

Hydraulic Engineering
Prof. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture- 32
Introduction to Open Channel Flow and Uniform Flow (Contnd.)

Welcome back. We are going to start with the last, last lecture. We saw what the Chezy equation and the Chezy coefficient was,

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Manning Equation

From series of experiments it was found by R. Manning that dependence on hydraulic radius R_h is not proportional to $R_h^{0.5}$ but $V \propto R_h^{2/3}$

- He proposed a modified equation for open channel flow

$$V = \frac{R_h^{2/3} S_0^{1/2}}{n}$$

(18)
 $V = C \sqrt{S_0} R_h$
Chzy

Eq. 18 is called Manning Equation and parameter n is called Manning resistance parameter

- n is obtained from Tables. Precise values are difficult to obtain
- Rougher the perimeter, larger the value of n

The slide also features a portrait of a man in the bottom right corner and two circular logos in the bottom left corner.

And now we are going to start with the Manning's equation. We found out that Chezy related the velocity proportional to R_h to the power half, where R_h is A / P area divided by wetted parameter and R_h to the power half was what Chezy derived. However, through a series of experiment a scientist called Manning found out that the dependence of the velocity on the hydraulic radius is not to the power 0.5, but V was proportional to R_h to the power $2 / 3$.

He therefore proposed a modified equation for open channel flow, where he said that V is proportional to $\frac{R_h^{2/3} S_0^{1/2}}{n}$ and this is equation number 18, and this equation is called the Manning's equation and parameter n is called Manning's resistance parameter, n is generally obtained from the table, which I will show you, the precise value of it is very difficult to obtain.

So, we generally use tables for finding those values of n . In your case, those values of n will be very standard or will be given to you at the time of the questions. So, an important thing to notice, the rougher the parameter is, the larger the value of n will be.

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Uniform Depth Flow

- Here
 - θ is very small. Bottom slope is very small.
 - Therefore $\sin \theta \sim \tan \theta \sim S_0$

$$\tau_w = \frac{WS_0}{Pl}$$

- Putting $W=YAl$ and Hydraulic Radius $R_h=A/P$

$$\tau_w = \frac{\gamma A l S_0}{Pl} = \gamma R_h S_0 \quad \text{Eq. 16}$$


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Uniform Depth Flow

- Open channels flows are mostly Turbulent
 - Reynolds number lies fully in turbulent regime
- Here, we draw analogy from Pipe flow for turbulent flow
 - For very large R_e , friction factor f for pipe flows is independent of R_e and dependent only upon relative roughness, ϵ/D
 - The wall shear stress is proportional to dynamic pressure $\rho V^2/2$ and independent of the viscosity.

$$\tau_w = K \rho \frac{V^2}{2}$$

K is a constant
 Depends upon pipe roughness



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Uniform Depth Flow

- Assuming similar dependence for high R_e open-channel flows, Eq. 16 can be written as

$$K\rho \frac{V^2}{2} = \gamma R_h S_0$$

$$V = C \sqrt{R_h S_0} \quad \text{Eq. 17}$$

- Constant C is Chezy Coefficient and Eq. 17 is called Chezy Equation
- Developed by French Engineer while designing canal
- C is determined from experiments
- Find the dimension ?? $\frac{L^2}{T}$

So, I will just go to the Chezy's. So, there is a correction here actually, so this is not A that is S_0 to the power half. So, I will rewrite this. Sorry, for this. So, what Chezy propose was, was this, same as what I had written before.

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Manning's n Table

Values of the Manning Coefficient, n (Ref. 6)

Wetted Perimeter	n	Wetted Perimeter	n
A. Natural channels		D. Artificially lined channels	
Clean and straight	0.030	Glass	0.010
Sluggish with deep pools	0.040	Brass	0.011
Major rivers	0.035	Steel, smooth	0.012
B. Floodplains		Steel, painted	0.014
Pasture, farmland	0.035	Steel, riveted	0.015
Light brush	0.050	Cast iron	0.013
Heavy brush	0.075	Concrete, finished	0.012
Trees	0.15	Concrete, unfinished	0.014
C. Excavated earth channels		Planed wood	0.012
Clean	0.022	Clay tile	0.014
Gravelly	0.025	Brickwork	0.015
Woody	0.030	Asphalt	0.016
Stony, cobbles	0.035	Corrugated metal	0.022
		Rubble masonry	0.025

