Hydraulic Engineering
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Lecture - 31
Introduction to Open Channel Flow and Uniform Flow (Contd.)

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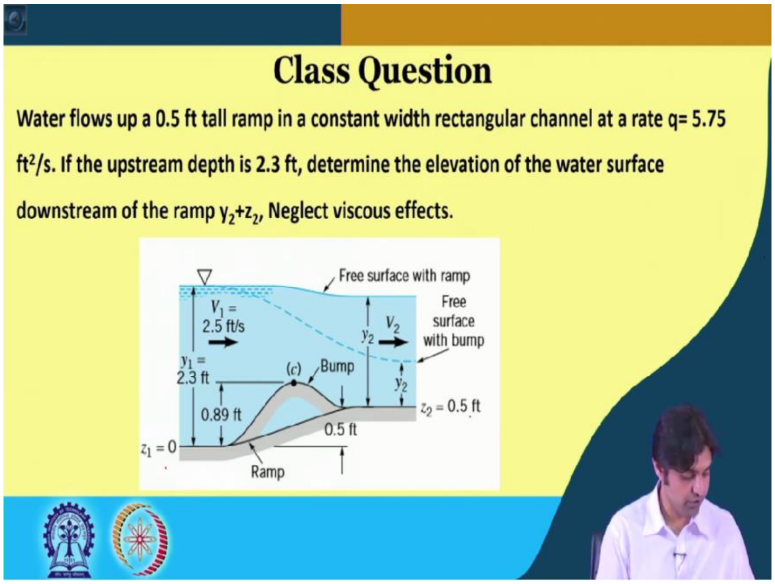

So, welcome back and we are going to start this lecture by solving the question which we just showed you last time in the last lecture. The question is, water flows up a 0.5 feet tall ramp, so this question has been taken from Munson, Young and Okiishi, but I would like to discuss it because it gives more a better understanding of the specific energy and all the concepts that you have read until now.

So, water flows up a 0.5 feet tall ramp in a constant width rectangular channel at a rate q, q is also in feet square per second. You do not have to worry very that much about the units but how this question is solved. So, if the upstream depth is 2.3 feet, so this is the upstream depth. This is the upstream depth 2.3. Determine the elevation of the water surface downstream of the ramp y 2 + z 2. So, we have to determine y 2 + z 2, we have to neglect the viscous effect, this is 0.5 feet and the flow rate q is given as 5.75 feet square per second, so do not worry about the any other unit that is given here.

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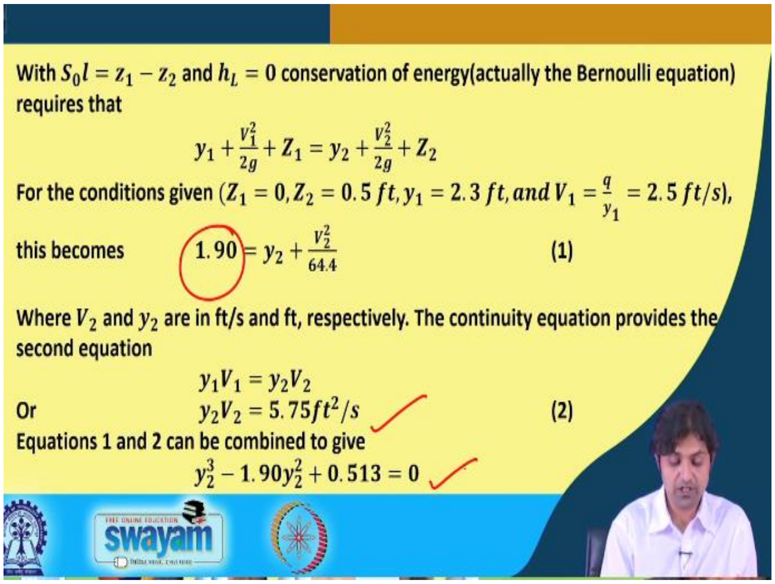

So, with this equation, S 0 l is equal to z 1 - z 2, and energy loss is equal to 0 that means conservation of energy. The Bernouli's equation will require that at 2 points we equate y 1 + v 1 square divided by 2 g + Z 1 is equal to y 2 + v 2 square / 2g + Z 2. For the conditions that are given, we know, Z 1 is equal to 0, Z 2 is 0.5 feet, if you go here, you see that Z 1 is equal to 0, Z 2 is at 0.5 feet, y 1 is 2.3 feets. Therefore, v 1 is given by small q 1 by y 1, which is q 1 because was given 5.75 and if you divide it by y 1, that is, 2.3 feet it gives 2.5 feets per second.

Therefore, the left hand side the value will turn out to be 1.90, y 2 is that we do not know and v 2 square also we do not know but other equations but other values we know. We have substituted the value of y all in this fts unit, where V 2 and y 2 to are in feets per second and feet respectively. Now, we also apply the continuity equation and this will give us a second equation. So, V 1 y 1 = V 2 y 2.

So, V 1 here you see, V 1 y 1, so y 1 V 1 or, you know, V 1 y 1 while because b is constant, the depth, the width is equal to V 2 y 2. So, y 2 into V 2 is equal to because we know y 1 V 1. y 1 we know 2.3 and V 2 also we know. Sorry, V 1 we also know, V 1 is 2.5 So, this becomes V 2 y 2 equals to 5.75 feet square per second, that is another equation that we have got. Now, equation 1 and 2 can be combined to give a cubic equation, if you do in terms of y you can find this equation 1 y 2 whole cube - 1.90 y 2 square + 0.513 = 0.

