Hydraulic Engineering
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Lecture-25
Dimensional Analysis and Hydraulic Similitude (Contd.,)

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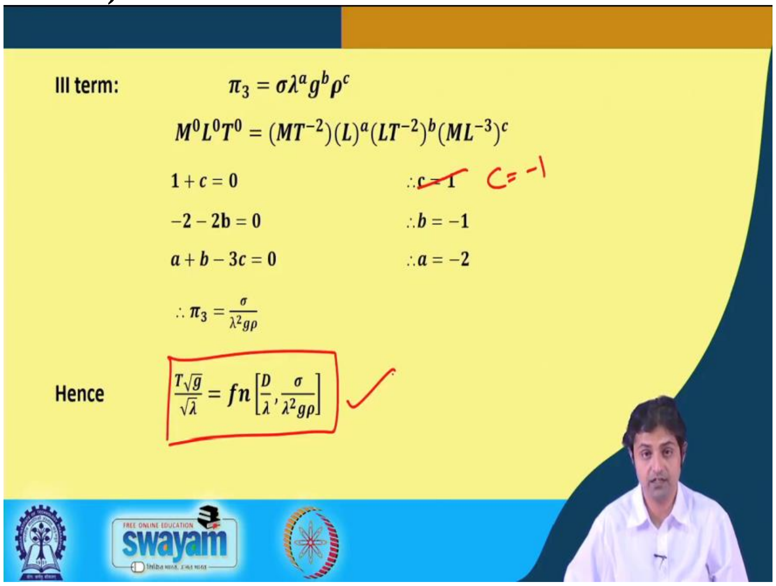

Welcome back. So, in the last lecture we concluded by solving question, by using Buckingham Pi theorem to obtain the time period, here. And now, we proceed back to our discussion of how to be able to choose the variables, in Buckingham Pi theorem.

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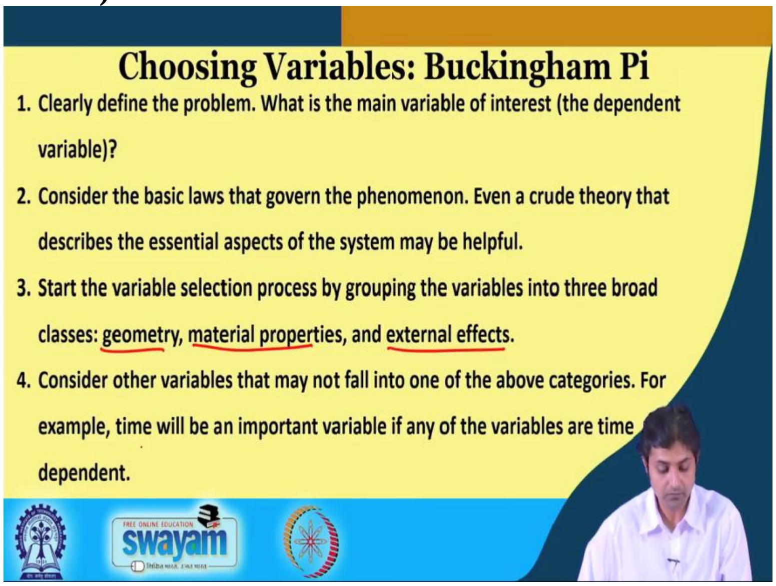

So, choosing variables, the first step is; clearly define the problem. And we have to understand, what is the main variable of interest or the dependent variable. In just the example that we are solved that the time period was the main, was the dependent variable. And in the pipe flow problem that we have done before there the pressure drop per unit length was the main variable of interest. Now, consider the basic laws that govern the phenomenon, even a crude theory that describes the essential aspects of the system may be helpful, in cases like this.

Now, you have to start the variable selection process by grouping the variables into 3 broad classes. With this we have already seen, one class is geometry the other is material properties and the third is external effects, that is. Please consider other variables that may not fall into one of the above categories. For example, time will be an important variable, if any variables are time dependent.

