumption is that the pressure force at either section is hydrostatic.

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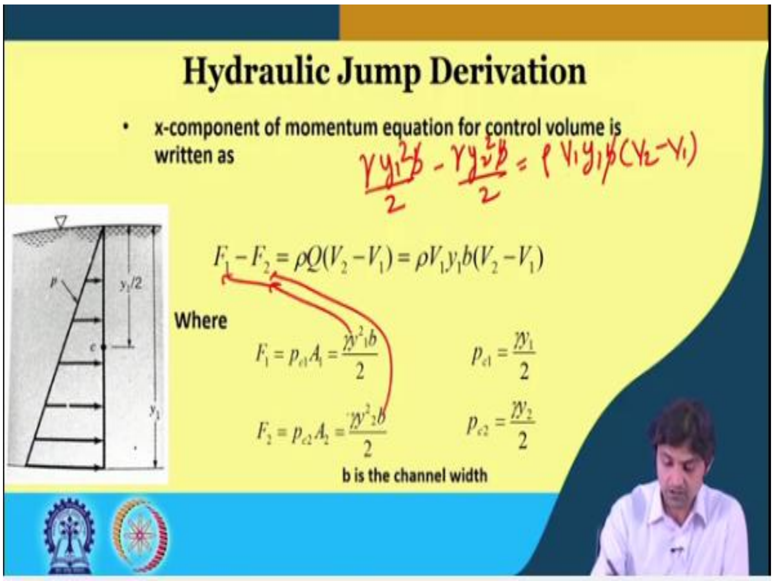

So, we have seen what the control volume is. So, the x moment, x component of momentum equation for control volume is written as, so F1 – F2, the difference in the hydrostatic pressure can be written as, $\rho Q(V_2 - V_1)$. So we go back to our original diagram and see what exactly we are talking.

I will take away this ink. So, this pressure F1 and this is also hydrostatic due to hydrostatic pressure. So, this F1 – F2 is the, this F1 – F2 will be $\rho Q(V_2 - V_1)$. How do we you remember, where did we get this equation from? Reynolds transport theorem or momentum equation that we derived in week 2. So, now, so rho Q, so Q is, Q can be written as, A1 V1 is equal to A2, sorry, A2 V2.

So, A1V1 is V1 y1 into b, where b is the width, multiplied and V2 – V1 is the change in the velocity, I mean, the difference of the velocities between section 2 and 1. So, instead of, so we can actually use this equation here. Now, we need to determine F1 and F2. So, how to determine F1, F2? This is a simplified diagram of the hydrostatic pressure that we have drawn, the force due to the hydrostatic pressure, if you remember.

Where, F1 can be given as, rho c1 pressure at the midpoint into A1, so this will be gamma y1 by 2 into y1b, you see here, 1/2 base into height. So, F1 will be $\frac{\gamma y^2{}_1 b}{2}$. Similarly, because pressure at c1 is gamma y1 by 2, similarly F2 is going to be pressure at c2 is, 1 and 2 denote the section in the control volume, where Pc2 is gamma into y2 by 2.

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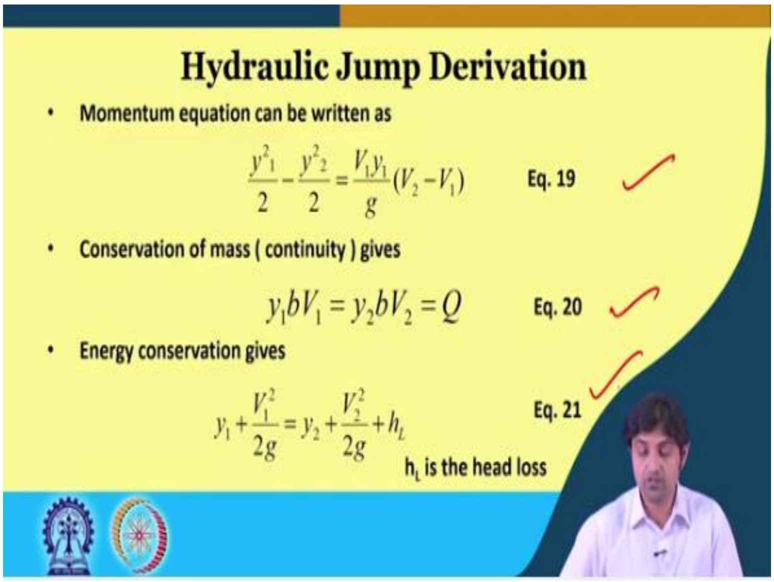

Here, b is the channel width, as I told you before. So, this equation, momentum equation, therefore, can be written as, so you see, so if you, let us try substituting it here and this one here. So, this will be gamma y1 square b by 2 - gamma y2 square b by 2 is equal to rho V1 y1 b into V2 - V1.