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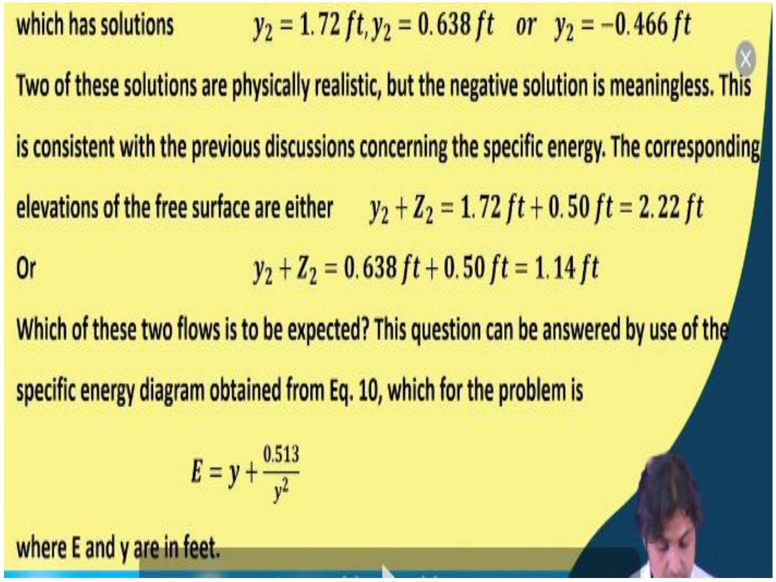

Now, if we solve this, we will get 3 solutions, y 2 is equal to 1.72 feet, y 2 is going to be another, another value is going to be 0.638 feet and this is a negative value. So, of course, we are going to neglect the negative values. So, 2 of these solutions are physically realistic, but the negative solution is meaningless. This is consistent with our previous discussion concerning the specific energy; there we also neglected the negative value.

The corresponding elevations of the free surfaces are either y 2 + z 2 is going to be one, I mean, if we take 1.72 as y 2 2.22 feet or if we take 0.638 feet then it will be 1.14 feet. So, actually, which of these flows is to be expected? The question can be answered by use of the specific energy diagram obtained from equation 10 which for the problem is. So, for this particular question, if we write specific energy, we have to make specific energy diagram for this equation.

This question is more for understanding, until this point is fine for calculations in numericals in the assignments and exams, but, I mean, the later discussion is a little intriguing, you know. So, for this particular question the energy the specific energy diagram E is equal to y + 0.513 / y square is something like this, where E and y both are in feets.

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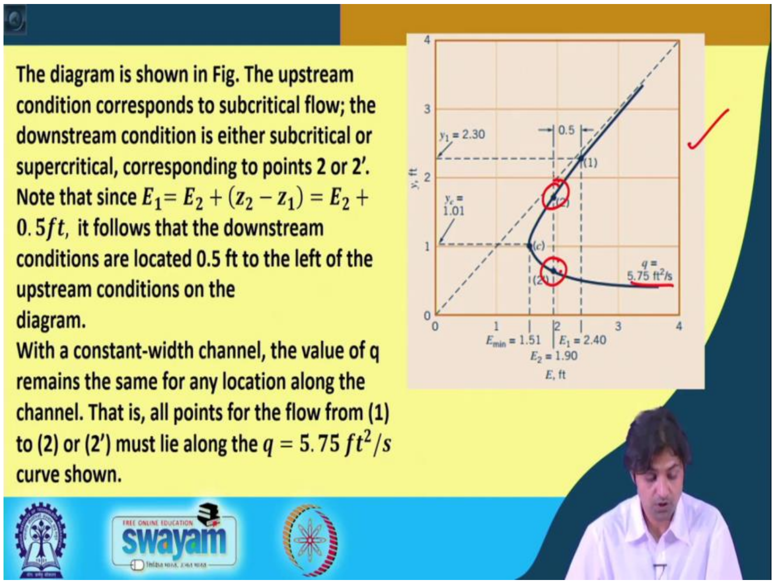

So, if we plot a diagram, we will get something like this, for this q which is 5.75 feet square per second. This is the diagram that we get, we plot we see that why critical is going to occur at 1.01 meter, the E minimum everything has been calculated if we plot this curve. So, it is, so the diagram is shown on the right hand side. The upstream conditions correspond to subcritical flow; the downstream is either subcritical or supercritical corresponding to the points 2 or 2 dash.

So, the values that we have got is either this one or this one, which is very true because this is the same discussion, which we had seen in the specific energy diagram. Now, if we note that since E 1 = E 2 + z 2 - z 1 or E 2 + 0.5 feet, it follows that the downstream conditions are located 0.5 feet to the left of the upstream conditions on the diagram. Now, with a constant width channel the value of q will remains the same that is true for any location along the channel. That is, all points for the flow from 1 to 2 or 2 dash.

So, this is 2 and this is 2 dash. I will just take away this. This is 2 and this is 2 dash that is all points for flow from 1 to 2 or 2 dash, must lie along this line. This is just the discussion of this curve actually going on.

