Hydraulic Engineering

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Lecture-26

Dimensional Analysis and Hydraulic Similitude (Contd.,)

Welcome back students. So, last 3 lectures we have studied Buckingham Pi theorem, solved some problems. Solved 3 different problems actually and see how do we choose the problems. There was at one point in the lectures where I told, what if we choose a different set of repeating variables. So, the topic related to that is called uniqueness of Pi term and that is what we are going to start with today.

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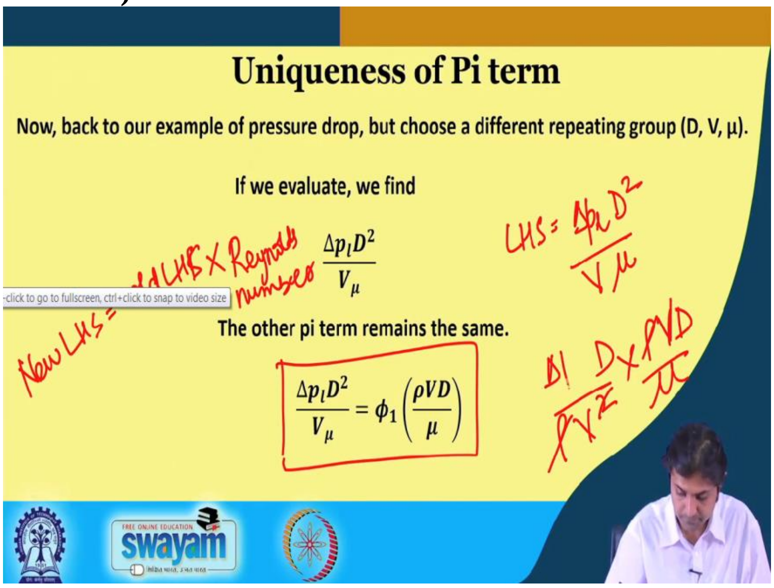

So, uniqueness of Pi term. So, now, if we go back to our example of pressure drop, but choose different repeating variable, D V and µ for example. If we evaluate, we will find that the first Pi term is going to be delta p l D square and µltiplied by V into µ. The other Pi term will remain the same on the analysis. So, we get a slightly different Pi term here. So, this is the another relationship.

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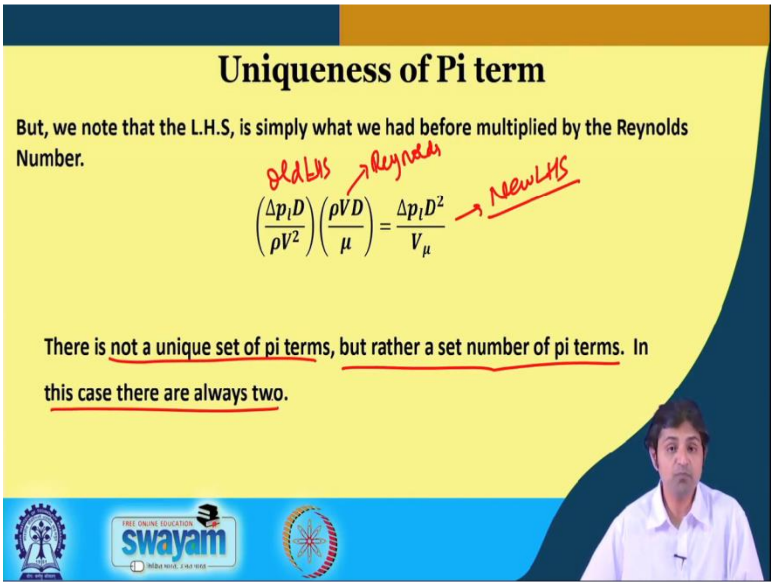

But, we note that the left hand side, so, the left hand side is this, delta p l D square. So, I will write it down, LHS actually is delta p l D square V into µ. And that is simple what we had before µltiplied by Reynolds number. So, you remember, what the last time, what the Pi term that we got was, we go back and find out, delta p l D / ρ v square µltiplied by Reynolds number ρ V D / µ.

So, this V and V gets cancelled, this D, D becomes, ρ and ρ gets cancelled. So, that is the correct. So, this new LHS actually is old LHS µltiplied by Reynolds number. So, it will become, delta p l .D ^{2} / V µ. So, we have actually seen that the LHS is simply what we had before µltiplied by the Reynolds number, you see, here. So, this is old LHS, this is Reynolds number and this is new LHS, a left hand side of the same equation.

So, the as conclusion is there is not a unique set of Pi terms, but rather a set number of Pi terms, in this case, there are always 2. So, there are no unique set of Pi terms, but a set number of Pi terms. So, that the number of Pi terms will remain fixed is 2, what will that be, that is not fixed.

