Hydraulic Engineering
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Lecture-16
Laminar and turbulent flow (Contd.)

Welcome back to this last lecture of turbulent flow and laminar flows where we are going to talk about turbulent flow in smooth pipes. Last time in the last lecture we had seen what a smooth and rough bed is based on the Reynolds particle Reynolds number Re*. Now we are going to continue over the turbulent flow in smooth pipes. So, if we refer to equation 18.

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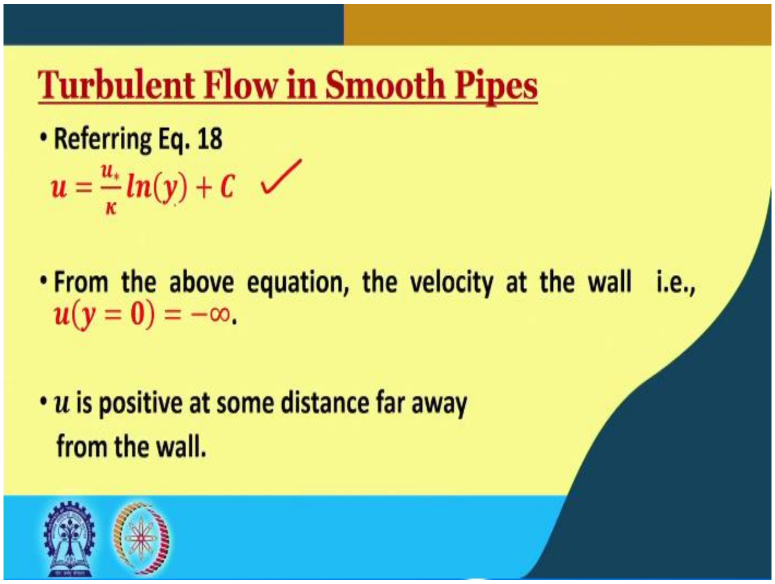

So, what we had found out was

$$u = \frac{u_*}{\kappa} ln(y) + C$$

. The equation 18, in this particular lecture, I mean, in this particular week's lecture. From the above equation, the velocity at the wall u at y is equal to 0 will be minus infinity, correct. If we put y is equal to 0. So, if we put ln 0, it will be minus infinity. u is positive at some distance far away from the wall.

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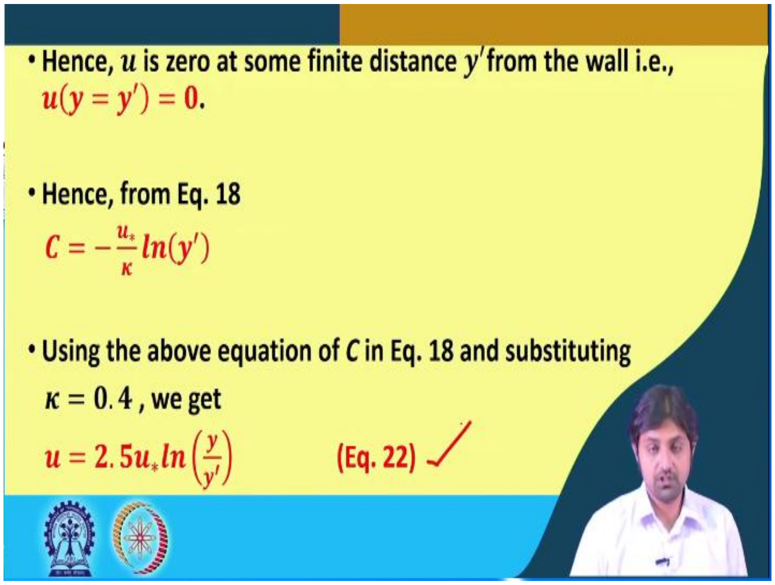

Hence, u is 0 at some finite distance y prime from the wall, that is, y at y prime is equal to 0. Hence, from equation 18, we can say that at distance y prime C will be minus u star by Kappa ln y prime. Because if we put y is equal to 0 it is going to be infinity. So, this logarithmic profile does not fit this. Therefore, we assume that u is 0 at some finite distance y dash y prime from the wall. And if you say the equation was this one, we assume there was u is 0 u star by Kappa ln and we say at this y prime it is 0 plus C. Therefore, C will be minus u star by Kappa 1n y prime.

Therefore, this is what we get, the same thing. Now, if we use this above equation in equation 18 and substitute Kappa is equal to 0.4 we will get

$$u = 2.5u_* ln\left(\frac{y}{y'}\right)$$

, where y dash is the point above the bed where the velocity is 0 and this is equation number 22.

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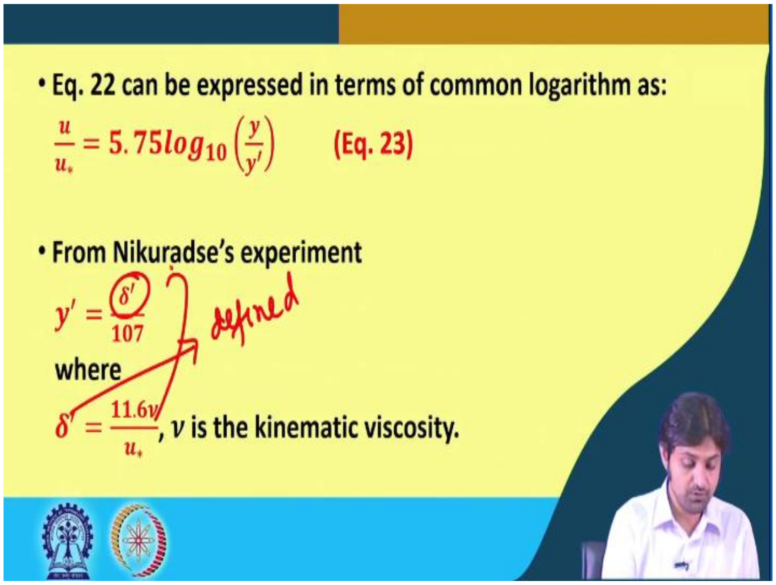

Now, this equation number 22, can be expressed in terms of common logarithm as, so, what we have done? We have done, converted ln into log, because the previous equation was in terms of natural log. We have converted into 10 log base 2 to the power 10 and we get

$$\frac{u}{u_*} = 5.75 log_{10}\left(\frac{y}{y'}\right).$$

Now, again from the Nikuradse's experiment, y prime actually has been found out to be delta prime, where this is a thickness of the laminar sublayer viscous sublayer by hundred and seven.

