Hydraulic Engineering
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Lecture- 13
Laminar and Turbulent flow (Contnd.)

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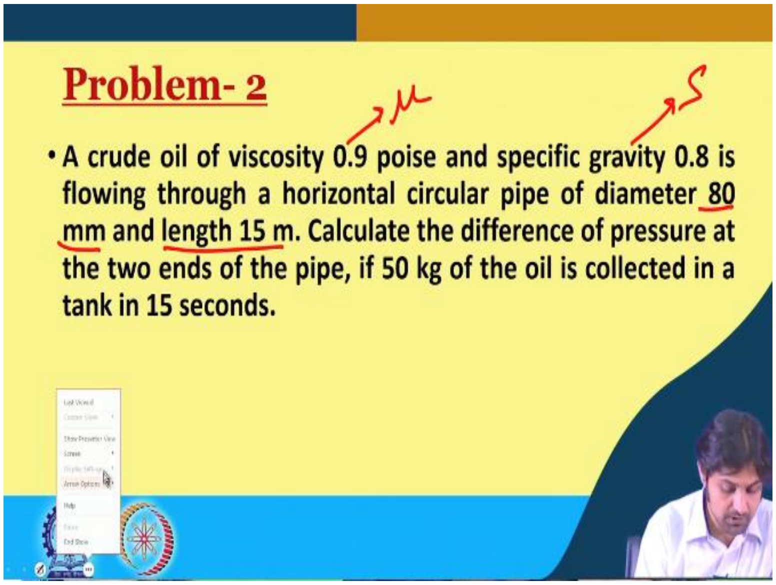

Welcome back to this lecture. As we were talking, we are going to start with another problem of laminar flow in pipes. The question says that the crude oil of viscosity 0.9 poise and specific gravity 0.8 is flowing through a horizontal circular pipe of diameter 80 millimeters and length of 15 meter. Calculate the difference of pressure at the 2 ends of the pipe, if 50 kilograms of oil is collected in a tank in 15 seconds. So, we are knowing this is mu, this is specific gravity S is 0.8. So, we know the diameter is 80 millimeters and the length is 15 meters. So, going back to, you know, we are going to have a white screen.

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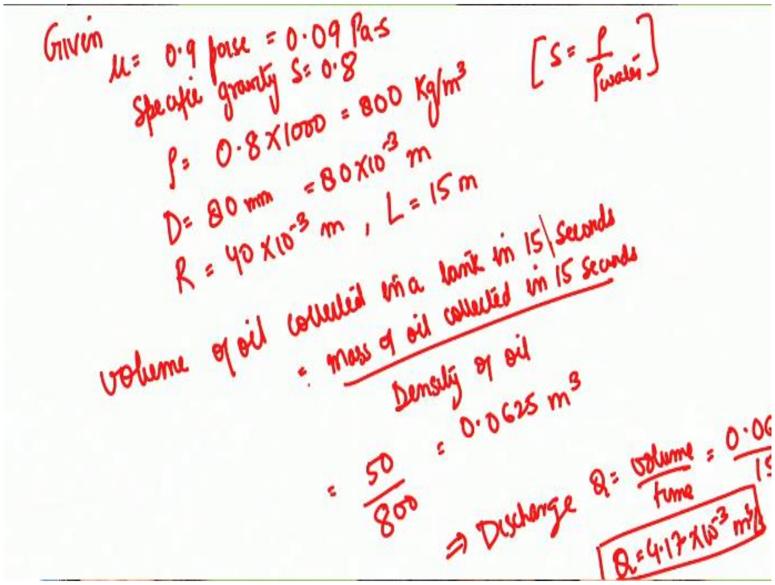

So, as always what we are going to do, we are going to write the things that are actually given. Given thing is mu is 0.9 poise or in SI unit is 0.09 Pascal seconds. We know it is specific gravity S is 0.8. Therefore, the density is given to be 0.8 and if we assume 1000 kilograms per meter cube density of water, so, the density of the fluid is 800 kilograms per meter cube, since, S is rho / rho water.

Diameter we know, it is 80 millimeters or 80 into 10 to the power minus 3 meter, R we know 40 into 10 to the power minus 3 meter and length L is given as 15 meter. So, these are the things that we already know from before. So, first we are going to do, we are going to calculate the volume of oil collected in a tank in 15 seconds is equal to mass of oil collected in 15 seconds divided by density of oil and that is going to be 50 divided by 800 that is 0.0625 meter cube.