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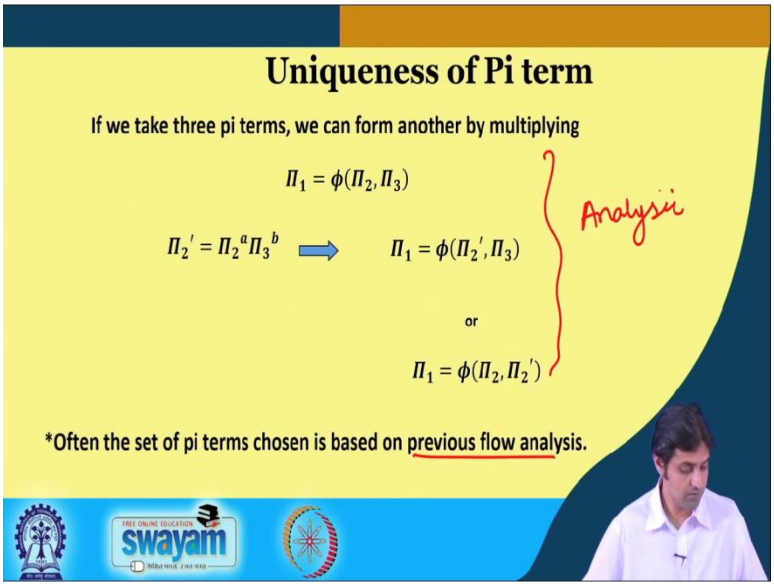

If we take 3 Pi terms, we can form another by µltiplying. So, if there is Pi 1 is equal to Phi Pi 2, Pi 3. So, we can form one by µltiplying Pi 2 to the power a, Pi 3 to the power b, fine, or Pi 1 is a function of Pi 2 dash, Pi 3, where this is Pi 2 and this is what exactly has happened in the last problem that we, last question that we were doing, or Pi 1 can be written in a function of Pi 2, Pi 2 dash because it already contains Pi 3, so, not a problem.

So, if this is the original case, we can always write Pi 1 is again a function of Pi 2, Pi 3 or Pi 2, Pi 2 dash and Pi 2 dash is the simple µltiplication to raise to the power a and b of Pi 2 and Pi 3, because this will also contain both Pi 2 and Pi 3, this will also contain Pi 2 and Pi 3. So, often the set of Pi terms chosen is based on previous flow analysis. So, what Pi terms will come is generally chosen by the previous flow.

So, when a flow comes, terms like Reynolds number, Froude number those things are very, very important. And because of those previous experiences, we try to modify our, you know, a Pi terms, dimensionless Pi terms using this analysis, you know, µltiplying with another dimensionless numbers and things like that, to obtain those well known set of dimensionless Pi terms.

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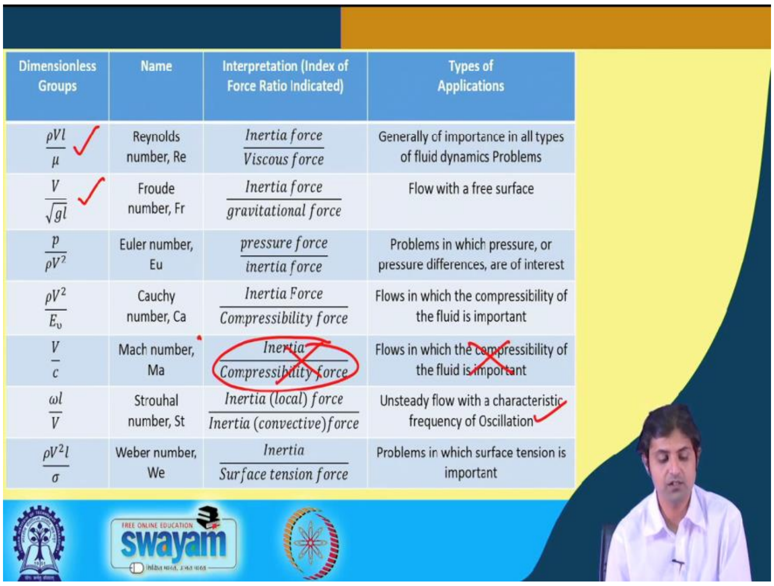

Dimensionless Groups	Name	Interpretation (Index of Force Ratio Indicated)	Types of Applications
$\frac{\rho V l}{\mu}$	Reynolds number, Re	$\frac{Inertia\ force}{Viscous\ force}$	Generally of importance in all types of fluid dynamics Problems
$\frac{V}{\sqrt{gl}}$	Froude number, Fr	$\frac{Inertia\ force}{gravitational\ force}$	Flow with a free surface
$\frac{p}{\rho V^2}$	Euler number, Eu	$\frac{pressure\ force}{inertia\ force}$	Problems in which pressure, or pressure differences, are of interest
$\frac{\rho V^2}{E_v}$	Cauchy number, Ca	$\frac{Inertia\ Force}{Compressibility\ force}$	Flows in which the compressibility of the fluid is important
$\frac{V}{c}$	Mach number, Ma	$\frac{Inertia}{Compressibility\ force}$	Flows in which the compressibility of the fluid is important
$\frac{\omega l}{V}$	Strouhal number, St	$\frac{Inertia\ (local)\ force}{Inertia\ (convective) force}$	Unsteady flow with a characteristic frequency of Oscillation
$\frac{\rho V^2 l}{\sigma}$	Weber number, We	$\frac{Inertia}{Surface\ tension\ force}$	Problems in which surface tension is important

So, as we said, there are some set of very famous, you know, dimensionless numbers. What are those? For most important is Reynolds number, the dimensionless groups is written as ρ V l / μ. And Reynolds number by definition is ratio of the inertia force by viscous force. It is generally of, I mean, it is very important in all types of fluid dynamics problems. Reynolds number will be there in any flow problems.

Another such quantity is Froude number. So, it is given by V under root g l and Froude number is the ratio of inertial forces by gravitational force. Reynolds number was inertial force by viscous forces. Whereas, Froude number is the ratio between inertia force and gravitational force, it is important in flows with a free surface. For example, when we will study open channel flow, we will see Froude number is an important quantity.

