Hydraulic Engineering
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Lecture – 14
Laminar and turbulent flow (Cond.)

Welcome back to this lecture of turbulent and fluid flow, sorry, laminar and turbulent flow. We are going to start with from that point where we left, that was, we are going to solve yet another problem on the laminar flow.

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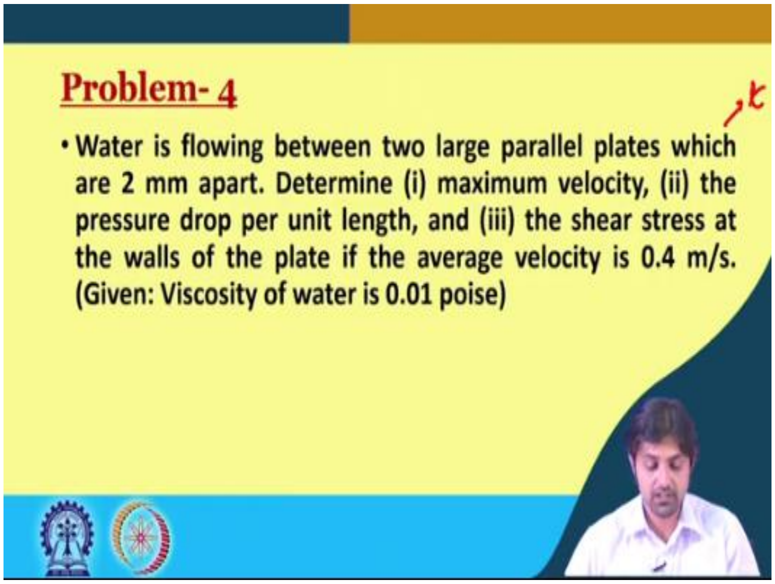

So, the question here is, water is flowing between 2 large parallel plates which are 2 millimeters apart. So, that is, means t is 2 millimeter. Determine the maximum velocity, the pressure drop per unit length, that is, dp dx and the sheer stress at the wall of the plate if the average velocity is 0.4 meters per second. And viscosity of water is given as 0.01 poise. So, to solve this, what I will go to the white screen and first write down what all things are given.

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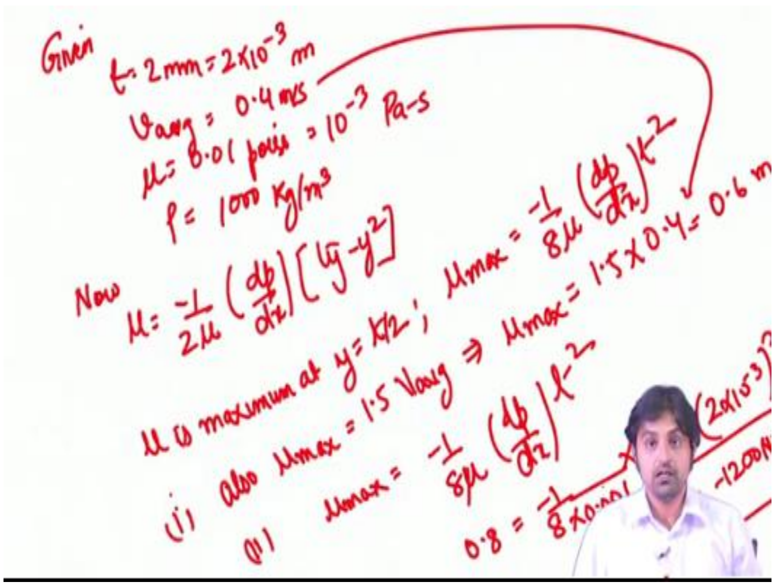

So, given thing is as I told you, t is 2 millimeters or 2 into 10 to the power minus 3 meter, the average velocity is also given 0.4 meters per second, mu is given as 0.01 poise which will be divided by 10 to obtain into 10 to the power minus 3 Pascal second, and at density of water is assumed to be 1000 kilogram per meter cube. So, we write the equation that we had derived. Now, u is written as minus 1 / 2 mu dp dx into t y minus y square, u is maximum at t is equal to, at y is equal to t / 2.

This means, u max is equal to minus 1 / 8 mu dp dx into t square. Also u max is equal to 1.5 V average. This implies that u max is equal to 1.5 and this is 0.4 from here and that comes out to be 0.6 meters per second. So, that is, the first, the second u max is also written as minus 1 / 8 mu into dp dx t square. So, if you substitute the value of u max, that is, 0.8 is equal to minus 1 / 8 into 0.001 and dp dx is what we need to determine and t is 2 millimeters. So, 2 into 10 to the power minus 3 whole square and this way we can obtain dp dx is equal to minus 1200 Newton per meter squared per meter, so, minus 1200 Newton.

So, yeah, it should be minus 1200 Newton per square per meter. I will take care. I think the sheet is being hidden by my photo. Anyways, I will proceed to the third subpart.

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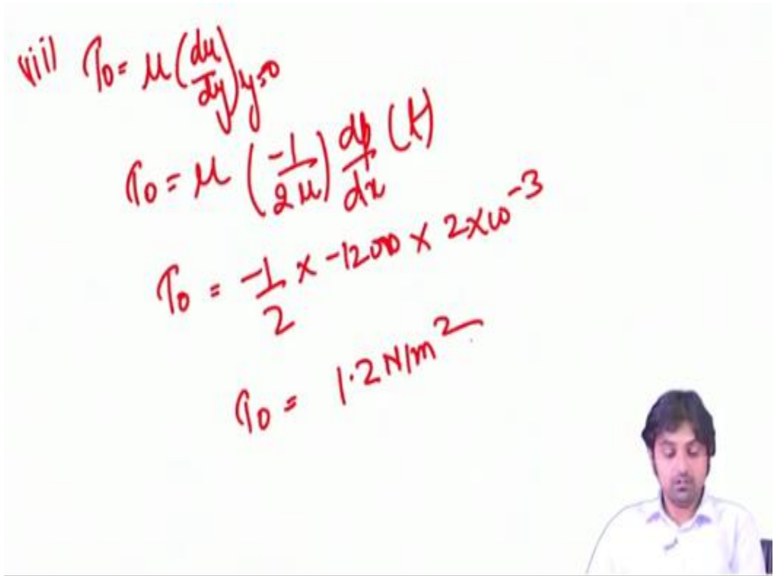