Therefore, delta bar can be written as, because this, I mean, this has been defined, not in this course but we know that delta prime is 11.6 nu by u star.

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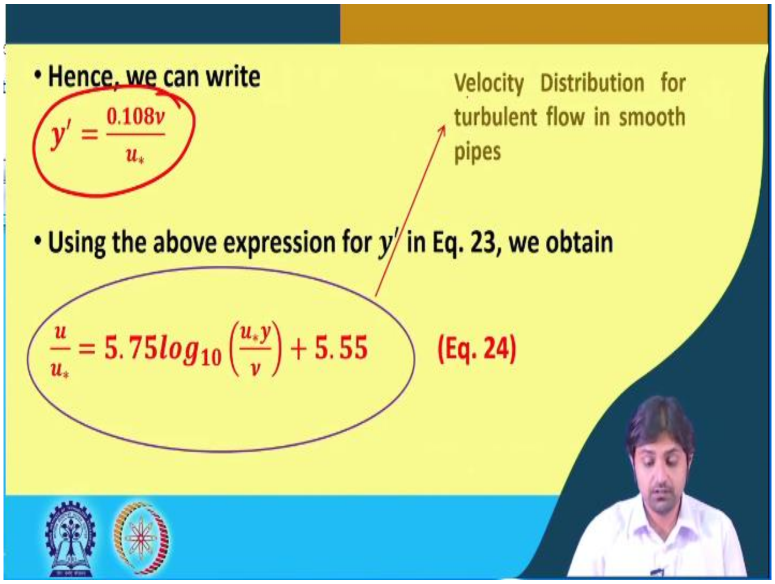

Therefore, we can write, so, if we use this and put it into this equation; we can write

$$y' = \frac{0.108\nu}{u_*}$$

, just simple substitution. And now, we can simply obtain, if we use this in equation number 18 or this particular equation, equation number 23, then we obtain

$$\frac{u}{u_*} = 5.75log_{10}\left(\frac{u_* y}{\nu}\right) + 5.55$$

, very simple. Now, this as you see, the velocity distribution for turbulent flow in smooth pipe. This is the velocity distribution for turbulent flow in a smooth pipe, where there is no irregularity.

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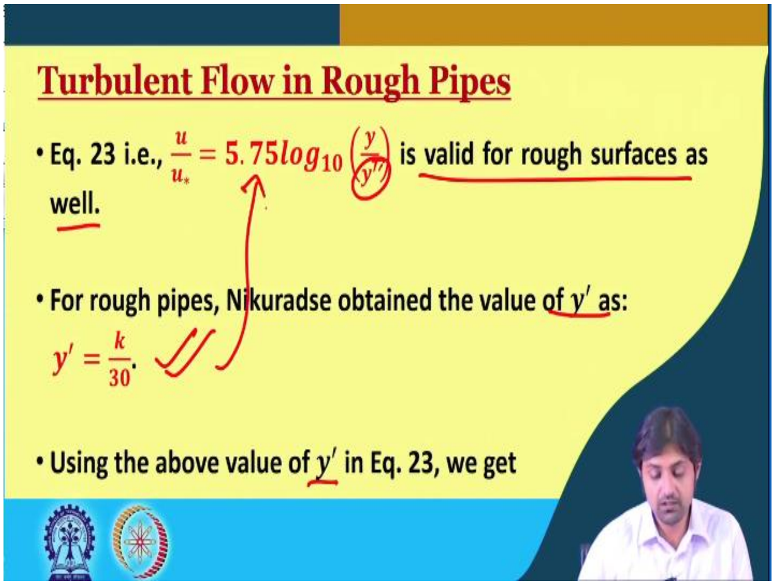

Now, what about the turbulent flow? So, it is better to note down this equation. Now, we have to see equation 23. So, actually this

$$\frac{u}{u_*} = 5.75 log_{10}\left(\frac{y}{y'}\right)$$

is valid for rough surface as well because all the approximation that we did was on this y dash. For rough pipes Nikuradse obtained the value of y prime as k/30. This is obtained by Nikuradse and if you substitute this y prime in equation number 23 this one,

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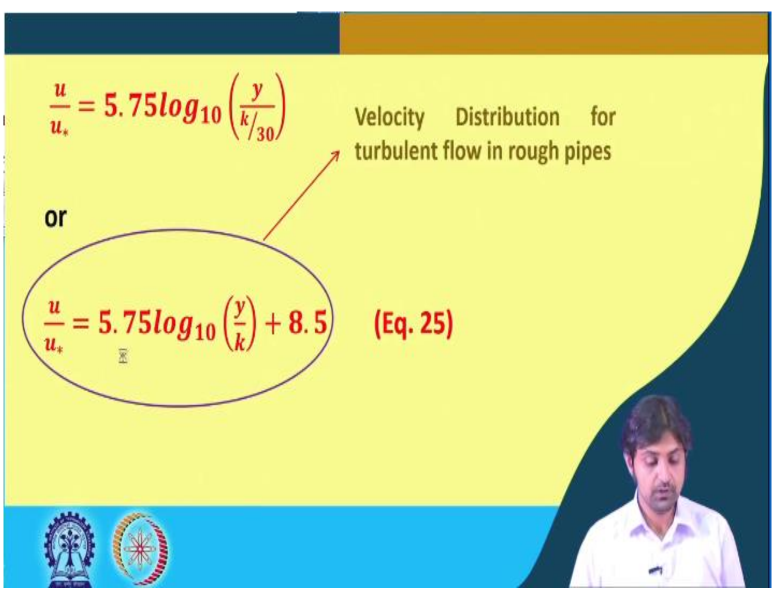

We can obtain,

$$\frac{u}{u_*} = 5.75 log_{10}\left(\frac{y}{k/_{30}}\right)$$

or if you take this 30 out, it is

$$\frac{u}{u_*} = 5.75 log_{10}\left(\frac{y}{k}\right) + 8.5$$

. Now, you see, this is the velocity distribution of turbulent flow in rough pipes. These coefficients are little different this k is different, more importantly, it also has some sort of logarithmic form, both smooth and the rough.