This is the volume of the oil that is collected in a tank in 15 second. Therefore, discharge Q is equal to volume by time equal to 0.0625 divided by 15, Q is 4.17 into 10 to the power minus 3 meter cube per second.

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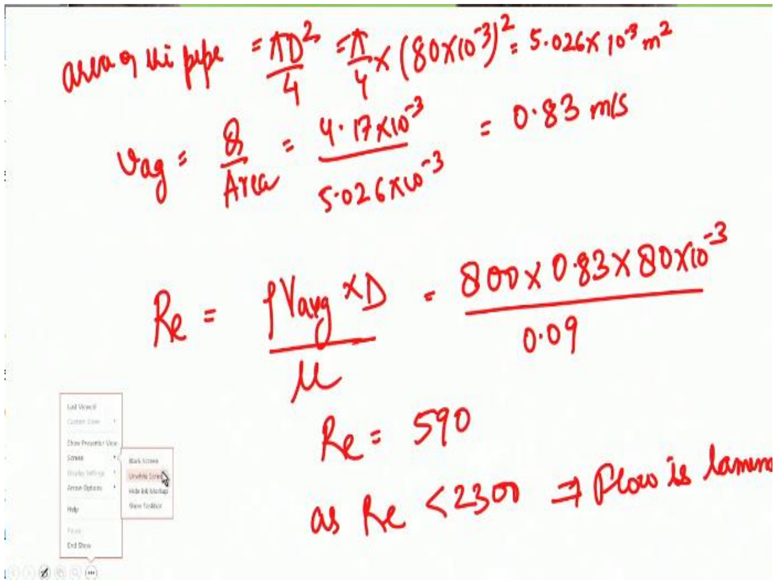

See, calculate the area of the pipe, area of the pipe is pi D square / 4 and, that is, pi by 4 into diameter was 80 into 10 to the power minus 3 and this gives us 5.026 into 10 to the power minus 3 meters square. So, the average velocity is Q by area and this will give us 4.17 into 10 to the power minus 3 divided by 5.026 into 10 to the power minus 3 is equal to 0.83 meters per second. Therefore, we will find the Reynolds number that is, rho V average into D / mu and this rho is 800, V average is 0.83, diameter is 80 into 10 to the power minus 3, mu is 0.09. This is actually very important to check and this comes to around 590. So, as Reynolds number is less than 23 the flow is laminar.

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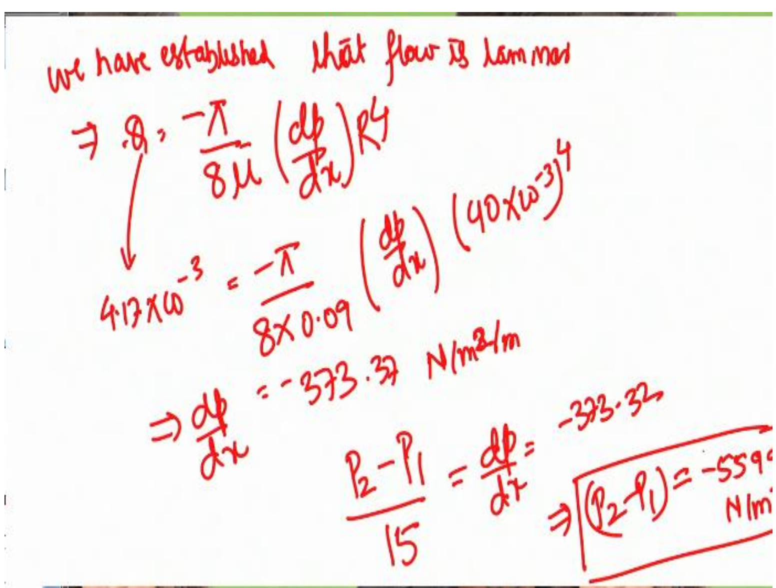

So, we are going to do the again white screen. So, we have established that flow is laminar, therefore, Q is going to be the formula that we have derived minus 8 minus pi divided by 8 mu dp dx into R to the power 4. So, Q value we already know, that is, 4.17 into 10 to the power minus 3 is equal to minus pi 8 into 0.09 dp dx into R, we know, 40 into 10 to the power minus 3 to the power 4, what we get is dp dx is equal to minus 373.32 Newton per meters square per meter.