I will erase all the ink on this slide and the third part is tau not is equal to mu du dy at y is equal to 0. So, tau not is mu du, we know u, so, we can differentiate that and obtain minus 1 / 2 mu dp dx into t and substituting the values mu is minus 1 / 2 into minus 1200 into 2 into 10 to the power minus 3. So, tau not is going to be 1.2 Newton per meter square. So, this is the solution to the question number 4. This was the third part. So, going back to the screen, so, this problem is solved now.

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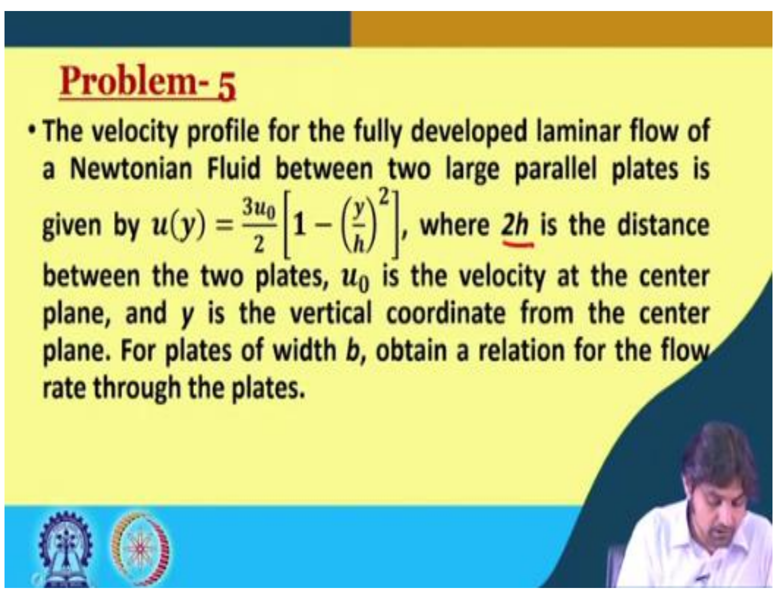

Now, we will solve one more problem, the last problem of the series of laminar flow around plates. It is a little inquisitive question. The velocity profile for the fully developed laminar flow of a Newtonian fluid between 2 large plate is given by, so, we have been given a new profile u y is given as 3 u not / 2 1 minus y / h whole square / 2, where 2h is the distance between 2 plates, u not is the velocity at the center plane and y is the vertical coordinate from the center plane.

So, more importantly, the frame of reference or x and y axis is not at the bottom but it starts from the middle between the plates. For the plates of width b, obtain a relationship of flow rate through the plates. So, what we are going to do, we are going to solve it and as always, we are going to use the white screen.

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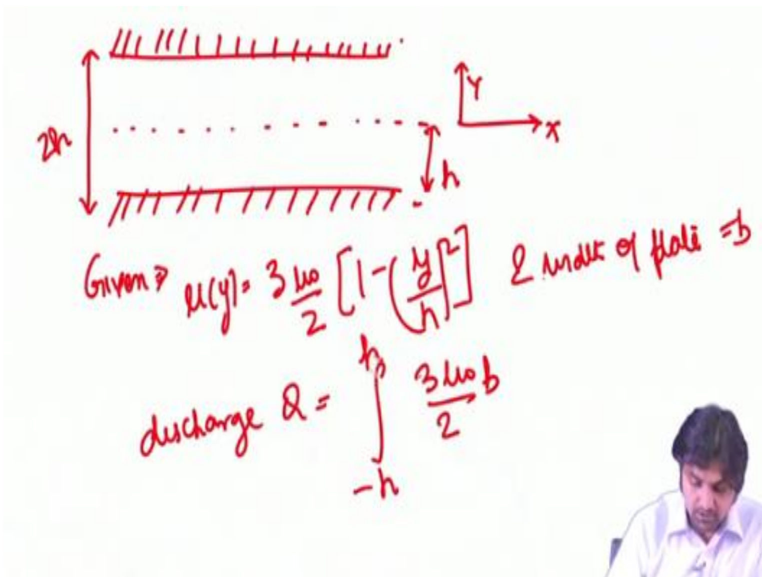

First of all it is more important to draw the plates and we draw. So, if this is the center line let us say, we are going to draw x axis in this direction, y axis is in this direction, this is h, the total thickness is 2h. So, the thickness width will be used when calculating the discharge. So, what we have been given.

Given is, u of y can be written as, 3 u not / 2 into 1 minus y / h whole square and width of plate is equal to b, this we have already been told. So, discharge Q is going to be, so, basically we have to integrate from minus h to h because this is 0 then this is minus h and this is plus h. 3 u not 2 we wil multiply, you know, or let me first write down the, I mean, this formula, then we will put in the values.

