Hydraulic Engineering

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Lecture – 10
Basics of fluid mechanics -II (contd.)

Welcome back, we are going to start this lecture by solving the practice problem, which where we ended our last lecture.

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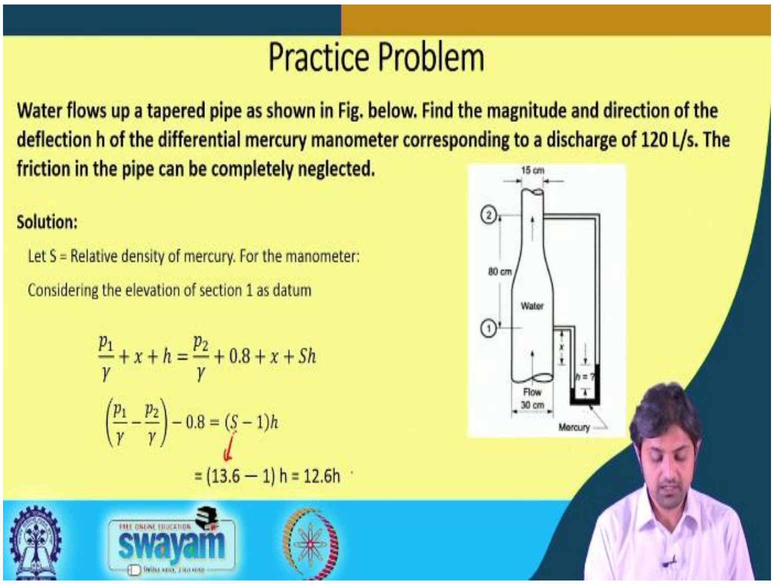

So, the problem goes like this, the water flows up a tapered pipe as shown in the figure below. Find the magnitude and direction of the deflection h of the differential mercury manometer corresponding to a discharge of 120 liters per second. The friction in the pipe can be completely neglected. The reason of neglecting the friction completely is, so that, we are able to apply, you know what, Bernoulli’s equation.

So, if we say that S be the relative density of mercury, so, if we have relative density of mercury S, we can relate all the densities to water using this relative density. So, for the manometer here, considering the elevation this section as the datum, we can write the, you know, we can write the Bernoulli equation as

$$\frac{p_1}{\gamma} + x + h = \frac{p_2}{\gamma} + 0.8 + x + Sh$$

.

This is from fluid statics, what we are doing is, we are equating the pressures here. So, p 1 / gamma + x you see, this here start from here we go down x + we go h down again, so, this is this starting at 1. Now, that will be equal to if we are able to find p 2 so, the p 2 / gamma + 0.8 that is because this is 80 centimeters plus x plus because this is pressure but the mercury is there no water so, instead of h we write Sh and this is how we get this equation.

So, we can write p 1 / gamma - p 2 / gamma - .8 because x x gets cancelled plus = S - 1 into h. So, this will become S is 13.6 that becomes 12.6 into h.

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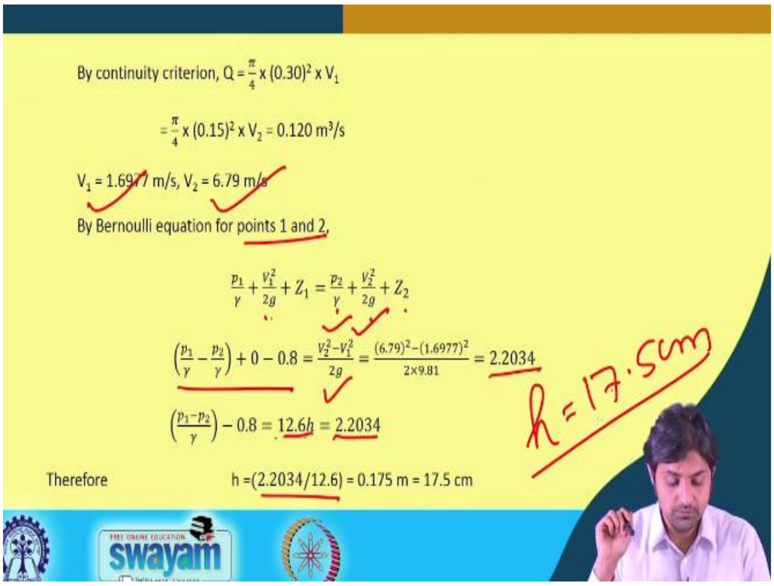

So, by continuity criterion Q is going to be you see, pi / 4 area A 1 V 1. So, V 1 we know, so, V 1 is the velocity at this section, area because this is 30 centimeters in diameter. So, pi d square / 4 into V 1 is q 1, so, d 1 pi / 4 into V 2. So, here, this is A 2, and this is V 2, this is A 1, this is V 1, so, V 1 from this method because Q is already given. What is the Q already given? 120 liters per second so, we can find V 1 and therefore, we also can find out V 2.

Now, by Bernoulli equation for points 1 and 2,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2.$$

So,

$$\left(\frac{p_1}{\gamma} - \frac{p_2}{\gamma}\right) + 0 - 0.8 = \frac{V_2^2 - V_1^2}{2g} = \frac{(6.79)^2 - (1.6977)^2}{2 \times 9.81} = 2.2034$$

Therefore, h can be found out as 2.2034 divided by 12.6 and that gives h is equal to 17.5 centimeters.