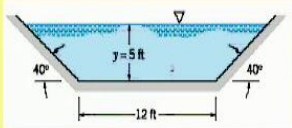
So, Manning's n table. So, this is the general value of the table. So, the wetted parameters, you know, if the natural channel is there, for a different type for clean and straight channels n is 0.030, for sluggish with deep pool it is 0.040, for most of the rivers it is 0.035. So, as you see, for different, you know, natural channels, for flood plains, for excavated earth channel an n has been found out. For artificially lined channels as well, you can actually control the roughness and prepare, for glass if you see, n is very less 0.010, brass is 0.011.

So, you know, with if, I mean, if here is a casted iron, if there is a wood, for the clay tile it is 0.014, for brick work. So, what normally you do is, you look at these tables and find out the value of n , some for asphalt lining is 0.016. You do not need to remember those values, but you should have an idea, for example, natural channels is around 0.035, glass is around 0.010. But in majority of the problems you should be given, if you are not asked to find out the roughness, you should be given what the Manning's parameter n is inside the question itself.

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Class Question

- Water flows in the canal of trapezoidal cross section shown in Fig. below. The bottom drops 0.42 m per 304 m of length. The canal is lined with new smooth concrete. Find
 - Area A
 - Wetted Perimeter P
 - Flow rate Q
 - Reynolds number Re
 - Froude Number Fr



Take 5 ft = 1.5 m and 12 ft = 3.6 m

We can look up values of
 $n = 0.012$

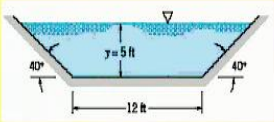
So, we start by doing a class question. And the question is the water flows in a canal of trapezoidal cross section which is shown in figure below. Let me see the where the figure is. We have to find the area A , wetted parameter, flow rate and Froude number for this case, you see, the channel is given here and this is trapezoidal cross section. And it says that the bottom drops 0.42 meter per 304 meter of length and the canal is lined with new smooth concrete. So, it is written new smooth concrete.

So, we can look up the value of n and after looking the value of n we know that it is 0.012 that is already given to us. And following are the things, this is a very classical example where you will learn how to do this type of problems, area we will find out, we will find out the wetted parameters, we will find out the flow rate, Reynolds number and in the end the Froude number as well. So, this you see, it is given in feet, so 5 feet and 12 feet

This problem has also been taken from Munson, Okiishi and Young. This gives a very clear-cut idea how to, you know, one of the most simple cases of finding uniform flow, hydraulic radius and the flow rate and everything, so okay.

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Class Question solution



$$A = 3.6 * 1.5 + 1.5 * \left(\frac{1.5}{\tan 40^\circ} \right) = 8.08 \text{ m}^2$$

$$P = 3.6 + 2 * (1.5 / \sin 40^\circ) = 8.26 \text{ m}$$

$$R = \frac{A}{P} = 0.99 \text{ m}$$

$S_o = \frac{0.42}{304}$
 $S_o = 0.0014$

So, we start. So, this is the figure, so area is going to be, you know, 3.6 into 1.5, so this is the, this area, because we have converted it already into meter square, plus this is half base into height and half base into height, so when you add this, this will become one into base into height, base is half, so height is, sorry, height is half and base is, so it becomes 1.5 divided by tan 40 and this will come out to be 8.08 meter square.

So, now, we need to know, we need to calculate parameter as well. Parameter the parameter that is wetted is going to be this length, plus this length, plus this length because this is the length which is, you know, wet so we can find this, plus 2 times this because this is equal to this. So, parameter this one is 3.6 in meters plus 2 into 1 of these length, this is going to be 1.5 divided by sin 40, so this is 1.5 divided by sin 40 will give us 8.26 meters.

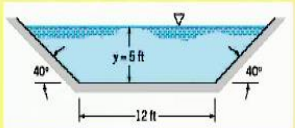
So, we have calculated the first 2 parts that we needed. Area is coming out to be 8.08 meter square and wetted parameter P is coming out to be 8.26 meters. So, hydraulic radius is going to be A / P and 8.08 divided by meter square almost 0.99 meter, almost. So, R is coming out to be 0.99 meters, so because it is a radius it is the dimension of length, so it is in meters. We have also

been told that the bottom drops 0.42 meters per 304 meter. Therefore, we have been indirectly given, slope S_0 , it drops 0.42 meter in 304 meters, so S_0 that comes out to be 0.0014.