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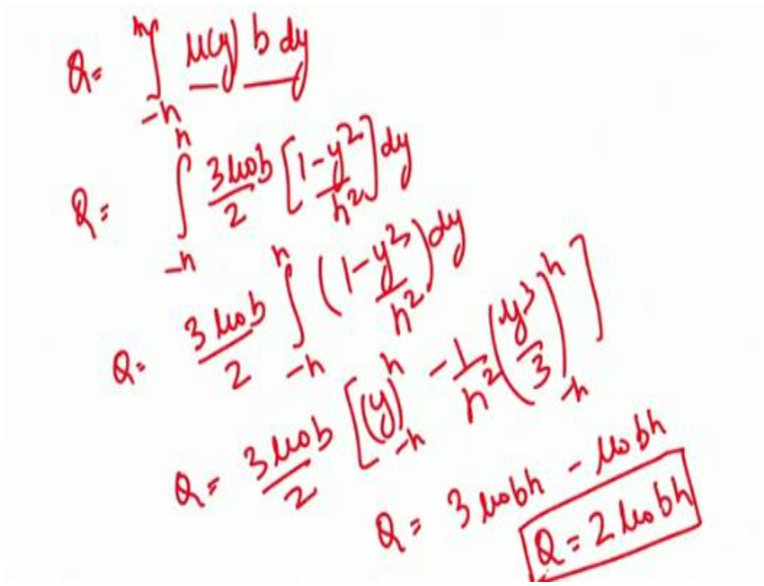

So, Q is equal to integral minus h to h u of y b dy. The discharge area into velocity. This is velocity this area and integrated over from minus h to h or when we can, sorry, so, minus h to h then it becomes 3 u not / 2 b into 1 minus y square / h square into dy. Now, we have to simply integrate it. So, Q is going to be, so, we will takeout whatever is constant, minus h to h 1 minus y square / h square dy or Q is equal to 3 u not / 2 b and after integrating it becomes y minus h to h minus 1 / h square y cube / 3 minus h to h and this will give 3 u not b h minus u not b h. So, Q will be 2.

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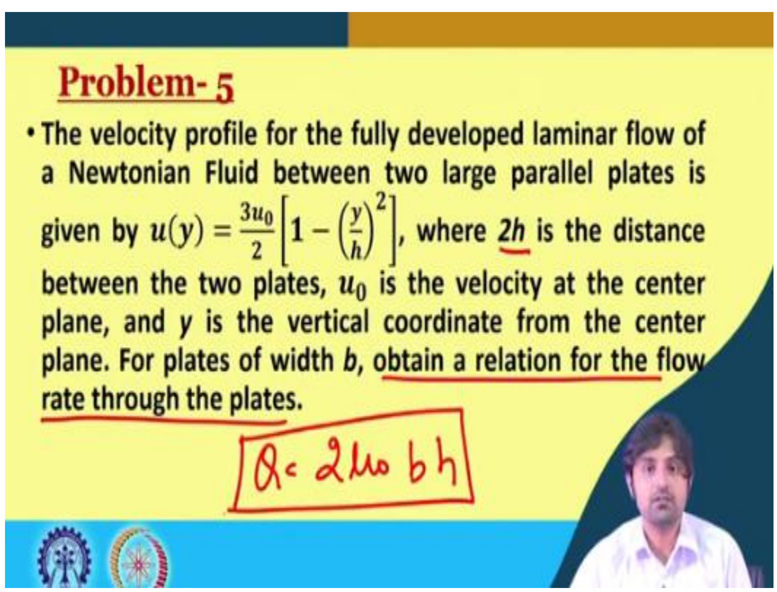

Now what we are going to do is we are going to the, so, we have to obtain a relationship for the flow rate through the plates and which we have got Q is equal to 2 u not b h for the problem that was there.

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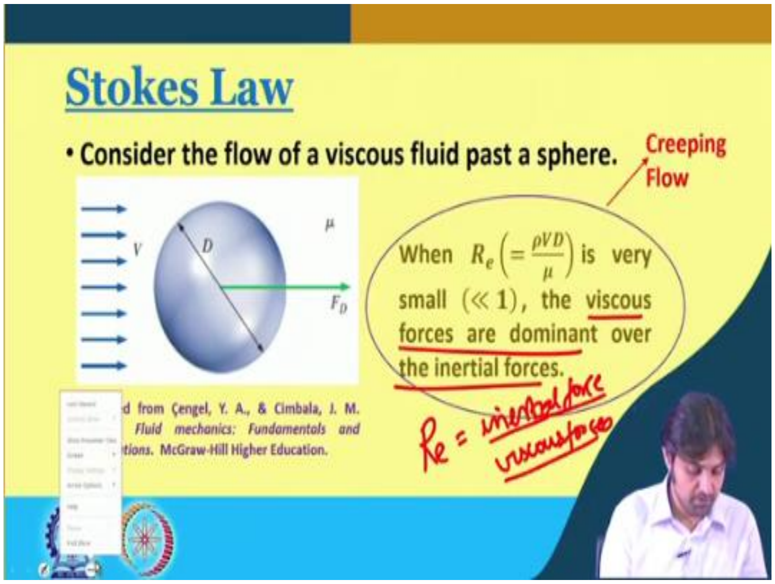

So, we now proceed to the another concept called Stokes law. So, we have to, you know, if we consider the flow of viscous flow past a sphere. So, this is the sphere and the, the flow is, there is a flow of velocity V coming to the sphere, the diameter of the sphere is D, mu is the kinematic fluid viscosity, then we will, you know, see that when the Reynolds number rho V D / mu is very small.

When this happens, then the viscous forces are dominant over the initial forces. But we already know that because Re is, in denominator what we have, inertial forces. So, if viscous forces will be dominant than inertial forces then Reynolds's number will be less than 1. This is very this, I mean, this can be seen from the equation as well. This is called creeping flow.