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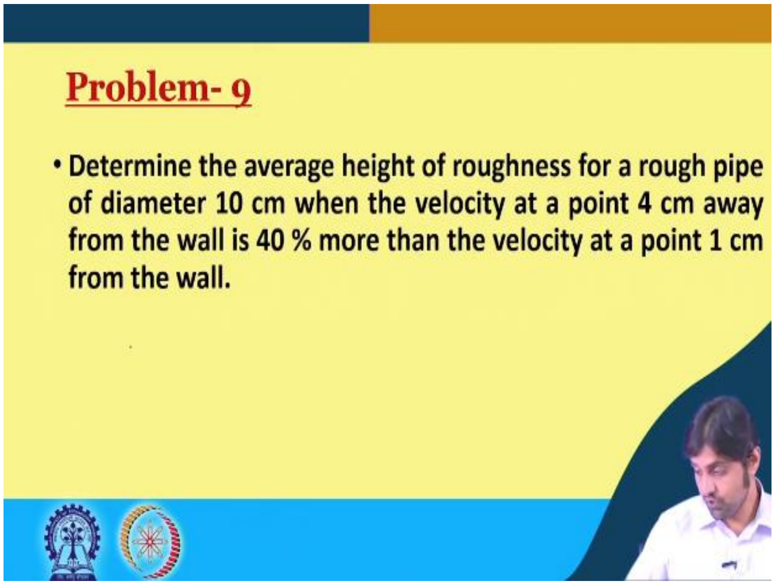

Now, it is a good idea to solve one problem here, on this particular concept, and the solving the problem will give more understanding now. You know, the question is, determine the average height of roughness for a rough pipe of diameter 10 centimeter when the velocity at point 4 centimeter away from the wall is 40 percent more than the velocity at a point 1 centimeter from the wall. So, diameter it says is 10 centimeter, when the velocity at a point 4 centimeter away is 40 percent more than the velocity at a point 1 centimeter away from the wall.

So, what we are going to do? We are going to assume our white screen back again and start solving by writing down what are given.

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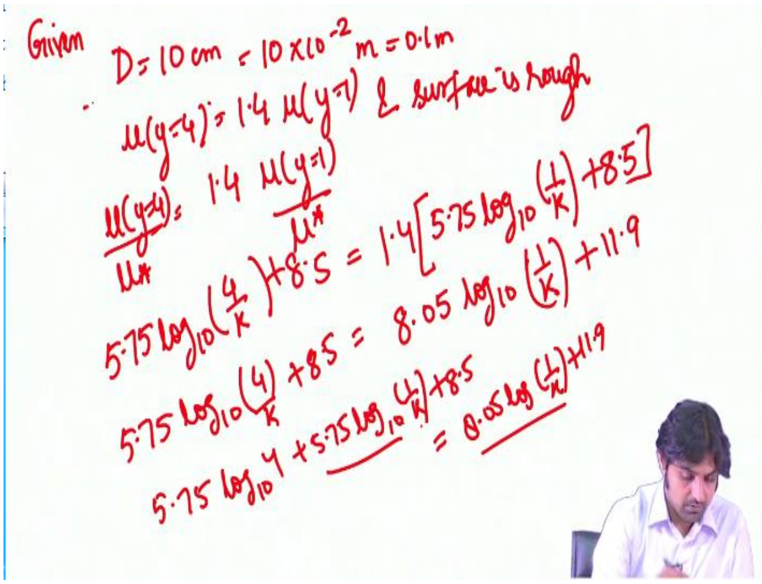

Given is, D is equal to 10 centimeters or 10 into 10 to the power minus 2 meters or 0.1 meter. It is given, u at y is equal to 4 centimeters or 4 is equal to 1.4 times u at y is equal to 1 and the surface is rough. This is what we have already been told. We can therefore, from our equations we can write, u at y is equal to 4 by u star can be written as, 1.4 u at y is equal to 1 divided by u star. So, what we do? We write this, so, we write on the left hand side, 5.75 log base 10, 4 by k, because it is rough, plus 8.5 is equal to 1.4 times 5.75 log base 10 kappa plus 8.5.

This is what is given, I mean, this is what we have know. So, we can write, this will remain as it is, 5.75 log base 10. So, log 4 by k plus 8.5 is equal to this, if you multiply it becomes 8.05 log base 10 one by kappa plus 11.9. So, we will split this log10, log4, I mean, log. So, we can write it, 5.75 log 4 plus 5.75log base 1 by k + 8.5 is equal to we can write 8.05 log 1 by k plus 11.9. And this you see, there is 1 by log 1 by k, here, log 1 by k here, and if you take it to the other side, I am going to use.

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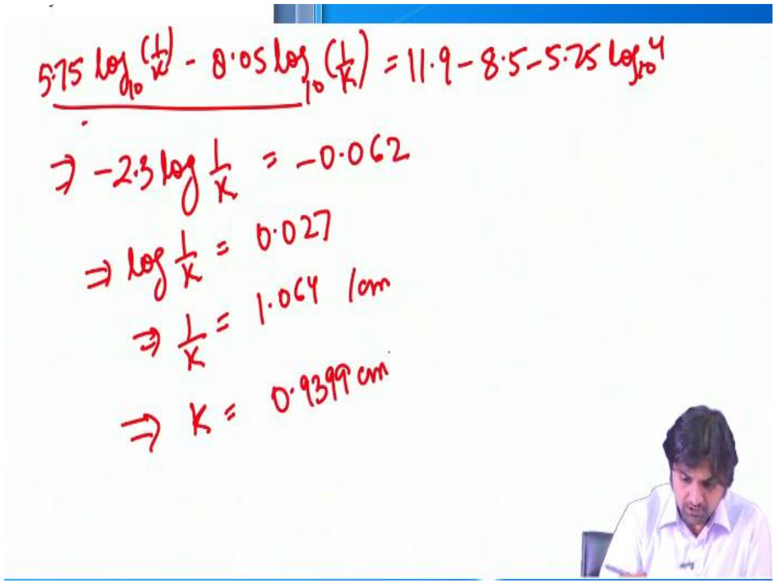