You see, we have already told you many times that S_0 is very, very small. So, here in a practical problem also, we find S_0 is very, very small. It is the order of 0.00 so .0014 to be specific, we will see in the next slide.

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Class Question solution



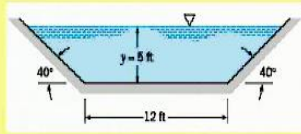
$$S_0 = \frac{0.42}{304} = 0.0014$$

$$Q = \frac{AR_h^{2/3} S_0^{1/2}}{n} = \frac{1}{n} (8.08)(0.99)^2 (0.0014)^{1/2} = \frac{0.30}{n} \text{ m}^3/\text{s}$$

So, S_0 is 0.42 divided by 304. Now, how to find the Q ? So, we know, V is $1/n R h$ to the power $2/3$ into S_0 to the power half implies Q is V into area. So, area into $1/n R h$ to the power $2/3$ S_0 to the power half, n we already know, n was, you know, for finished concrete n was 0.012 that we already looked up from the table, area we have already calculated 8.08 meter square, $R h$ also we calculated, $R h$ was 0.99 and S_0 also we calculated was 0.0014. So, if we substitute in these values, we are going to get $0.30/n$. Now, n we have not yet substituted.

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Class Question solution



$$n = 0.012$$

For finished concrete, See Table

$$Q = \frac{0.30}{0.012} = 25 \text{ m}^3/\text{s}$$

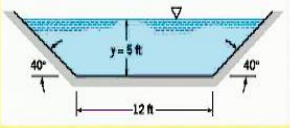
$$Re = \frac{RhV}{\nu}$$

But, n for finished concrete is 0.012, so the Q comes out to be 25 meter cube per second, approximately. So, we have calculated. We have used the Manning's equation and calculated area, we have calculated parameter, we have calculated the discharge Q for this channel, for a uniform flow condition. So, the next question says we have to calculate the Reynolds number. So, Reynolds number is given by $Re = \frac{RhV}{\nu}$ and this is the hydraulic radius.

So, this is the characteristic length, as we as I told you in the beginning, it mostly it is the height but in open channel flow it is hydraulic radius, I mean, hydraulic radius, not Re , it is R , Rh into V / ν . So, R we have calculated, V we know already, so V is Q by area and ν is fixed quantity for water.

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Class Question solution



$$R_e = \frac{R_h V}{\nu} \quad R_e = \frac{0.99 * (25 / 8.08)}{0.13 * 10^{-5}} = 2.36 * 10^6$$

$$F_r = \frac{V}{\sqrt{gy}} \quad F_r = \frac{3.1}{\sqrt{9.8 * 1.5}} = 0.808$$

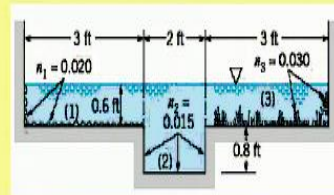
So, R_e is $R_h V / \nu$, this is an important formula that you must remember. For open channel flow this is a Reynolds number. And if we calculate, we substitute in the values, R_h is 0.99, V is Q / A and ν is 0.13×10^{-5} and on calculation, it gives 2.36×10^6 , so this is turbulent flow. As we, I mean, as this we have discussed this that most of the flows in principle in open channel flow will be turbulent. Froude number is $\frac{V}{\sqrt{gh}}$ or gy , y is the depth.

So, V we already know, we have calculated $25 / 8.8$ is 3.1 under root $9.8 \times y$ is the water depth, so 5 feet comes out to be 1.5 meter and this comes out to be 0.808, so it will be a subcritical flow but anyways. So, I have put it also in the slides, so it is 3.1 divided by 9.8×1.5 , that is, 0.808.