So, we have used continuity equation, we have used Bernoulli equation, we have also used fluid statics and using all those 3 we have found out the value of h the value of h here. So, this completes our Bernoulli equation however, we are going to continue with the final topic of the basics of fluid mechanics that is called fluid dynamics. So, we have read about fluid statics, we have read about elementary fluid dynamics that is Bernoulli equation,we have read about fluid can kinematics, we have read about properties of fluid and this we are going to see so that, we are able to solve the, you know, momentum equations for example, in the fluid flow. This, I mean, these all topics are very briefly taught, because you have already done in a lot of detail in your fluid mechanics second year course.

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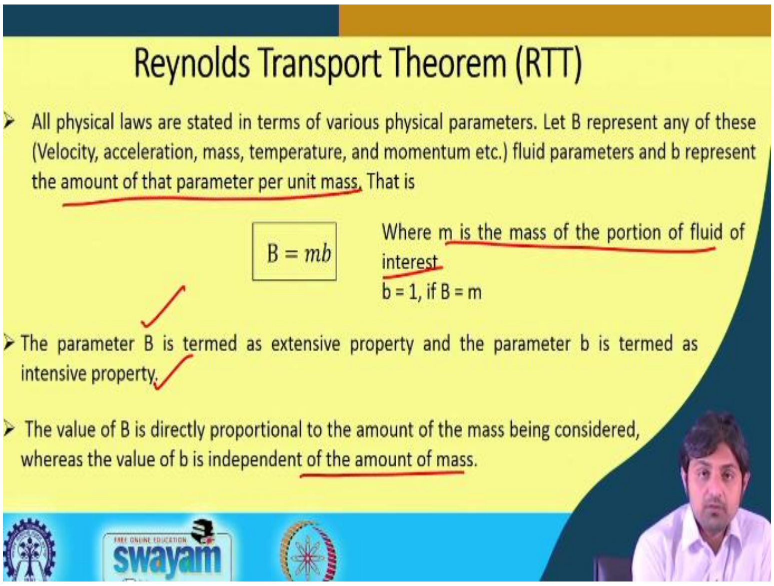

So, to start fluid dynamics, one of the most important theorems that generally is not taught in fluid mechanics is Reynolds transport theorem. So, what we are going to do, we are going to derive this Reynolds transport theorem in a little bit more detail. So, all physical laws are stated in terms of various physical parameters. So, if B represents any of these, these parameters can be velocity, acceleration, mass, temperature, momentum, anything etc.

So, what it says is, let B represent any of these fluid parameter capital ‘B’ and small ‘b’ represent the amount of that parameter per unit mass, that is, B is equal to m into b, where m is the mass of the portion of the fluid of interest, b will be 1 if B is equal to m. So, B is the amount of that parameter per unit mass. The parameter B, capital B is termed as extensive property and the parameters b is termed as intensive property.

Because it is the parameter per unit mass, the value of B is directly proportional to the amount of mass being considered. Whereas, the value of a b is independent of the amount of mass because by definition, it is the amount of the parameter per unit mass b.

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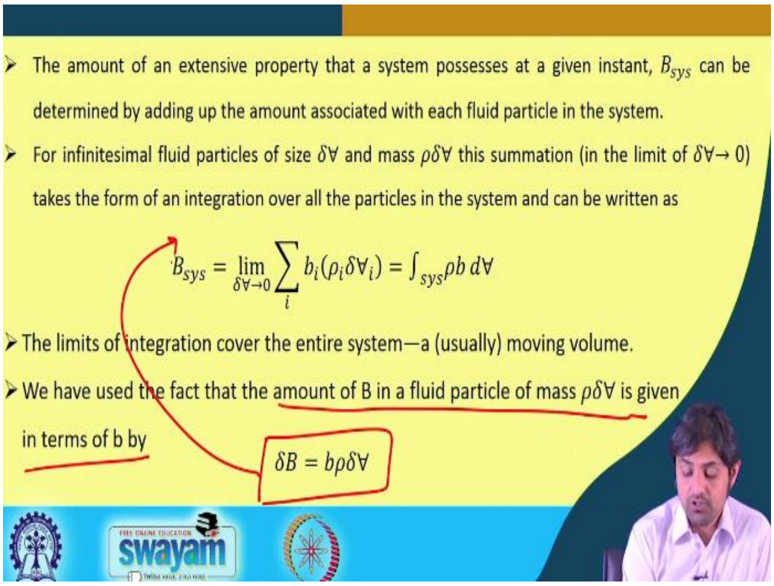

The amount of an extensive property that the system possesses at a given instant B, sys so, basis is the amount of the extensive property that a system will have at any given instant. And how that can be found out? It can be determined by adding up the amount associated with each fluid particle in the system, it is very simple. So, B system can be calculated by summing up different B s, you know, capital B, capital B of each particle.

If, for an infinitesimal fluid particles of size delta V and mass, rho delta V, this summation takes the form of an integration over all the particles in the system. So, if we start considering the fluid particles, small particle of size delta V and we tend the limit to 0, this B sys will be integration of all the particles of the system. So, B system is summation of bi rho i so, it is the summation of the mass. Correct.

So, this can be written as, rho b da, you see, da or dV whatever, sorry, not da, its rho b dV. Because that is the B, B was a small m into small b. So, M is mass is rho into dv and multiplied by b this is the B system this is the integration. I hope this is getting clear to you a little. This limits of the integration cover the entire system. So, it can be a moving volume as well. So, I am taking this was not to, you know, give you difficult questions in the exam or the test.