So, we are going to bring it on the other side, 5.75 log 1 by k base 10 minus 8.05 log base 10 1/k is equal to 11.9 minus 8.5 minus 5.75 log base 10 4. So, this will become, the left hand side will become, minus 2.3 log 1 by k is equal to minus 0.062. And therefore, log 1 by k is equal to 0.027, which implies, 1 by k is equal to 1.064 per centimeter, which implies, k is equal to 0.9399 centimeter.

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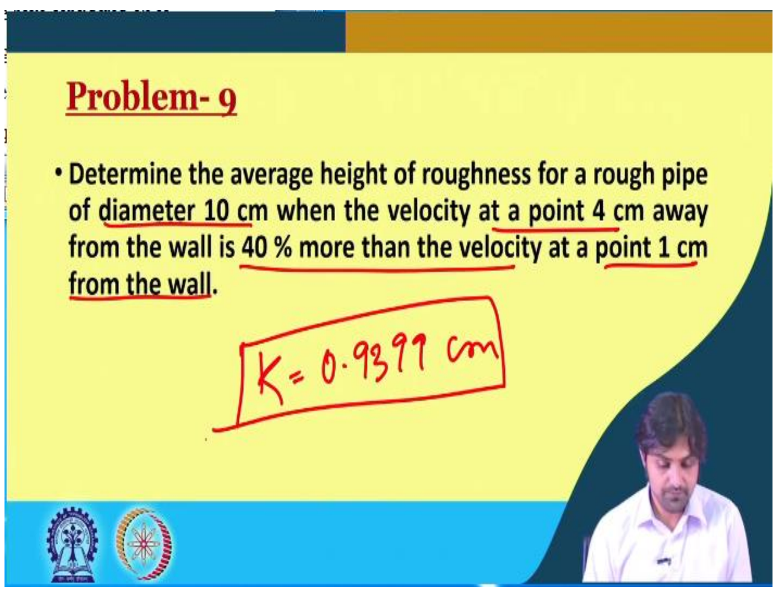

So, this was what was asked. So, k is coming out to be 0.9399 centimeter. This is the answer and how to solve? We have already done that in the sheet.

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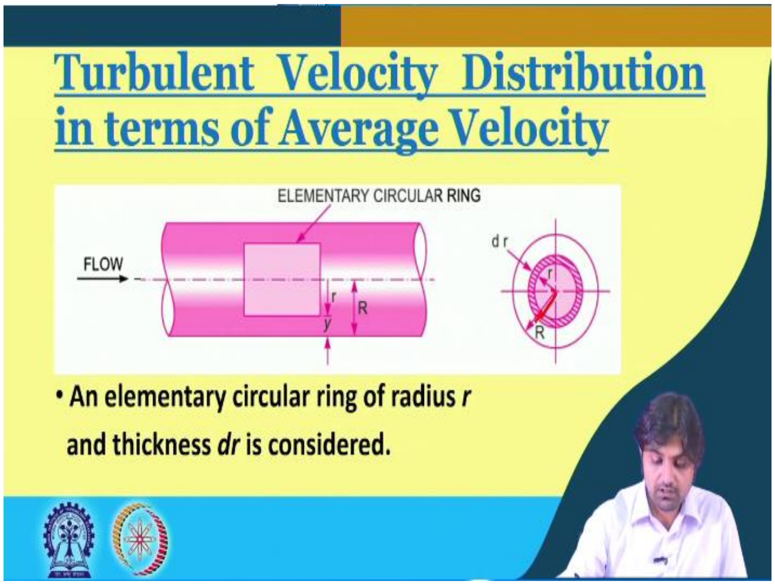

Now, we are going to talk about turbulent velocity distribution in terms of average velocity. So, there is a flow and there is an elementary circular ring here, as you can see, this is of radius R. So, we have an elementary circular ring of radius r and thickness dr which we have considered. So, radius small r and thickness dr, that is, what we have considered.

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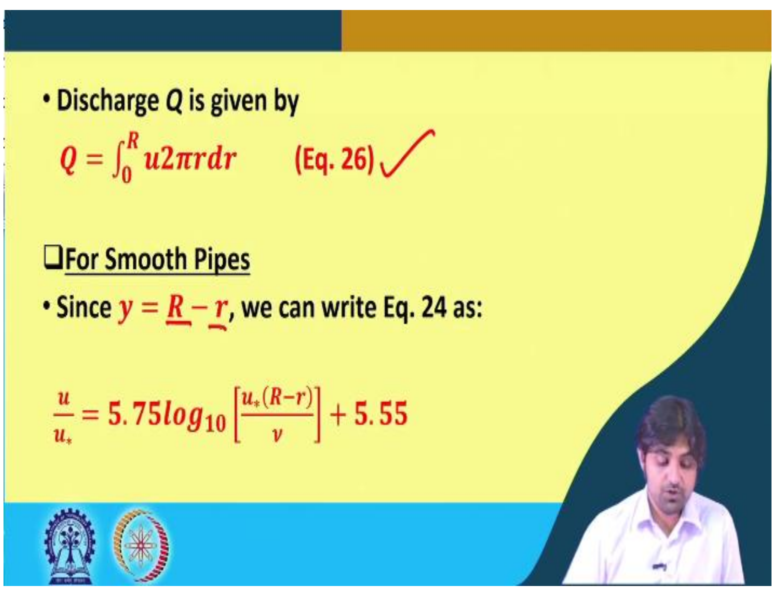

Discharge Q is given by

$$Q = \int_0^R u2\pi r dr$$

. Now, can we calculate it for smooth pipes? Yes, since, y is equal to R minus, capital R minus small r, we can write, equation 24, you know, if you do not remember the equation 24, I will take you to equation 24. So, this was equation 24. Equation 24, u by u star can be written as 5.75 log 10 u star R minus r by nu plus 5.55 and that we have to substitute in Q.