There is another number, that is, Euler number, which is E u, given by p / ρ V square, it gives the ratio between the pressure force and the inertia force. What type of applications this is important to? It is with problems in which pressure or pressure differences are of interest. There is another number called Cauchy number. Cauchy number is given by ρ V square / E v, it is the ratio of the inertia force by the compressibility force. So, Cauchy number is important in flows in which the compressibility of the fluid is important. So, we assume water is incompressible, but there are fluids like air where the compressibility is important and in those type of flows Cauchy number is important.

There is another very famous dimensionless group is called Mach number. It is given by V / c, you know and so, this is not correct it is related to the sound. So, this is the velocity, fluid velocity by the speed of sound. And this is required in applications like aeroplanes. So, sorry for this error here, typing error.

There is another number of calls Strouhal number. Strouhal number is given by omega l / V. It is the ratio of the inertial forces, local forces by inertia convective forces. It is important in unsteady flow with a characteristic frequency of Oscillation. This also plays an important role. The finally, there is one number called Weber number W e, which is the ratio between the inertia and the surface tension forces. And it is very important, this number is important in which surface tension is important.

So, you have, I mean, I have gone through these types of applications where such flows are important. When you get a question or when you are doing experiments, please see the type of experiments you are doing and in which category they are falling. And while doing the dimensional analysis, you should keep in mind which number to arrive at. For example, in any flow problems best is to obtain the Reynolds number and Froude number. Froude number when the gravity plays a role. For example, open channel flow. But Reynolds number will likely be there in all the fluid flow related problems.

So, this is a brief, you know, overview. I will go a little bit more detail into some of the important numbers that I have discussed.

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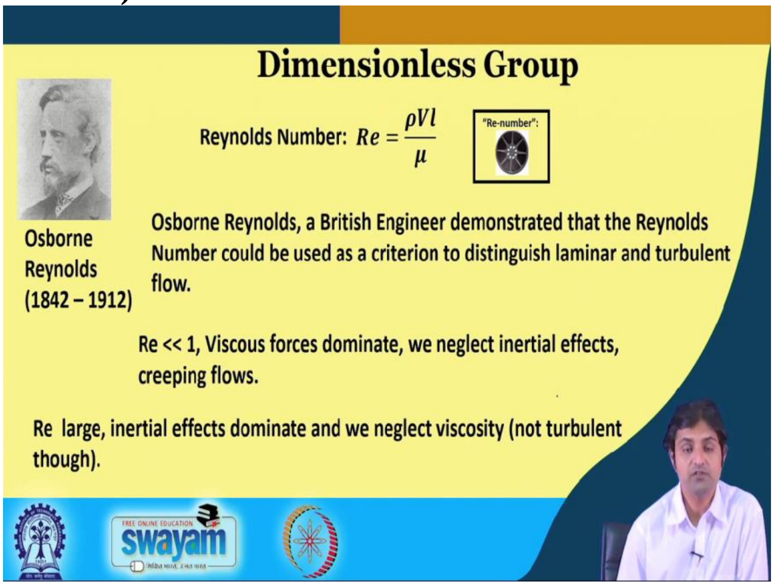

So, Reynolds number, here, is given by ρ V l / μ. This was given by Osborne Reynolds. So, Osborne Reynolds was a British engineer who demonstrated that Reynolds number could be used as a criterion to distinguish laminar and turbulent flow. If Reynolds number is very µch less than one, viscous forces dominate and we neglect the inertial effects and this is called creeping flows, which we have already seen, when we were doing the Stokes derivation. If Reynolds number is large, inertial effects dominate and we neglect viscosity.

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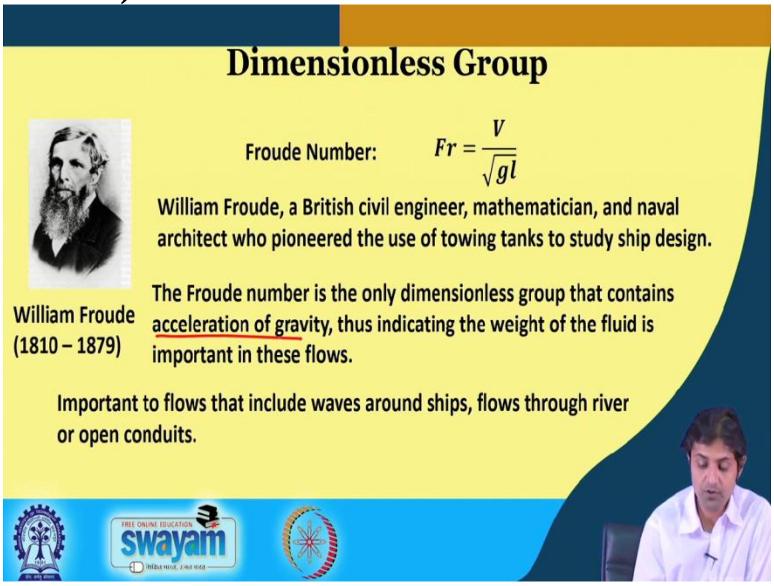

The other is Froude number, this is given by V under root gl. William Froude he was a British civil engineer and mathematician who pioneered the use of towing tanks to study ship design. The Froude number is only dimensionless group that contains acceleration of gravity, thus indicating that the weight of the fluid is important in these flows. So, gravity is very important. This is important to flows that include waves around ship, flows through river or open conduits, as I said, open channel flow. You have been talking about it.