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Class Question

- Water flows along the drainage canal having the properties shown in Fig. If the bottom slope $S_0 = 0.3/152 = 0.002$. Estimate the flow rate when depth is $y = 0.24 + 0.18 = 0.42$ m



Take 1 ft = 0.3 m

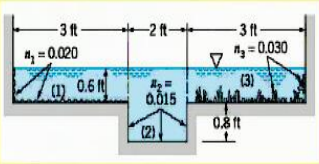
- Also obtain effective manning parameter n_{eff}

So, you see, we have found out the all the questions, I mean, all the things that we wanted to. So, we go to solve another question now. It says that, water flows in a channel shown in figure below, this is the figure, at a rate Q is equal to, no, so water flows along a drainage channel having the properties shown in this figure. If, the bottom slope S_0 is equal to 0.3. So, we have already been given S_0 . Estimate the flow rate when depth is y_0 , I mean, depth is 0.24 plus 0.18 that is 0.42.

So, we have to take one foot is equal to 0.3 meter. The whole configuration is given actually. So, how do we solve this? We also have to obtain an effective Manning parameter, see because this channel is composed of 3 effective, you know, at different n ones, you see. So, in this part the Manning's roughness is 0.20, in this one it is 0.015, so n_3 0.030. So, we have to find an effective Manning parameter n_{eff} . So, how do we solve this problem?

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Class Question



Doing one sample calculation for A, P and R; Say cross section 2

$$A_2 = 0.6 * (0.24 + 0.18) = 0.26 \text{ m}^2$$

$$P_2 = 0.6 + 2 * 0.24 = 1.09 \text{ m}$$

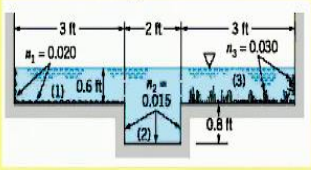
$$R_{h2} = \frac{A_2}{P_2} = \frac{0.26}{1.09} = 0.238 \text{ m}$$

We will go step by step. So, first, we redraw the diagram here and we divide the cross section into 3 sub sections and the total discharge is going to be $Q_1 + Q_2$, this is section 1. Here, it will be Q_1 , it will be Q_2 and it will be Q_3 . So, we find separately. So, now, for each cross section Q_i is going to be, we use the Manning's formula, A_1 into R_h to the power $2/3$ into S_0 and n_i . S_0 is not as common for all that we have seen, S_0 was given here 0.002. So, we find q_1 , q_2 and q_3 . So, best is that we do one simple calculation of A and P, let us say we do it for cross section number 2.

So, we are going to see this calculation. So, area A_2 is going to be 0 point it is the width 0.6 into the total height is $0.24 + 0.18$, so area is going to be 0.6 multiplied by 0.42 0.25 meter square, A_2 . Similarly, the wetted parameter is going to be only this one, plus this one, plus this one. So, this is 0.6 meters or 2 feet and this one is 0.8 and if you multiply it into 0.3 it will be 0.24 and we have two of those 1 and 2. So, 2 into 0.24 and this gives us 1.08 meter. So, A_2 was 0.26 meter square and this was 1.09. So, hydraulic radius is going to be 0.26 divided by 1.09, it comes out to be 0.238 meter.

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Class Question



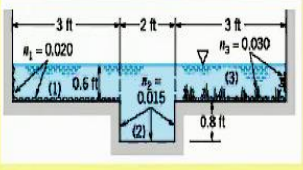
Listing them in a Table

i	A_i (m ²)	P_i (m)	R_{hi} (m)	n_i
1	0.16	1.09	0.15	0.020
2	0.26	1.09	0.24	0.015
3	0.16	1.09	0.15	0.030

So, now if we list each of them, we did the calculation for number 2, you should be able to do the calculation for section number 1 and 3. So, we find out A_1 , P_1 , hydraulic radius and n_1 is given. So, similarly in all the 3 ways, you can list all 3 of them, in this table and the value has been written. You verify it after the lecture.

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Class Question



The total flow rate is $Q = Q_1 + Q_2 + Q_3$

$$Q = 0.002^{0.5} \left[\frac{0.16 * 0.15^{\frac{2}{3}}}{0.020} + \frac{0.26 * 0.24^{\frac{2}{3}}}{0.015} + \frac{0.16 * 0.15^{\frac{2}{3}}}{0.030} \right] \text{ m}^3/\text{s}$$

$Q = 0.46 \text{ m}^3/\text{s}$

So, the total flow rate is $Q_1 + Q_2 + Q_3$, so with those values, you know, area multiplied by R_h to the power $2/3$ and this is S_0 because see, the what the formula says, divided by $1/n$, so this is n , Manning's n , which are different for different, this is S_0 , this is areas A_i , n_i and this is R_{hi} and you sum it and you will find out that $Q = 0.46$ meter cube per second.

