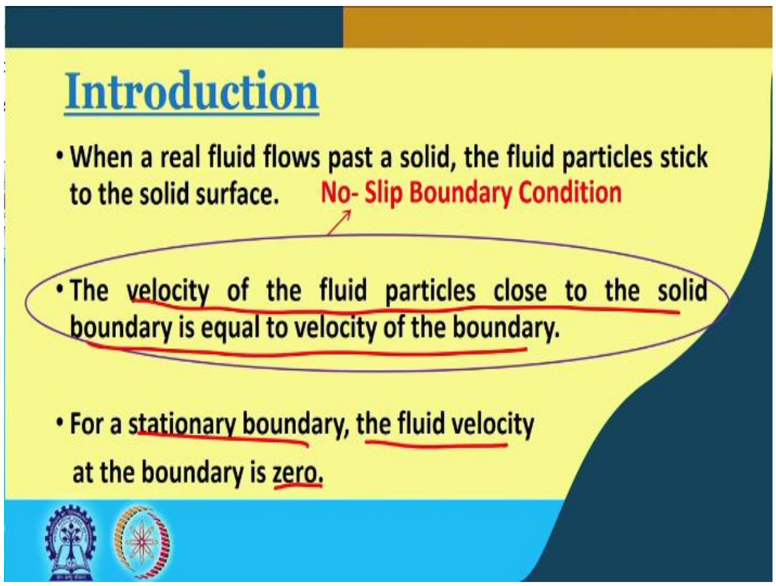
Hydraulic Engineering
Prof. Mohammad Saud Afzal
Department of Civil Engineering,
Indian Institute of Technology – Kharagpur

Lecture-17
Boundary Layer Theory

Hello, everyone. So, this week we are going to study about the boundary layer theory. This is the week 4 of hydraulic engineering course. So, we are going to go to straight to the slides now.

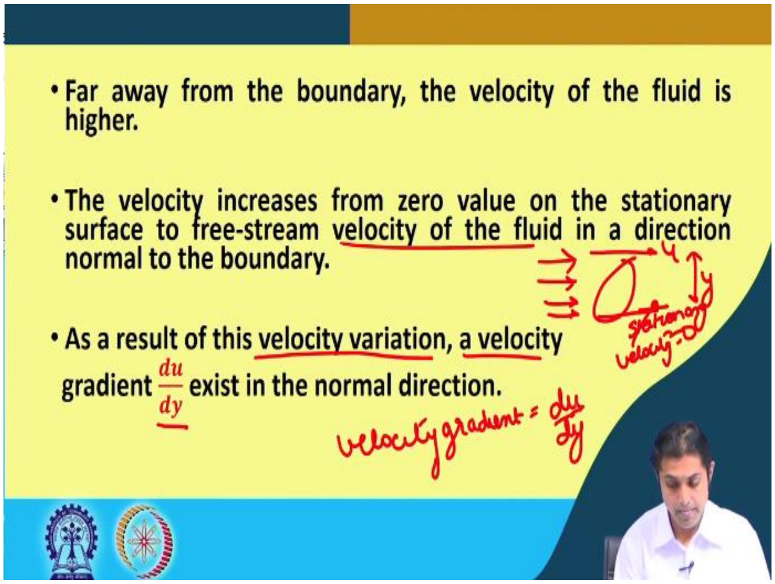
(Refer Slide Time: 00:38)



So, when a real fluid flows past a solid, the fluid particles stick to the solid surface. So, that is one of the phenomena that happens. And the velocity of the fluid particles close to the solid boundary is actually equal to the velocity of the boundary and this phenomenon is called the no slip boundary condition. For a stationary body, the fluid velocity at the boundaries going to be zero because as we said in this point above that the velocity of the fluid particles close to the solid boundary is equal to the velocity of the boundary.

So, if the boundary is stationary then the fluid velocity at the boundary is going to be zero. So, actually this is the most commonly used no slip boundary condition that we use in a viscous fluid flow. What is viscous fluid flow? A small detail which we have already talked about before but will look into more details in the upcoming lectures in coming weeks.

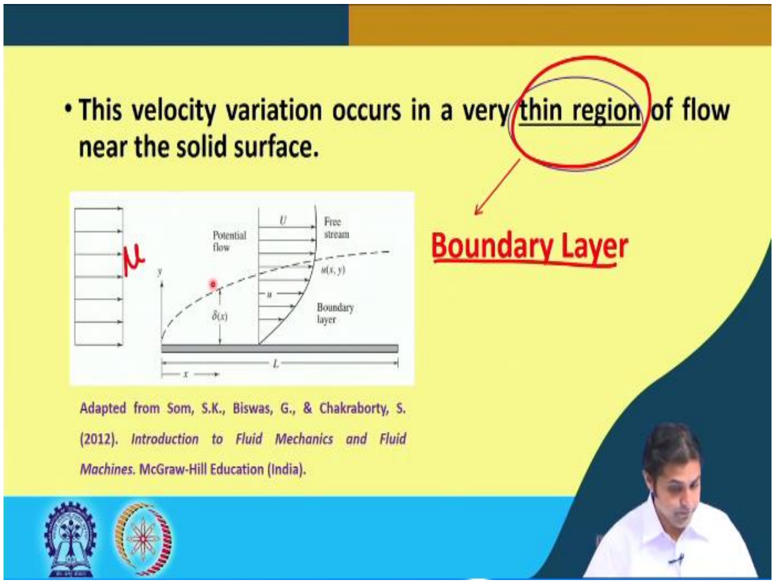
(Refer Slide Time: 01:55)



So, what happens is, far away from the boundary, the velocity of the fluid is actually higher. And the velocity increases from zero value on the stationary surface to free-stream of the velocity, free stream velocity of the fluid in the direction normal to the boundary. So, if this is the case that happens when the surface is stationary. If it is moving then the velocity at the boundary is going to be the velocity of the surface that it is on.