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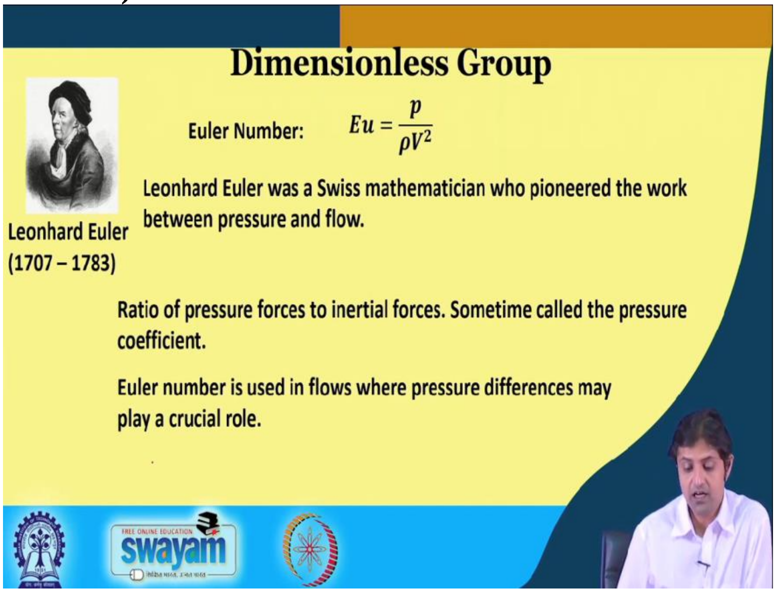

Euler number. So, it was given by Leonhard Euler and he was an, Leonhard Euler was a Swiss mathematician who pioneered the work between the pressure and flow. So, Euler number is the ratio of pressure forces to inertial forces. Sometimes this is also called the pressure coefficient. Euler number is used in flows where pressure differences may play a crucial role.We have already discussed this.

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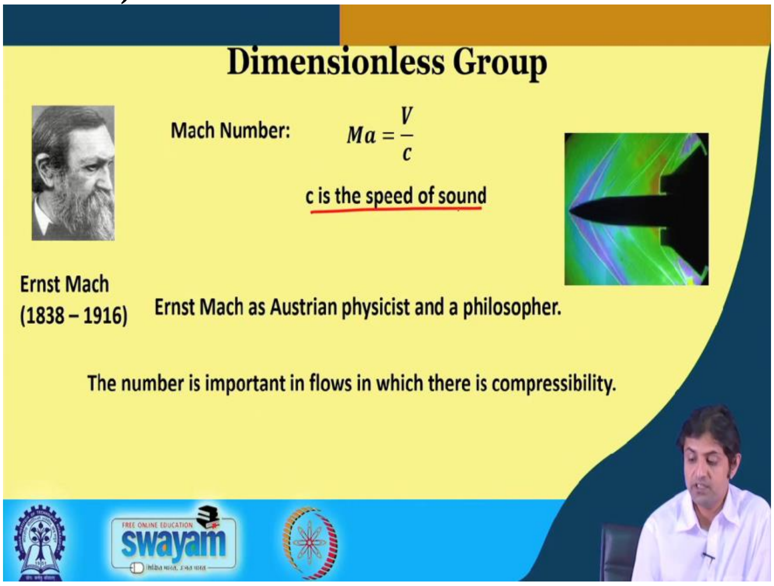

Mach number is the ratio of V / c. Here, c is the speed of sound and it was given by Ernst Mark as Austrian, he was an Austrian physicist and philosopher. The number is important in flows in which there is compressibility, you know, because it has to do something with the speed of sound as well.

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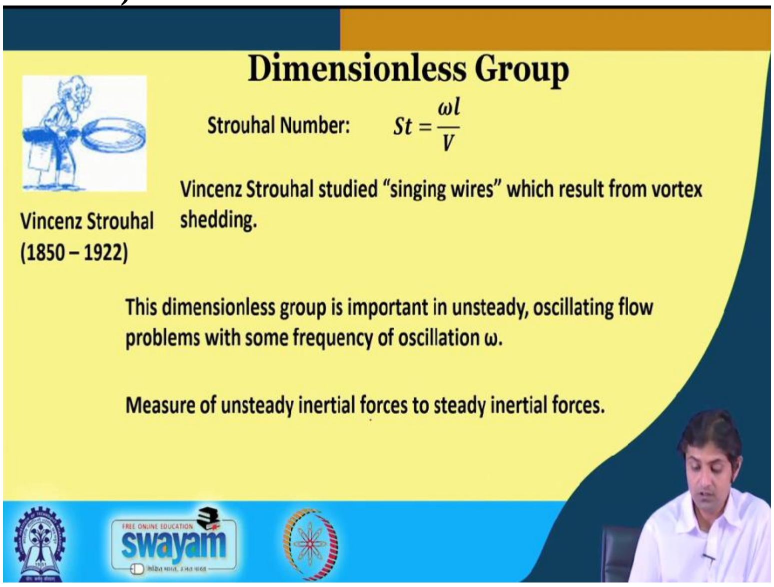

Strouhal number given by Vincenz Strouhal, he studied "singing wires" which result from vortex shedding. This dimensionless group is important in unsteady and oscillating flow problems. And this oscillating flow if say for the frequency of oscillation omega and this is how this is given it is it gives a measure of unsteady inertial forces to steady inertial forces. And this is quite an important, you know, number.

