**Hydraulic Engineering**
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**Lecture-30**
**Introduction to Open Channel Flow and Uniform Flow (Contd.,)**

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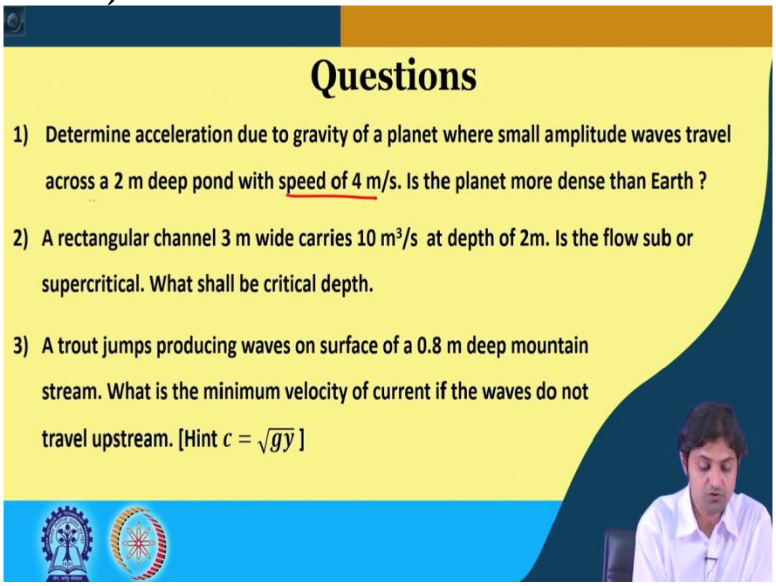


Welcome back. In the last class, we stopped the lecture by looking at the questions. So, there were 3 questions that we have to solve. And we start this lecture by going to solve these questions, you know. So, the first question, again just going back is, determine the acceleration due to gravity of a planet where small amplitude waves travel across a 2 meter deep pond with speed of 4 meters per second. So, you know, so the speed of the wave is given, c is given and we have also been given the depth.

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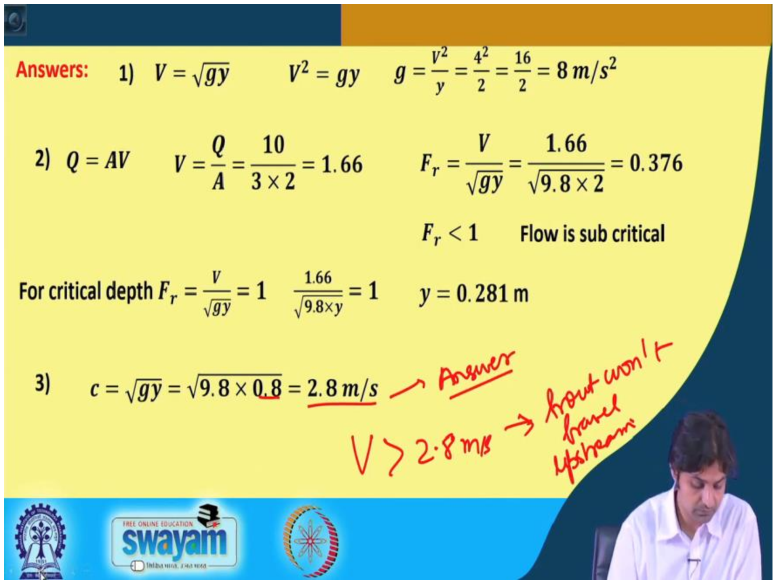


So, the way it is solved is, the first question, we know, V or c is equal to under root gy. So, V square can be written as g into y, squaring both side. Therefore, if we want to find out g, it will be V square / y, V or c. So, V was given as 4 meters per second and y was given as 2. So, this gives us an acceleration of 8 meters per second squared. Of course, the density, there are many other things that the density depends upon but if they say the acceleration due to gravity is less than 10 it should be less dense than the earth, I mean, very crude approximation.

This is not a very clear answer, but more importantly finding out this acceleration due to gravity was important. Second question, it has told us to ask, it has asked us to find out the flow regime. So, we know that Q is given by area into velocity. Therefore, the we have been given the discharge; we have been given the area. So, area is 3 into 2 and the Q was given as 10. If you go and look, go back and look at the problem, it says that the rectangular 3 meter wide, so this was the width, carries 10 meter cube, so this was discharge, at the depth of 2 meter, this was d.

Now, if this is asking if the flow is subcritical, supercritical. So, Froude number is given by V under root gy. So, V we have already found out using the discharge, it was 1.66 from here, g is 9.8 and depth is given as 2 in the question itself. So, this comes out to be 0.376, which means Froude number is less than 1 and therefore, the flow is subcritical, very simple to solve. Now, going, so and the second part was it we have to find out critical depth for the same flow condition.

So, see the same flow conditions is there V is not going to change. The only thing that we can change is y. And for the critical depth Froude number has changed from 0.376 to 1, therefore y will change. So, if we use this equation, V / under root gy is equal to 1, therefore substituting V will remain same 1.66, g is 9.8 and y we have to find out. Using this equation we can find out y is 0.281 meter so it has to be less shallow. Earlier the depth was 2 meter, here we have to make it 0.28 meters.