This means, this is the dp dx and it is constant, so, the pressure difference per unit length is minus 373.37. If we assume P 2 pressure at one end and P 1 at other and length is 15 this should be equal to dp dx equal to - 373.32 implies P 2 – P 1 the pressure difference between the both end is - 5599 Newton per meter square. So, this is the final calculation.

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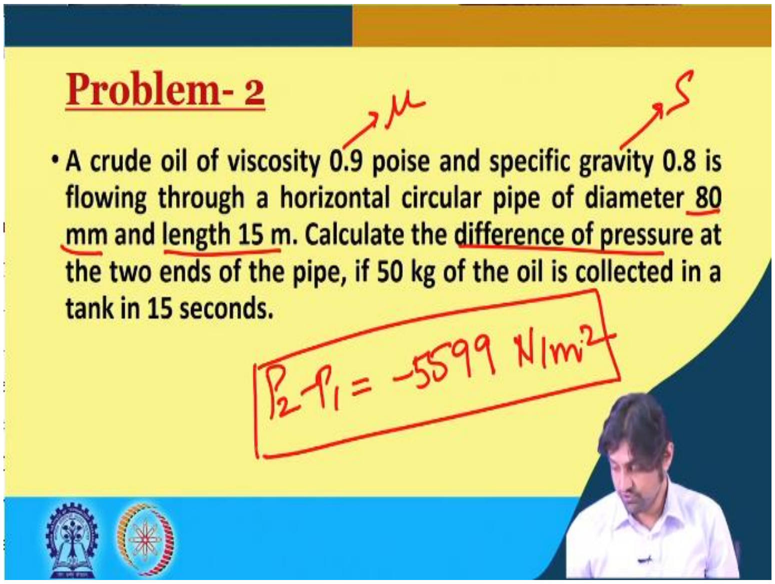

This was the difference of pressure. So, that came out to be finally and right here, P 2 - P 1 came out to be minus 5599 Newton per meters square. So, this is how we solved a detailed solution. Let me take down this ink.

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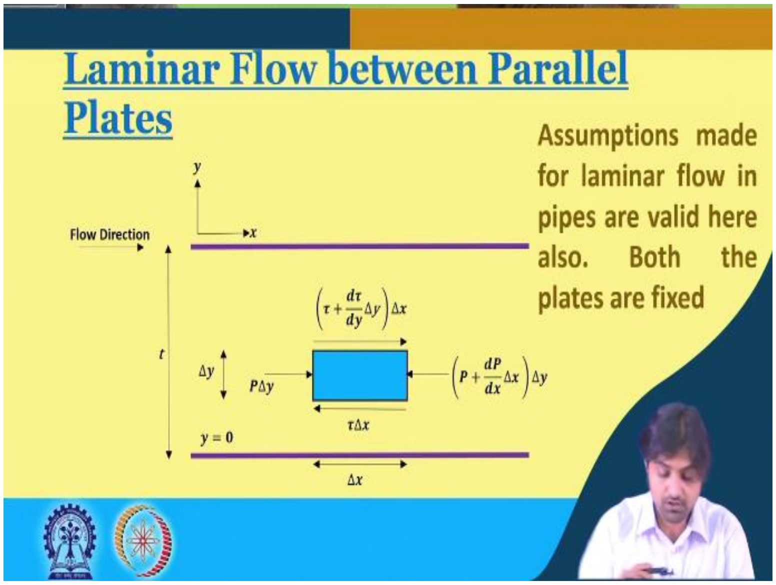

So, now we have done the laminar flow through the circular pipe. Now we are going to see the laminar flow between the parallel plates. So, this is a parallel plate. So, whatever assumptions that we have made for laminar flowing pipes are valid here also. In this case both the plates are fixed plate and this is also fixed plate. As you can observe, this is x direction and this is y direction.

The thick the distance between the plate is given by t, the flow is in this direction positive x axis and what we have assumed is an element having the thickness this length is delta x and this is delta y this is the direction. And as we have written all the forces in a similar fashion as we did it for the pipe flow. So, if the pressure is P here, so, the force by per unit meter is P delta y from this side, if the pressure if there is a pressure gradient dP dx.

So, over the length delta x the pressure will become P + delta P delta P / delta x into delta x. This will be multiplied by the area delta y into 1 because we have assumed a unit length on the other direction. Similarly, tau delta x force is the force per unit meter per unit length. And if we assume that the shear stress is increasing in this direction by delta tau delta y, so, this is the force in the other direction.