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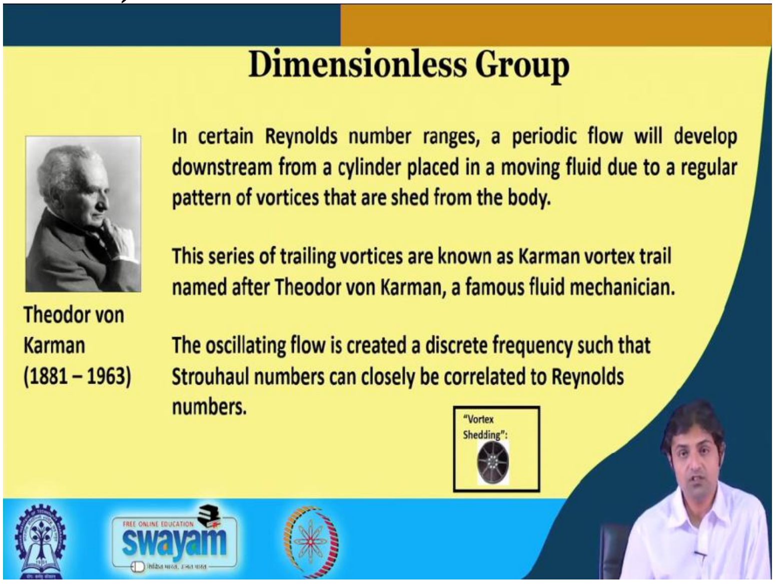

So, in certain Reynolds number ranges, a periodic flow will develop downstream from a cylinder placed in a moving fluid due to a regular pattern of vortices that are shed from the body. The series of trailing vortices are known as Karman vortex trail named after Theodor von Karman, a famous fluid mathematician and the oscillating flow is created a discrete frequency such that Strouhaul numbers can closely be correlated to Reynolds number, you know.

So, in vortex shedding, this is Strouhaul number is very important and this was given by Theodor von Karman is a very, very famous name, we have heard about him in the last week's lecture as well.

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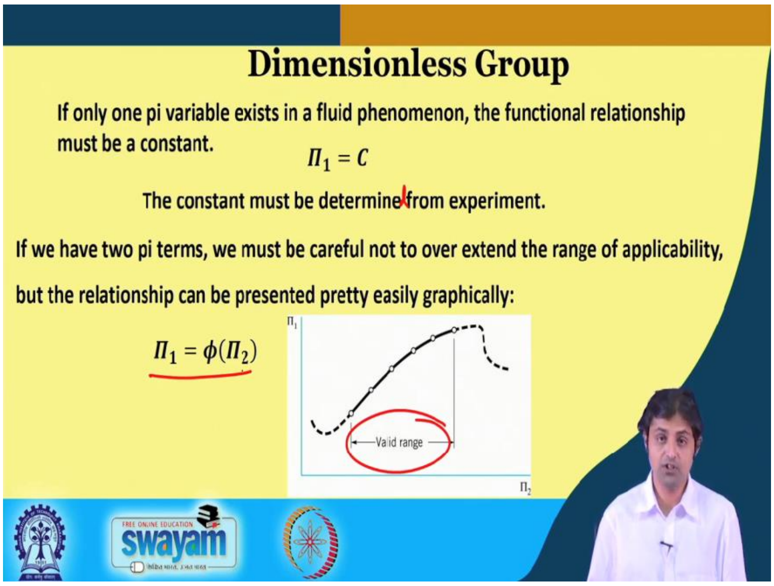

So, coming back to the dimensionless group, suppose, if only one Pi terms exist in a fluid phenomenon, the functional relationship µst be a constant, that is true. Suppose, if there is Pi1 and normally, if there is Pi 1, Pi 2, we simply write, Pi 1 is equal to function of Pi 2. But if there is only one, then Pi 1 µst be constant that is simple. So, Pi 1 is equal to c and this constant µst be determined from the experiments.

If we have two Pi items, we µst be careful not to over extend the range of applicability, but the relationship can be presented pretty easily graphically. So, Pi 1 is a function of Pi 2 simple, something like that, you know, and there will be a valid range, For example, we can say, this is the valid range. So, we, I mean, this is applicability while you conducting experiments, you know, would be knowing what the valid range is, we simply cannot go from say, for example, minus infinity to plus infinity.

There should be some, you know, logical range and finite range in which these relationship like Pi 1 is equal to Phi function of Pi 2 would be applicable. Other places maybe the theory that is behind that starts to break. I mean, break means it starts to fail, does not hold true in that regime.