So, before starting the third question we should now revise. I mean just relook what the third question is. It says that the trout jumps producing waves on surface of a 0.8 meter deep mountain stream. What is the minimum velocity of current if the waves do not travels upstream? So, just going back, so what I am going to do, so basically, there is a velocity V of a stream and there is speed c of wave.

So, we have to find V, such that, it is greater than c. So, first half I will find out the speed of the wave which is dependent only on depth. Depth is given 0.8 meter. So, this comes out to be 2.8 meters per second. So, if the stream speed is greater than 2.8 meters per second, this means the trout would not travel upstream. So, that is the answer.

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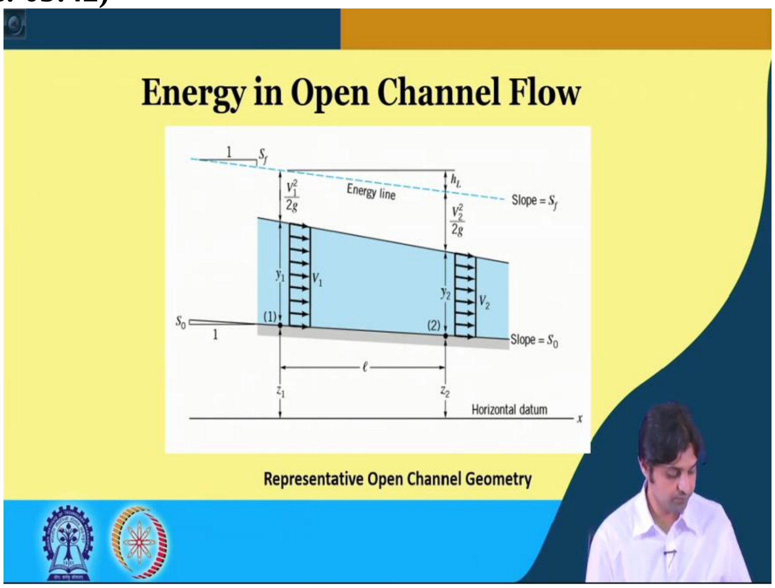


So, now, starting with a brand new sub topic here. What is the total energy in open channel flow? So, coming back from classification of open channel flow to the development of surface solitary waves due to a disturbance because free surface can distort, we are going to see what the energy in open channel flow is. Here, we have assumed, I am going to go into more detail but first try to explain you a figure.

So, this is the horizontal datum, this is any level and our channel is like this, you know, and the bed slope. So, all the surfaces in the world might not be horizontal, it could be at a very small angle to the horizontal and let us say that small angle slope of the channel is let us say S not will be a very small value but anyways to be able to consider there is a slope S not. If at cross section 1, the depth of the flow is y 1 and the velocity here is V 1 and at section 2, the depth is y 2 and the velocity is V 2.

We have indicated what the, you know, because of speed V 1, this is the energy line and other stuff, you know, V 1 square / 2g, V 1 V 2 square / 2g, you know. And this is the total head and if going from one point to the other point, the head loss is at h L. We will come to these terms again and this is something S f that we will come back to again what this slope S f is. S f is generally the slope of the energy that is lost. But that we are going to come very soon now. So, this is actually a representative open channel geometry.

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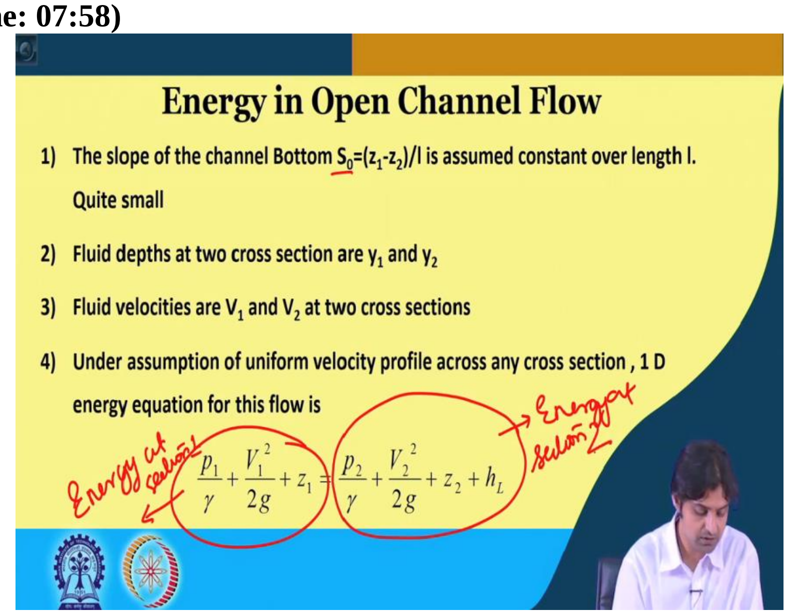


So, going back again, I was saying that all the channel will have a small slope. So, the slope of the channel bottom S 0 can be actually found out. How? If the height of this particular point is z 2 and this is z 1, then S 0 will be z 1 - z 2 and if this is the length of the channel that we have considered, it will be z 1- z 2 / l. And we assume that the slope of this channel S 0 is assumed constant over this entire length and this actually is very small S 0. As I said the fluid depth at 2 cross sections are y 1 and y 2. The fluid velocities are V 1 and V 2 at 2 cross sections.