So, as you can see, pressure forces acting in this direction, pressure forces acting in this direction, on the bottom side, this is the shear forces acting in negative x direction and here it is acting in positive direction.

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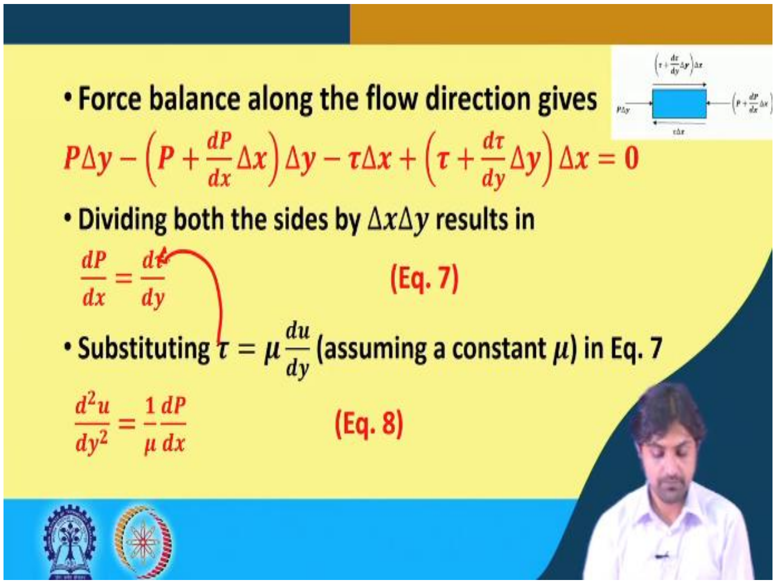

So, with this in background, we start to write the force or we do the force balance analysis along the flow direction. So, for your convenience I have made the free body diagram that is there, I have just reproduced it in the top corner. You see, P delta y, that is, force per unit length minus this one here and this again is in the negative direction tau delta x. So,

$$P\Delta y - \left(P + \frac{dP}{dx}\Delta x\right)\Delta y - \tau\Delta x + \left(\tau + \frac{d\tau}{dy}\Delta y\right)\Delta x = 0$$

Now, if we divide both sides by delta x into delta y, that is, area, what we are going to get is, so, this becomes P delta x delta y and this also become P delta x into delta y whole square. So, this will get cancelled here, here. So, if you do these cancellations what we are actually left with is dP dx here and d tau dy, rest of the things will get cancelled. So, this is one important equation that we are going to get.

So,

$$\frac{dP}{dx} = \frac{d\tau}{dy}$$

. Now, this is again a laminar flow assumption and also we will also assume that the mu the viscosity is constant. So, it will be

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dP}{dx}$$

. This is the equation that we get after we put in this, this is equation 8.

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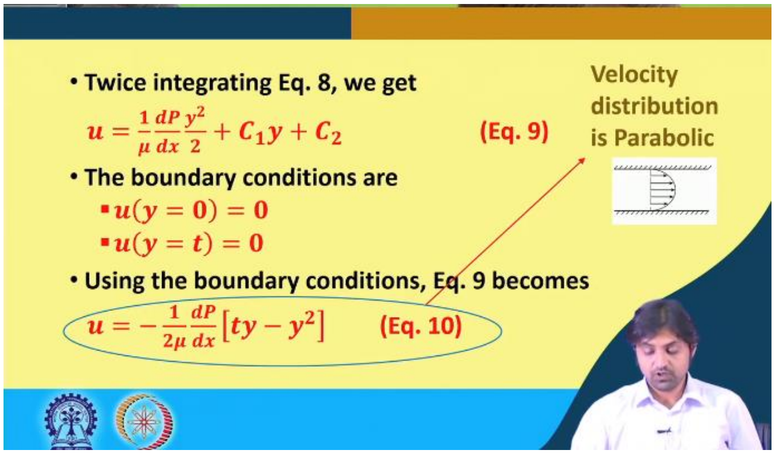

Now, because here it is double integral, so we will integrate it twice. So, we will get

$$u = \frac{1}{\mu}\frac{dP}{dx}\frac{y^2}{2} + C_1 y + C_2$$

, because we have assumed y, you see, y is in this direction. So, this is the equation we get. Now, the boundary conditions are u at y equal to 0 if the plate is fixed. So, due to no slip condition the velocity will be 0 also at the thickness at the other end after the distance t.