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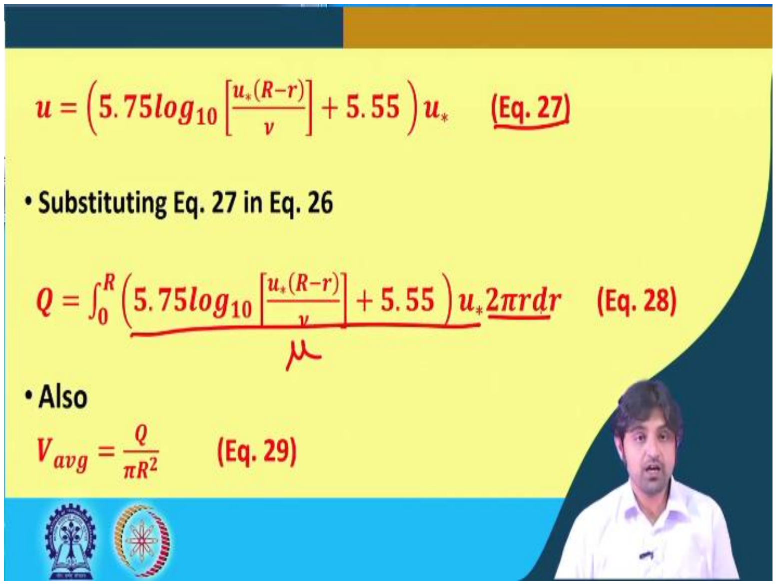

Or we can also write, u is equal to 5.75 log base 10 u star R minus r by nu plus 5.55 y into u star. So, what we did? We took u star that side. Now, if you substitute this equation 27 in equation number 26, so, it will become integral to 0 to R so u this is u into 2 pi r dr, very simple. Also we can write, V average is Q by pi R square. So, we can directly go for the average velocity.

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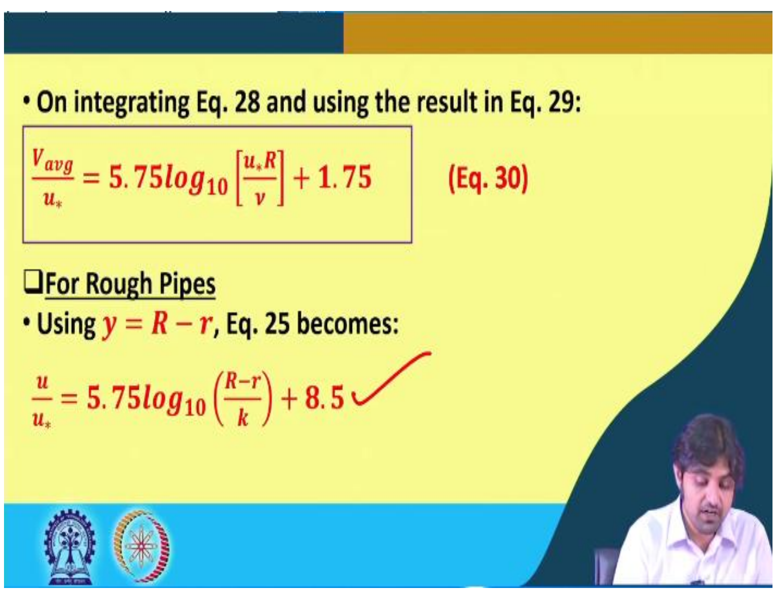

If we integrate this equation number 28 and using the results in equation 29, that is, dividing by pi R square for obtaining V average, we can get,

$$\frac{V_{avg}}{u_*} = 5.75 log_{10}\left[\frac{u_* R}{\nu}\right] + 1.75$$

My request to you is that you please try to integrate this equation, it is very simple, it is class 12th 11th and 12th Mathematics. But you please integrate and try to obtain the equation number 30.

Now, for rough pipes similarly, we will use the equation that we have got for the rough pipes and we use the still the same y is capital R because we are calculating the distance from the wall capital R minus small r. So, we are going to get u, we have this equation that we have derived equation number 25, that was, R minus small r, so, instead of, y by R, y by k we have, R minus small r by k.

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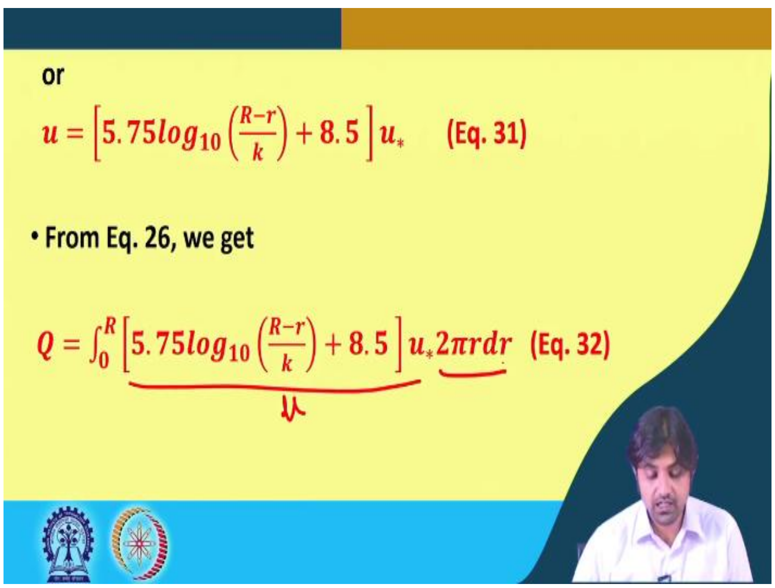