Under the assumption of uniform velocity profile across any cross section, we assume that the velocity profile is uniform and not like the boundary layer that we have seen in our previous week's lectures. So, under the assumption of uniform velocity profile across any section one dimensional energy equation for the flow is we say that $\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1$, this is energy at section 1 is equal to $\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$.

So, if we assume, if there is a head loss h L, then this is if we account for energy loss then our energy should be conserved. If we know how much energy has been lost and we add it to the system, so this is energy at section 2.

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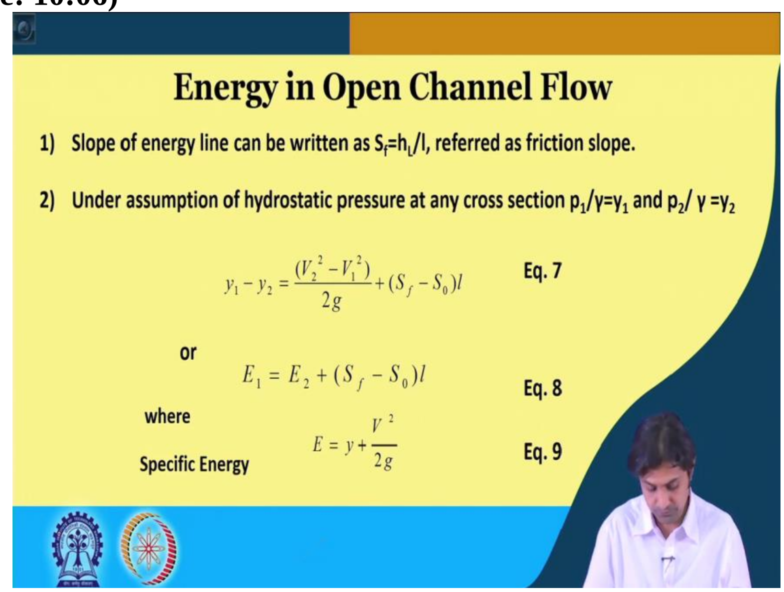


Actually, this is another way of writing, we can also write the slope of energy line. So, we had said that if h L is the head loss or energy loss, from going from point 1 to point 2, and if we divide it by length L, we can also write is, write it in terms of slope. So, if we say, S f is the slope of then we say it as h L / L. where L is the length of the channel in the consideration and this is called friction slope. So, under the assumption of hydrostatic pressure at any cross section, if we assume hydrostatic pressure, p 1 / gamma can be written as y 1, under hydrostatic.

And $\frac{p_2}{\gamma}$ is written as y 2. So, under this one and this one our energy equation from this changes to, so $\frac{p_1}{\gamma}$, this is $\frac{p_1}{\gamma}$, this is $\frac{p_2}{\gamma}$, V 1 was on this side on the left hand side while going it becomes negative on the right hand side and same, the same about S 0. Because, okay, I think I should go back here and try to do this on the slide. It would be much more easier. So, p 1 / gamma was y 1. This is y 2. So, $\frac{p_1}{\gamma}$, see I am writing the left hand side, it is becoming y 1.

And we bring this one on this side, the p $\frac{p_2}{\gamma}$ on the other side becomes y 1 - y 2. Let us concentrate, we also write $(V_2^2 - V_1^2)$. So, V 1 we have brought on this side + z 2 - z 1 + h L. So, I am writing y 1 - y 2 is equal to $(V_2^2 - V_1^2)$ / 2g, z 2 - 1. So, you remember what this is, z 1 - z 2 was S 0. So, it becomes minus of S 0 l and h L is S f into L. So, it becomes y 1 - y 2 is equal to $\frac{(V_2^2 - V_1^2)}{2g} + (S_f - S_0)l$, same as given in the next slide. This is the equation, simplified equation that we get.

And this we call as equation number 7 or in other words we can say, so at energy at 1, energy grade line at 1 is E 1, which is y 1 + V 1 square / 2g and on the right hand side E 2, so if we say E 1 is equal to y 1 + V 1 square / 2g and E 2 is y 2 + V 2 square / 2g, then we get E 1 is equal to E 2 + S f - S 0 into L. Equation at where E is a specific energy. What is this E? We call E as specific energy, which is given by y + V square / 2g at any point. You know, at any point where the specific energy is given by y + V square / 2g.

So, this is a new concept called specific energy to which you have been introduced now. It is the sum of the depth. So, the depth and the kinetic energy head. And this is equation number 9 that we are talking about.