So, b will cancel from both side and this gamma can come down, so it will become, this gamma can come down here, so Rho by gamma is going to be g. So, using this we have simply written y1 square - y2 square is equal to V1y1 by g into V2 - V1. So, this is one equation that we get. So, if we use the conservation of mass that is the continuity equation, it will give us y1b1V1, A1 V1.

So, A1 V1 and this is A2, this we have already actually written before also, is Q and the energy conservation, so we have applied momentum conservation, we have applied continuity, now we will write energy conservation. So, between point 1 and 2, $y_1 + \frac{V_1^2}{2g}$ is equal to $y_2 + \frac{V_2^2}{2g} + h_L$, this is the energy loss because in the rapidly varied flow, there is going to be some energy loss, because of the transition from one flow regime to the other or because you see it is a very complex process, so we assume there is going to be an energy loss.

So, as I told, h_L is the head loss, so we have 1 equation, 2 equation and 3 equation. The matter is we have to solve this, so that we are going to get some, you know, some definitive results for y2 and y1.

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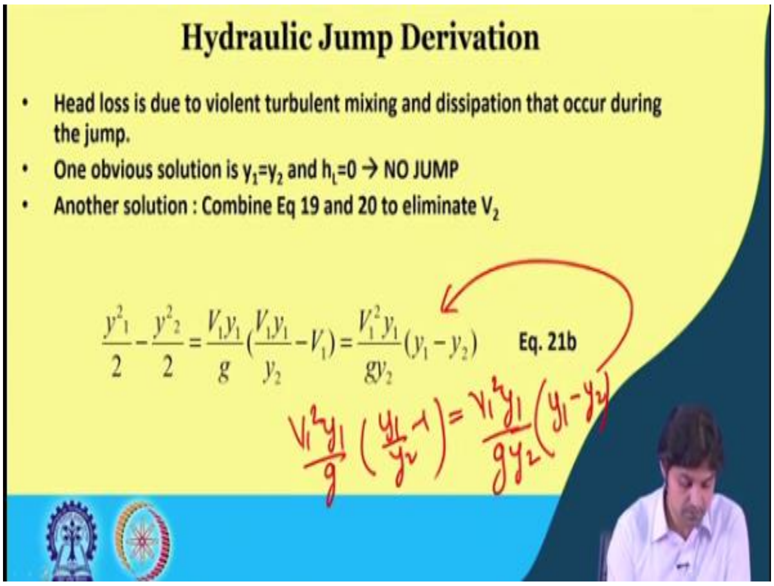

So, it is, we know, that head loss is due to violent turbulent mixing and dissipation that occurred during the jump. As I said the phenomenon of the hydraulic jump is quite complex, it is phenomenon where violent turbulent mixing so, vigorous turbulent mixing is there and because of that dissipation of heat, I mean, dissipation in terms of heat will occur, so energy is going to be lost during this jump.

For the equations written behind, you know, one of the obvious solution is y1 is equal to y2 but this implies that there is not going to be head loss. Therefore, implying that there is no jump, so this is a not an acceptable solution to us. Because we are assuming that there is going to be a jump. So, one of the solution is y1 is equal to y2 which we neglect. In a second case, another solution is, if we combine equation 19 and 20 to eliminate V2, you go back and see, what equation 19 and 20 is, we can eliminate V2 and write everything in terms of V1.

So, using these 2 equation, we can write, see, this is equation number 19, sorry, yeah 19. So, y1 square - y2 square by 2 is the same as left hand side, V1 y1 by g is equal to, so here you remember, there was a V2 term, this one, this term, so using the continuity equation that y1 V1 is equal to y2 V2. So, we have written V2 is equal to y1 V1 by y2.

So, instead of V2, we have used the equation number 20 here, and this will become, so we can take 1 V1 outside as well and y2 also, so I will just. So, how to go from here to here? I will just solve it out, y1, sorry, V1y1 by g and let us take from this side, V1 outside, this becomes y1 by y2 - 1 and this is equal to, now try to multiply up and down both by y2, so we multiply y2, both side.