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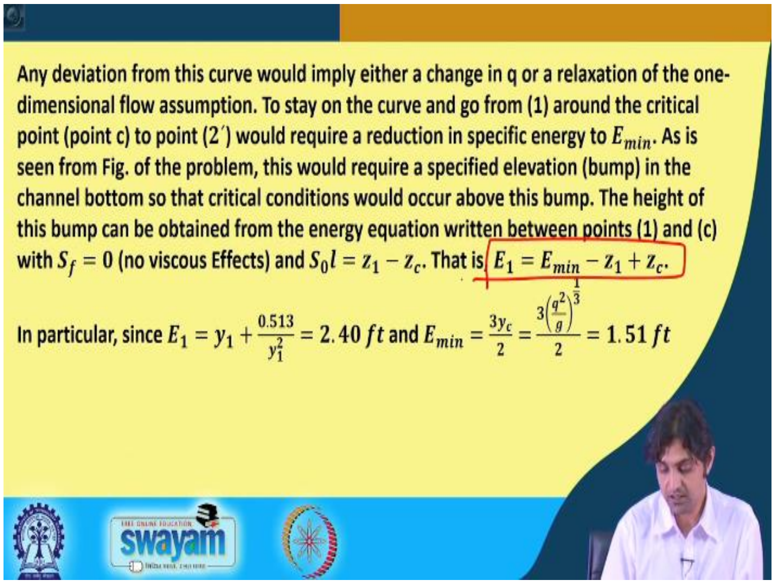

Any deviation from this curve would imply either a change in q or a relaxation of one dimensional flow assumptions. To stay on the curve and go from one, so this is the point one; the critical would require a reduction in specific energy to E minimum. As is seen from figure of the problem, this would require a specific elevation bump in the channel bottom so that the critical conditions would occur above this bump.

The height of this bump can be obtained from the energy equation written between point one and c with S f is equals to 0. So, E minimum is going to be 1.51 feet and the top of this bump would need to be Z c - Z 1 is equal to E 1 - E min, that is, you see, E 1 is going to be E min - Z 1 + Z c critical.

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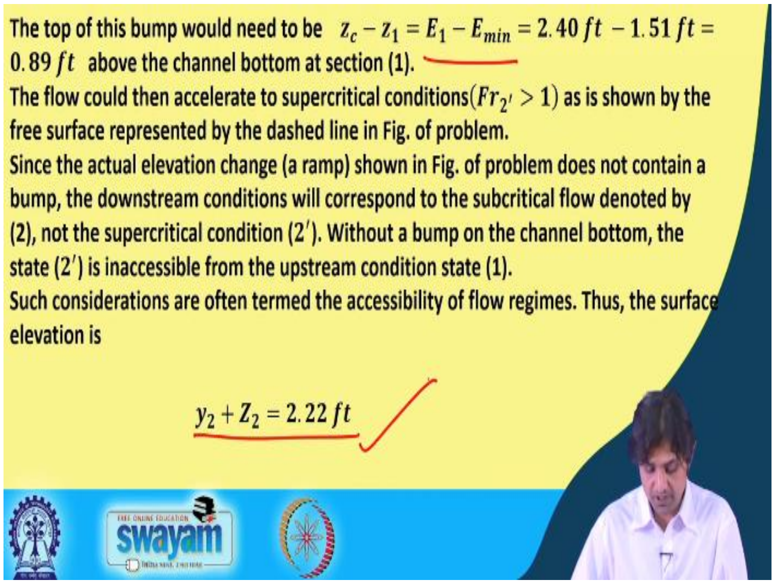

So, Z c - Z 1 will be E 1 - E min, that is, 0.89 feet above the channel bottom at section 1. The flow could then accelerate to supercritical conditions as is shown by the free surface represented by the dashed line. Since the actual elevation change shown in figure of the problem does not contain a bump, the downstream condition will correspond to the subcritical flow denoted by 2, not the supercritical condition 2 dash. Without a bump on the channel bottom, the state 2 dash is inaccessible from the upstream condition state at 1.

Such considerations are often termed the accessibility of flow regimes. Thus, the surface elevation is 2.2. So, because of this discussion y 2 + Z 2 is not going to be 1.68 feet that was there you see here. If you had followed this description, I mean, if you just understand, so this comes to be y 2 + Z 2 is coming out to be 2.22 feet and not 1.14 feet. But I am not expecting you to go to such a detailed discussion. If you have understood well and I mean, fine.

Otherwise even if you have not understood this particular discussion about the bump and other things that is also okay, because what you need to know, how to calculate the specific energy, the minimum energy, the critical depth, is the flow supercritical or subcritical and other things like that.