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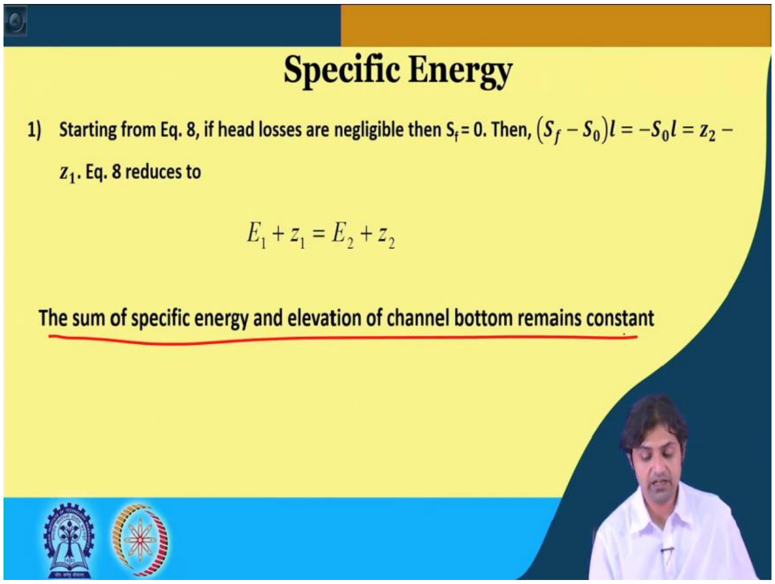


So, going to, I mean, more detail into specific energy if we start from equation number 8. What was equation number 8? It is E 1 is equal to E 2 + S f - S 0 into L. If head losses are negligible, then S f is equal to 0 that is true, then S f - S 0 in that will become minus S 0 into L or z 2 - z 1 c. Then equation 8 can be written as $E_1 + z_1 = E_2 + z_2$. How? We are we I will just show you, it is better to show. So, equation number 8, if S f is 0, and S 0 is given as z 2 minus, z 1 - z 2 / L.

So, E 1 is equal to E 2 + - S 0 into L and S 0 is given as z 1- z 2 / L. So, S 0 L, so E 1 is equal to E 2 + minus of z 1 - z 2. So, E 1 is equal to E 2 - z 1 + z 2 and if you bring z 1 on this side, it becomes E 1 + z 1 is equal to E 2 + z 2. So, this is the result that we have got, which indicates that the sum of specific energy and the elevation of the channel bottom remains constant, if the head losses are negligible in a frictionless case.

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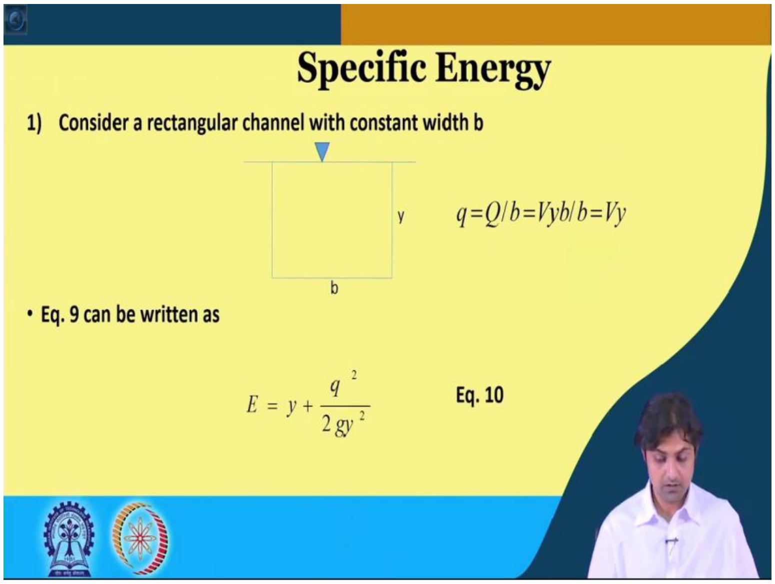


Now, for the specific energy, we consider a rectangular channel with a constant width of b, something like this and having height y. So, q is going to be Q / b discharge per unit width, Q is given by velocity into y into b. So, q is going to be V is the velocity into the depth. Now, using this equation 9. What was equation 9? So, equation 9 was E is equal to y + V square / 2g. And if we use this thing in equation number 9, y will remain y and instead of V we write q / y. So, we can write q square / 2 g y square.

So, in terms of y and q, we have written E is equal to y + q square + 2 g y square. If you see, this is a cubic equation in y, anyways, we will come to it and this is written as equation number 10.

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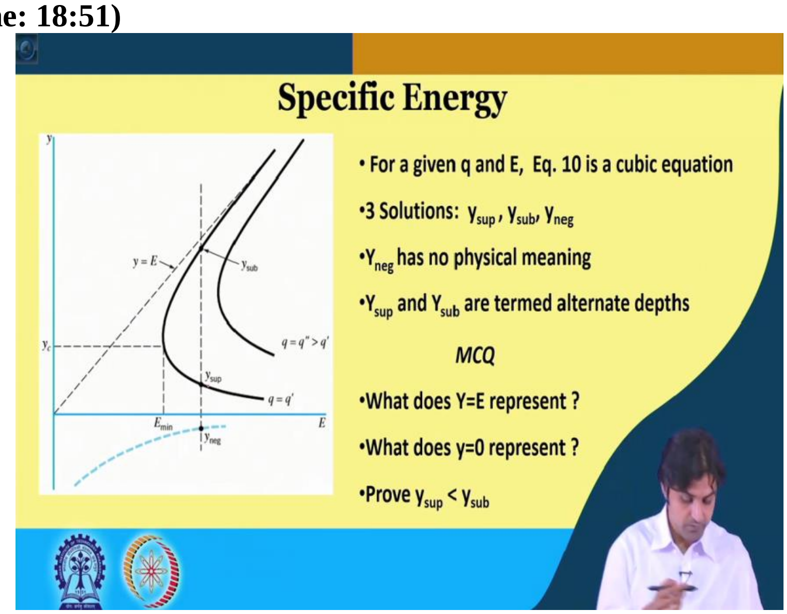