So, because of this phenomenon what happens is there is a velocity variation and this gives a velocity gradient du / dy which exist in normal direction. So, in case of say, for example, this is the boundary because of no slip and this is stationary and suppose there has been a velocity coming in, so, we are not going to talk about let us say, in between now but we say, suppose here is a velocity u and this distance is y and, of course, because this is stationary at this point, the velocity is going to be zero. So, the velocity gradient to be du / dy.

(Refer Slide Time: 03:42)

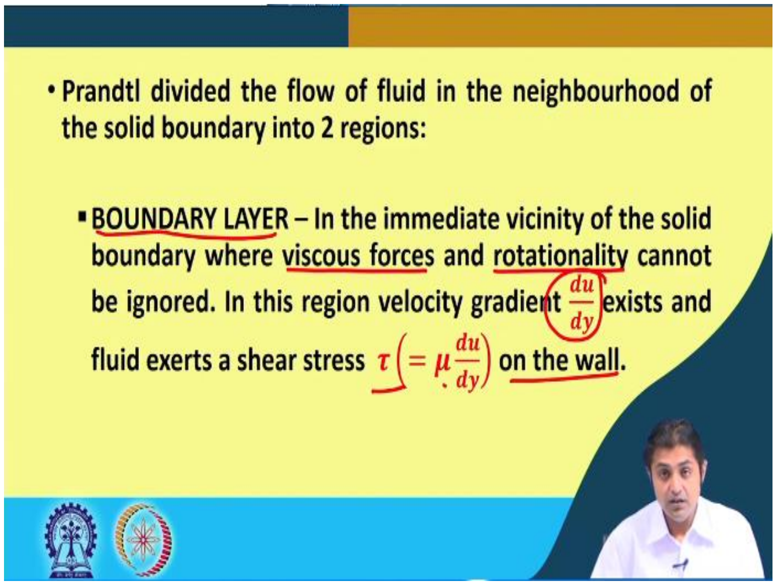


This velocity variation occurs in a very thin region of flow near the solid surface. So, far away from the solid boundary the velocity is going to be the velocity with which the flow was actually coming. So, this whole phenomenon occurs in a very thin region and this layer, this thin region is called the boundary layer. This is the diagram which has been adopted from S. K., Som's book of Fluid Mechanics introduction to Fluid Mechanics and Fluid Machines.

So, what happens is, if there is a flow that has suppose the velocity u that is coming and there is a flat plate of length L here, what happens is, when it encounters there starts developing a thin region. So, I will show you, so, this is the thin region and above this thin region the velocity will remain u, but below that the velocity will start varying. It goes from zero at this point to say, for example, u at this point and this is the existence of the velocity gradient, this where the velocity gradient is going to exist.

Not saying it is going to be constant du / dy but there exists a non zero velocity gradient and this thickness we are going to talk about it later. This is at any distance, this is the boundary layer thickness.

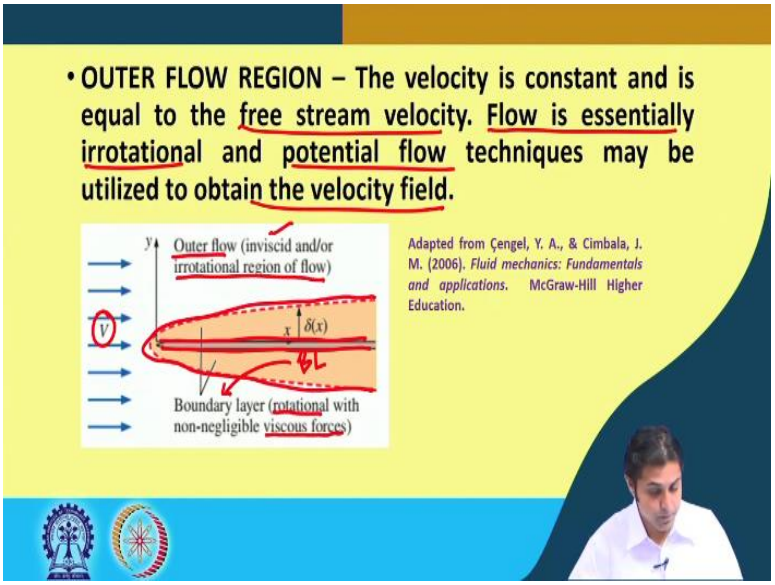
(Refer Slide Time: 05:29)



So, Prandtl actually divided the flow of the fluid in the neighbourhood of the solid boundary into 2 regions, to have a more simplified look. So, one is called the boundary layer. So, it is in the immediate vicinity of the solid boundary where the viscous forces and rotationality cannot be ignored. So, there are 2 things that cannot be ignored in the boundary layer, the viscous forces and the phenomenon of rotationality. We have seen in the previous lectures that most of the flow in the water is considered irrotational.

We are going to explore that into more detail, when we study the viscous fluid flow. So, as I said in the boundary layer, these are the things that cannot be ignored and in this region there will be a velocity gradient equal to du / dy. And therefore, the fluid will exert a shear stress tau on the wall which we assume to be equal to mu du / dy, where mu is the eddy viscosity.