So, this becomes, so we still, V1 y1 by g, so this becomes V1 square. Now, y1 by y2 into y2 by, no, we do not multiply, so this one is okay, V1 square y1 by g by y1 by y2 – 1, this we have, so we take y2 outside, so V1 square y1 by g and we take y2 outside, so this is y1 minus this becomes y2 and this is same as this equation.

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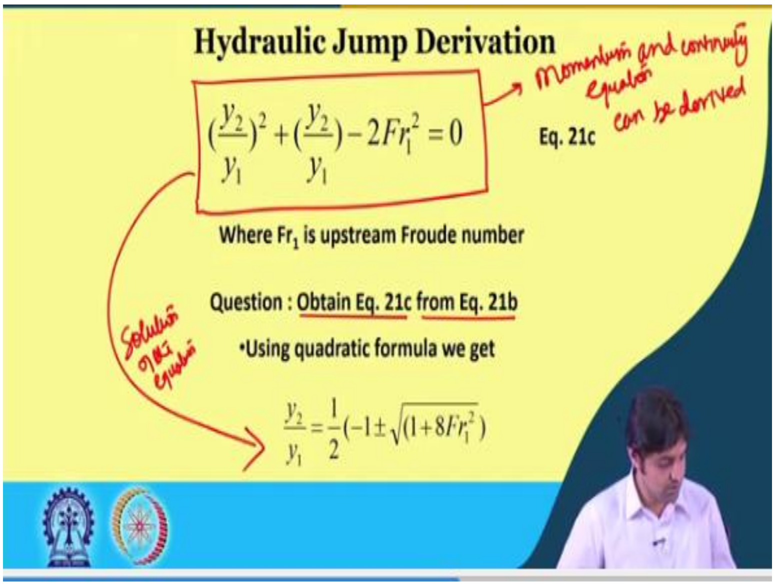

So, this equation, you see, can be rewritten as, $(\frac{y_2}{y_1})^2 + (\frac{y_2}{y_1}) - 2Fr_1^2 = 0$. You see, because Froude number 1 is V by under root gy1, V1, so Froude number 1 whole square is V1 square by gy1. So, we try to put V1 square by gy1 in this equation. So, if we multiply by 1 here, it becomes y1 square, so this can be written as, this part, can be V1 square by gy1 into y1 square by y2 into y2 – y1, so this is Froude number 1 whole square.

And when this goes, this goes inside, this becomes 1, y1 by y2 and let us and yeah, so this y2 whole square when multiplied here, will also be divided here, so this will become y1 by y2 whole square and this becomes minus 1 by 2 is equal to Froude number whole square y1 by y2 whole square into 1 – y1 - y2 and this on rearranging will come out to give this equation.

So, this is a simple equation, which using the only the momentum continuity can be derived. So, now you see, this is a quadratic equation in y2 by y1 and Fr1 here, is the upstream Froude number. So, there is a question which says obtain equation 21c from equation 21b. So, very simple, I have showed you most of the steps before, here in this lecture. How to do that?

You have to just do some rearrangement, so you try this at home and we can discuss that. So, the for, so this is a quadratic equation and this will give us a result $\frac{y_2}{y_1} = \frac{1}{2}(-1 \pm \sqrt{(1+8Fr_1^2}\,)$. So, this is the solution of the equation.

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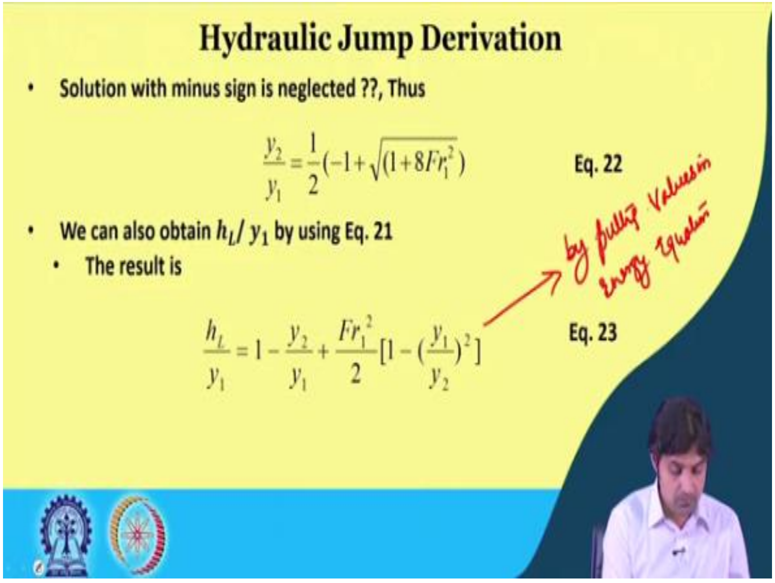