So, for this particular case, if our equation number 10, if we plot equation number 10, this is the type of curve that we get. Where, E specific energy has been plotted against y, for specific q. So, we have fixed the q and try to plot E as a function of y, and this is what we get, we are going to go on it one by one. So, for a given Q and E, so if we choose a q and E equation 10 is a cubic equation in y, so for that there are going to be 3 solutions. y super, y sup, y sub and y negative.

So, if you take on one curve, there is will be 3 y. This will be 1, this will be 2 and this will be 3. So, actually y negative because this is negative y, this will have no physical meaning, y negative will have no physical meaning, y sup and y sub are termed as alternate depth. So, this one and this one are actually called alternate depths. Because it is cubic equation we get 3. One is not possible and the 2 are called the alternate depth.

Do not worry about sup and sub and negative for now. Now, my question to you is, I have some question. What does this line represent, y is equal to E? Secondly, what does y is equal to 0 represent? Where is y is equal to 0? y remains 0 all the time, so this is the line on E axis and we also have to prove that y sub is greater than y sup. Of course, we can see this, but we have to show.

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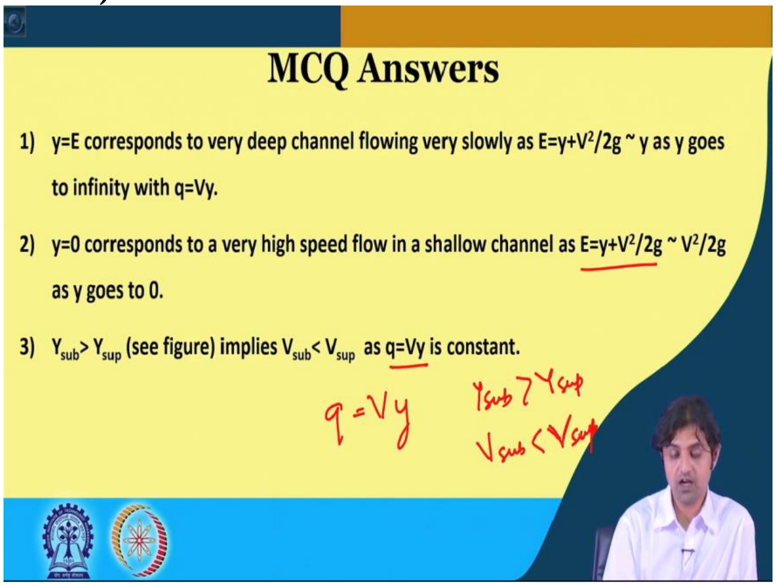


So, the answer is y is equal to E corresponds to a very deep channel flowing very slowly. Why? Because E is written as y + V square / 2 g and this will go to y, as y goes to infinity, because, you know, in a deep channel y is very deep. Secondly, y is equal to 0 corresponds to a very high speed flow in a shallow channel, because if you see, E is given as y + V square / 2 g, because in a shallow channel this term will turn out to be V square / 2g, as y goes to 0.

And the third is y sub is greater than y supercritical, see the figure, because this implies that velocity of supercritical is greater than velocity or subcritical as q is equal to V y is constant. So, q is given as V into y. So, from the figure, we can see that y sub, if y sub is greater than y sup, you know, then V sub has to be less than, because q is equal to V y is constant, I mean, it is constant.

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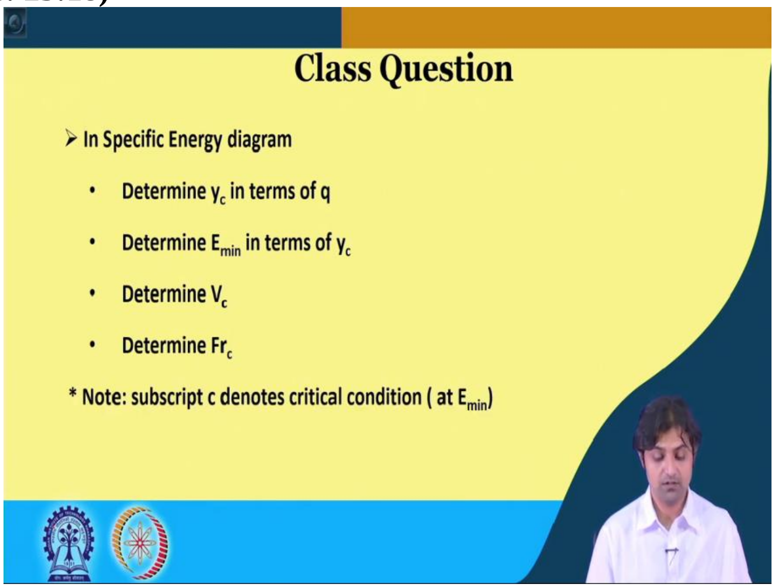


Now, the class question is, in specific energy diagram, determined y c in terms of q, also determine E min in terms of y c and determine V c, determine Froude number. So, we are let us go and see what E min and those things on the diagram is. So, this is the diagram this is energy minimum. This is y critical, so this should correspond to critical flow depth, this is y is equal to E and things like that.