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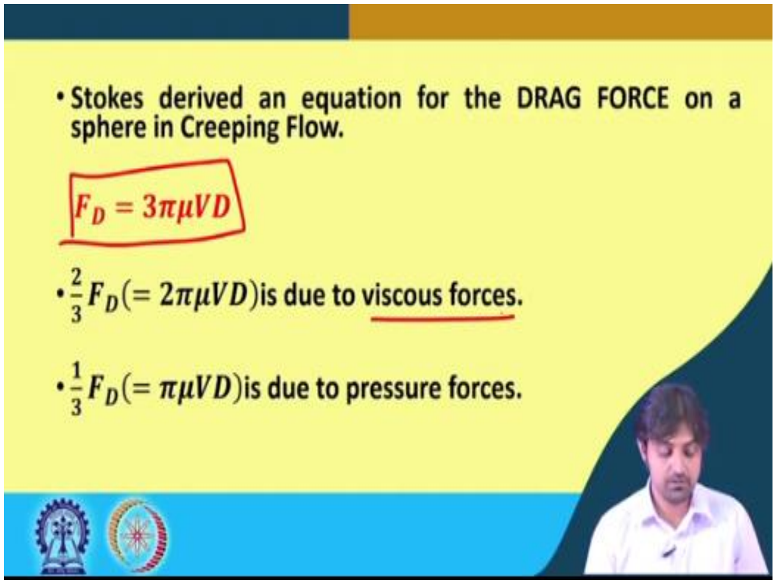

So, now Stokes derived an equation for drag force on the sphere in creeping flow. So, the derivation of the Stokes, I mean, this drag force is outside the scope. So, I am what I am going to give you is, I am going to give you directly the result. So, the drag force on the sphere is given by $3\pi\mu VD$, I think, you should all should memorize it, out of which two-third of this, that is, 2 pi mu VD is due to viscous forces and 1 / 3rd contribution is due to the pressure forces. So, this is important.

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So, after this we are going to calculate the terminal fall velocity. So, if there is a sphere, you know, there is when it falls through a liquid, there is the weight acting and there is a drag force acting against its motion direction. We are going to see how we arrived at the terminal fall velocity.

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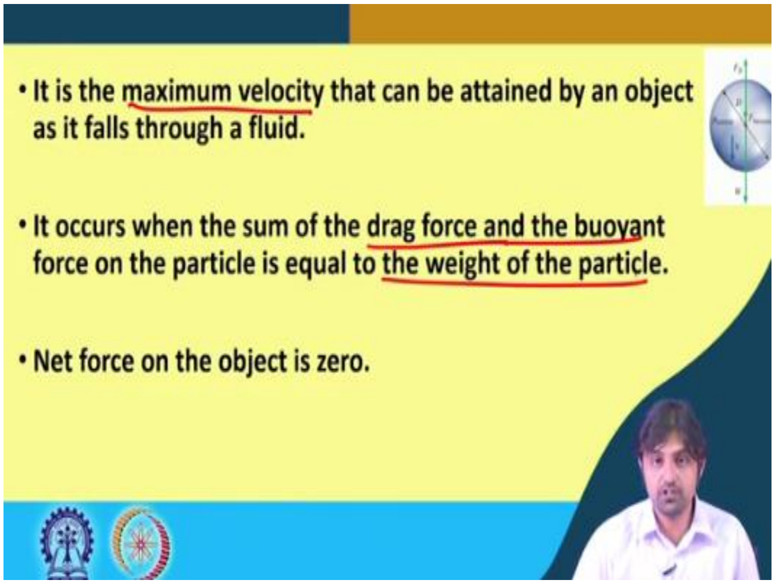

So, for the sake of easiness, I have put a figure here, on the right side top corner, where you can see the sphere. So, what is terminal velocity? It is the maximum velocity that can be attained by an object as it falls through a fluid. It occurs when the sum of the drag force and the buoyant force on the particle is equal to the weight of the particle. The drag force and the buoyant force on the particle is equal to the weight of the particle. Because both drag force and buoyant force will try to take it upward and weight will try to bring it downward. Therefore, the net force on this particular sphere or object is 0.

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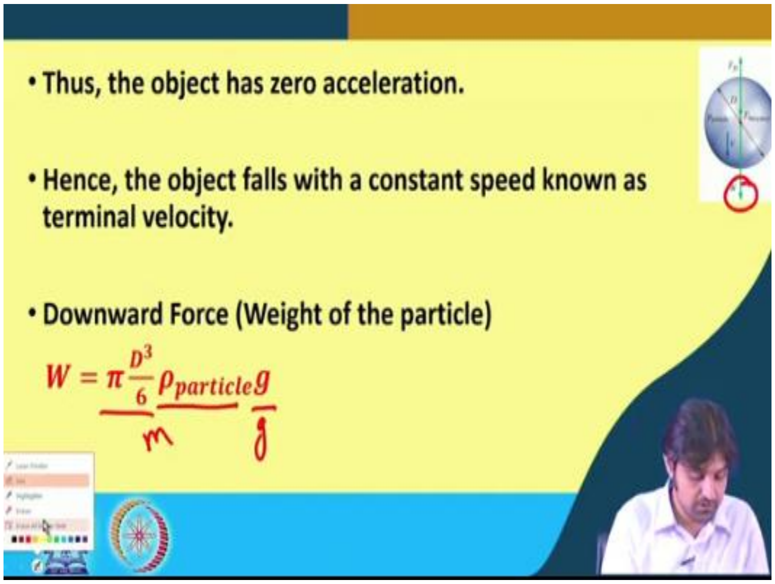

Thus, the object has 0 acceleration, as the object falls with a constant speed, known as terminal velocity. This phenomenon is what results in a sphere having a terminal velocity in a fluid. So, the weight of the particle is downwards, so, downward forces weight of the fluid, weight of the particle and this is equal to density multiplied by volume is mass, and g is acceleration due to gravity. So, weight is mass m g. For a sphere it is $\pi D^3/6$ is the volume and rho is the particle the whatever the sphere is made of. So, I will just, so you do not have confusion, this is the W.