Also because of no slip condition the velocity is going to be 0. And if we use these boundary conditions in equation 9, we get

$$u = -\frac{1}{2\mu}\frac{dP}{dx}[ty - y^2]$$

. So, I will just quickly try to tell you how it is done. We know at so because so that you are able to repeat it at home. So, what we say, u at y is equal to 0 0. So, in this equation, we know, at y is equal to 0, so, put this u is 0 at 1 / mu dp dx, y is 0 square / 2 + C 1 into 0 plus C 2, this will give C 2 = 0.

And if you do u at y is equal to t so u at y is equal to t is also 0. So, 1 / mu dP dx and y here is t, so t square / 2 + C 1t because we already found out C 2 is 0. This is how you will obtain C 1. C 1 is going to be minus 1 / mu dP dx t / 2 and you substitute T and C 1 and C 2 in these equations and this is how you end up in this y. Sorry, this u. So this is the final equation for you after substituting the boundary conditions. This is also something that you can try to remember because it is quite simple. Here, also the velocity distribution is parabolic, you see, ty - y square, parabolic in nature. This is how it looks.

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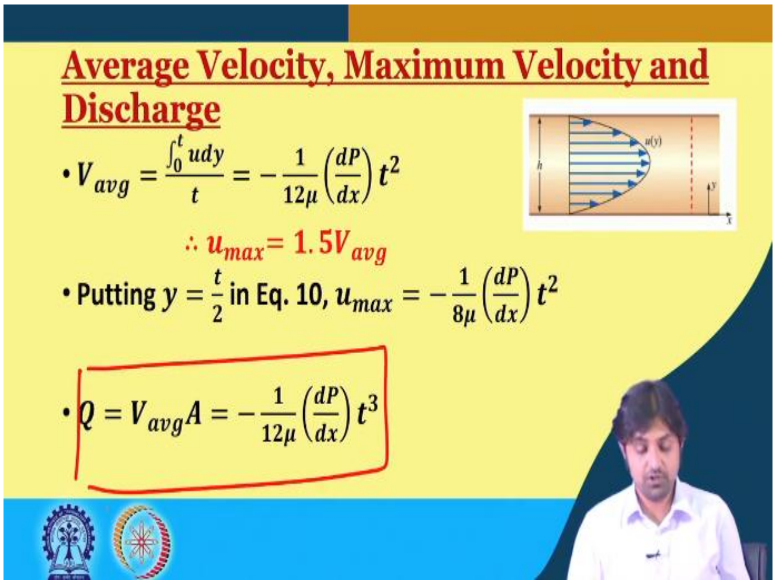

Now, repeating the same procedure of average velocity, maximum velocity and discharge. The average velocity is again given by integral 0 to t udy / t and if you put u as obtained in the last slide here, what we are going to obtain is $-\frac{1}{12\mu}\left(\frac{dP}{dx}\right)t^3$. I would say please do these as homework exercise. If you have any problem, message me on forum.

And you not only this V average, but I request you to also do this for the flow through the pipes that we covered in the last lecture. Now, you see, the velocity profile is parabolic. This means the maximum velocity again is at the center line, which is y is equal to t / 2 and if you obtain y is equal to t / 2 in the equation that we obtained in the equation number 10, then we can get u max, which comes out to be

$$u_{max} = -\frac{1}{8\mu}\left(\frac{dP}{dx}\right)t^2$$

. My recommendation is also try this, and if you are unable to, contact me on the forum.

So, if you divide u max / V average, what we get is, actually u maximum is 1.5 times V average, In the pipe flow it was, in pipe flow it was u max was 2 times V average. Here, it is 1.5 times to V average and Q is simply V average into A. So, V average we already know, 1/12 mu and the area is simply t because we are considering the per unit length. So, that we multiply the per unit length t, so, that becomes t square become t cube. So, these are some of the formulas that you must be remembering.