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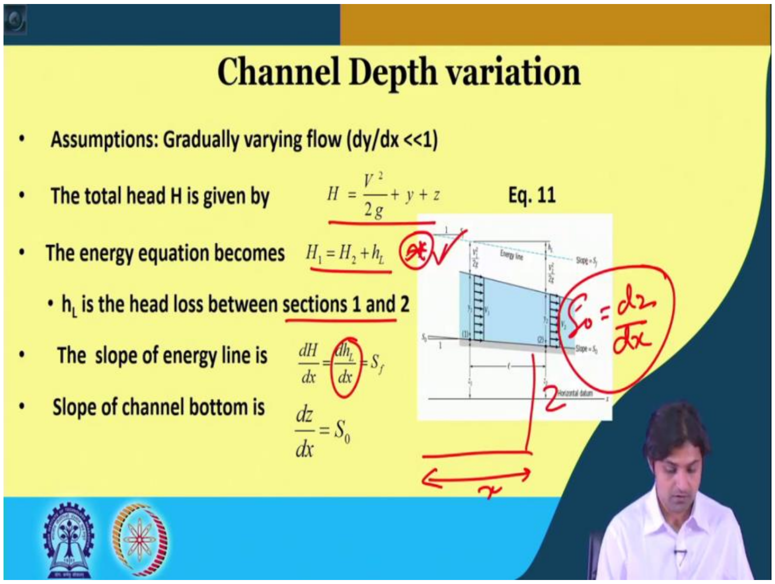

I think, we can proceed now, to a topic that is called channel depth variation. So, for Channel depth variation the general assumption is that, it is a gradually varying flow, that is, dy / dx is less than 1. We have studied this type of flow, gradually varied flow, uniform flow and rapidly varying flow. So, for channel depth variation, we assume that the flow is gradually varying. Therefore, the total head H is given by $\frac{V^2}{2g} + y + z$. This is equation 11.

Then the energy equation becomes, if we assume the total head H 1 because there was no loss enough going to point 2, this $H_2 + h_L$, where h $_L$ is the energy loss, where h $_L$ is the energy loss or the head loss between section 1 and section 2. The slope of energy line is, so this is the equation or this is the equation, so we say, if you use this equation here, we say that dH / dx is $\frac{dh_L}{dx}$.

Because other things if you see, delta H, if you divide by delta x, this equation, star equation, then we get dH / dx is equal to $\frac{dh_L}{dx}$ which is equal to S_f, because this is S $_f$. So, slope of the channel bottom is dZ / dx. Z, if Z is the height the datum, you know, so it becomes dZ / dx that is the slope that is what we had seen, in terms of x and this was the Z, the Z, datum Z. So, it becomes dZ / dx is the slope. Okay, I will just take all of this. So, if you go back to this diagram again. So, you see, this equation and this equation when hold.

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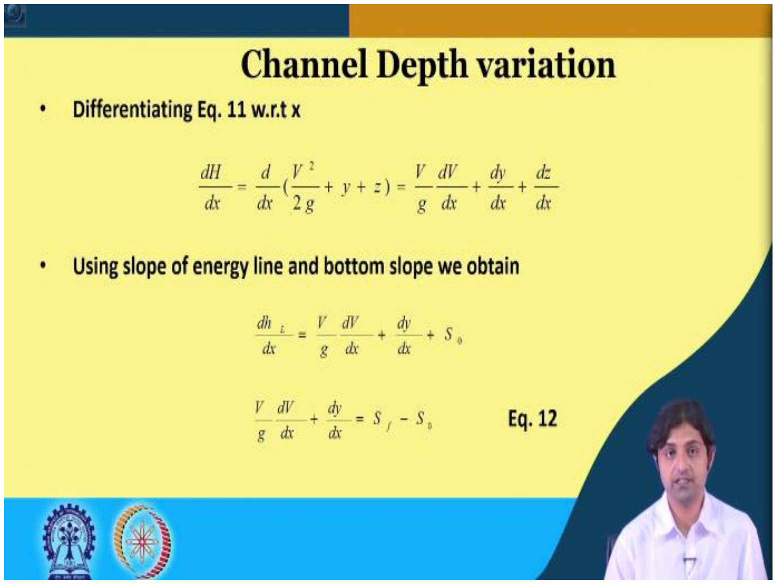

Now, if you differentiate 11, so equation number 11. This equation with respect to x, what are we going to get? So dH / dx is equal to d dx of whole H, this is H. So, differentiating one term by term so it will become 2V / 2g so it will becomes $\frac{V}{g}\frac{dV}{dx} + \frac{dy}{dx} + \frac{dz}{dx}$. So, using the slope of energy line and the bottom slope we can obtain, so dh / dx is also dh L / dx from the previous slide.

So, dV dx is the same and dy / dx is the same, but dz / dx can be written as as S_0. What is this value? This is S f . So, V / g dv dx + dy / dx is equal to S f, S 0 can be taken on the left hand side. So, we can, we are able to get $\frac{V}{g}\frac{dV}{dx} + \frac{dy}{dx} = S_f - S_0$ or this is called equation number 12, in our current slides.