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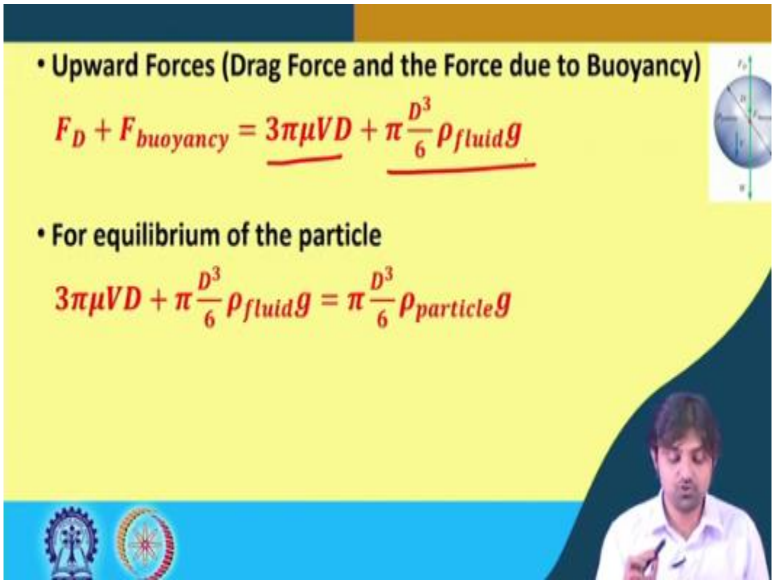

What is going to be the upward force? It is the drag force and the force due to buoyancy. So, let us say, F D is the drag force and F buoyancy is the buoyancy force. As we have already seen 2 couple of slides back, Stokes derived, the drag force as 3 pi mu V D. And the buoyancy force is the weight of the liquid displaced by that sphere. So, it will displace the weight of the liquid that will be displaced will be the volume of the sphere, but into the density of liquid. That is what we have written, rho fluid and this is acceleration due to gravity.

So, F D plus F buoyancy is 3 pi mu V D plus pi D cube / 6 rho fluid into g. Now, for the equilibrium of this particle these 2 forces should be balancing. So, F D this plus F buoyancy is equal to W for example.

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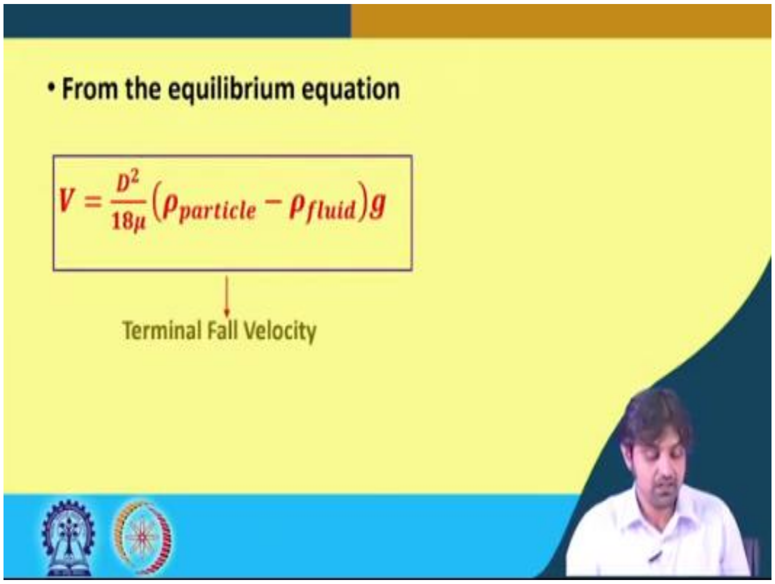

So, from the equilibrium equation we can simply rearrange these, because V is the term that we get from Drag Force equation and this is the terminal fall velocity. So, this is an important equation

$$V = \frac{D^2}{18\mu}(\rho_{particle} - \rho_{fluid})g$$

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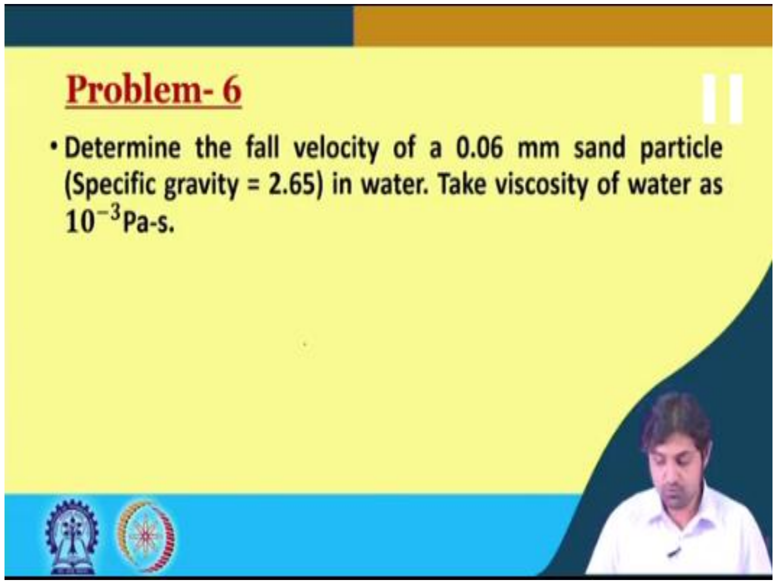

Now, we will solve a problem. The question says; determine the fall velocity of a 0.06 millimeters sand particle, its specific gravity 2.65 in water. Take viscosity of water as 10 to the power minus 3 Pascal second. This is as simple a problem as it can get. Here, you are given the diameter of the particle, we have given the gravity. Therefore, you know, the density of the particle, water properties you already know, which has actually viscosity been given already to you.

It is 10 to the power minus 3 Pascal second. By this time you need not have to be given the viscosity of water all the time, you must have an idea how much that viscosity is.