So, we are going to solve this question. Look at the question once again. So, determine y c in terms of q, determine E min in terms of y c, determine V c, determine Froude number at critical depth, so everything is about critical depth.

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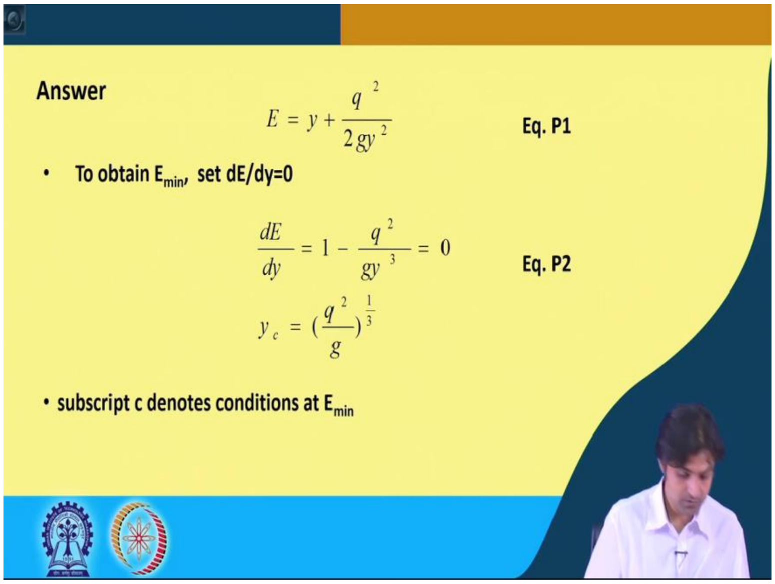


So, then we start with the equation that we already know. So, we know this equation, E is equal to y + q square divided by 2 g y square. This is equation number 1. Now, if we want to obtain E minimum we have to set dE / dy is equal to 0. And if we set dE / dy is equal to 0 then it becomes one - q square - 2gy cube, something like that, or 1 - q square / gy cube is equal to 0. I will take down this. So, dE / dy is 1 - q square / g y cube is equal to 0.

So, to solve this, and this will give the corresponding y that is going to come from this equation is going to give us y critical. So, we can write q square is equal g y cube. So, that means, y cube is g, sorry, y cube can be written as q square / g. That means y is given as q square / g to the power 1 / 3 and because it is critical, we write at critical. So, that is y c. So, that is the first thing that we obtain, y c in terms of q, this is equation 2.

Subscript c here, denotes conditions at E minimum. At E minimum we say critical condition. Now, if you substitute y c into equation number 2, so if you substitute y c in equation number 2. Sorry, y c in equation number 1. So, E will be y + q square / 2g and now, y c we have got q to the power 2 / 3 into g to the power 1 / 3. No, we can actually use the value of q square. Sorry, erase all ink.

So, we can use E is equal to y c + q square can be written as g y c whole cube divided by 2gyc whole square. So, g and g will get cancelled, y c cube will get cancelled with square. So, E will be y c + y c / 2. That is 3y c / 2 and let us see our solution at the back, 3 y c / 2.

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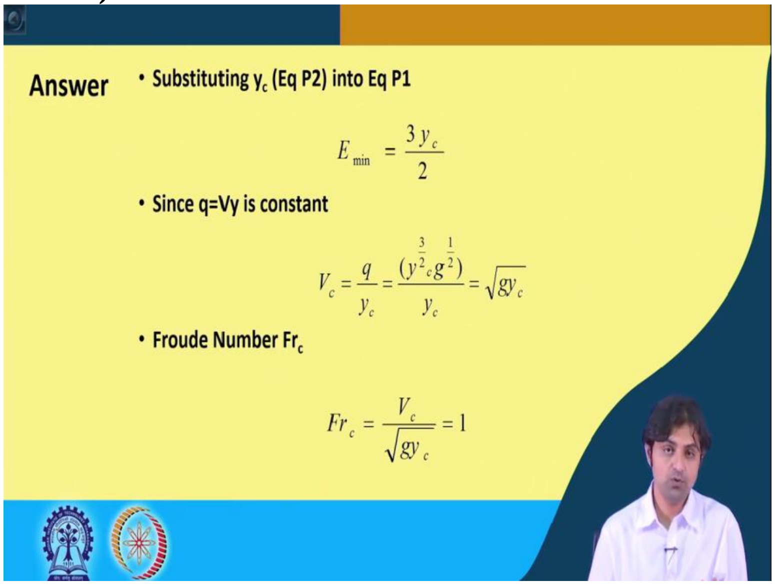


So, E min is going to be 3 y c / 2 and since q is equal to V y is constant we can obtain V c. So, this is going to be q, we already know in terms of y c and g. So, we are going to get at critical under root g y c, which we already know at Froude number is equal to 1, Fr c is equal to, now Froude number is V c / under root gy c, so that is going to be 1, but anyways, we will calculate. So, Froude numbers it is coming to be 1. So, actually we have only looking at the curve without assuming that it was already a critical flow, you know, we have derived this.