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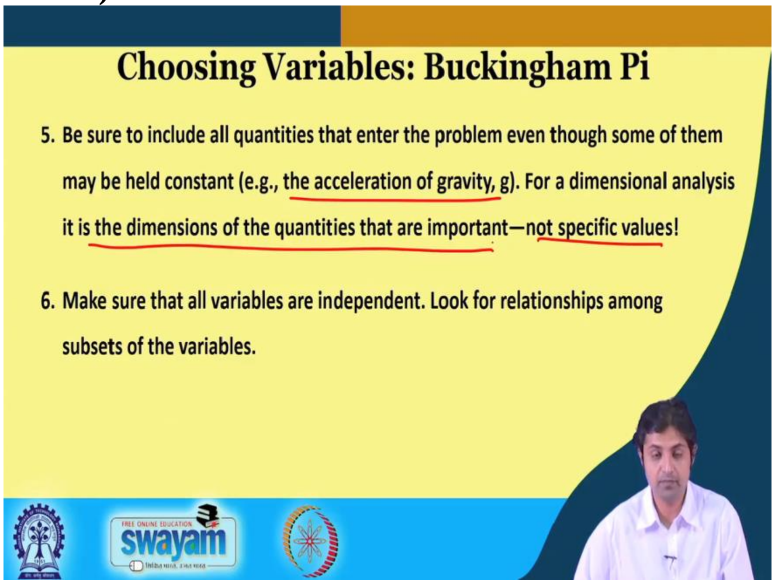

You have to make sure to include all the quantities that enter the problem even though some of them may be held constant. For example, acceleration due to gravity, but this should go into the problem. For a dimensional analysis it is the dimensions of the quantities that are important, not specific values. So, do not miss any of those even though there are constants. Sometimes we have the tendency of missing the constant, you know, values.

For example, as it is mentioned here the acceleration due to gravity, but that has to be taken into account because it is the dimension of the quantities that are important and not their values. You have to make sure that all those variables are independent. Look for relationship among subsets of those variables.

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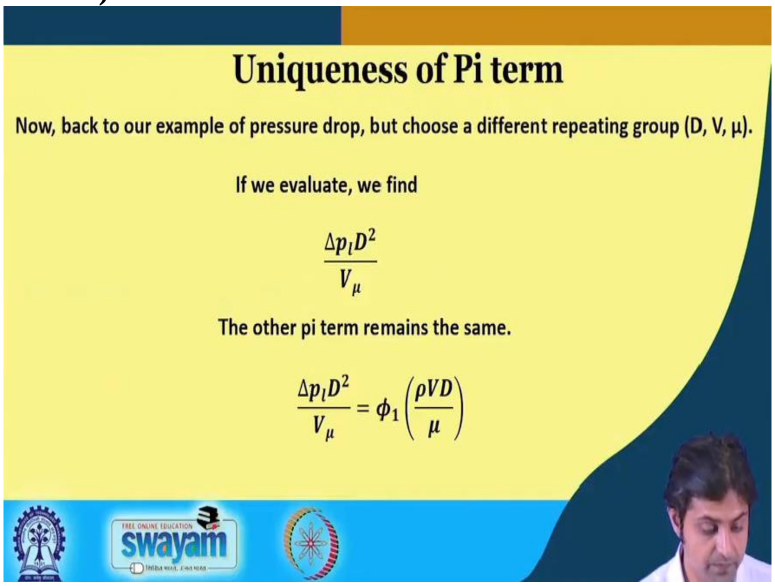

So, another topic that is uniqueness of Pi term, the Pi term should be very unique. So, before actually going to this I would like to, you know, go and solve one problem here that I have included towards the end.