(Refer Slide Time: 06:45)



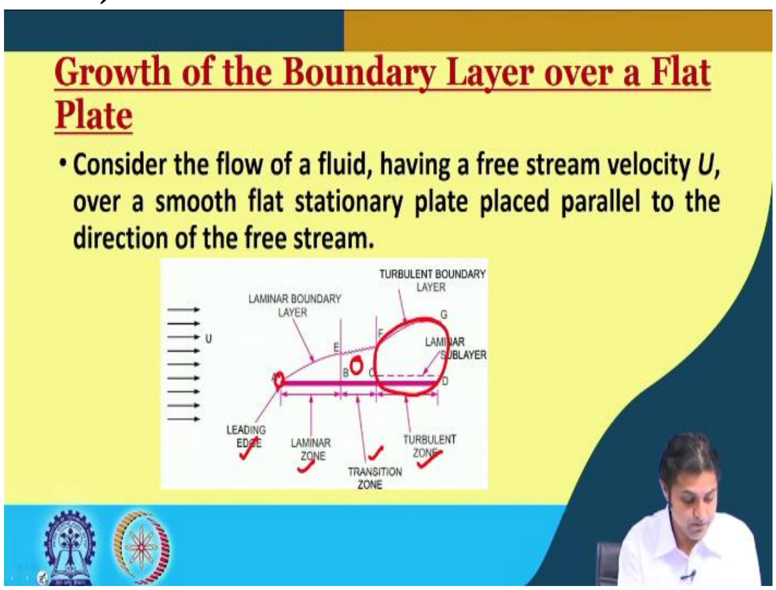
Then the second region is called the outer flow region. The velocity is constant here and is equal to the free stream velocity. As I told you in the initial slides that above this boundary layer the velocity will not be effected, it will be the same as this free stream velocity that was the flow was coming with whatever velocity it had before the same will be maintained above this boundary layer.

So, all the phenomenon that is happening is in this boundary layer. Boundary layer is a very, very important phenomenon actually in oceans or rivers. All the phenomenon, such as, the sediment transport or the transport of phytoplanktons, Norway is it occurs in this particular region, that is, called the boundary layer. However, above that, that is, outer flow region and the velocity remains unaffected.

So, in this there is no viscous forces that we consider, and the flow is essentially irrotational. And therefore, the potential flow techniques may be utilized to obtain the velocity field. So, we have talked about potential flow in the previous slide, in the previous lecture, where we studied about the streamlines and the potential flow in a topic called fluid kinematics. So, this figure has been taken from Cengel, the name of the book is Fluid Mechanics: Fundamentals and Applications from McGraw-Hill Higher Education.

Here, it shows, that the flow is coming with a velocity V and there is a plate here, you see, this is the plate and if the flow will passed over it, this region, that is, enclosing the flat plate is actually boundary layer, as indicated here, and the flow here is rotational, as we have seen, and with some finite viscous forces. So, viscous forces will exist in this region and the region outside is outer flow where the flow is inviscid or there is no viscosity or friction and we can consider irrotational region of the flow here.

(Refer Slide Time: 09:12)

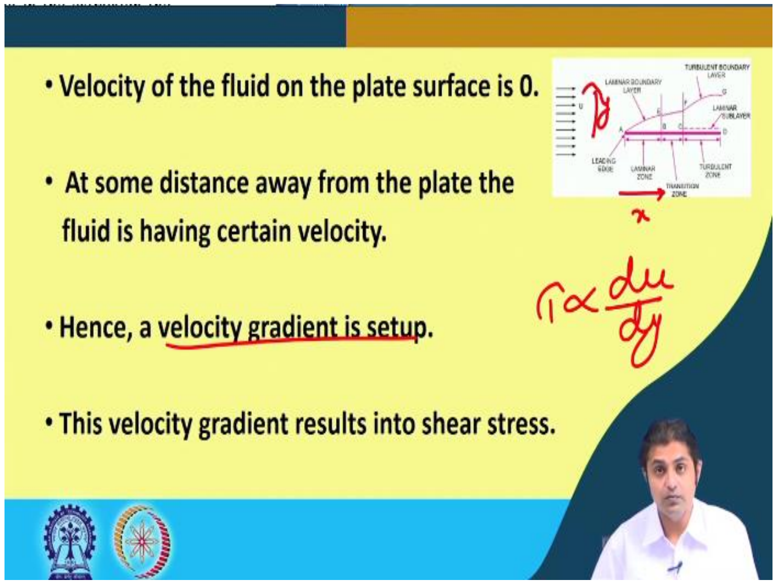


Now, after this brief introduction, we will talk about how the boundary layer grows over a flat plate. This is the most simplified object that you think of is a flat plate above when the flow will passed over it how the boundary layer will grow above that. So, to do that we have to consider the flow of the fluid having free stream velocity U and that happens over a smooth flat stationary plate placed parallel to the direction of the free stream. So, something like this.

So, I will explain the terminology here. This is the velocity, the free stream velocity where the flow is coming, this is a velocity U and it encounters a plate here. So, this is the plate. I will take away the ink now. And the because the flow will strike the plate here first, that is, called the leading edge and this is point A. So, after that I will come to see why this is laminar zone, why this is transition zone, why this is turbulence zone, but just to show you in the diagram.

So, the first few, I mean, units over the flat plate is going to be laminar, that is, the laminar zone. After which this the flow in this region is going to be a transitional zone and after that, this is a turbulent zone where the entire flow will become turbulent.