But this is to make you feel and appreciate how these momentum equations that we are going to see in the fluid dynamics, how do they have their origin. So, the Reynolds transport theorem is very important in that aspect. So, for doing that we have used the fact that the amount of B in a fluid particle of mass rho delta V is given in terms of b by this is what we have assumed, in getting this delta b is equal to b rho delta V.

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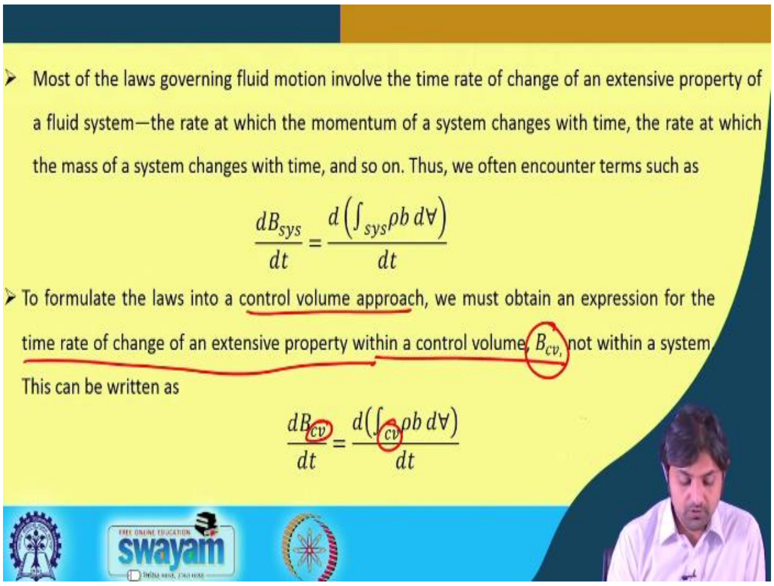

So, most of the laws governing fluid motion involve the time rate of change of an extensive property of a fluid system, the rate at which the momentum of a system changes with time, for example or the rate at which the mass of the system changes with time and so on. Thus, we often encounter terms such as dB sys dt. So, not only B system is important with that is the summation of so, B summation B sys was integral system rho b d V.

So, this is important but not only this, the derivative of this B sys this is also important because in many times we require that mass rate of change of mass or rate of change of momentum. So, those things are required so, that is why it is important to know these derivatives. To formulate the laws into a control volume approach we must obtain an expression for time rate of change of an extensive property within a control volume B cv and not within a system.

So, we have to obtain the rate of change of extensive property in a control volume. And so until now what we have been doing is we have written this extensive property in terms of a system, but to be able to obtain laws and formulate laws we should be writing that not in terms of system, but in terms of control volume,. So, the same thing we can write d dt of B cv is the differential of so, this from system goes to control volume, the same equation.

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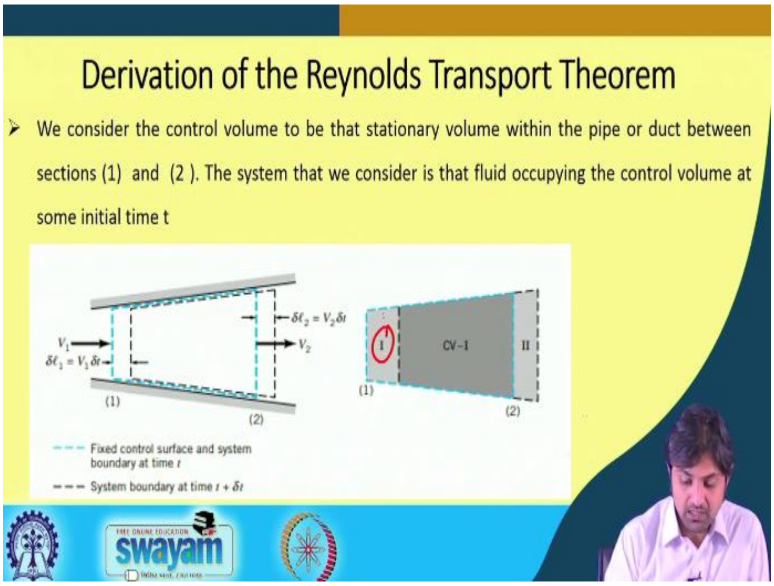

So, now we are, I mean, this is the prelude to but now we are going to derive Reynolds transport theorem. We consider the control volume to be that stationary volume within the pipe or duct between section 1 and 2. So, this figure have been taken from Munson, Young and Okiishi, but yeah, so, you can refer to that book for the derivation of Reynolds transport theorem as well. So, again repeating we consider the control volume to be that stationary volume within the pipe or duct between section 1 and 2.

This is section one, this is section 2 or in this figure section 1 section 2. The system that we consider is that fluid occupying the control volume at some initial time t. So, this blue line here, I mean, this line I will just take the laser pointer, so, this blue line is the fixed control surface at time t. However, after time delta t that is at t is equal to t + delta t our system boundary has moved from here to here. So, we have divided it into 3 regions 1, 1 control volume 1, 2.