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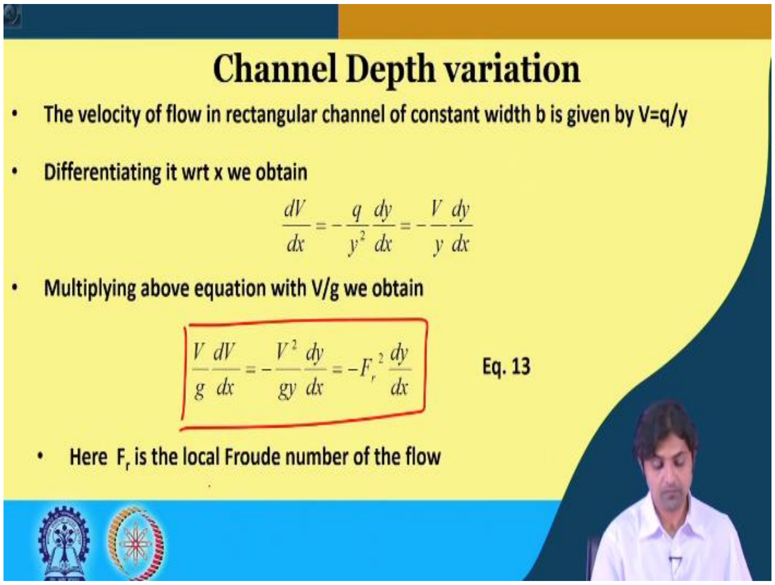

The velocity of the flow in rectangular channel of constant width b is given by, I mean, V is small q / y. If we differentiate this also with respect to x, we can obtain dV / dx is equal to $-\frac{q}{y^2}\frac{dy}{dx}$ and if you put back V, q is V y. So, if you put this in this equation, we get this one, so it will become - V / y dy / dx. So, if we multiply the above equation with V / g, this equation by V / g, we are able to obtain this equation.

So, V / g dV / dx, from here, and this also by V / g this will become - V square / gy dy / d x. and this V square / gy is Froude number whole square. So, actually we get V / g dV dx is equal to - Froude number whole square into dy / dx. I should take down all the ink. So, here F r is the local Froude number of the flow. So, here F r is the local Froude number of the flow, so equation number 13. So, you see equation number 13 is this and this is equation number 12.

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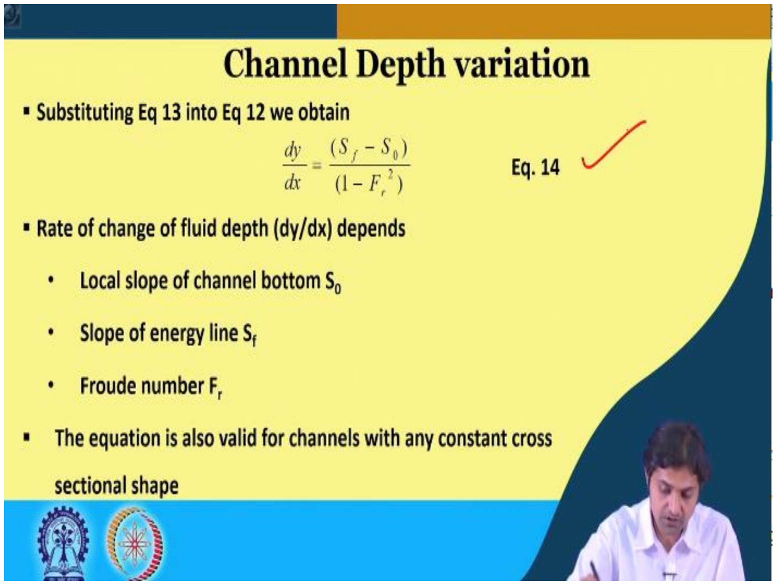

Proceeding forward, if we substitute equation 13 into equation number 12, we can simply write, see, going to equation number 13. So, first we should know equation number 12. So, this is equation number 12, this value we already know. What is that? V / g into dV D x is equal to - Froude number square into dy / dx, minus Froude numbers square into dy / d x. So, we know this, minus Froude numbers square into dy / dx.

So, if we take dy / dx common, it will be 1 - Froude number whole square is equal to S f - S 0 implies dy / dx is going to be S f - S 0 divided by 1 - Froude number whole square. So, and this is exactly what we obtain, dy / dx is going to be $\frac{(S_f - S_0)}{(1 - F_r^2)}$. So, rate of change of fluid depth dy / dx depends upon, so if you look closely at this equation the rate of change of fluid depth or dy / dx.

Now, if you remember in the beginning we related all the type of flows like uniform flow or rapidly varied or gradually varied flow with this value of dy / d x. So, this dy / dx depends upon the local slope of the channel bottom, which is called S 0, it also depends upon the slope of energy line S f and it also depends upon the Froude number or Fraud number F r. Now, this equation is also valid for channel of, is valid for channels with any constant cross sectional shape.

We derived this equation for a rectangular channel. However, it is valid for channels with any constant cross sectional shape. So, we have obtained the equation for dy / dx. Now, we will utilize this equation to study different dy by dx.

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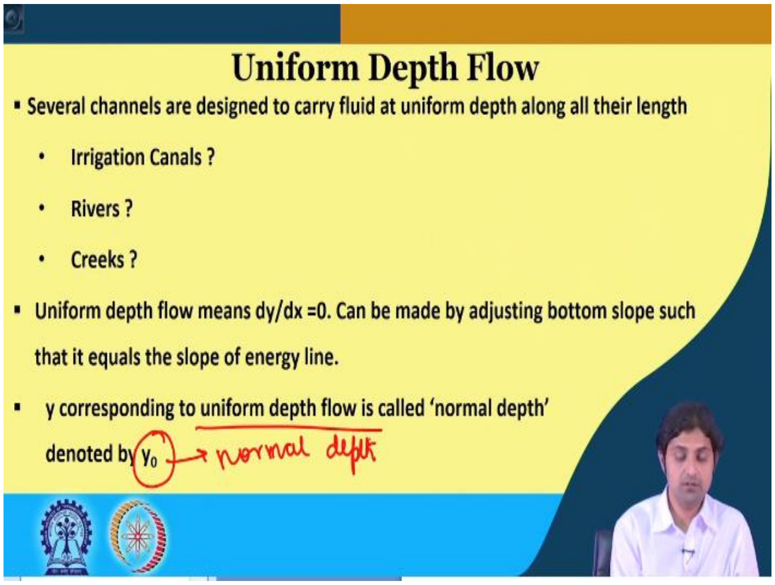