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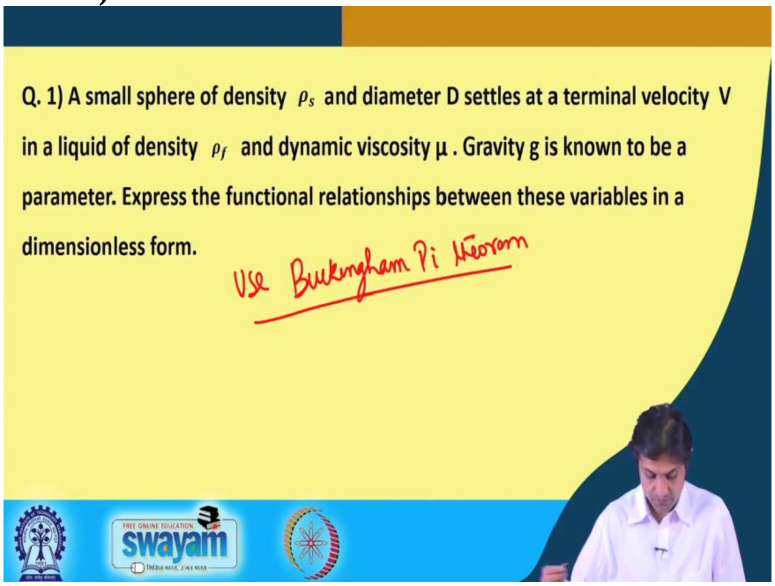

So, the question is, there is a small sphere of density ρ s and diameter D, that settles at a terminal velocity V in a liquid of density ρ f and the dynamic viscosity μ gravity g is known to be a parameter. Express the functional relationship between these variables in dimensionless form. So, what are we going to do? We will use Buckingham Pi theorem. I just want to solve this problem so that, you know, you are able to practice a little more. So, what we are going to do is going to white screen.

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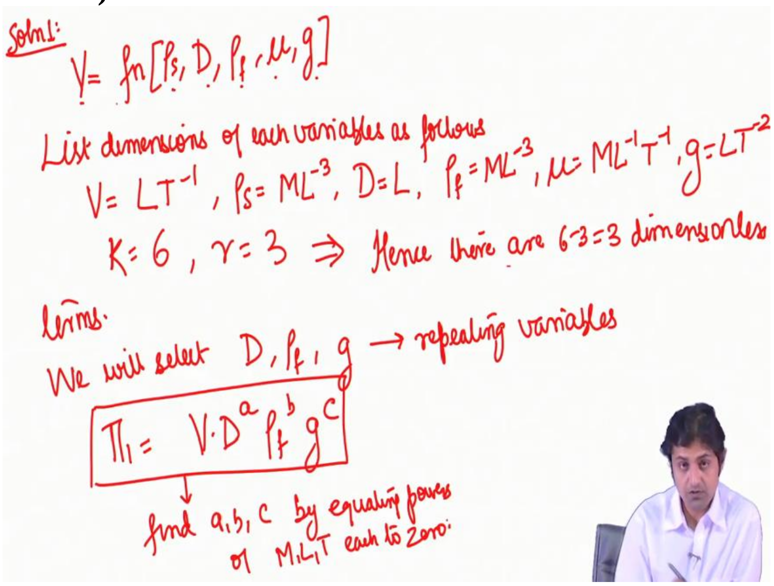

So, like the problem last time everything is given. So, it is given the terminal velocity V is a function of, what are function, ρ s, D, ρ f, μ and g. See, if we have missed any variable. So, it says, small sphere of density ρ s and diameter D settles that a terminal velocity V in a liquid of density ρ f, dynamic viscosity μ and g. So, first step is that we have to list the dimension of each variables. So, V is written as, LT – 1, ρ s will be written as ML - 3.

I will do it very slow so, that you are able to follow, diameter is has a dimension of L, ρ f will have the same dimension as ML – 3, μ will have the dimension of ML - 1, T – 1, and g will have LT - 2. So, we have listed the dimensions of each variables as follows. So, K is 1, 2, 3, 4, 5, 6, so k is 6 and r is basic dimensions if you see, M is also appearing, L is also appearing, T. So, r is 3, which implies hence, there are 6 - 3 = 3 dimensionless terms. So, these following some of the initial steps we have obtained that there are going to be 3 dimensionless Pi terms.

Next step is selecting the repeating variables. We will select D ρ f and g as repeating variables. So, first Pi term can be written as, so what terms are remaining? V is remaining, ρ s is remaining and μ is remaining. So, the first term is going to be V into D to the power a, ρ f to the power b and g to the power c. So, this is the first Pi term. Now, we have to find a, b and c by equating powers of M, L, T each to 0 and this we will do in the next page.