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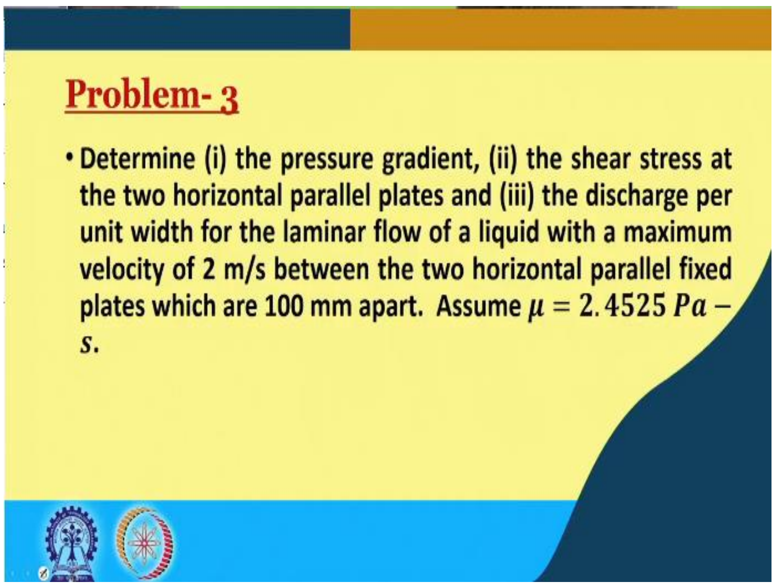

Now, we will solve one problem from this So, the question is determine the pressure gradient, the shear stress at the 2 horizontal parallel plates and the discharge per unit width of the laminar flow of a liquid with maximum velocity of 2 meters per second between the 2 horizontal parallel fixed plates, which are 100 millimeters apart. We assume that mu is equal to 2.4525 Pascal's per second. So, this any to solve this any problem what we do is we simply, you know, write first the things that we know from before.

So, I will just start writing down. First we write out the things that are given. Given is the flow is laminar, u max is given as 2 meters per second, thickness is given as 100 millimeters or 0.1 meters, mu is given as 2.4525 Pascal second. Now, u max is 1.5 V average. This is which we have found out implies V average is going to be 2 divided by 1.5, that is, 1.33 meters per second.

So, for laminar flow between 2 parallel plates, V average is equal to – 1 / 12 mu dP dx into t square.

We simply start putting in the values of V average, – 1 / 12 mu is 2.4525, as given in the question, dP dx we do not know and thickness we know 0.1 square. This will on solution if you use the calculator you will get dP dx is equal to - 39142 Newton per meter square per meter. So, this is the first part, I mean, where we first thing that we do is we obtain dP dx. Now, what I do is choose the white screen again.

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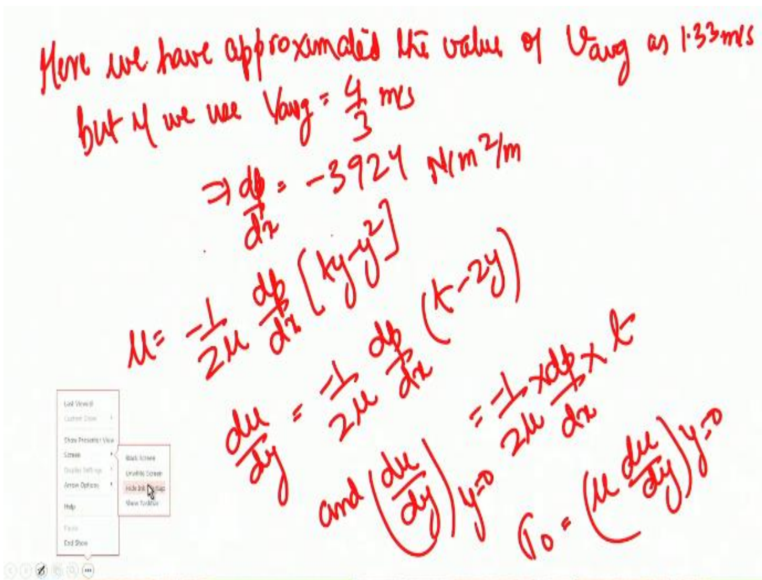

For the another part. So, here we have approximated the value of V average as 1.33 meters per second. But if we use V average is equal to 4 / 3 which is 1.33 meters per second we can get, I mean, there is I am just trying to show there will be slight difference, of course, not that much, but it will be minus 3924 Newton per meter square per meter, I mean, very, very, almost not very difference. So, now we know that u is minus 1 / 2 mu dP dx into ty – y square.