So, we neglect the solution with the minus sign. Let me just, so we neglect the minus sign, therefore, we get final solution as, y2 by y1 is equal to 1/2 – 1, see because negative solution is neglected because this will give a negative value of y2, which is not possible. So, that minus sign is neglected so this is the final solution. And what we, so using energy equation after we have got y2 by y1, we can obtain the head loss h_L.

And this is completely, so if we are able to find y2 by y1 from this equation, see what happens is when the condition at upstream is always known, we would most of the time know what the discharge is, what the velocity is, where the height of the, you know, water is at the upstream end. We would want to find out the situation after the hydraulic jump has occurred. So, in most of the cases y2 and V2 and those things are unknown.

So, y1 is known, using this formula, we can also find y2 or y2 by y1, so this is known, this is known, Froude number at 1. So, if we know everything at section 1, so Froude number at 1 is also known. We have already find y2 and y1, so head loss can be calculated using this formula and this formula has been got, has been derived by putting values in energy equation, simple.

So, I think this one, these 2 formulas are very important to remember. So, I would suggest you to remember those, these formulas because they would prove very helpful in the questions about the hydraulic jump.

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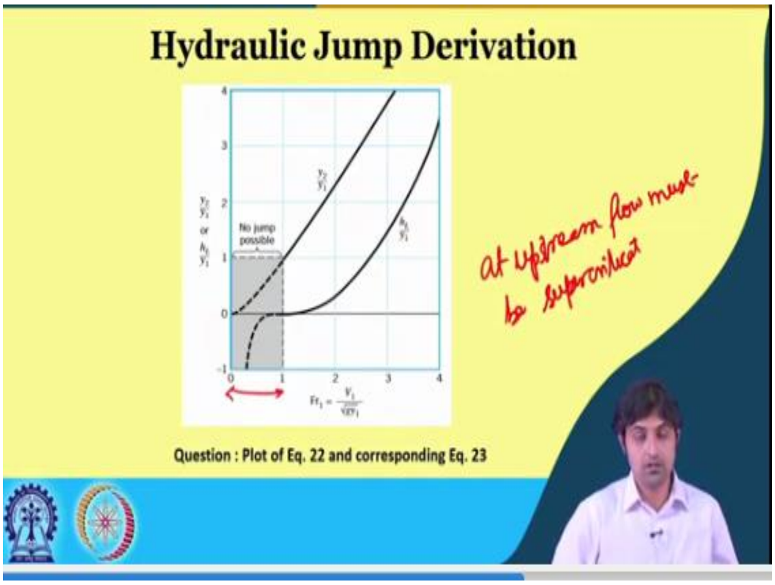

So, this is a graph that has been drawn. So, this is a graph showing, y2 by y1 against the Froude number at 1. So, if you keep plotting, this is hL by y1, this is an area where there cannot be any possible jump, this area. And if you note this particular one, there is no possible jump when Froude number is less than 1. So, that means at upstream flow must be supercritical.

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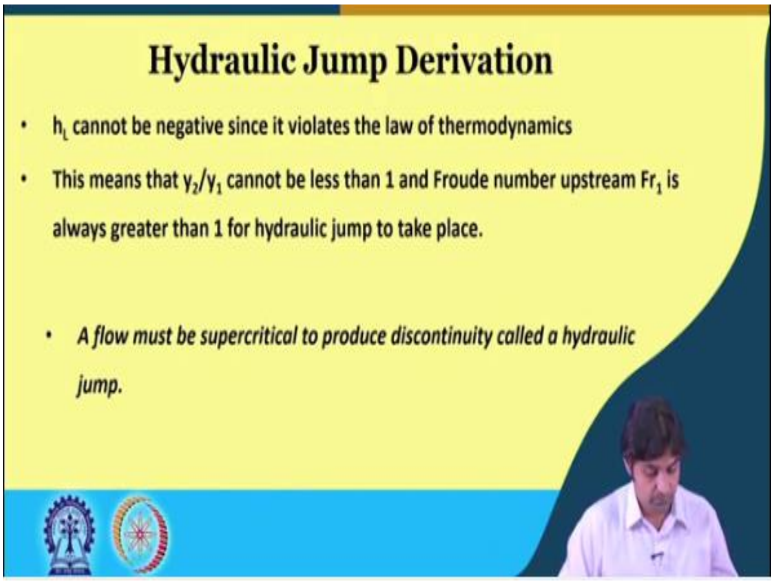