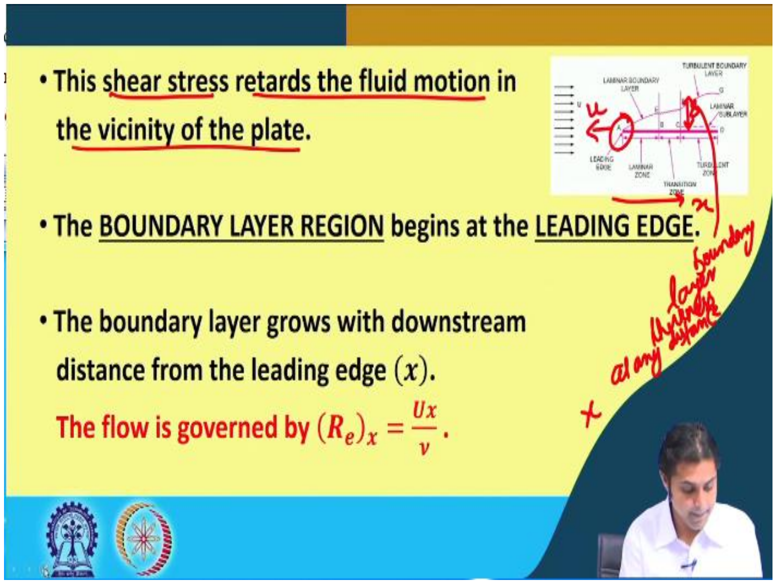
(Refer Slide Time: 11:05)



So, now we have put the drawing here. So, that will be of help when we start writing some of the equations. So, as I told you before, the velocity of the fluid on the plate surface is zero because of no slip condition, as the plate is stationary. But at some distance away from the plate the fluid is having certain velocity, that velocity we do not know for now. But at some distance away, suppose this distance, for example, that is, away from the plate, by this much, it will have certain velocity.

Therefore, because of existence of this velocity, a velocity gradient is setup. This is X direction, let us say, this is Y direction. And this velocity gradient results into shear stress because for existence of shear stress. Shear stress is proportional to du / dy, the rate of change of velocity with the distance normal to the flow.

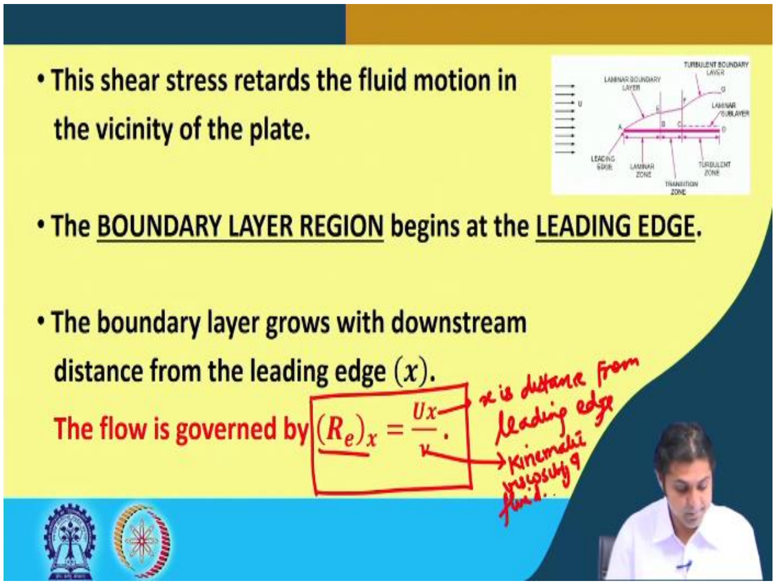
(Refer Slide Time: 12:20)



Now, the presence of this shear stress, what it does is, it retards the fluid motion in the vicinity of the plate. So, this, I mean, in the existence of the velocity gradient will induce shear stress and that is going to slow down the fluid motion near the plate. And the boundary region therefore, will start occurring at the leading edge because that is the first point where the velocity is zero. And, as soon as, we go over and above it there will be certain velocity.

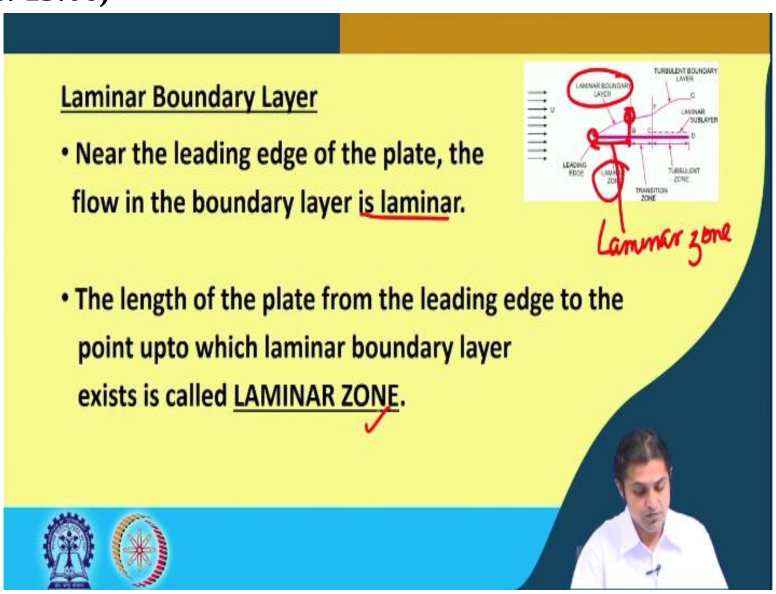
So, that is the first point. Before that any point will have the velocity U. The boundary layer grows with downstream distance from the leading edge x. So, this is the direction x. So, as soon as, we start moving in the x direction, the boundary layer will keep on growing in thickness. So say, let us say, this is delta, that is, the boundary layer thickness, at any distance x.