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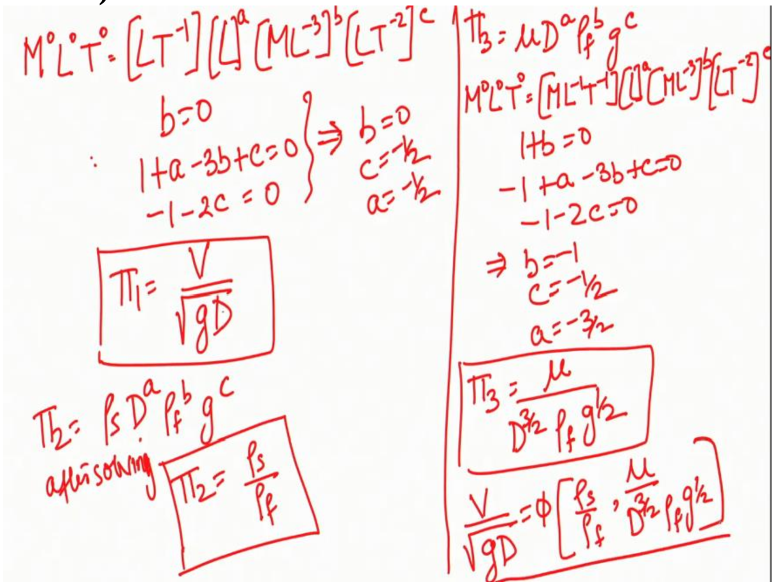

So, what we write is M to the power of 0, L to the power 0 and T to the power 0 is equal to LT - 1 this is because of velocity L to the power a, this is because of D, ML - 3 to the power b because of ρ f and g is LT - 2 to the power c. So, if we solve this, we see there is only one term for M. So, equating M we will get b is equal to 0. Secondly, we will get 1 + a - 3b + c is equal to 0 and thirdly, - 1 - 2c is equal to 0.

This all together is going to give me, b is equal to 0, c is equal to minus half, and a is equal to minus half. So, the first Pi term that we are going to get is V is to the power, V will be V, b is 0. So, what we get is V under root g D, so this is the first Pi term. The second, Pi 2 will be the second non repeating variable, which was ρ s. And similarly D to repeating variable will remain the same, D ρ f and g D to the power a, ρ f to the power b and g to the power c.

So, if we solve, you should do this after solving, we can simply get Pi 2 = ρ s / ρ f. You can solve this using the same procedure given here. Now, the third term, Pi 3 will be the last repeating variable, which was μ D to the power a, ρ f to the power b and g to the power c. So, M to the power 0, L to the power 0, T to the power 0 = ML - 1, T – 1, L to the power a, ML - 3 to the power b, actually I am just writing down, to the power c.

So, this will give us, 1 + b = 0 - 1 + a - 3b + c is equal to 0 - 1 - 2c is equal to 0 and this will after solving it will give, b as minus 1, c as minus half and a as minus 3 / 2. So, the Pi 3 is going to be, μ divided by D to the power 3 / 2, ρ f g to the power half. So, we have got Pi 1, Pi 2 and Pi 3. So, finally the functional relationship will be Pi 1 V by under root gD is a function of Pi 2, Pi 3. So, ρ s / ρ f, μ divided by D to the power 3 / 2 ρ f g to the power half.

This is one of the, I mean, this is one of the examples where we have solved this. So, one another problem that we can actually go and do this. So, yes. So, another question which we are going to solve in Buckingham Pi theorem is question number 6.