So, head loss cannot be negative, since it violates the law of thermodynamics. We go back to this equation, so this head loss, it cannot be below this, because it violates the law of thermodynamics. So, this means, y2 by y1 also cannot be less than 1 and therefore, the Froude number upstream Fr1 is always greater than 1, for hydraulic jump to occur.

So, important message that you must remember is that Froude number at section 1 will always be greater than 1, if hydraulic jump is supposed to occur. So, if the Froude number is less than 1 upstream, there cannot be, there is no way hydraulic jump will ever occur. Therefore, the flow must be supercritical to produce discontinuity called a hydraulic jump, a very important conclusion.

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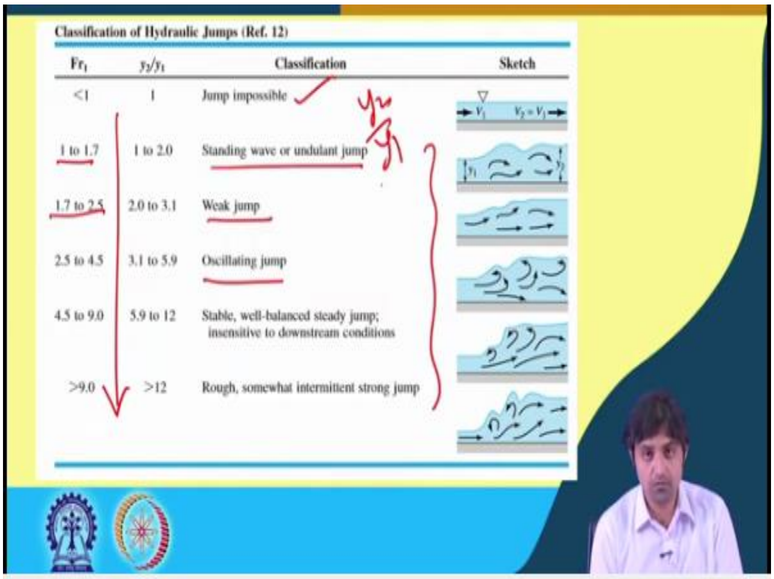
Classification of Hydraulic Jumps (Ref. 12)

Fr_1	y_2/y_1	Classification	Sketch
<1	1	Jump impossible	
1 to 1.7	1 to 2.0	Standing wave or undulant jump	
1.7 to 2.5	2.0 to 3.1	Weak jump	
2.5 to 4.5	3.1 to 5.9	Oscillating jump	
4.5 to 9.0	5.9 to 12	Stable, well-balanced steady jump; insensitive to downstream conditions	
>9.0	>12	Rough, somewhat intermittent strong jump	

So, these are the classification. So, if Froude number is less than 1, y2 by y1 is equal to 1, jump is impossible. Now, these are, I mean, you should not worry about this much, you know, but at the undergraduate level, but here for information, if the Froude number at upstream is 1 to 1.7, the standing wave or undulant jump, if it is between 1.7 to 2.5, it is called a weak jump. So, as, you know, Froude number at upstream keeps on increasing, the jump becomes more violent and violent, that is an important message that you must remember.

So, you do not need to worry about what is the value of Froude number for oscillating jump or what is the Froude number for, you know, balance steady jump. Main thing is for Froude number less than 1, there is no jump possible and secondly, if the Froude number at upstream section keeps on increasing, there the while, I mean, the rough the jump is going to become more and more violent.

Other than that 2 formulas, y2 by y1, the depths y2 and y1, in terms of Froude number at location 1 is important and also the head loss. But head loss you can always if, you know, y1 and y2, you can always use the energy equation to find out the head loss.