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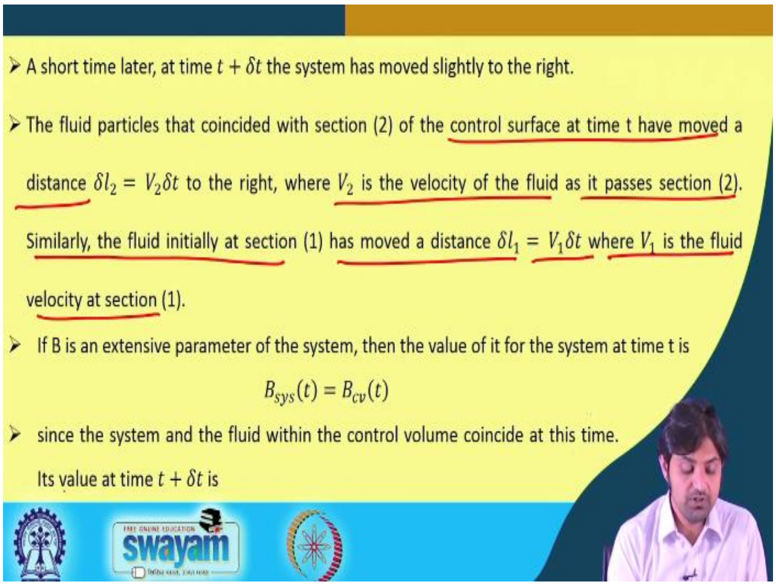

So, at short time later, at time t + del t the system has moved slightly to the right, you see, here, as we talked about, this has moved from here to here. The fluid particles that coincided with section 2 of the control surface at time t, have moved distance. So, the distance that has moved is V 2 into delta t, you see, here, the if this had a velocity V 2, this would have moved by V 2 delta t and this would have moved by V 1 delta t, where V 2 is the velocity of the fluid as it passes section 2. Similarly, the fluid initially at section 1 has moved a distance delta l 1 is equal to V 1 delta t where V 1 is the fluid velocity at section 1. This I have already shown you in the previous figure. If B is an extensive parameter of the system, then the value of it for the system at time t is B system is equal to B control volume.

Since the system and the fluid within the control volume coincide at this time, its value at time t + delta t is, see, again I am repeating here the system and the fluid within the control volume both are coinciding at the beginning. So, B system is equal to B control volume that is true. Now, if we want to find out the value of B system or B control volume at t + delta t

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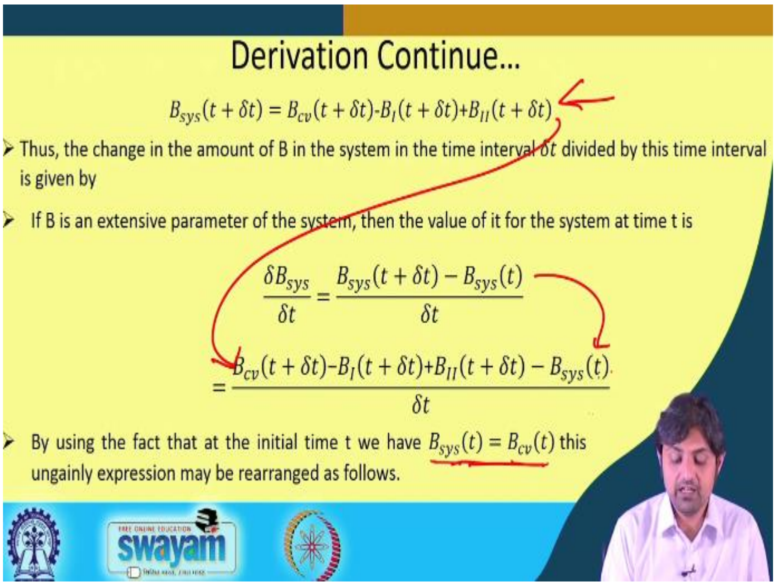

so, B system at t + delta t will be B of control volume t + delta t - B of 1 at t + delta t + B 2 at t + delta t. So, now taking you back what actually is B 1 and B 2. So, you see, this was 1, this was 2, because this has moved. Now, what earlier it was a part of B system. So, this is simply we write, B cv t + delta t - this has moved, but this has already proceeded forward so, it has come into the system of B, B system.

Thus, the change in the amount of B in the system in time interval delta t divided by this time interval is given by if B is an extensive parameter of the system, we can write

$$\frac{\delta B_{sys}}{\delta t} = \frac{B_{sys}(t+\delta t) - B_{sys}(t)}{\delta t}$$

$$= \frac{B_{cv}(t+\delta t) - B_I(t+\delta t) + B_{II}(t+\delta t) - B_{sys}(t)}{\delta t}$$

So, B system at t + delta t is coming from here, - B system at t, this remains same. By using the fact that at initial time t we have B system t is equal to B control volume t. This is what we have seen in the last slide, we can rearrange this. How? We see in the next slide.