So, du / dy will be minus 1 / 2 mu dP dx t - 2y and du / dy at one plate will be equal to minus 1 / 2 mu into dP dx into t. For tau not at the wall is going to be mu du / dy at y is equal to 0. So, tau not is going to be, this tau not I will solve on the next sheet so, sorry, first let me do the eraser. So, I am choosing another sheet now, white screen.

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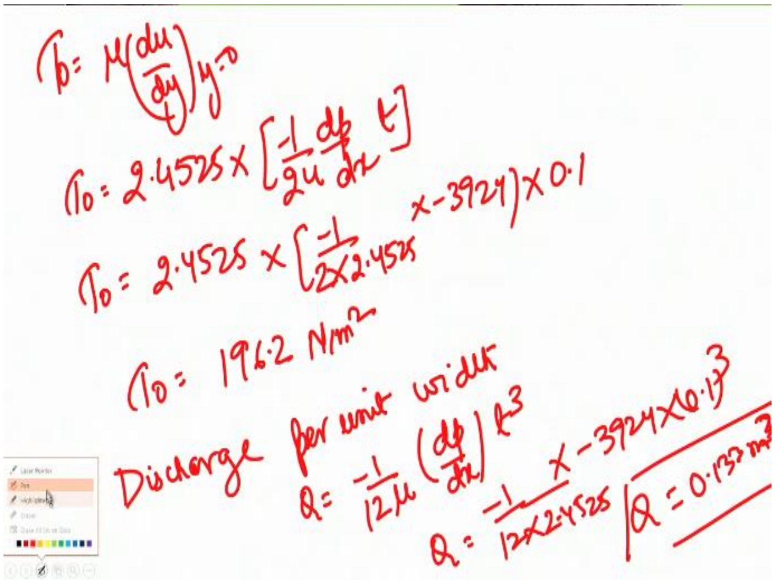

So, tau not is going to be, writing that equation again mu du / dy, at y is equal to 0, which we have already actually calculated. Therefore, tau not is going to be 2.4525 into - 1 / 2 mu dP dx into t or tau not is equal to 2.4525 into - 1 / 2 into 2.4525 and dP dx was - 3924 into 0.1. So, tau not is coming out to be 196.2 Newton per meter square. Now, discharge per unit width Q is – 1 / 12 mu dP dx into t cube. Therefore, Q is going to be minus 1 / 12 into 2.4525 into - 3924 into 0.1 whole cube. So, Q is 0.133 meter cube per second. So, I will erase all the ink here.

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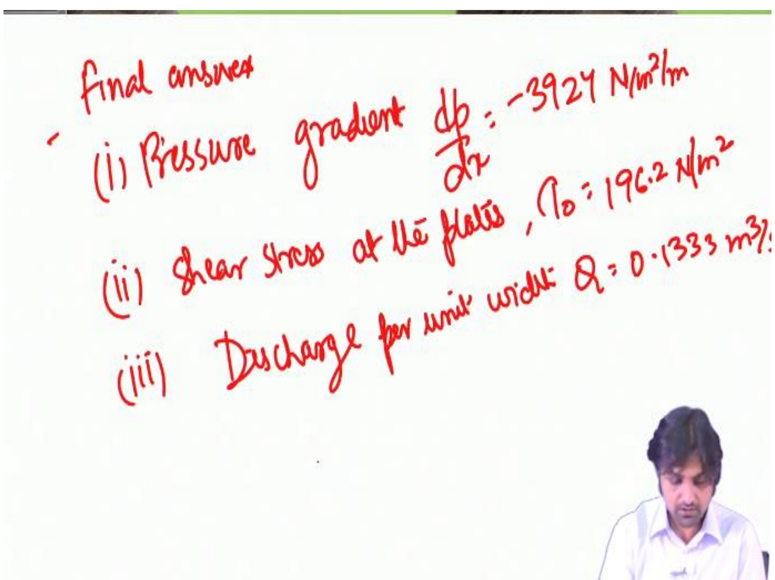

So, final answers is going to be one is the first one is pressure gradient, that was, required dP dx came out to be minus 3924 Newton per meter square per meter. Second one was shear stress at the plates, that is, tau not equal to 196.2 Newton per meter square. And the third one was discharge per unit width Q is equal to 0.1333 meter cube per second. So, these are the final answers. So, this was the problem that we have solved now.

There is one more problem, but I think it is nice time to end this lecture here. And we will resume our next lecture by solving this problem. Thank you so much, and I will see you in the next lecture. Bye.