So, Froude number at this particular point came out to be 1. So, this is the point where the flow regimes are separated, I will just go back to this. So, we have calculated this just looking at by plotting the figure, you know, that is why this was subcritical and this was super critical. There will be a similar curve like this here as well, there will be a negative here, there will be a sub here, sup and E min. So, for each q this is a different case.

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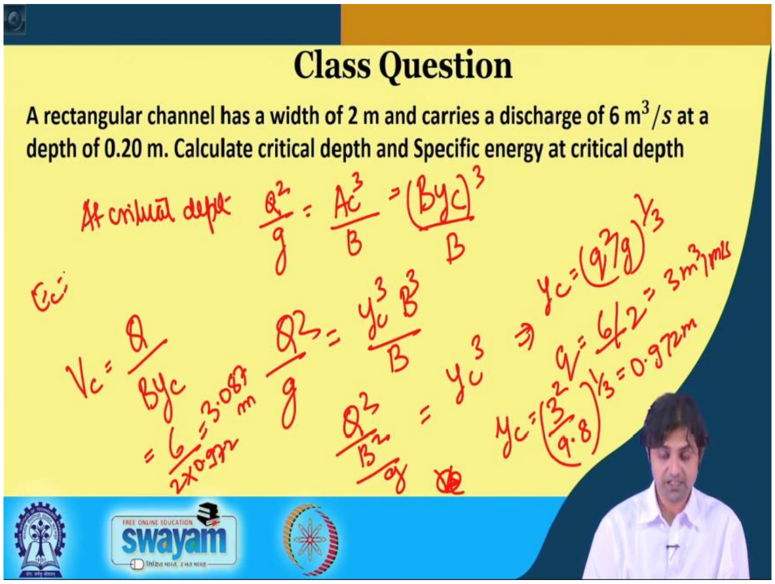


So, we will solve a small question. The question is, a rectangular channel has a width of 2 meter and carries a discharge of 6 meters cube per second at a depth of 0.20 meter. Calculate the critical depth and specific energy at critical depth. So, we have already seen that at, so if we write in terms of Q, we can write actually A c, you know, in terms of area, this if this is the B, you know, this is the channel width. And A c is, you know, Ac simply is the area, so we write By c. So, not b y c, a whole cube / B.

Hence, Q square / g is equal to y c whole cube B cube / b. Therefore, we can write Q Square / B Square / g is equal to y c whole cube. I mean we already know y c is q square / g to the power 1 / 3. So, q we already know from this question, Q we know, so 6 divided by 2 per unit width is 3 meter cube per meter per second. Simply, y c we can get, y c is q square is 3 square divided by 9.8 to the power 1 / 3 is 0.972 meter.

And velocity is, so 6 divided by 2 into 0.972 and that comes out to be 3.087 meters or specific energy is 3 / 2, you know, you can either do 3 / 2 y c or just you can, you know V c, you know y c, I will just erase everything.

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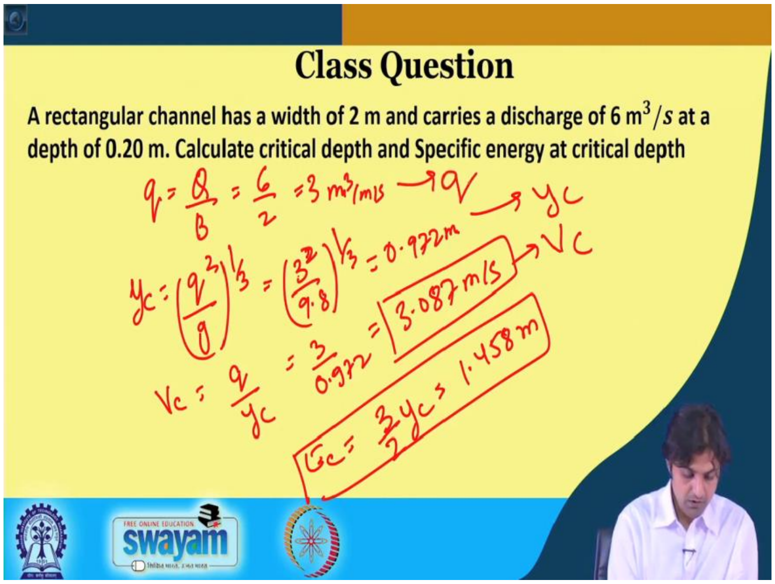


And now, simply try to, you know, do it with the knowledge that you have. So, in the derivation we know that Q / B, so 6 / 2 is 3 meter cube per meter per second. And y c by our formula is q square / g to the power 1 / 3, that is what it came. So, substituting the value, so 3 square / 9.8 to the power 1 / 3 and this gives 0.972 meter. V c is the q / y c because that was the equation that we are using.