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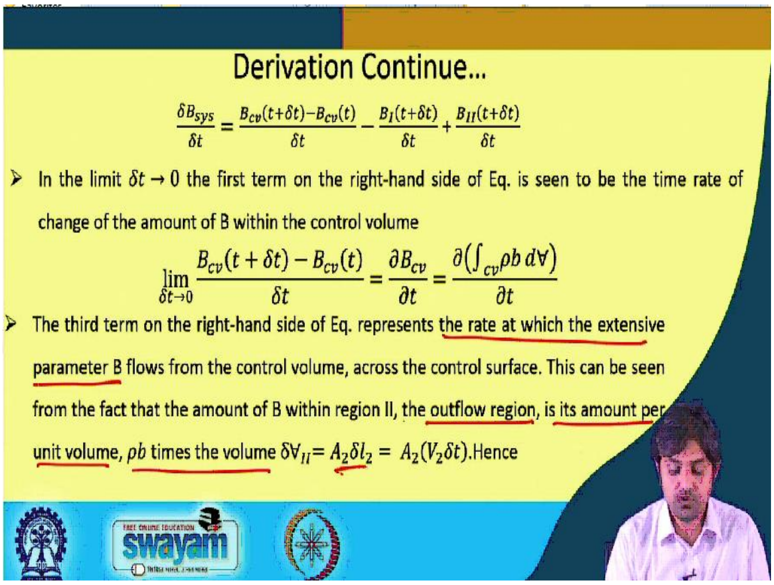

So, delta B system by del t became B cv t + delta t - this 1 became this, and this is the 1 that is here, in the next slide here, - B 1 t + delta t divided by delta t + B 2 t + delta t divided by delta t. So, in the limit t tends to 0 the first term on the right hand side of equation is seen to be the time rate of change of amount of B within the control volume, so, this one. So,

$$\lim_{\delta t\to 0} \frac{B_{cv}(t+\delta t)-B_{cv}(t)}{\delta t} = \frac{\partial B_{cv}}{\partial t}$$
$$= \frac{\partial(\int_{cv}\rho b\, d\forall)}{\partial t}$$

this we have seen in the previous slide.

The third term on the right hand side of equation, so, the third term on the right hand side of the equation represents the rate at which extensive parameter B flows from control volume, across the control surface. This can seen from the fact that amount of B within region 2, the outflow region, is its amount per unit volume rho b times the volume, that is, delta V 11 is equal to A2 into delta l 2.

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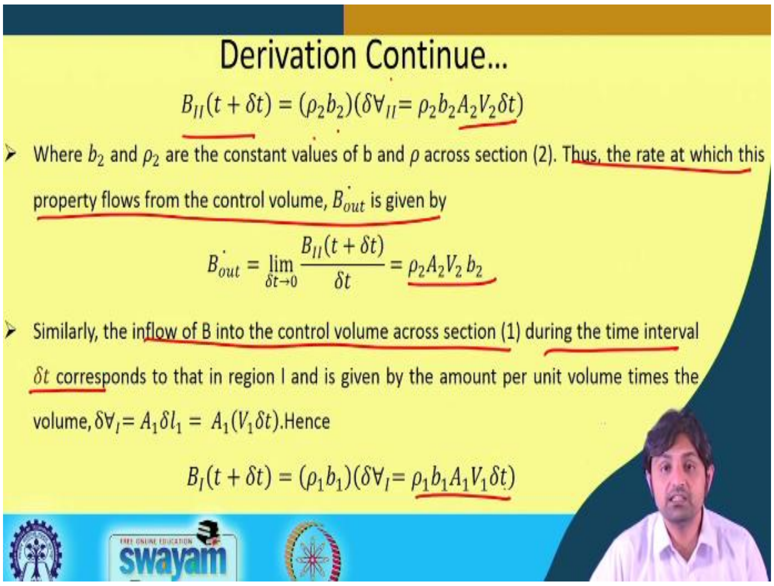

Hence,

$$B_{II}(t+\delta t) = (\rho_2 b_2)(\delta\forall_{II} = \rho_2 b_2 A_2 V_2 \delta t).$$

So, where b 2 and rho 2 are the constant values of b and rho across section 2. Thus, the rate at which this property flows from the control volume B out is given by B out is given as limit delta t tends to 0 B 2 at t + delta t divide by delta t or we can simply write rho 2 A2 V2 b2, correct simply the total mass that is flowing out of the cross section area 2.

Similarly, there will be an inflow of B into the control volume in the section 1 during the time interval delta t and that will be equal to rho1 b1 A1 V1 delta t.

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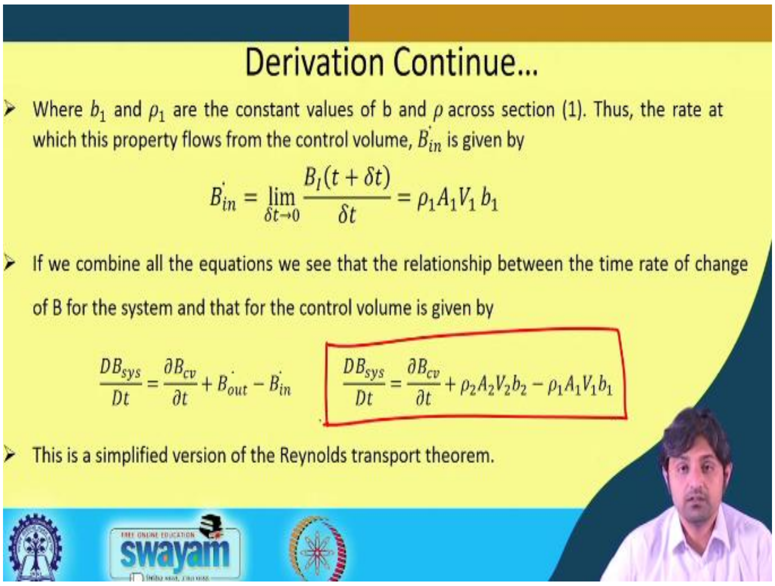

So, now putting this all together where are b 1 and rho 1 are the constant values of b and rho cross section 1. Thus, the rate at which the property flows is given by rho1 A1 V1 b1. If we combine all the equations we see that the relationship between the time rate of change of B for the system and that for the control volume is given by

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_1 A_1 V_1 b_1$$

this is 1 equation that we obtain.