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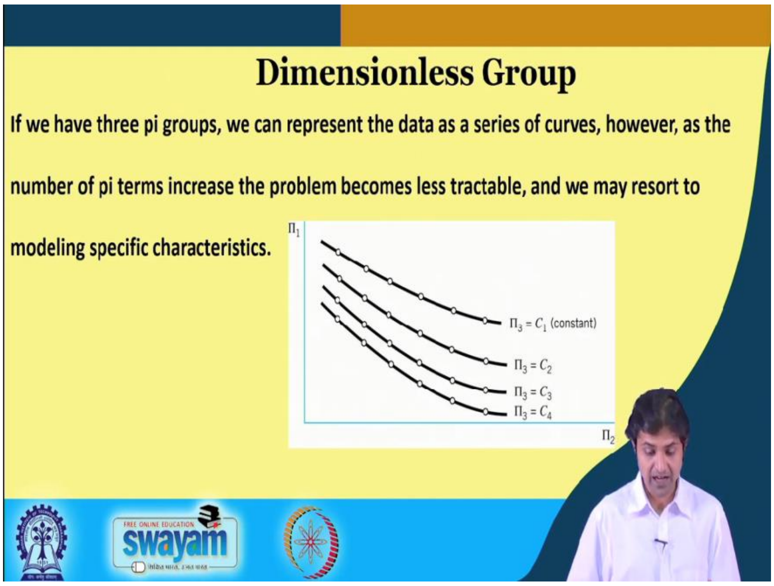

If we have 3 Pi groups we can represent the data as a series of curves, however, as the number of Pi terms increase the problem becomes less tractable and we may resort to modelling specific characteristic. So, with 1, with 2 we had only one curve but if the number of dimensionless group increases, we will have more number of curves, something like this, you see, Pi 1 as a function of Pi 2, when Pi 3 is equal to c 1, when Pi 3 is equal to one constant, when Pi 3 is equal to another constant when, you know.

So, things like this can happen. So, I think before going, I think this actually concludes our dimensionless groups and, you know, and we are going to start now with another topic that is called similitude.

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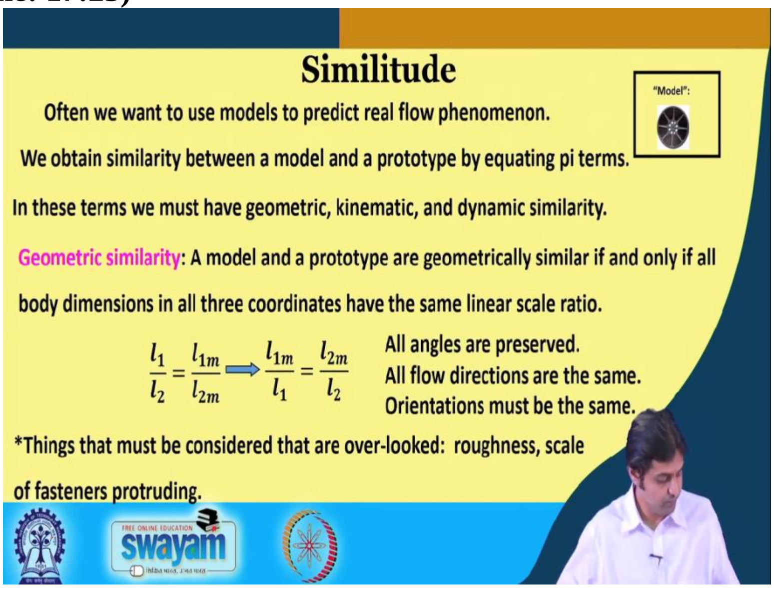

So, we will give some, you know, basic idea of what a similitude is. So, why similitude is needed? So, often we want to use models to predict real flow phenomena. So, this is a model, for example, we obtain similarity between a model and a prototype by equating the Pi terms. That is how we do it in real life. In these terms we μst have geometric, kinematic and dynamic similarity.

What is a geometric similarity? A model and a prototype are geometrically similar if and only if all the body dimensions in all the 3 coordinates have the same linear scale ratio. For example, if there is a ship 100 meter long, 100 meter wide and let us say10 meter deep and if we have to have a geometric similarity in a lab, and we can create. So, what we have do? We will create one meter long, one meter wide and set say 10 centimetres deep ship and this is called the having the same linear scale ratio. Here, linear scale ratio was 100, the example which I just spoke.

So, l 1 / l 2 this is prototype, this is model. Or exactly, as I said l 1 m by l 1 is equal to l 2 m / l 2, a model ship prototype. In this case, if we do that, all the angles will be preserved, all flow directions are going to be the same and the orientations will also be the same. So, we do not have to worry about anything else. So, this is geometric similarity. So, things that μst be considered that are overlooked. So, generally there are some of the things that are overlooked when we try to do that, one is roughness and was scale of fasteners protruding.

So, basically roughness, roughness is like friction at the bed, for example, in the rivers. So, those things are generally overlooked, because that also has to be scaled. But unfortunately, we cannot scale them, it is very difficult to manipulate friction at the bed. It is very difficult to find the stones of such order and things like that. And also in the ship, you know, friction due to the ship surface. Those are generally overlooked.

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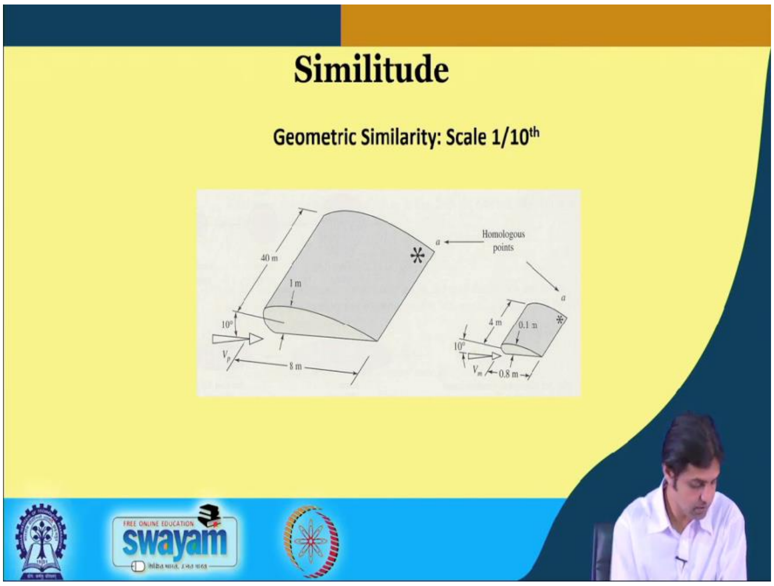