Or we took u star that side and substituted this u in the equation for Q. So, we integrate it from 0 to R, this is u into 2 pi r dr and using the equation 32, we will get a similar equation

$$\frac{V_{avg}}{u_*} = 5.75 log_{10}\frac{R}{k} + 4.75$$

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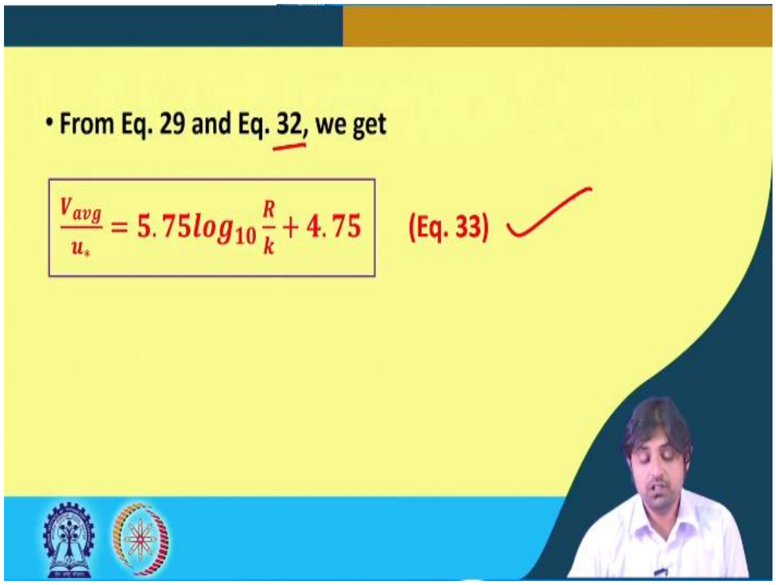

So, what is this? This is the average velocity divided by the frictional velocity for the turbulent pipe flow case.

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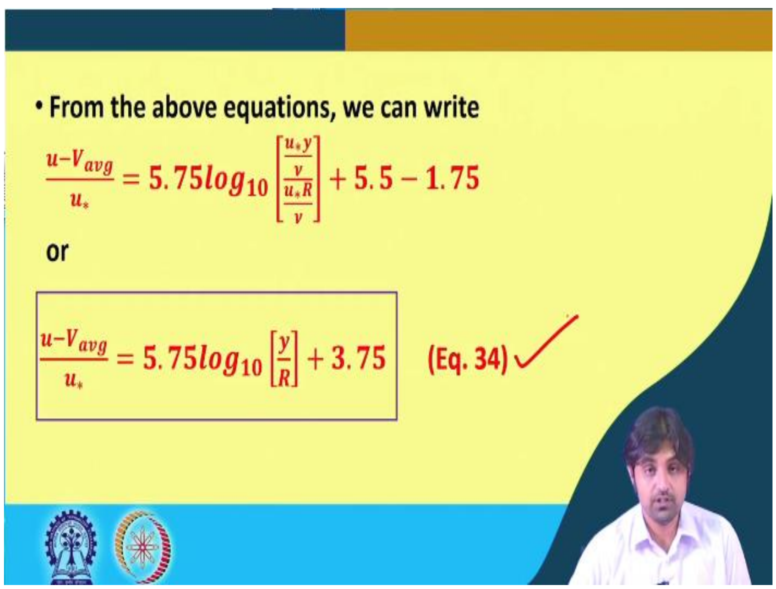

Now, the difference of the velocity at any point and the average velocity for smooth pipes. For smooth pipes, we have seen that the u by u star we had, we came, we derived this equation, correct. And we also just now we saw that V average by u star is going is this equation and therefore, from the above equation we can simply subtract these two equation and we are able to find u minus V average by u star. So, we do this equation, this minus this, this minus this. So, it will be u star is common, so, it will be u minus V average by u star is equal to 5.75 log to the base 10 u star y by nu u star R by nu.

So, nu and nu can get cancelled, it will be y by R or we can say, u minus V average by u star is equal to 5.75 log y by R plus 3.75. So, this is an important equation again. So, all these you either you remember or you can actually derive it. It is very simple, starting from the, starting from the basic logarithmic velocity profile.

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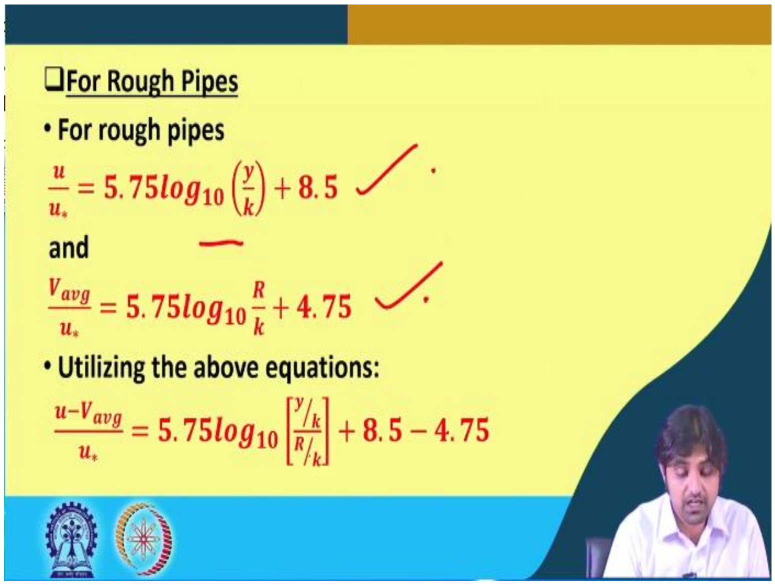

Now, we did it for smooth pipes. Now, for rough pipes, we have an equation and we also obtained V average by u star. We do the same procedure, we subtract this equation, we subtract this equation from this equation. So, this minus this and utilizing the above equation, we get u minus V average by u star. So, we have got y by k divided by R by k plus 8.5 minus 4.75