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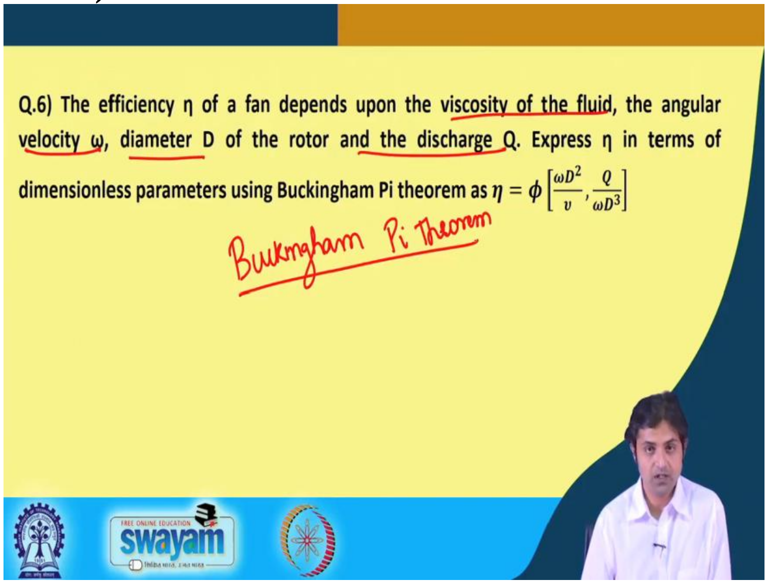

So, it says that, the efficiency eta of a fan depends on the viscosity of the fluid, the angular velocity omega, diameter D of the rotor and the discharge Q. Express eta in terms of dimensionless parameters using Buckingham Pi theorem as eta is a function of omega d square nu / Q / omega DQ. So, here also we are going to use Buckingham Pi theorem. So, as always, we will go on a white screen.

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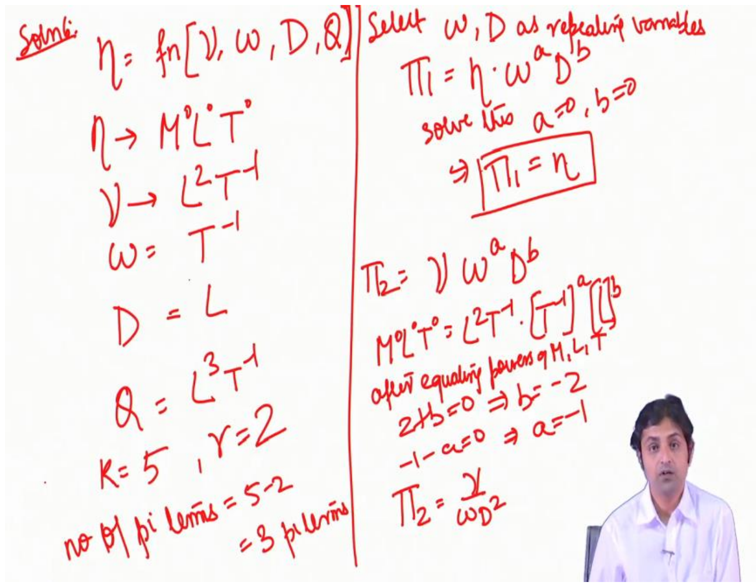

So, you see, the first step is solution 6 as in the question number 6 that we have here. So, this eta this is the efficiency of the fan is a function of, what all variables have been told, viscosity nu, omega the angular velocity omega, diameter D of the rotor and discharge Q. So, this is the first relationship that we obtain. The second what we do is we write the dimensions. So, dimension is M to the power 0, L to the power 0, T to the power 0.

Nu is L square T - 1, omega is angular velocity so it is going to be T - 1, the diameter is of course L and Q is L cube T - 1. So, these are the basic dimensions that we have written. And here, how many dimensions are there? k is 1, 2, 3, 4, 5. And how many basic dimensions have been used here? So, you see, in nu, omega, D and Q the dimensions that have been used is L and T, there is no mass, it is M to the power 0 in efficiency.

So, it is only L and T that we have been used. So, r is going to be 2. So, number of Pi terms, this is an example also for you to showing that r is not always 3, 5 - 2 is 3 Pi terms. So, our next step always is, we have to choose the repeating variables. So, the number of repeating variable should be near same as r. So, select, we are selecting omega and D as repeating variables. So, first Pi term is very obvious.

If we follow our regular procedure, what is the regular procedure? We µltiply the first non repeating with omega to the power a, D to the power b. And if we solve this we get a is equal to 0 and b is equal to 0 implies the first Pi term, implies the first Pi term is going to be eta itself ((), very simple, you see that. Now, this second Pi term will be the leftover. So, we can choose the non repeating variable, one we have already chosen eta is non-repeating variable, the second will be nu.