V c is q / y c. So, 3 divided by 0.972 is equal to 3.087 meters per second. E c is equal to, so everything you know, V c is known, y c is known and this is q. So, this is how you will solve this question. So, just going at critical depth.

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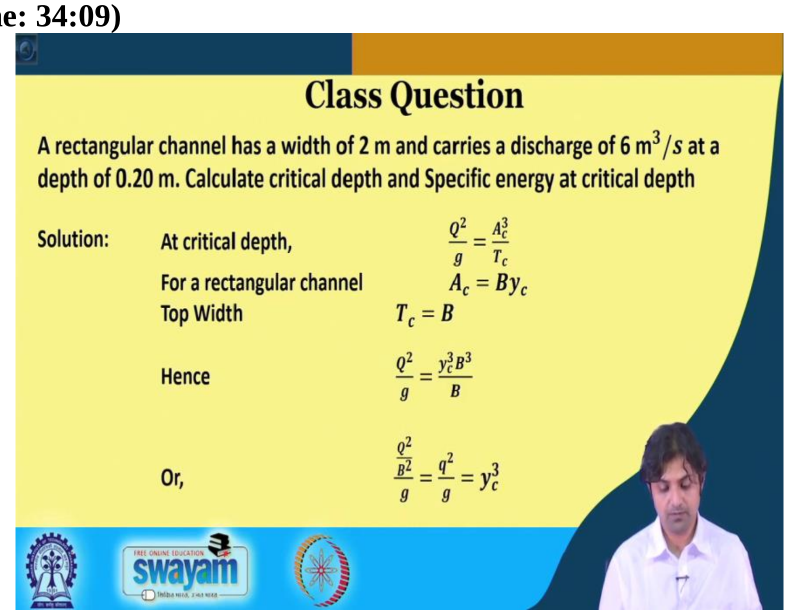


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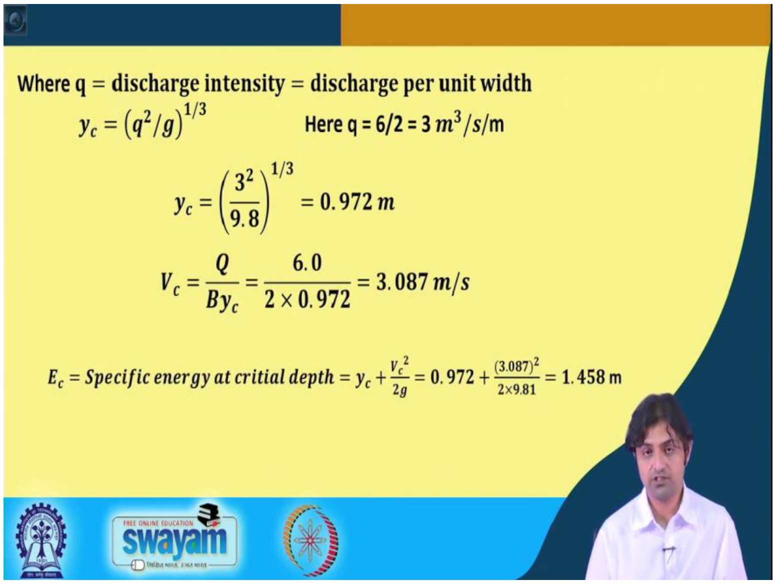


It is a general approach. So, I have included these solutions so that it is easy for you to, you know, when you get the slides you can actually see, so this is a different method of solution.

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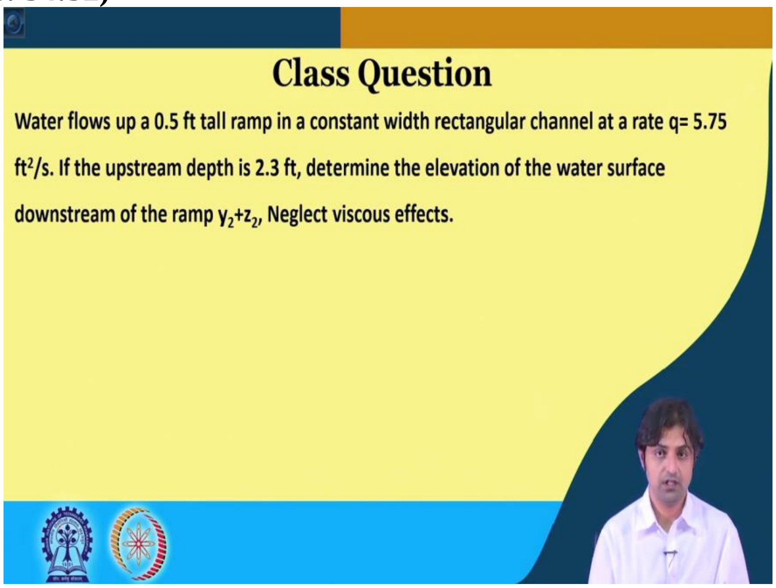


So we have another question, but we will like to stop at this point right now and start our next class by solving this question. Thank you so much.