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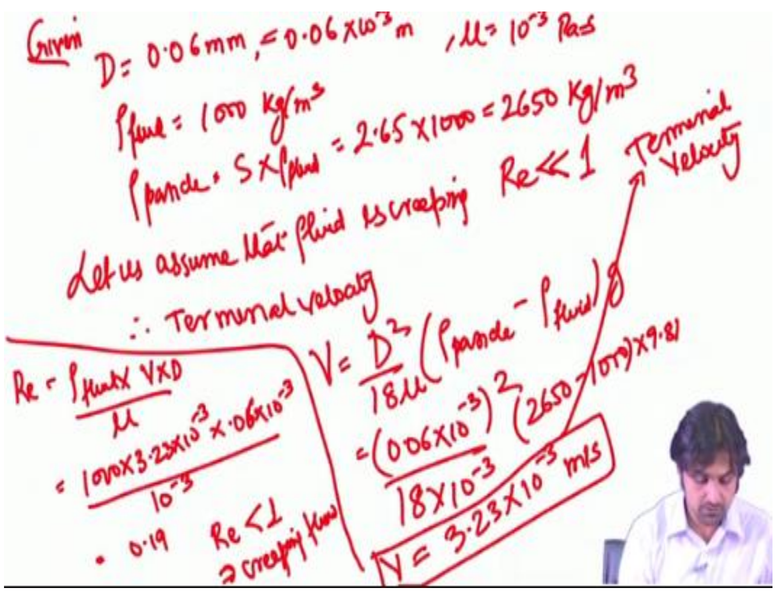

So, as all the other problems what we are going to do is, we are going to have a white sheet and start writing the things that are given first. So, we have been given D is equal to 0.06 meters or, sorry, millimeters or 0.06 into 10 to the power minus 3 meter, mu is given as 10 to the power minus 3 Pascal second, rho of fluid is given as because it is water so, we can take 1000 kilogram per meter cube, rho particle will be S into rho fluid. So, S is given as 2.65 into 1000 and that gives 2650 kilograms per meter cube.

So, we have to assume, let us assume that, fluid is creeping, that is, Reynolds number is very much, not very much but less than 1. Therefore, terminal velocity V, as we have derived is written as D square / 18 mu rho particle minus rho fluid / g and this 0.06 into 10 to the power minus 3 whole square divided by 18 into 10 to the power minus 3 2650 minus 1000 into 9.81.
This gives, V is equal to 3.23 into 10 to the power minus 3 meters per second. So, this is an important value that we have got now, V. So, please note it down.

But we have to check if our assumption of creeping flow is correct or not. So, for doing that, what we can do is, in here in this slide itself, so, R e can be given as rho fluid into V into D / mu. So, 1000 into 3.23 into 10 to the power minus 3, D is 0.06 into 10 to the power minus 3 divided by 10 to the power minus 3. So, this is 0.6. And this comes approximately to 0.19. So, our Reynolds's number is less than 1, therefore, it is creeping flow. So, our this is the terminal velocity. So, we are going to go back to the slide. So, this is the problem that we have solved.

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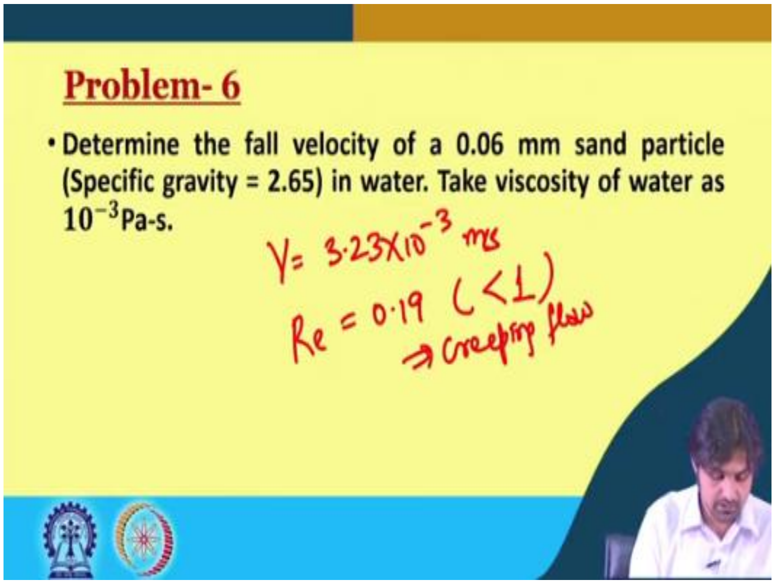

So, we have got V is equal to 3.23 into 10 to the power minus 3 meters per second and after checking the Reynolds number it came out to be 0.19 which is less than 1, therefore, creeping flow and we would have applied the terminal velocity.

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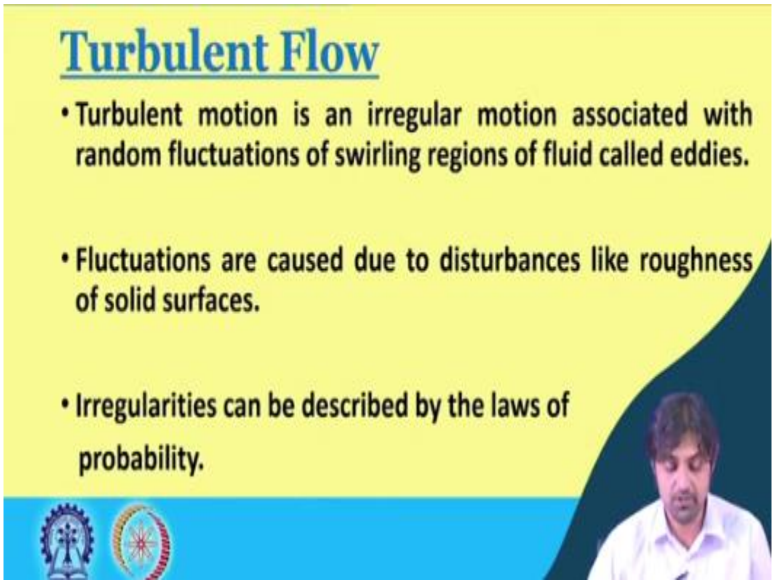

So, we are now moving slowly to the phenomenon of turbulent flow. So, turbulent motion is an irregular motion which is associated with a random fluctuation of swirling regions of fluid called eddies. These fluctuations are caused due to disturbances like a roughness of solid surface. Suppose a fluid is moving and it counters a roughness a rough surface. So, the fluctuations in the velocity will happen, will arises, will be produced and these are the causes of turbulent motion. Irregularities can be described by different laws, one of which is law of probabilities.