Nu omega to the power a and D to the power b or in other terms, we can write M to the power 0, L to the power 0, T to the power 0 is equal to L square T - 1 into T to the power minus 1 whole power a and D is L to the power b. And after equating powers M, L and T, what we get is 2 + b is equal to 0 implies b is equal to minus 2 and minus1 minus a is equal to 0 implies a is equal to minus 1. So, our second Pi term is going to be nu / omega D square. So, that is the second Pi term.

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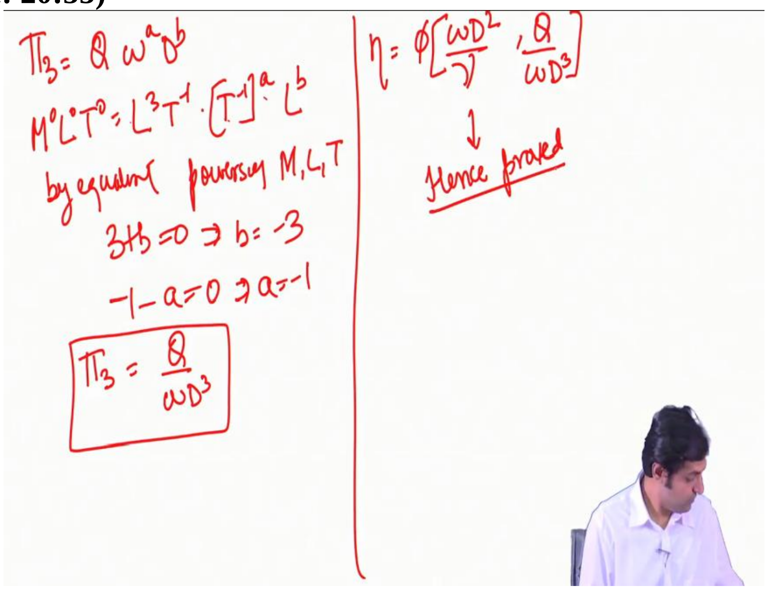

So, now we are going to have the third Pi term. So, Pi 3 can be written as, so, the last one is L 3 T - 1, so that is Q, omega to the power a and D to the power b or we can write M0 L0 T0 = L 3 T - 1 into T - 1 to the power a and L to the power b and by equating power of M, L, T, we get, 3 + b is equal to 0 implies b is equal to minus 3, minus 1, minus a is equal to 0 implies a is equal to minus 1. Therefore, the third Pi term is going out to be Q / omega D cube.

So, the final expression is going out to be eta as function of omega D square by nu, or, I mean, it is 1 / Pi 2 and Q / omega D cube. Hence, so, I will go back and solve the last problem for this one, Buckingham Pi theorem.

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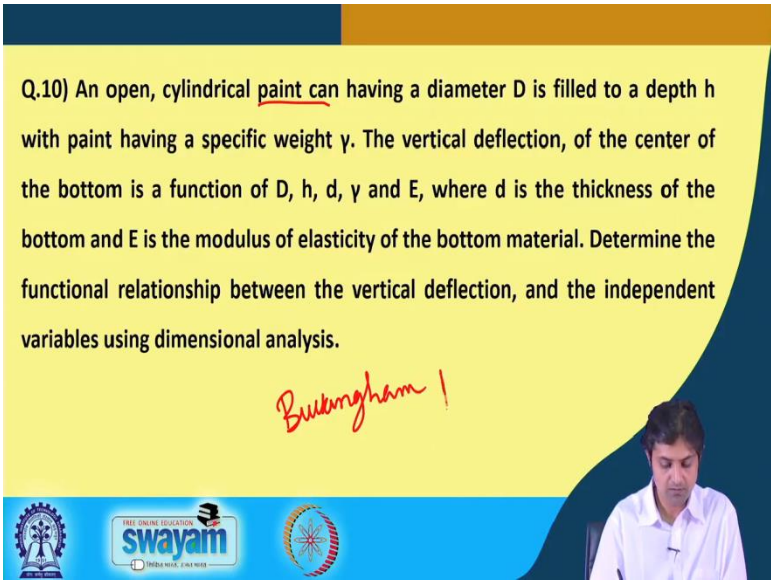