(Refer Slide Time: 13:51)



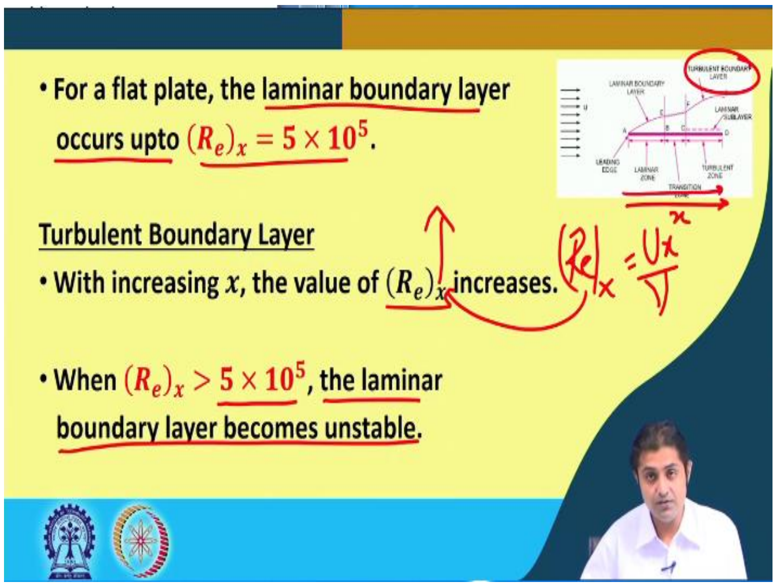
So, one of the important parameters that you have read in the fluid flow is Reynolds number. So, there is going to be a Reynolds number associated with this type of phenomenon. That is the occurrence of or the development of the boundary layer which is given by, as given here, R e at a distance x, is given by, Ux / nu. Where u is the free stream velocity here, x is the distance from the plate, a leading edge and nu is the kinematics viscosity of fluid.

(Refer Slide Time: 15:06)



Now, what is laminar boundary layer? We are going to continue, this is the laminar zone, so we are going to concentrate on that. Near the leading edge of the plate the flow in the boundary layer is laminar. This is important to note. And the length of the plate from the leading edge to the point upto which laminar boundary layer exists is called laminar zone. So, this is the leading edge here. So, the length of the plate from the leading edge, to the point, where the laminar boundary layers, so here, the laminar boundary layer is existing until this point, as indicated in this diagram. So, this distance is called the laminar zone, this one here.

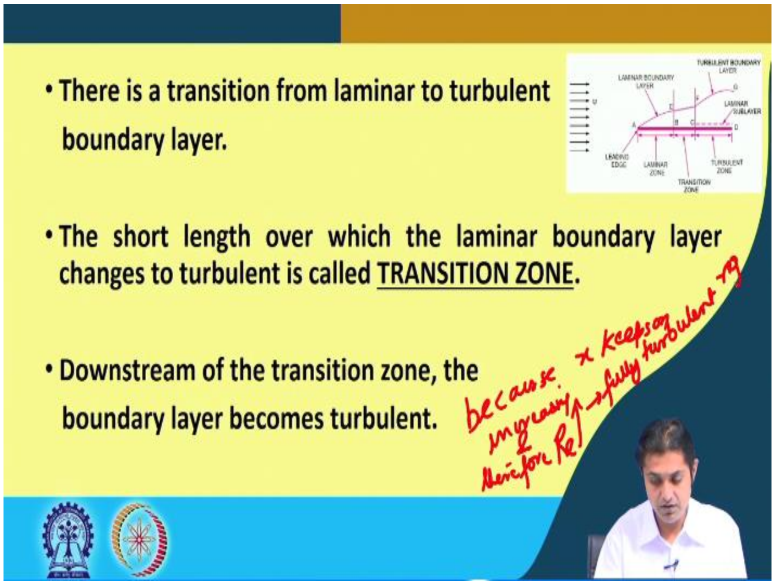
(Refer Slide Time: 16:03)



So, for a flat plate it has been found out that the laminar boundary layer occurs up to Reynolds number of 5 into 10 to the power 5. Lot of people have done research, experimental results confirmed by numerical analysis and this value has come up. So, if the Reynolds number is less than 5 into 10 to the power 5 for a flow over a plate the laminar boundary layer will occur up to that particular point. And then there is a turbulent boundary layer here, a little bit information on that. Actually with increasing x as we have seen, Reynolds number as x increases.

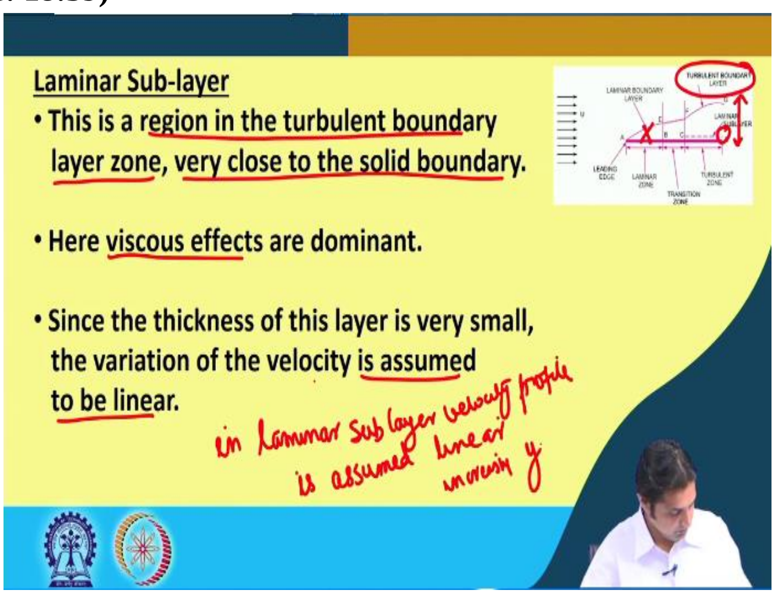
As we saw that Reynolds number as a function of x for ux / nu and x is in this directions. So, as you keep on moving in this direction, x is going to increase and therefore, the Reynolds number will increase. Now, when the Reynolds number increases to more than 5 into 10 to the power 5 the laminar boundary layer becomes unstable. Unstable means, the velocity will have some fluctuations.