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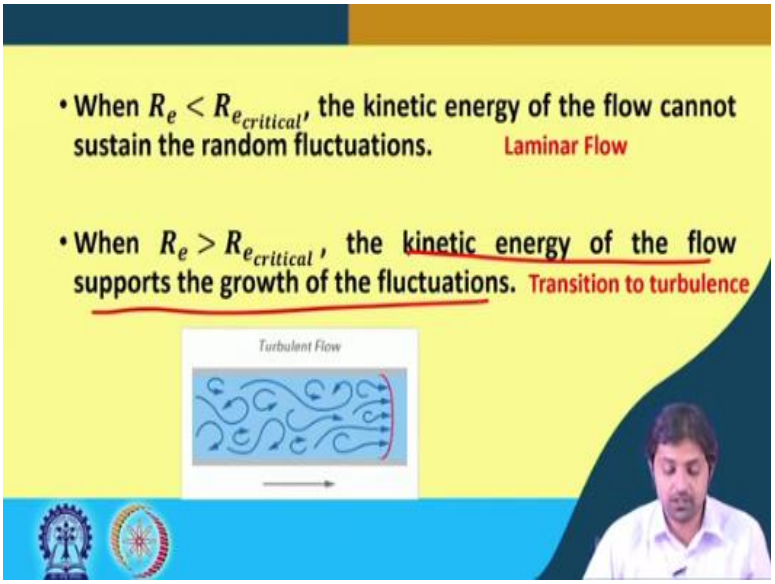

Important to know is, when the Reynolds number of the flow is less than Reynolds number, which is a critical Reynolds number, for if you remember, in the pipe flow it was 2300, the kinetic energy of the flow cannot sustain the random fluctuations and then this is called the laminar flow. When Reynolds number is more than the critical Reynolds number for example, in pipe flow it was 4000 the kinetic energy of the flow supports the growth of the fluctuation.

This is a different way of seeing what a laminar and turbulent flow is or that is called transition to turbulence or turbulent. So, it no longer remains laminar. So, this is how the velocity, you know, vectors or the eddies in the flow looks like, these are turbulent flow.

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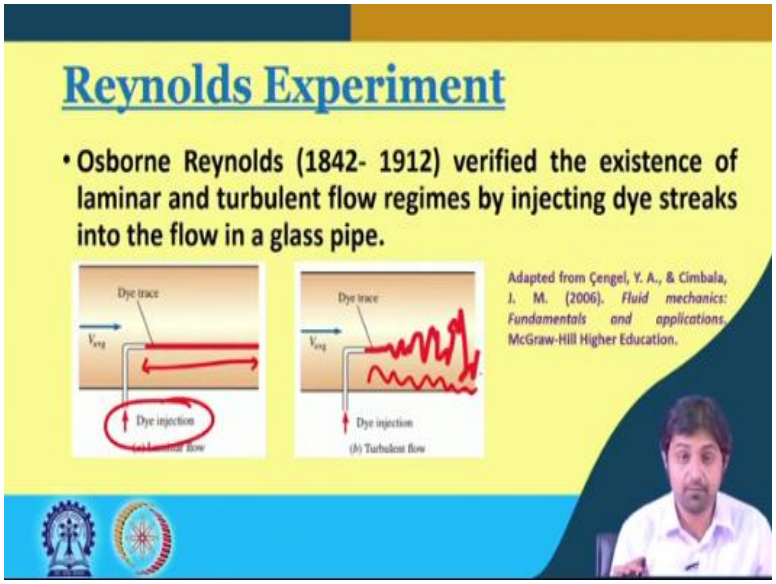

So, when we talk about the turbulent flow there is an important mention about Reynolds experiment. So, Osborne Reynolds he verified the existence of laminar and turbulent flow regimes by injecting dye streaks into the flow in a glass pipe. So, what he did was, he introduced the dye tracer and dye was injected from here. And, you know, then there were certain velocity the dye had a movement like this because the fluid became colored. But when he increased the velocity the dye tracer started having the fluctuations like this, as you can see, and this was a clear indication of the existence of laminar and turbulent flow regimes.

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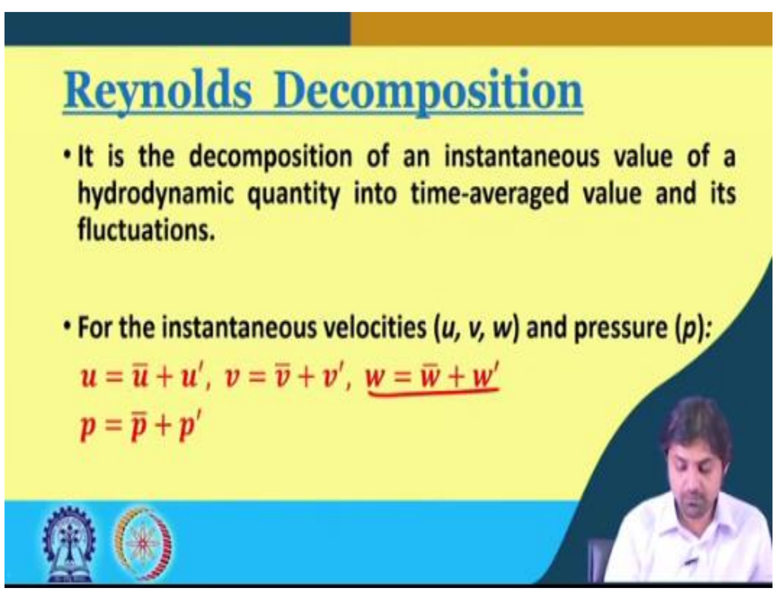