So, geometric similarity scale 1 is to 10th has been shown here. If you see, this object, this is a wink. So, you see, this length is 40 meters here and this has been brought down by 10 times therefore this length is 4 meter. I will go one by one and show you. So, this was 8 meters, this long, so, this is 0.8 meters, so, also a ratio of 10. So, this was 10 meters thick. Sorry, 1 meter thick and 10 times less is 0.1 meters thick.

So, you see, all the 3 dimensions, we have given the scale ratio of 1 by 10th and the angle was 10 degrees, which is preserved here as 10 degrees, you see that. And all the directions and the flow directions, I mean, that does not really, you know, change. So, what we have done is we have done the geometric similarity, 1 is to 10th in all the 3 dimensions, length, breadth and height.

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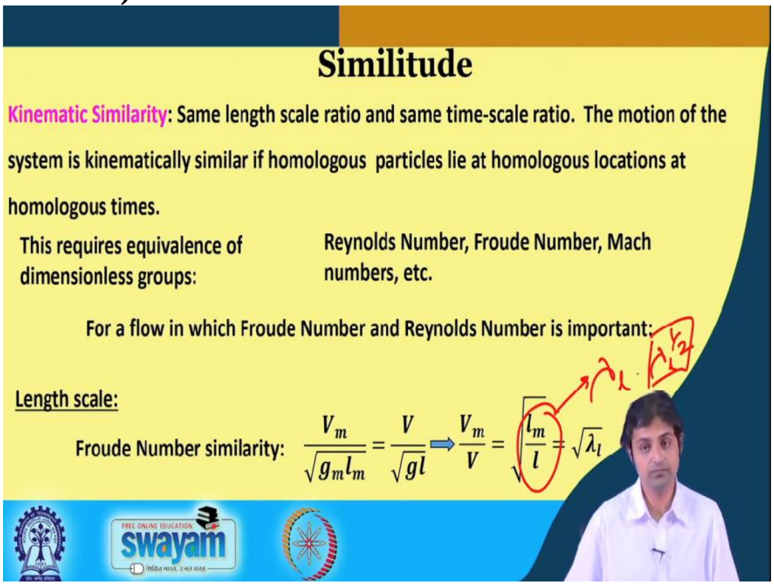

Now, there is another term called Kinematic similarity. Kinematic similarity means Kinematic is related to speed. So, this means, if we have the same length scale ratio and same time scale ratio. The motion of the system is kinematically similar if homologous particles lie at homologous location at homologous times.

And to be able to do that if homologous particle lie at homologous location at homologous times, basically, having same length scale ratio and same time scale ratio, to be able to do that this requires equivalence of dimensionless group, now which one is for Kinematic similarity, Reynolds number should be equated, Froude number should be equated and numbers like Mach numbers etc, because that has something to do with the kinematics of the flow. So, for a flow, let us see, a flow in which Froude number and Reynolds number is important.

Length scale. So, for Froude number similarity, we can write, V m by under root g m lm. This is Froude number in model is equal to V by under root g l and this is Froude number in prototype, let us say, or, you know. This will give us, V m / V. So, V m, V will come this side and it will become g and g will get cancelled because g will remain the same, wherever, you know, on earth we are at the same height. So, V m / v is equal to under root lm / l. If we say, the ratio of l m / l is lambda l. So, for the Froude numbers similarity, the length scale will have to depend upon the ratio of lambda l to the power half.

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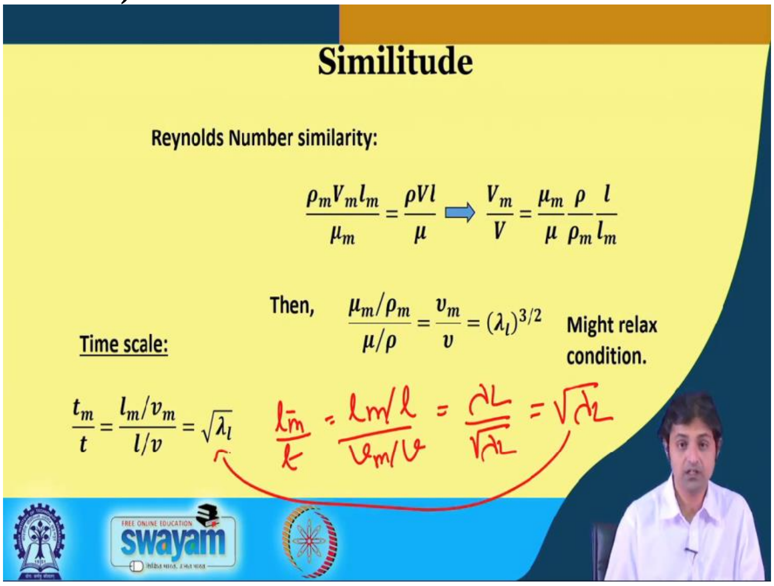

Now, for Reynolds number similarity, we have ρ m V m lm / μ m is equal to, so this is model and this is prototype, so, we do, V m / V is equal to μ m / μ, ρ / ρ m, l / lm. So, then we call, we divide μ m / ρ m it becomes kinematic viscosity in model, where it divided by kinematic viscosity in prototype. So, this is I will just, because we have skipped something, here, so you will not understand. So, V m / V we already know, what we have got from the Froude number similarity.