(Refer Slide Time: 17:28)



And then there is a transition from laminar to turbulent boundary layer. So, this is the transitional zone here. So, this short length over which the laminar boundary layer changes to turbulent is called the transition zone, indicated by this distance here. Now, the downstream of the transition zone, the boundary layer becomes turbulent because x keeps on increasing and therefore, Reynolds number increases leading to fully turbulent region.

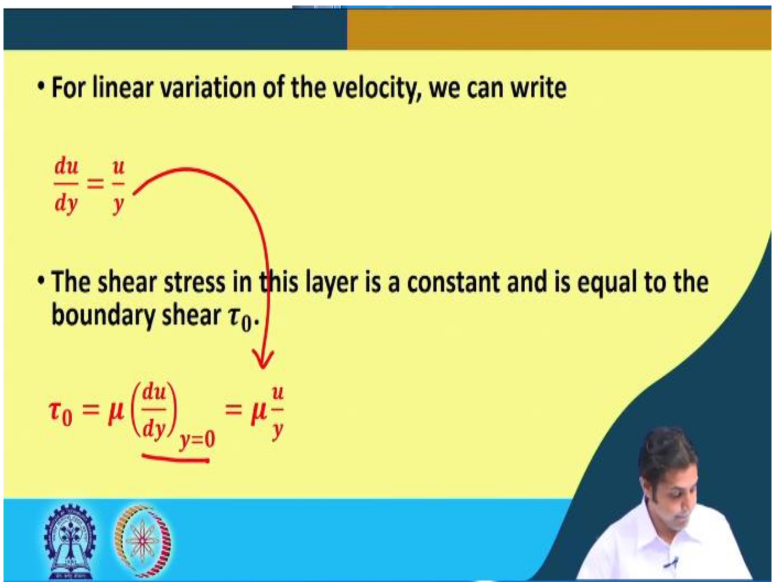
(Refer Slide Time: 18:39)



Now, as you see in this diagram, there is something called laminar sub-layer. And what is that laminar sub-layer? This is a region where the turbulent boundary layer zone and it is very close to the solid boundary. So, basically it is a region in the turbulent boundary layer zone. So, this is this does not happen here, but it happens in the turbulent boundary layer and it occurs very close to the solid boundary and here, because viscosity will play an important role.

Therefore, the viscous effects are dominant; they are much more than the other type of forces. Since, the thickness of this layer, as we can see, this is very, very small compared to this, the variation of the velocity can be assumed to be linear. So, in laminar sub-layer velocity profile is assumed linear. Linear with respect to what? With the distance increasing distance linear, that means, with increasing y. And we also assume that there is has a constant velocity gradient. So, the velocity gradient du / dy is constant.

(Refer Slide Time: 20:49)



Therefore, for linear variation of velocity, we can write,

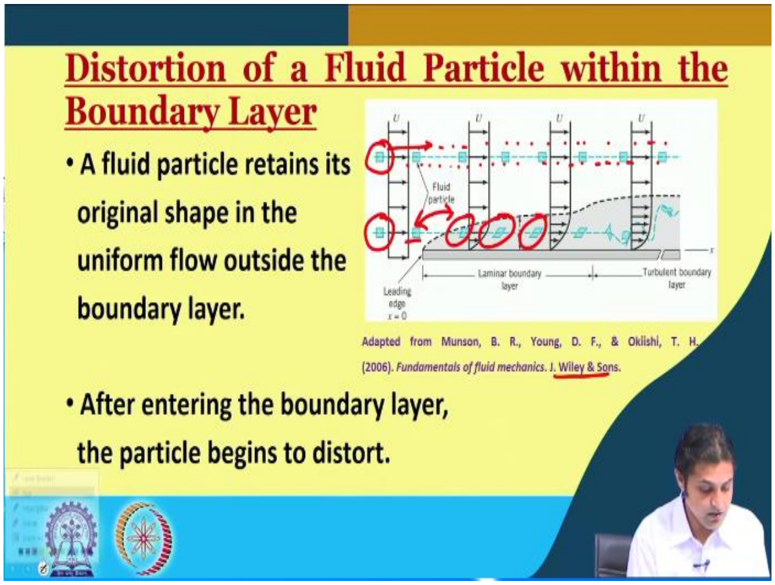
$$\frac{du}{dy} = \frac{u}{y}$$

.

Now, the shear stress in this layer is constant and is equal to the boundary shear stress given by tao not, as we have already been using not for the tau not, for the shear stress near the wall this is also is it is like a sort of a wall only this is the boundary. And therefore, we can write

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = \mu \frac{u}{y}$$

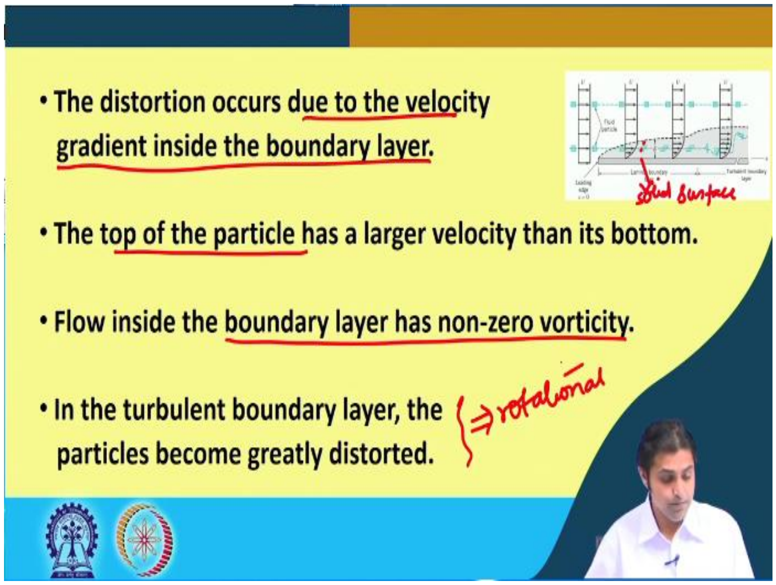
(Refer Slide Time: 21:45)