Now, this equation above equation this is a very simplified version of the Reynolds transport theorem and this we have derived from the scratch the most basic thing.

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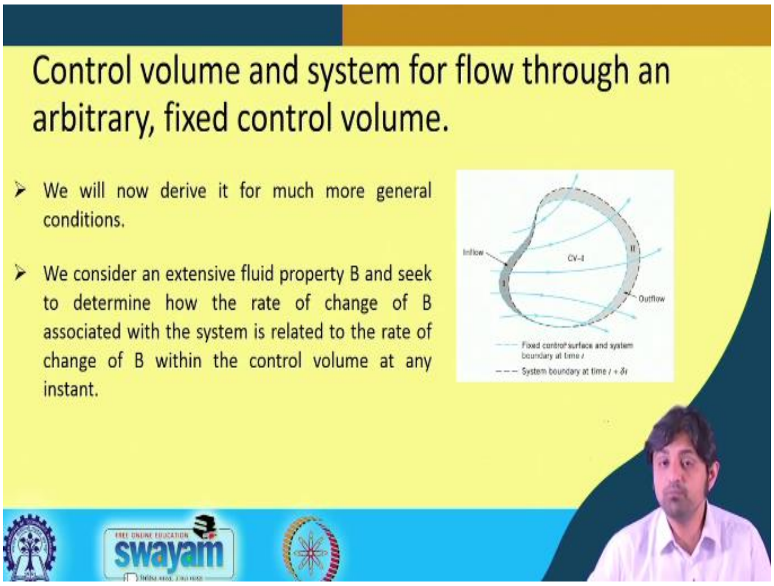

Now, the next step is we have to find this for control volume and system for flow through an arbitrary fixed control volume, this is arbitrary. Now, we will derive it for more general conditions. So, this is the general condition. So, this is in flow is happening in this there is this is outflow area and this is the original the control volume. The fix control wall control surfaces the system boundary at time t is equal to t, and system boundary at time t + delta t, So, we consider an extensive fluid property B and seek to determine how the rate of change of B associated with the system is related to the rate of change of B within the control volume at any instant. Same procedure but for the any arbitrary surface.

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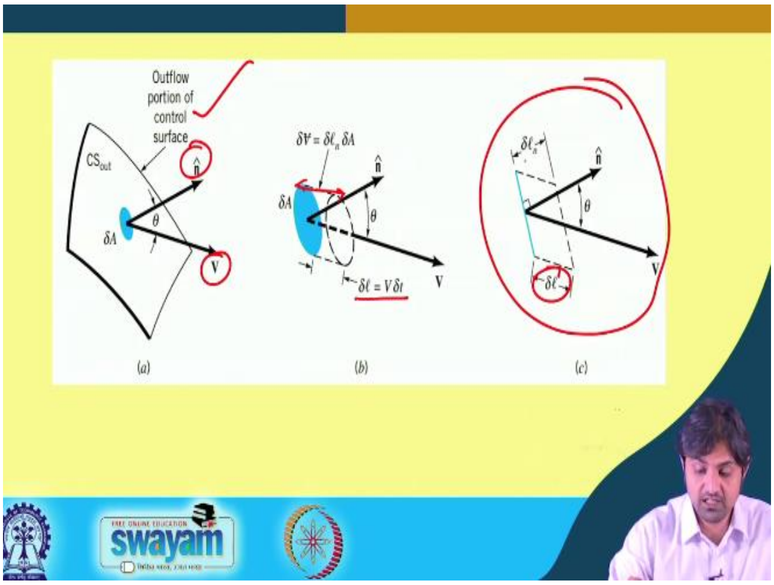

So, in general, the control volume may contain more than 1 inlet and 1 outlet. A typical pipe system for example can contain several inlets and outlets as shown in this figure for example, you know, can be more than that or even less than that so, we have to find a general equations. So, so, this is the outflow portion of the control surface this 1 here. This is cross sectional area out, this is delta A, this is the V velocity and this is the normal surface, this is the velocity and this is the theta.

Now, in time delta t this has progressed V delta t and this length will be delta l. So, this length that have progressed is delta ln into delta A, so, a more clear figure. This is the one that gives the more clearer picture, this is normal to the this is the velocity and this makes an angle theta with the normal direction. So, the component of delta l in a normal direction is given us delta ln.