So, first thing that comes to mind is uniform depth flow. So, we know that there are several channels are designed to carry fluid at uniform depth along their channel, for example, irrigation canals, rivers, creeks. And uniform depth means the rate of change of y with respect to x is equal to 0, from equation number 14. So, if dy / dx is equal to 0 that means it is a uniform channel and this can be made by adjusting the bottom slope bottom slope.

Bottom slope is Z 0 because if we are going to design say an irrigation canal or something we can obviously change the angle of the, you know, the slope of the canal can be made by adjusting bottom slopes such that it equals the slope of this energy line for roughness, you know that can be known and we can calculate S f So, if S f is equal to S 0, then we are going to have uniform flow conditions and then this y, which will correspond to this uniform depth flow is called normal depth and it is denoted by y not. So, important thing to note is, that the y which corresponds to a uniform depth flow is called normal depth and it is denoted by y not, normal depth.

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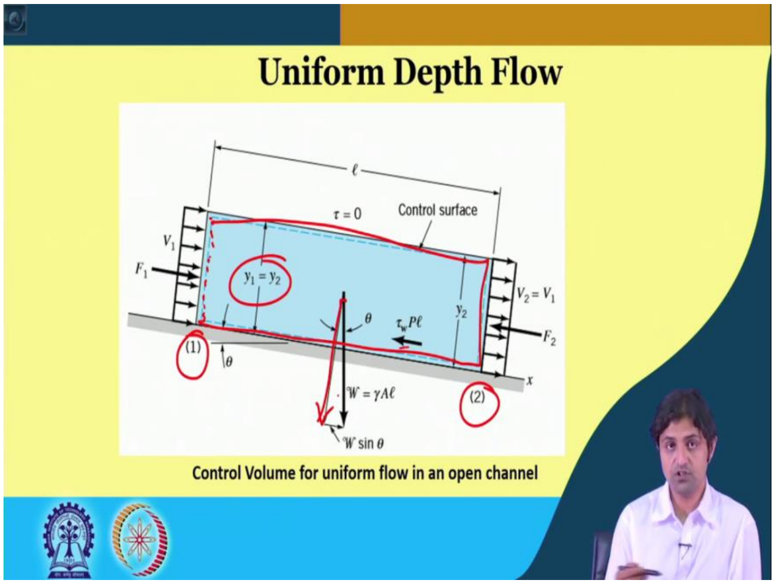

So, now, this is a cross section and we are seeing the control volume for uniform flow in an open channel. You please, we get because we are going to derive this. So, if you look at this figure very carefully, you see this is the uniform section 1 here. This is a uniform section 2, f 1 is the force, which we are going to calculate, hydrostatic force v 1 is the velocity or depth is y and y 1 and y 2, but since this is uniform depth, y 1 is equal to y 2.

This is the control surface given by this one and this is the if the bed shear stress is tau w and this theta is the slope, you know, the angle of the bed slope, if the weight is W, one of the components will be acting perpendicular to the, you know, surface and one will be acting along the surface. So, using the force balance and continuity equation we are going to derive something now, so, let us go and see.

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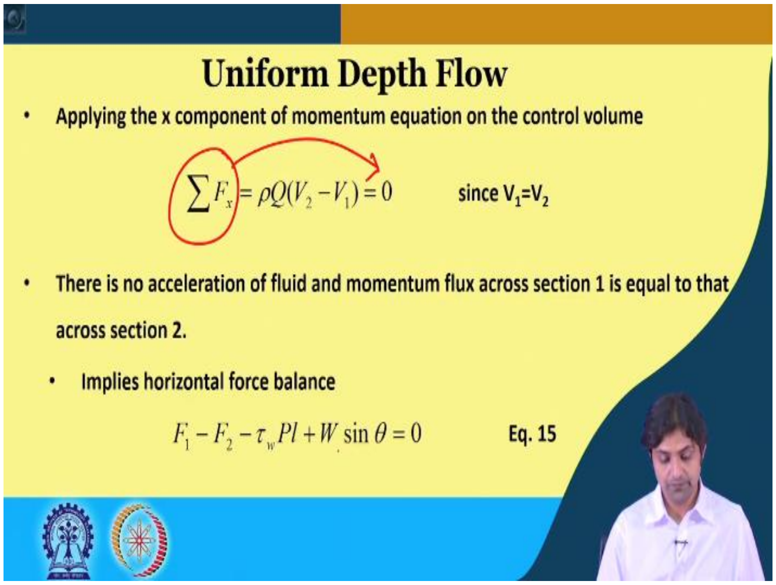

So, if we apply the x component of moment equation on this control volume. So, what we are going to see is sigma effects, that the net force is going to be rho Q v2 - v1, mass flux into v2 - v1, v2 is the velocity here and v 1 is here. And that is going to be 0, because v 1 is equal to v 2 in uniform flow, because we have taken the same y 1 is equal to y 2. So, the sum of the forces is going to be 0. There is no acceleration of the fluid and the momentum flux across section 1 is equal to that across the section 2.