Now, we will talk about another phenomenon, that is, distortion of a fluid particle within the boundary layer. What happens? So, this figure has been taken from Munson Young and Okiishi's Fundamentals of Fluid Mechanics published by Wiley and Sons. So, let me just, so, what it says is that the fluid particle retains its original shape in the uniform flow outside the boundary layer. That is very true, because outside the boundary layer there are no effects and the fluid particle, this is the fluid particle above the boundary layer, this is the fluid particle that is going to be in the boundary layer.

So, when it moves in this direction, there is no problem at all because all the points will have equal velocities. However, this particle here, after entering the boundary layer the particles begin to distort, as you can see. This is where it starts distorting and when it goes it distorts more, it distorts more, depending upon what the velocity gradient but there is definitely is a distortion from this point to this point, as soon as it enters the boundary layer.

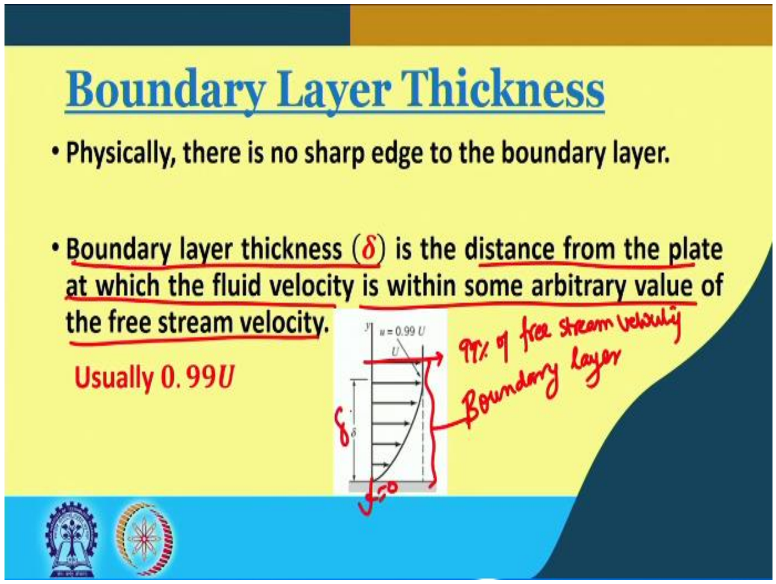
(Refer Slide Time: 23:24)



So, the why this distortion occurs? This distortion occurs due to the velocity gradient inside the boundary layer. How? Because the top of the particle has a larger velocity then its bottom. So, this point here, the bottom will have lesser velocity than at the top because this is more closer to the solid surface the lower, the surface which is closer to the solid surface here.

Therefore, the flow inside the boundary layer has a non zero vorticity. Because what does vorticity causes? It causes rotation and this is what we see, the fluid particle will rotate. And this happens because of the differential velocity at the top and the bottom surface. And what we can see is, in the turbulent boundary layer the particle becomes greatly distorted. So, in the boundary layer the flow is rotational.

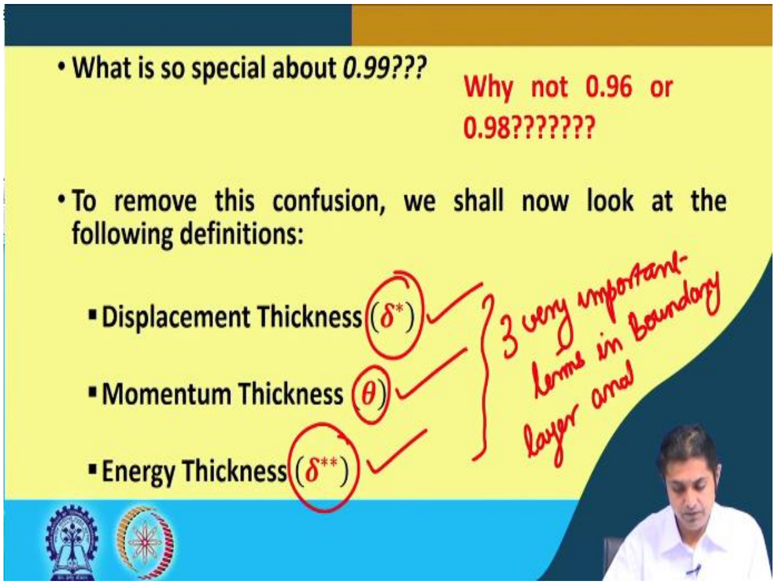
(Refer Slide Time: 24:59)



Now, we are going to see, what the boundary layer thickness is. In real sense, physically, there is no sharp edge to the boundary layer. Now, the boundary layer thickness is the distance from the plate at which the fluid velocity is within some arbitrary value of the free stream velocity. So, this is an important term, boundary layer thickness delta. We have seen similar term in some slides before. And what is this boundary layer thickness? It is distance from the plate, the flat plate over which we have considered the flow.