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Class Question

If entire channel cross section were considered as one flow area, then

$$A = A_1 + A_2 + A_3 = 0.59 \text{ m}^2 \quad P = P_1 + P_2 + P_3 = 3.27 \text{ m}$$

$$R_h = A/P = 0.180 \text{ m}$$

$$Q = \frac{A R_h^{2/3} S_0^{1/2}}{n_{eff}} \text{ m}^3/\text{s} \quad n_{eff} = \frac{A R_h^{2/3} S_0^{1/2}}{Q} = \frac{0.59 * 0.18^{2/3} * 0.002^{1/2}}{0.46} = 0.0179$$

So, if now we have found out everything for entire, we have found out Q . Now, if we assume that if entire cross-section were considered as one flow area then the total area is going to be $A_1 + A_2 + A_3$, which we have already computed in the tables, if you add it will come to be 0.59 meter square. The whole parameter if you count $P_1 + P_2 + P_3$, it is going to come to 3.27 meter, from the table, you find it from table and same is from the table and add and you get these values, the table that we have written. So, effective hydraulic radius is A / P and it comes out to be 0.180 meter.

Now, Q can be written as $A R_h$ to the power $2 / 3$, S_0 to the power half divided by n effective. So, this is the effective, Q which we have already calculated Q total, area total which we have already calculated, P we have already we know, so R_h , sorry, so this R_h will be this one here. S not is constant, same for all. So, n effective is going to be $A R_h$ to the power $2 / 3$ into S_0 to the power half divided by Q , so A is 0.59 into 0.81 to the power $2 / 3$ into 0.002 to the power half divided by 0.46.

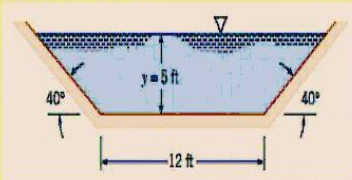
So, n effective came out to be 0.0179, so this is this lies between n_2 , n_1 and n_3 . So, this question we have solved. So, what we did? We had a complex cross section, in that complex cross section there were 3 parts, part 1, part 2 and part 3. So, for each part we found out areas A_i , we found out parameter I_i , we found out hydraulic radius R_{h_i} , we found out Q_i and then we summed all the discharges to find the total Q in that section. With this we will move on to next question.

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Class Question

Water flows in the channel shown in Fig below at a rate of $Q = 10.0 \text{ m}^3/\text{s}$. The canal lining is weedy. Determine the depth of the flow. Given $n = 0.030$

Solution:
In this instance neither the flow area nor the hydraulic radius is known, although they can be written in terms of the depth, y . Hence, the bottom width is $(12 \text{ ft}) (1 \text{ m}/3.281 \text{ ft}) = 3.66 \text{ m}$ and the area is



$$A = y \left(\frac{y}{\tan 40^\circ} \right) + 3.66y = 1.19y^2 + 3.66y$$




That will give us more practice, so the question is that, the water flows in a channel, as shown in this figure below, at the rate Q is given here, 10 meter cube per second. It says, the canal lining is weedy and its Manning's roughness is given. As I said that most of the time it will be provided to you and now the question is, determine the depth of the flow. It is a very, very simple to calculate, you see, it is again given in terms of feet, the reason is that most of the irrigation thing earlier was in this feet system square., so the generally the question.

But we use the conversion system actually and we are going to solve this. So, the cross section is given, we will be able to find out the area and if we assume, you know, actually we have to find this y , which actually is given here, the solution is given. But with these values, Q is equal to 10 meters cube per second and n is equal to 0.030; this length is given 12 feet. We have to calculate the y .

So, how to attack this problem? Very simple to do, so in this instance, neither the flow area nor the hydraulic radius is known, although they can be written in terms of depth y . Hence, in terms of meter, this is actually the bottom width is 3.66 meters and the area can be written as, you know, 3.66 into y plus half base into height plus half base into height, these are symmetric. So, we can write, half plus half 1, so y into $y / \tan 40$. This one is $y / \tan 40$ because angle is given to us.