Implies there is a horizontal force balance given by F x is equal to 0. So, simply writing, F 1 - F 2, F 1 is the force from section 1 hydrostatic force, F 2 is the force that is the force from the other side, that is, section 2, because the force acts normal, so it will be in the opposite direction at F 2, if the shear stress at the bed is tau w that will be acting against the flow so it will be in the negative direction. And the weight component of sin theta will be in x direction and that will be towards the flow direction.

Just taking you back to the, you see here, F 1 - F 2 - tau w Pl and this is the component mind, this is W sin theta that is supporting the flow, this is the equation. Now, we need to know what F 1, F 2 and another parameters are.

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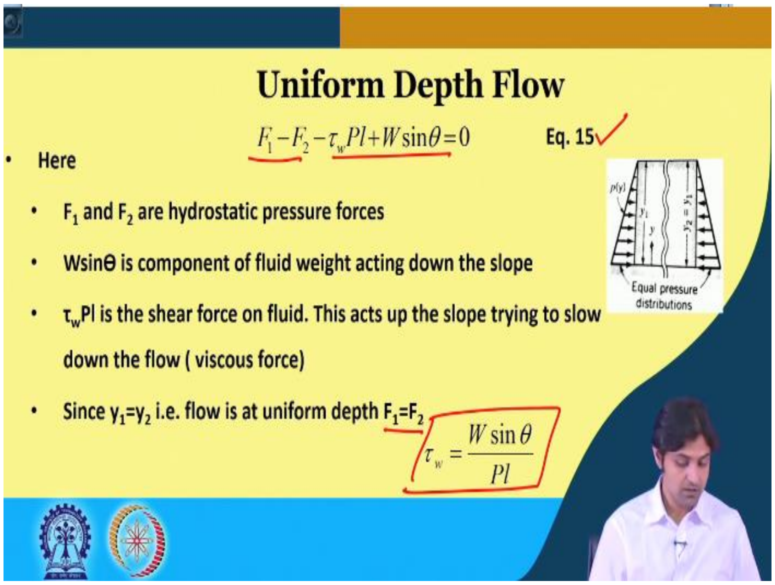

So, again rewriting the equation, $F_1 - F_2 - \tau_w Pl + W\sin\theta = 0$ and this is equation number 15. Here, F_1 and F_2 are hydrostatic pressure forces, as we have already told, W sin theta is the component of fluid weight acting down the slope, tau w Pl is the sheer force on fluid, this acts up the slope trying to slow down the flow, this we have already talked about. This happens because of the viscous forces.

Since, y 1 is equal to y 2, flow is at uniform depth. So, that means, F 1 is at uniform depth, F 1 = F 2. If, y 1 = y 2, F 1 = F 2. True. So, if F 1 = F 2, this gets cancelled out, so tau w Pl will be W sin theta. So, tau w can be written as W sin theta / P into l.

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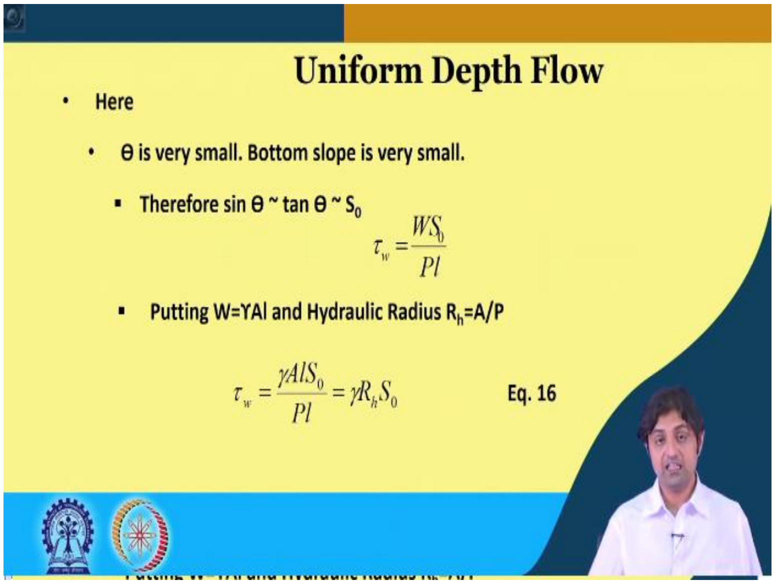

So, here theta is very small that means bottom slope is very, very small. Therefore, sin theta can be written as tan theta and that can be written as bottom slope S 0. So, we can simply write

$$\tau_w = \frac{W \sin\theta}{Pl}$$

. So, instead of sin theta we have written S 0. Now, if we put the weight as gamma A into l, so weight is gamma A into length l. And if we say now we have defined something called hydraulic radius, which is called A by the parameter P.