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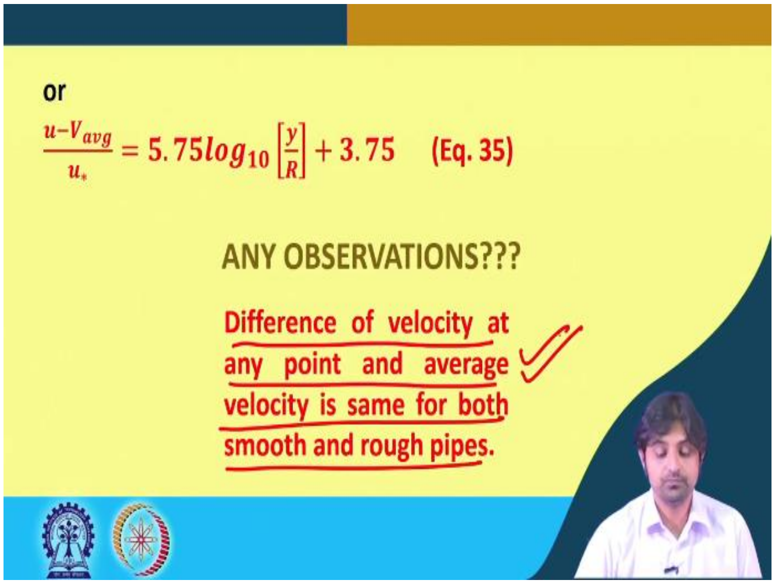

Or we can simply get u minus V average by u star is equal to 5.75 log to the base 10 y by R plus 3.75. Is there any observation? If you see for either for smooth or for rough this difference came out to be the same. So, the observation is the difference of velocity at any point and average velocity is same for both smooth and rough pipes. This is important to know that.

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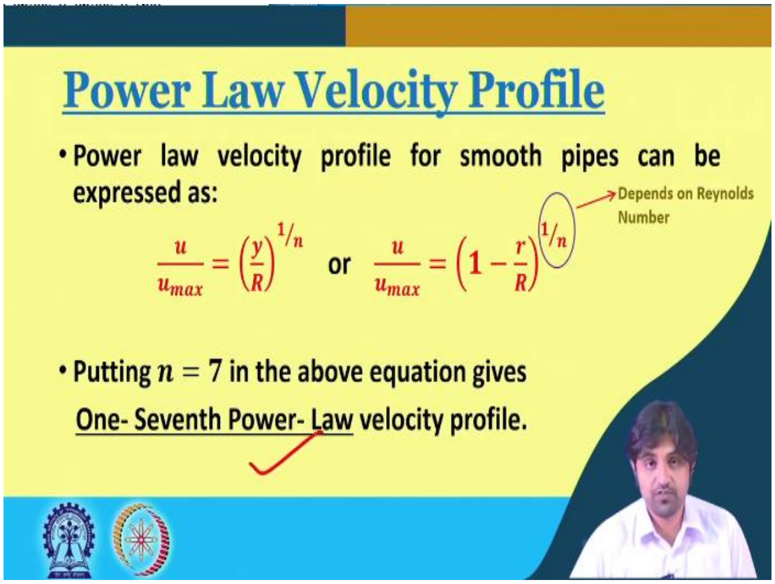

Now, about the power law velocity profile, a little bit on that. So, the power law velocity profile for smooth pipes can be expressed as, it is, u by u max can be written as y by R to the power 1 by n. This is the power law velocity that was given or we can always write in terms of R because y is, y is R minus small r and that you substitute here, you will end up in this equation. So, this 1/n depends upon the Reynolds number, putting n is equal to 7 in above equation gives one-seventh power law velocity profile. This is one of the very famous velocity profiles.

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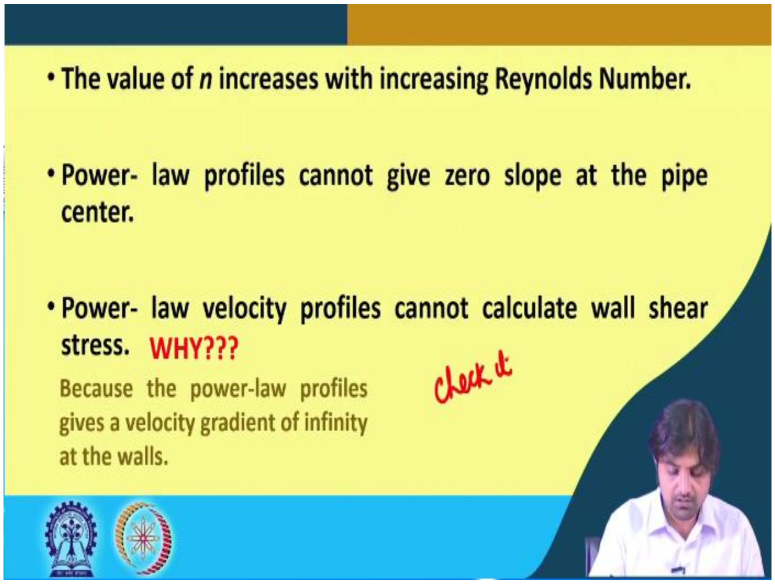

The value of n will increase with increasing Reynolds number. Power law velocity profiles cannot give 0 slope at the pipe center. So, power law of velocity profiles also cannot calculate the wall shear stress. Why? Because the power law profiles gives a velocity gradient of infinity at the walls. That you can try, by substituting r is equal to, small r is equal to capital R. You can check it check it at.

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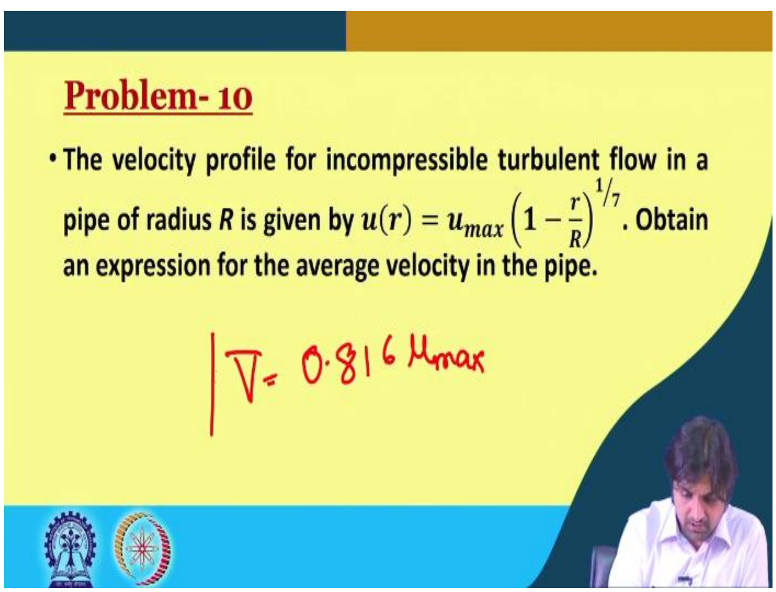