One of the important composite, I mean, one of the important components of the turbulent flow is the Reynolds decomposition. What is that? It is the decomposition of an instantaneous value of a hydrodynamic quantity into time-averaged value and its fluctuation. For example, we are going to describe, but just to tell you, if there is a velocity u an instantaneous velocity, it can be broken up into 2 components; average value plus whatever deviation it has from the average value and those deviations are called fluctuations.

For example, at any point in time, if you measure a velocity, let us say, 2 meters per second, and the average velocity of that flow is 1.8 meters per second, then the fluctuation is going to be 0.2 meters per second, this is just a very crude example. So, here we see for the instantaneous velocity u, v and w and pressure P, u can be written as,

$$u = \overline{u} + u'$$

. This is the average, bar denotes average, prime denotes fluctuations. So, bar denotes average values and prime which is the like this denotes fluctuate, w is

$$w = \overline{w} + w'$$

, p as

$$p = \overline{p} + p'$$

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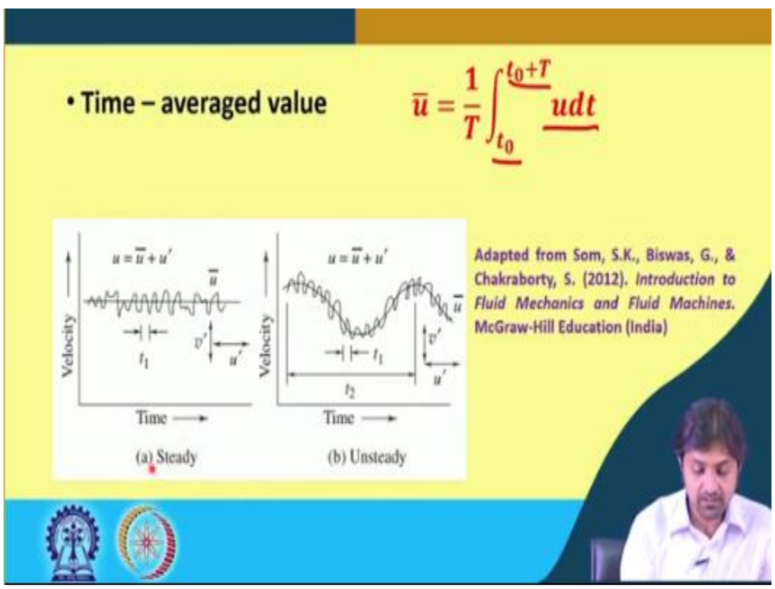

What are the time-averaged values? Time-averaged value is, if we start from beginning or time T is equal to 0 and integrate over t0 + T u dt and divide it by the whole time we get the average time-averaged value. We have done this previously when we were deriving the laminar flow we have used time-averaged values. So, suppose this is a turbulent flow this one here So, I am going to use laser pointer.

So, this is turbulent, you know, like this and by calculation if we say that this is the, the straight line here, is the u bar then the deviation of at any point is called the fluctuation. Therefore, u is u bar plus u prime. If we consider this point then the fluctuation is in the other direction. This is also the same, this is steady and this is unsteady, it is changing in time.

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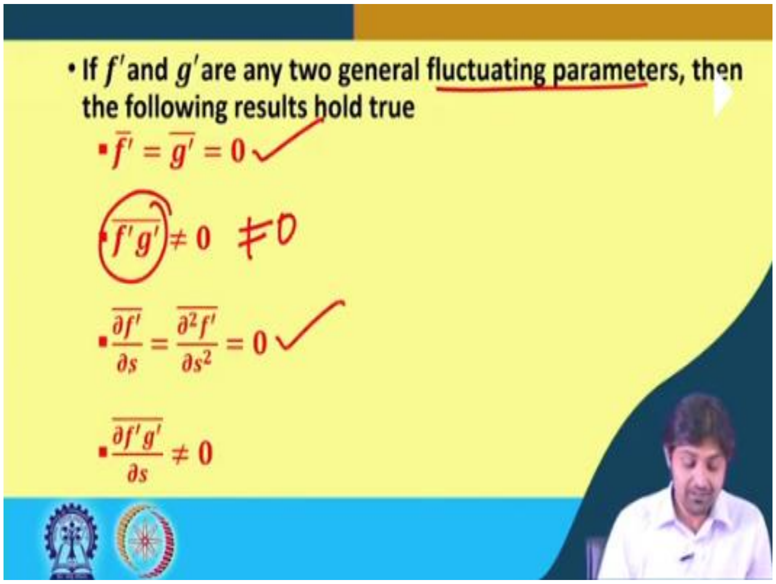

So, we are going to define some maths here. If f' and g' are any two general fluctuating parameters, we have seen u prime, v prime, w prime and p prime then the following result holds true. So, if we take the average of the fluctuating component it will be 0. If we multiply both fluctuating component and then take the average it will never be 0 it will be non 0. Also, if we differentiate those fluctuating component or double differentiate it and take the average alone this will be 0. And if we do the differential of multiplication of these 2 quantities, then it will never be equal to 0. These are the Reynolds condition.

So, I think this is a nice place to stop before the next lecture. We are going to start with shear stress in turbulent flow in the next class. Thank you so much