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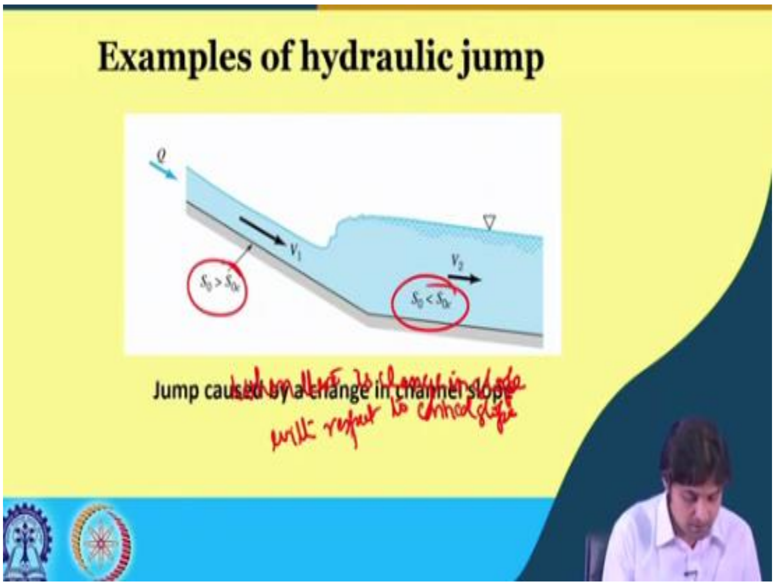

So, this is some example of hydraulic jump, you know. So, there is a constriction and when it, you know, falls down, so when S0 is greater than S0c and this the water depth increases and so basically, when there is change in slope, with respect to critical slope. So, this means, when the slope of the bottom was greater than the critical slope and it transits to a region where the slope becomes less than the critical slope, then the hydraulic jump will occur.

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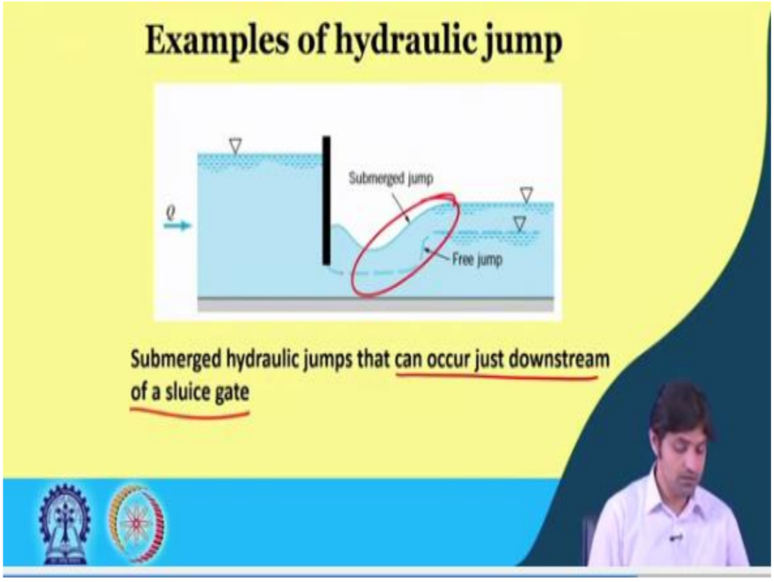

So, as I said, this is a jump which is caused by change in the channel slope. So, this is the submerged hydraulic jump that can just occur downstream of a sluice gate. So, in this, depending also on the flow conditions, you know, I mean, if many things will depend upon the velocity and other things but this can occur just downstream of a sluice gate, here.

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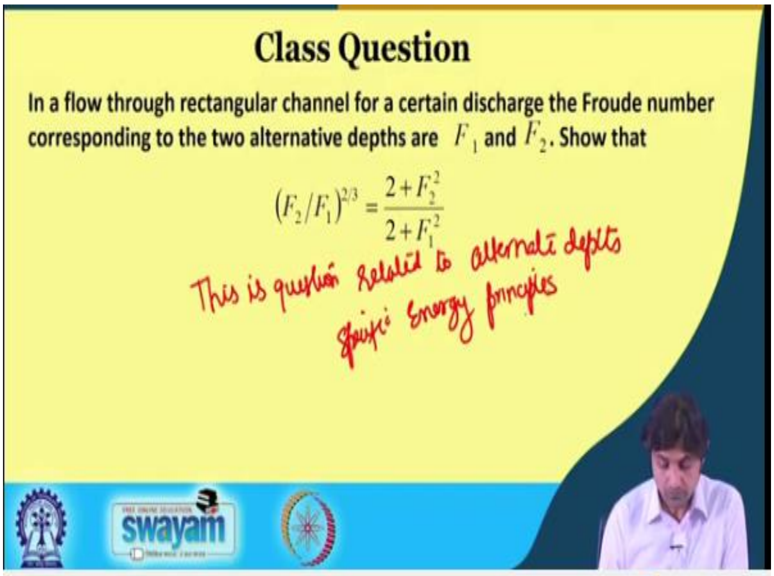