Now, we are going to solve one of the problems. So, the velocity profile for incompressible turbulent fluid in a pipe of radius R is given by u of r as u max into 1 minus r by R to the power 1 by 7. Obtain an expression for the average velocity in the pipe. Hear by chance, or by, you know, you get 1 by 7 power law. So, how to attack this type of problems and actually you can be given any such profile and you should follow the same procedure as I am going to do now, so that, you are able to calculate the average velocity in the pipe. So, how to do that? We simply go to white screen.

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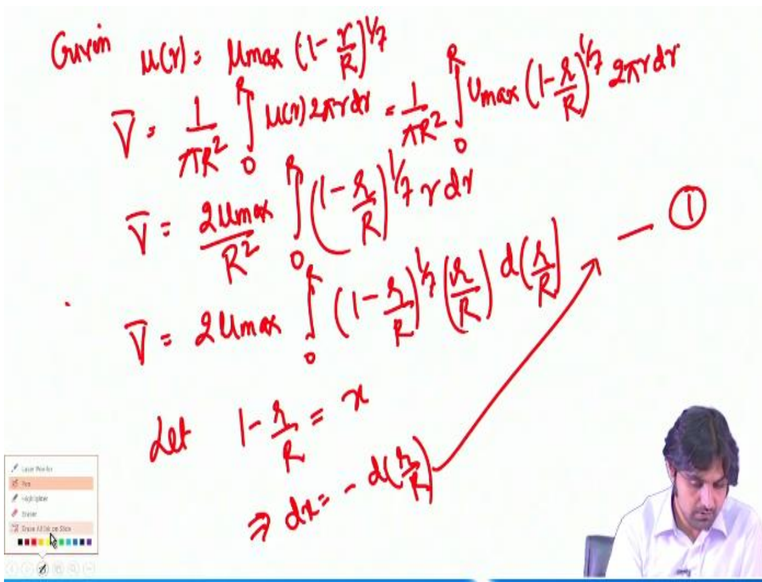

And we write given, u of r is given as u max into 1 minus r by R to the power 1 by 7. So, the average velocity profile will be simply 1 pi divided by the entire area 0 to R u r 2 pi r dr or we can write, 1 by pi R square integral 0 to R u max into 1 minus r by R to the power 1 by 7 2 pi r dr. Therefore, we can take out u max outside and we can also take 2 pi also outside.

So, this pi, pi gets cancelled, 2 comes out, so, it became 2 u max by R square because pi gets cancelled integral 0 to R 1 minus r by R to the power 7 r dr or what we can do is, V bar is 2 u max integral 0 to R, we will keep this one as 1 by 7, and we can write r by R and d r by R and call it one. Let, 1 minus r by R as x, this means, dx is equal to minus d r by R. And so, if we substitute this, in this equation, what we can do? I will start from,

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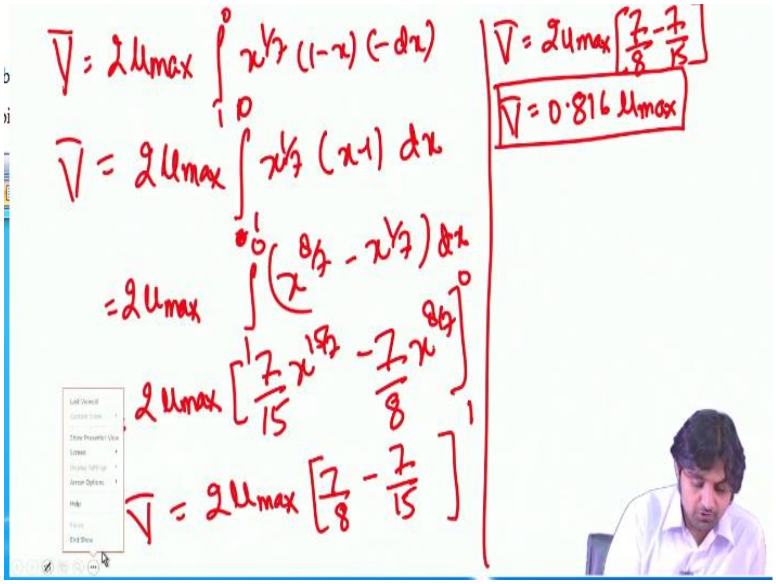

So, using the, we write V bar is equal to 2 u max integral x to the power 1 by 7 one minus x minus dx. So, that is from 1 to 0. So, it will be 2 u max integral 0 to 1 x to the power 1 by 7 x minus 1 dx or 2 u max x to the power 8 by 7 minus x to the power 7 dx, that is, 1 to 0. These limits have to be checked 1 to 0. So, it was 1 to 0, 1 to 0, 1 to 0.

So, V prime can be 2 u max into 7/15 x to the power 15 by 7 minus 7/8 x to the power 8/7, the limits from 1 to 0 for both side. So, V bar is 2 u max and we substitute one as 1 and one as 0. So, it will be 7 by 8 minus 7/15. So, V bar will be written by 2 u max into, just copying the same thing from the last line, 7/15 and V bar is going to be 0.816 u max. So, this is actually a general way to calculate the average velocity. So, does not matter what the profile is, this is how you are going to proceed. So, an expression I will write it down here as well is 0.816 u max .

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So, these are the references, as I have already shown you in the book, I mean, also in the introduction slides. And so, this week's lecture on the laminar and turbulent flow is finished. I will see you next week with another set of lectures on hydraulic engineering. Thank you so much. Have a nice weekend.