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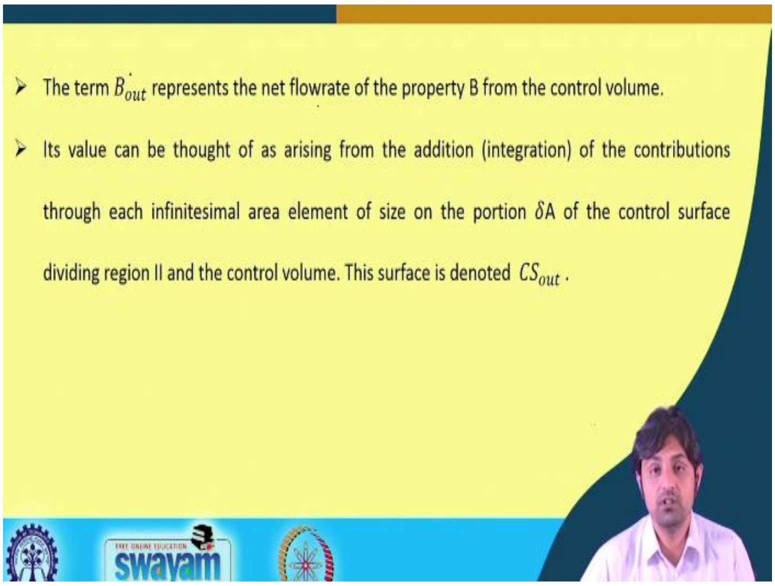

The term B out represents the net flow rate of the property B from the control volume, you see, its value can be thought of as arising from the addition of the contributions through each infinitesimal area element of size on the portion del A of the control surface dividing region 2 and control volume this surface is denoted as CS out, that is what we talked about, I was showing on the finger.

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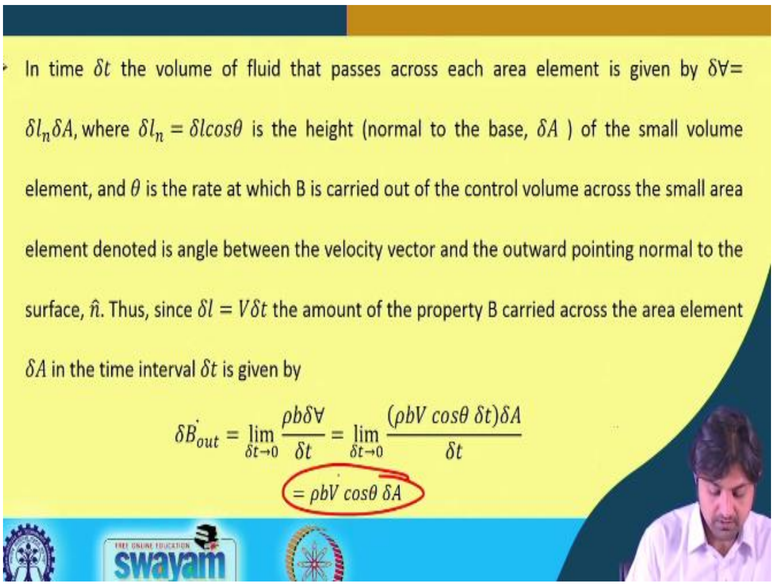

So, let us suppose, in time delta t the volume of fluid that passes across each element is given by delta ln into delta A, where delta ln is delta l cos theta, in the previous figure, is the height of the small volume of element and theta is the rate at which B is carried out of the control volume across the small element denoted, is angle between the velocity vector outward pointing normal to the surface and so theta is the, you know, the angle between the velocity and the n cap.

Thus, since delta L is equal to V delta t, the amount of property B carried across the element area delta A, in the time interval delta t is given by simple same thing b rho V cos theta del t into delta A, same now instead of V is V cos theta, direction. The rate at which B is carried out of the control volume across this small element area delta A denoted by delta B out is so, so, delta B out is given by rho b delta A delta t or rho b V cos theta delta t delta A / delta t, that is, rho b V cos theta delta A. So, this is the main, you know, this is the delta B out.

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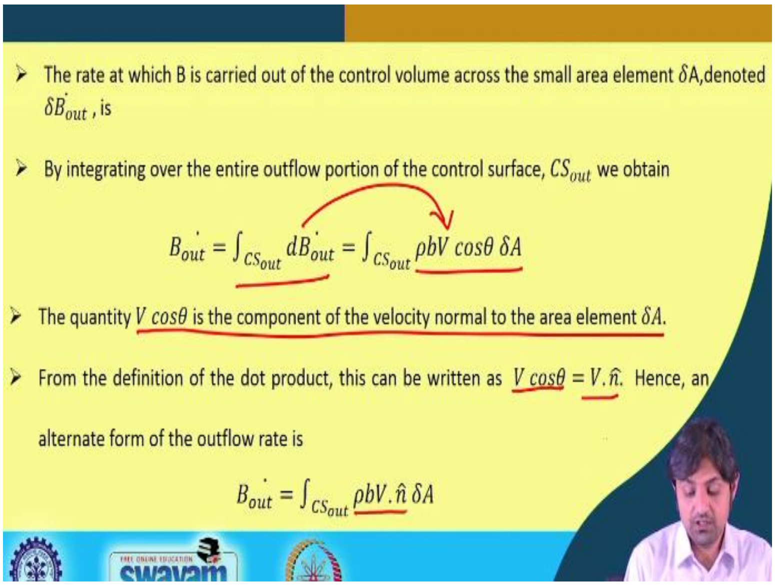

The rate at which B is carried out of the is denoted by B out so, by integrating over the entire outflow portion of the control surface we obtain B out as integral of dB out as the rho b So, we substitute this rho b V cos theta delta A. The quantity V cos theta is the component of the velocity normal to the area element delta A, from the definition of the dot product, this can be written as

$$\dot{B}_{out} = \int_{CS_{out}} \rho b V.\hat{n}\, \delta A$$

Hence, an alternate form of the outflow rate is rho b V dot n cap multiplied by delta A.