So, there is a question number 10. This has, you know, so and, I mean, the question says, an open cylindrical paint can have so, an open cylindrical paint can, one this is one, paint can having a diameter D is filled to a depth h with paint having specific weight gamma. The vertical deflection of the center of the bottom is a function of D, h, d, gamma and E, where d is the thickness of the bottom and E is the modulus of elasticity of the bottom material. Determine the functional relationship between the vertical deflection, and the independent variables using dimensional analysis.

So, this is a classical example of, I mean, we can actually go, you know, a little fast here, because in the 2 questions we have clearly laid down, you know, the procedure. So, but you see, even in such complex cases, you know, where lot of terms, new terms are there, we can use, and we can use Buckingham Pi theorem for this again. So, how we do this is, we go to the.

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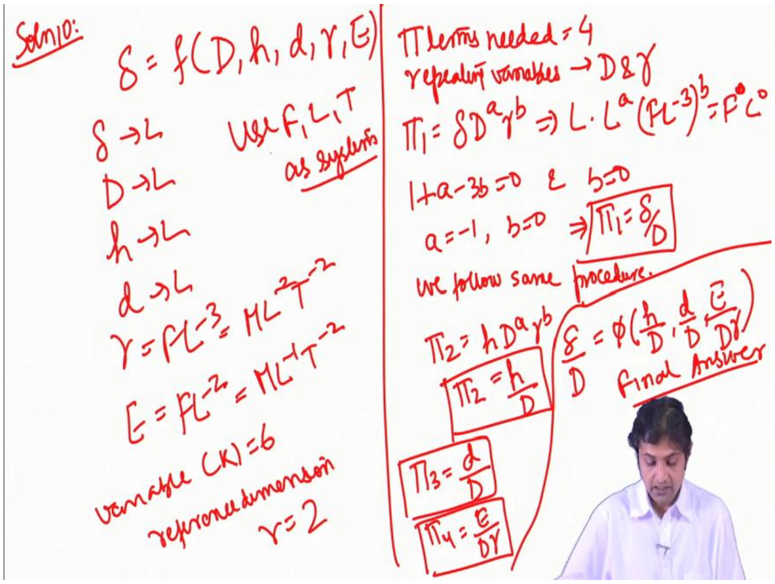

So, we have seen that, according to the solution, so, vertical deflection delta is a function of D, c h, d, gamma and E. And we can simply write the dimension, dimension here is length L, D is also length and h is also length L, d is also length L, gamma is FL - 3 or we can write ML – 2T – 2, E is FL - 2 or ML - 1 T - 2. So, we have now, we have to determine the number of required Pi terms here. So, for now we will use F L and T, so, this term.

So, we are going to use F L and T as our system, not MLT as system. So, how many variables? k = 6, or reference dimension r is how µch? You see, in FLT system, it has only F and L. So, that means 2. So, number of Pi terms is equal to 4. Since, there are only 2 reference dimensions repeating variables, let us choose D and gamma, one representing L and one representing both F and L.

So, the first Pi term, D to the power a, gamma to the power b or if we write in terms of, you know, L into L to the power a, F L - 3 to the power to b is equal to F to the power 0, L to the power 0. So, this will give, 1 + a - 3b is equal to 0 and b is equal to 0, resulting in a is equal to minus 1 and b is equal to 0. Thus, implying that Pi 1 is equal to lambda by d. If we follow same procedure, we are going to get, Pi 2 as h, D to the power a, gamma to the power b thing and we get Pi 2, you know, as h / D.

And Pi 3 will be given as, d / D and Pi 4 is going to be E / D gamma. And therefore, we can simply write, lambda by D is a function of h by D, d by D, E by D gamma. So, you can see how we have used the Buckingham Pi theorem to solve our problems for dimensional analysis. And this and with this I will conclude the lecture for now and I will see you in the next lecture. Thank you so µch.