So, there is one question, in a flow through a rectangular channel for a certain discharge the Froude number corresponding to 2 alternative depths are F1 and F2. We have to show that, F2 by F1 raised to the power 2 by 3 is equal to 2 + F2 whole square divided by 2 + F1 whole square. What do you think this is a question related to the hydraulic jump, yes or no? So, this is a question related to alternate depths. So, basically specific energy principle. So, I will, you know, quickly solve this one, white screen.

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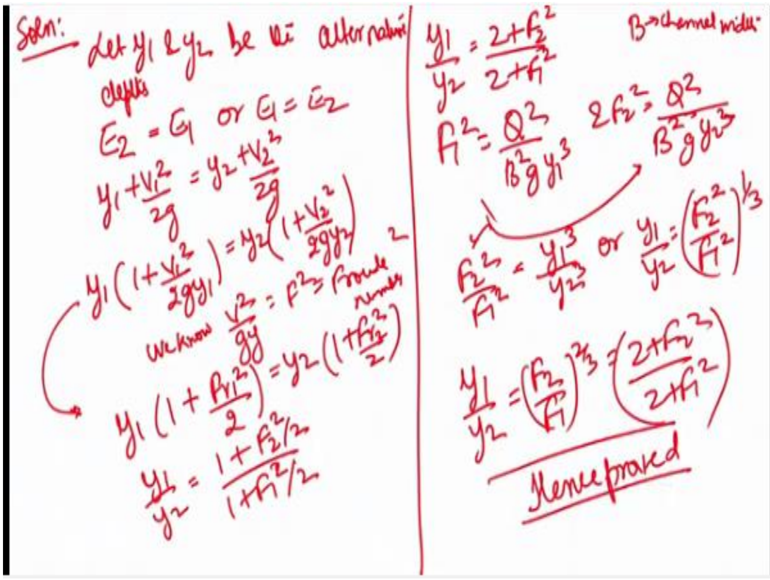

So, let y1 and y2 be the alternative depths. So, you remember, this alternative depths was a concept that was given in specific energy and specific energy, in that case is, should be equal, at section 2 and section 1. So, let us write, E1 or E1 is equal to E2, or same thing.

So, y1 + V1 square by 2g is equal to y2 + V2 square by 2g, so we will take y1 common, so it will become 1 + V1 square by 2g, y1 is equal to y2 1 + V2 square by 2g y2. And we already know, V square by g is equal to Froude number whole square. So, this equation, you know, y1 1 + Fr1 by 2 Fr2 by 2. So, basically, y1 by y2, can be written as, 1 plus, if Froude number we just write F2, F2 square by 2 F1 square by 2, or y1 by y2 is equal to 2 + F2 whole square.

So, F1 and F2 we already know. So, instead of, you know, Q, I mean, we can, we also know that F1 square is Q square divided by B Square gy1 cube and F2 square is equal to, same thing, Q square divided by B Square gy2 cube, just instead of V we are writing, in terms of Q and B is the channel width. So, using this one, you know, we can, if we use these 2 equations here and we can write, F2 square by F1 square, using these 2 here, will be equal to y1 cube by y2 cube or y1 by y2 is equal to F2 whole square divided by F1 whole square to the power 1 by 3.

So, this was cube, so y1 by y2 is equal to F2 by F1 to the power 2 by 3 is equal to 2 by F2 square and this was what we had to prove. So, this was the thing that we had to prove. So, basically, we started with specific, equating specific energy. So, that was step 1 and step 2 was writing the Froude number in terms of Q and obtaining y1 by y2, from here, step 2 and obtaining y1 by y2, from step 1 and equating those will give us the required.

So, this was the thing that we had to prove. So, this was one very simple which was not about hydraulic jump. The questions about hydraulic jump we will start doing in the next, starting from the next class. So, I think this is enough for today and we will start our, probably the last lecture on open channel flow next time. Thank you so much. See you in the next class.