What was V m / V? Under root lambda l and this is 1 / lambda l. So, we can write, µ m / ρ m is nu m divided by µ / ρ, nu is equal to lambda l, µltiplied by this, this will go here, into lambda l. So, nu m / nu comes out to be lambda l to the power 3 / 2. So, this might be confusing, that is why I just showed you how this we get. So, now, if we have to have the same time scale ratio so time scale will be l m / V m divided by l m / V.

So, what it is going to be? lm / l. So, let us say, lm / l divided by V / V m. So, this is under root lambda l divided by, this is. Sorry, lm / l m is lambda l and this is, so V m / V. Let me just take this one again, so that it is, you know. So, tm / t, can be written as, l m / l divided by V m / V. So, lm / lm is lambda l and V m / V from the previous slide we found out was. So, this is how we get, this here.

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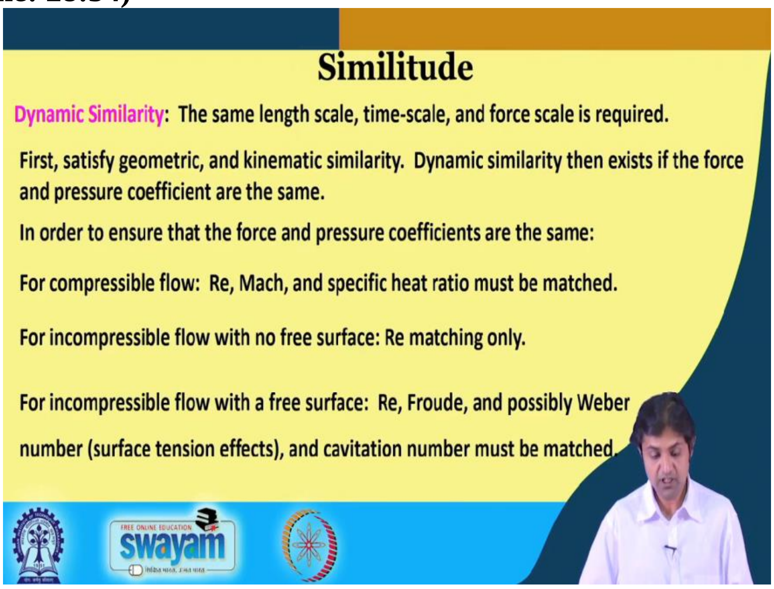

Now, the dynamics similarity. So, in dynamics similarity we have the same length scale, time scale and force scale. So, all the length scale, timescale and all the 3 scales are required to have dynamic similarity. First, satisfy the geometric and kinematic similarity. And then, dynamic similarity then exist if the force and pressure coefficients are the same. In order to ensure that the force and pressure coefficients are the same.

For example, in compressible flow; Reynolds number, Mach number and specific heat ratio µst be matched, for incompressible; Reynolds number match is good enough and for incompressible flow with the free surface; Reynolds number, Froude number and also Weber number; in case if there is surface tension, µst be matched.

So, for all the practical problems of hydraulics, for the dynamic similarity we have to match the Reynolds number and Froude number, I mean, if exists there is Weber number, than Weber number as well. But practically, these 2 numbers µst be matched for, you know, dynamic similarity.

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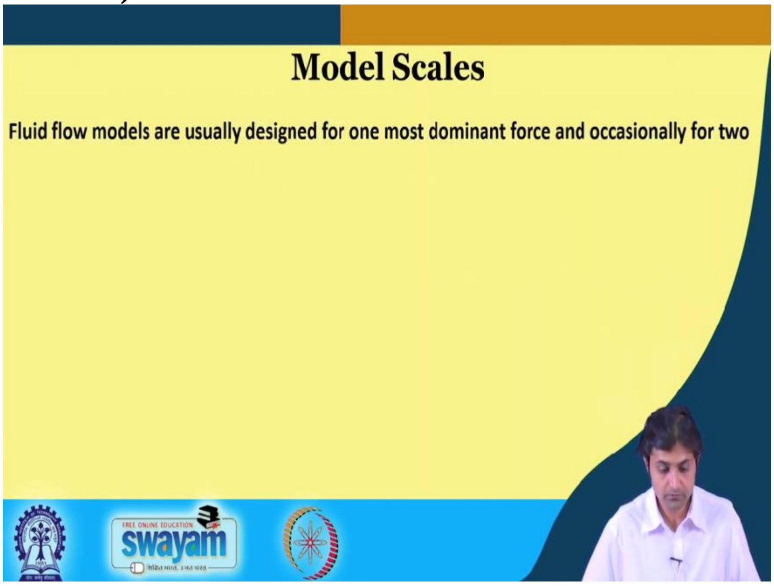

So, I think the end of the similitude, we are going to start with model scale in our next lecture. But I think this is a nice point to stop and we resume and we build up upon the similitude and study the model scales and solve some questions on the slides. And also, if time permits, you will solve some problems on the white board as well. And we will start with this topic here, model scales. And before that, I will see you in the next class. Thank you so µch.