P, if you see we have always constantly been writing about P and you would be wondering what that P actually is, right from the beginning. So, you know, the shear stress tau w will act on the entire parameter. Which all parameters? That are the parameters which is wetted by the liquid. So, all those area where the water is, because the, so that is called the wetted parameter P. So, the parameter or the length across the entire, that is called the perimeter, which is, you know, wetted by the liquid.

So, if we put this term, so tau w will be gamma A l S 0 divided by P into l. So, l and l can get cancelled and we have defined already A / P, this A / P as R h. So, we can simply write, gamma R h into S 0 for uniform flow. So, shear stress will be gamma R h S 0. And this is an important equation that is called equation number 16.

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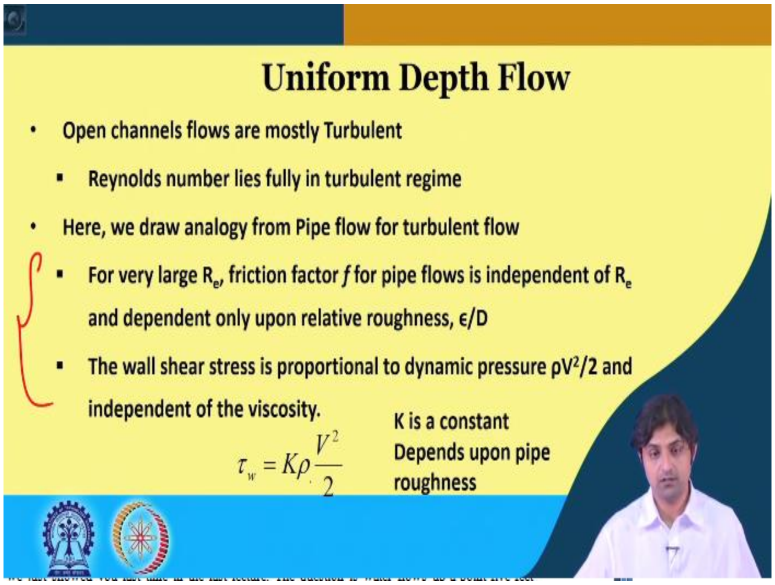

So, open channels are actually mostly turbulent and the Reynolds number lies fully in the turbulent regime. So, here what we are going to do and actually we are going to see the analogy from the pipe flow, which we will actually study as well. So, for a very large Reynolds number, friction factor f for pipe flow is independent of Reynolds number that is what we know. So, for very large Reynolds number, friction factor f for pipe flow is independent of Reynolds number and dependent only upon the relative roughness, epsilon by D.

And the wall shear stress is proportional to dynamic pressure rho V Square / 2 and is independent of the viscosity; we know this from pipe flow. And we, we as in the scientists' community had applied this analogy to open channel flow. So, using this analogy we can say that tau w can be written as K into rho into V square / 2g, K is a constant and it depends upon the pipe roughness, but this is just an assumption.

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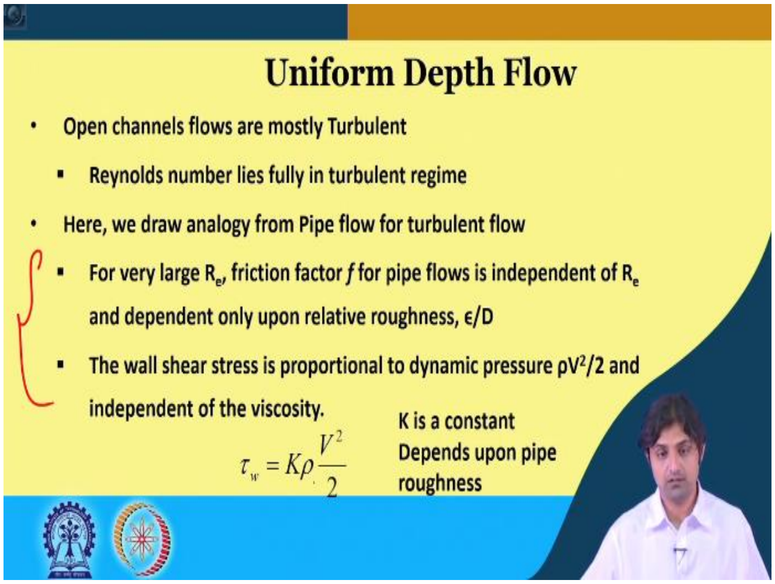

So, assuming similar dependence for high Reynolds number in open channel flows equation 16. You see, this equation 16 here, can be rewritten as, this one we already found, right hand side of equation, left hand side we substitute by $K\rho\frac{V^2}{2}$ and we get something like V is equal to C, another constant C. So, we write gamma / K rho, 2 gamma divided by K rho whole root as C. And this equation is the Chezy's equation and C is called the Chezy coefficient. That is an important equation.

So, V the velocity can be written as C under root R h S 0, where R h is the hydraulic radius. It was developed by a French engineer named Chezy while designing the canal, C is generally determined from the experiments. Now, you can find the dimensions yourself, you have done the dimensional analysis. If somebody is not able to, you can ask that to me in the forum. However, I erase this.

So, now, there is, you know, something called Manning equation. We have studied the Chezy's equation. We will be ending the class, this lecture right now and we will start our next lecture by discussing what a Manning's equation and Manning's number is. Thank you so much for listening. I will see you in the next lecture.