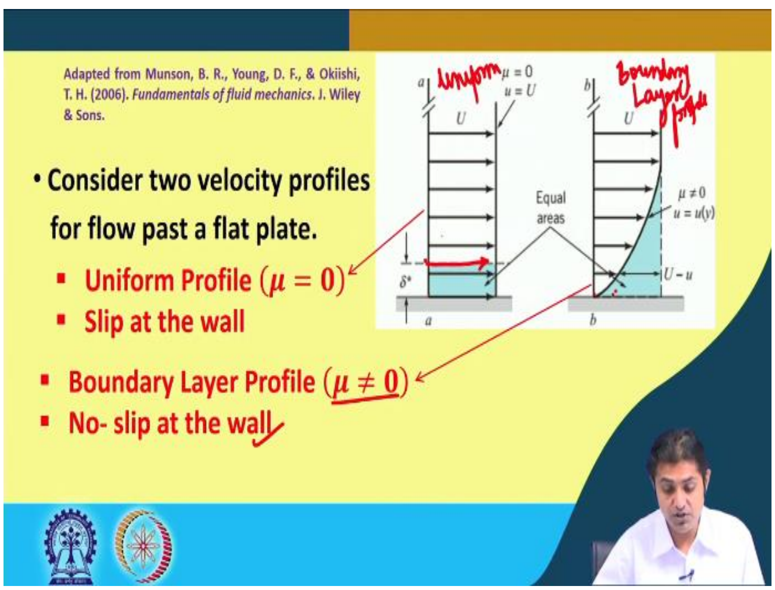
So, distance from the plate at which the fluid velocity is within some arbitrary value of the free stream velocity. Ideally, at the top of this boundary layer, the velocity should be equal to the free stream velocity, normally, at the top. So, this is the boundary layer. So, because of this is the velocity is going to be zero and at one point we have to consider where the boundary layer. And so we assume, when the velocity reaches almost 99% of free stream velocity, the boundary layers cease to exist above that. And that thickness is called the boundary layer thickness.

(Refer Slide Time: 26:55)



So, what is so special about 0.99? Why not for 0.96 or 0.98? To remove this confusion, we will now look at some of the definitions. Some of the definitions is displacement thickness, given by, delta star, very important term, in this particular module of hydraulic engineering. This is another thing called momentum thickness that is called theta. And then there is something called energy thickness which is given by, delta double star. So, these 3 are very important terms in boundary layer analysis.

(Refer Slide Time: 28:06)

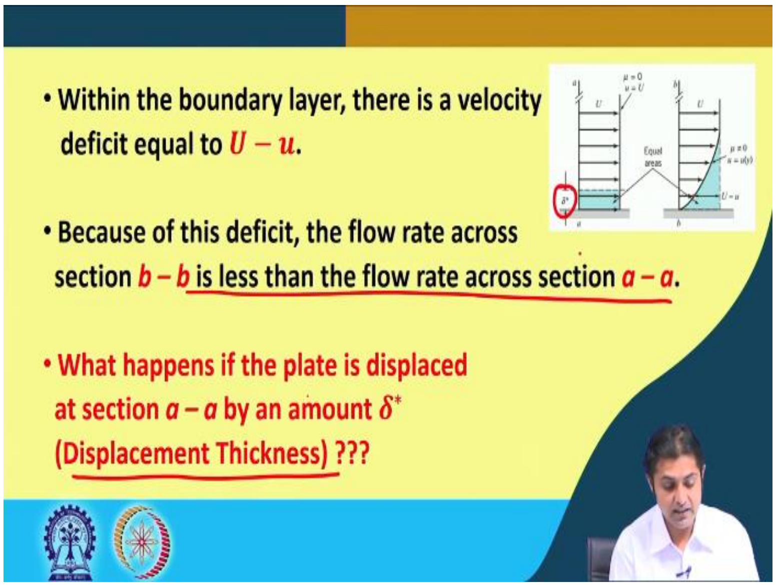


So, we consider 2 velocity profiles for flow past a flat plate. So, this is flat plate here. This is the 1 velocity profile, this is 2 and both has equal areas. And this figure has been taken from Munson, Young and Okiishi Fundamentals of Fluid Mechanics. So, as you can see, I will explain these terms, as it comes. So, this is a uniform profile, where mu is zero. So, there is no viscosity and there is going to be slip at the wall, this place.

It is not, I mean, here, there will be no slip condition. The second is the boundary layer profile, this was a uniform profile and this is the boundary layer profile. Here, there will be some viscosity and mu is not equal to zero and there is going to be no slip at the wall. No slip at the wall means, the fluid velocity just above the plate is going to have the same velocity as the plate. In this case, since, the plate is stationary, the fluid particle just above this, will have a zero velocity.

So, that is why, the velocity profile, you see, it is zero here. In profile, in the uniform profile, at the beginning there exist some, the u is here, there is the uniform velocity u, it is not zero.

(Refer Slide Time: 30:14)



So, within the boundary layer there is a velocity deficit. So, this is the boundary layer. So, U capital U is the uniform velocity profile, here also the free stream velocity is same. But say, at this distance if the velocity is u, then the deficit of the velocity that is, happening is U minus u. u here, is the velocity at a point in the boundary layer. So, this is the velocity deficit equal to. Because of this deficit, the flow rate across the section b - b. So, this is the section b – b. I am going to remove. Sorry.

So, because of this deficit, the flow rate across the section b - b is less than the flow rate across section a – a, that is, correct. Because the velocity here is limited, I mean, less than this U / u. So, the velocity in the boundary layer is lesser than the uniform velocity profile by U minus u. Suppose what happens, if the plate, my question to you is, what happens if the plate is displaced at section a - a by an amount delta dash? So, here, this delta dash is called the displacement thickness. With this we will see in the upcoming lecture.

So, the question at which we are going to end this lecture today is, what happens if the plate is displaced at section a - a by an amount delta dash. So, this is enough for today and we are going to start the next lecture by answering this question. Thank you so much. See you in the next lecture. Bye.