So, in terms of y, we are writing this will come, area is going to come as 1.19 y square divided by 3.66y, where A and y are in square meters and meters respectively. We have assumed it in terms of meters. The perimeter, perimeter, so this length is already known 3.66,

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where A and y are in square meters and meters, respectively. Also, the wetted perimeter is

$$P = 3.66 + 2 \left(\frac{y}{\sin 40^\circ} \right) = 3.11y + 3.66$$

so that

$$R_h = A/P = \frac{1.19y^2 + 3.66y}{3.11y + 3.66}$$

Where R_h and y are in meters.

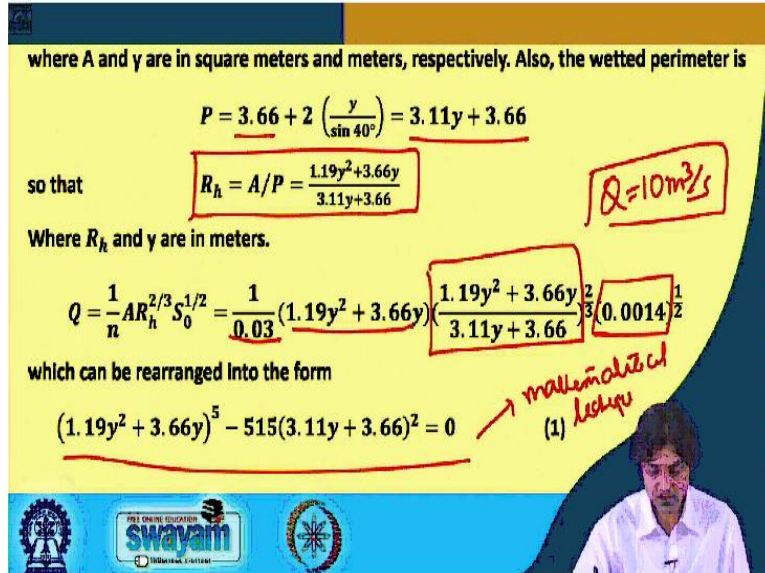
$Q = 10 \text{ m}^3/\text{s}$

$$Q = \frac{1}{n} A R_h^{2/3} S_0^{1/2} = \frac{1}{0.03} (1.19y^2 + 3.66y) \left(\frac{1.19y^2 + 3.66y}{3.11y + 3.66} \right)^{2/3} (0.0014)^{1/2}$$

which can be rearranged into the form

$$(1.19y^2 + 3.66y)^5 - 515(3.11y + 3.66)^2 = 0$$

mathematical technique (1) technique



plus 2 multiplied by, because this is one and plus 1, so this will be, if this is y this is going to be y / sin 40. And this is what we have done in terms of y, we have also written the parameter 3.11 y + 3.66. So, the hydraulic radius will come, area by parameter that is 1.19 y square + 3.66 y divided by the parameter. The discharge according to the Manning's is given by, 1 by n, n is given as 0.03.

Area we have written in terms of the in the previous slide we have found out, this is R_h to the power 2 / 3, this is R_h and S_0 is given as well 0.0014 to us. Now, if we, and what is the value of Q that we are going to use? That is Q is 10. It is a complex, you know, calculation but if you solve using this, you will get an equation like this. So, you can use mathematical techniques. This is not a question that will be given to you in exam or, you know, test but this is just to demonstrate how things can be found out.

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The solution of Eq. 1 can be easily obtained by use of a simple root finding numerical technique or by trial-and-error methods. The only physically meaningful root of Eq. 1 (i.e., a positive, real number) gives the solution for the normal flow depth at this flowrate as

$$y = 1.5 \text{ m}$$

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So, the solution of equation can, one can be obtained easily by use of simple root-finding numerical technique or by trial and error methods. The only physical meaning roots of equation one give the solution for the normal flow depth at this flow rate as, so this will satisfy. And the technique is trial and error, for example and this type of questions, where trial and error, it will not be given in your test for this particular course.

But it is as I said, it is important, so that you should be aware of this type of question. If the calculation is simple, definitely it can be given. But here, the calculations in the end becomes so complex, you are unable to solve it using the calculator. So, this is the final value of the normal depth. So, I will close this lecture, here and we will resume with the next questions and topics in the next lecture. Thank you so much.