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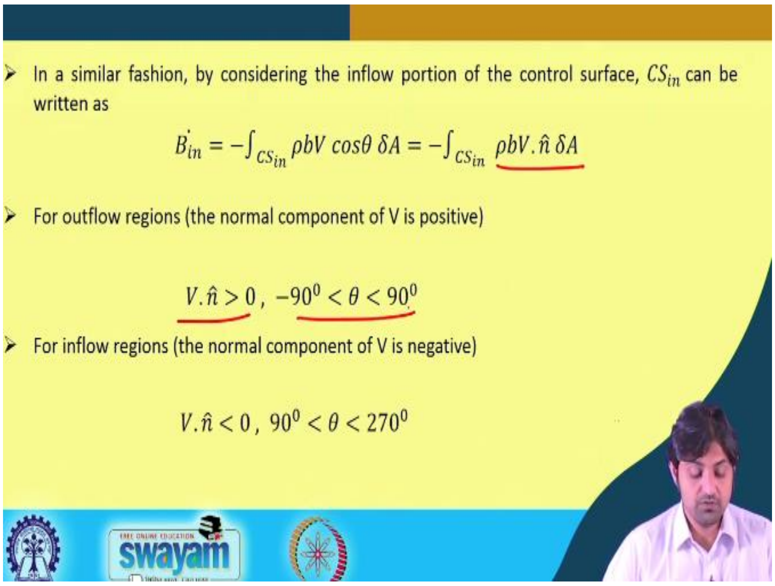

In a similar fashion, by considering the inflow portion of a control surface CS in can be written as because it was negative sign rho b V dot n cap del A. So, for outflow region just remember V dot n cap is greater than 0 and theta is between 90 degree and - 90 to 90. So, for inflow region the normal component of V is negative and for out flow region it is positive. So, V dot n cap is less than 0 and theta varies from 90 to 270 degree.

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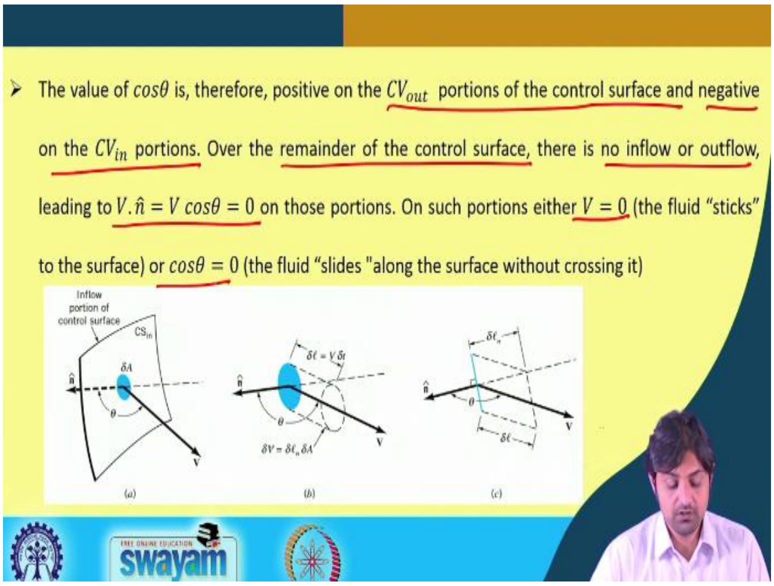

So, value of cos theta is therefore, positive on the CV out portions of the control surface and negative on the CV in portions over the remainder of the control surface, there is no inflow or outflow leading to V dot n cap is equal to 0 on those portion. On such portion either V equal to 0 or cos theta is equal to 0.

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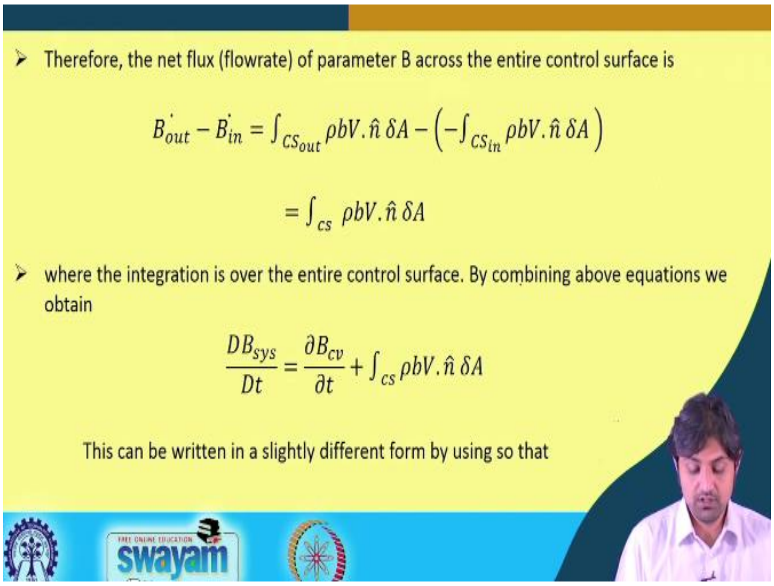

Therefore, showing the cross section in therefore the net flux of the parameter B across the entire control surface can be written as B out – B in is equal to CS out rho b V dot n cap delta A, almost the same expression just the cross sections are different or we can also write in over the entire cross section because at other places, this component is 0. There is only a contribution at outside and inside which can be taken care of from this expression.

So, this is where the integration is over the entire control surface. By combining above equations we obtain

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \int_{cs} \rho bV.\hat{n}\,\delta A$$

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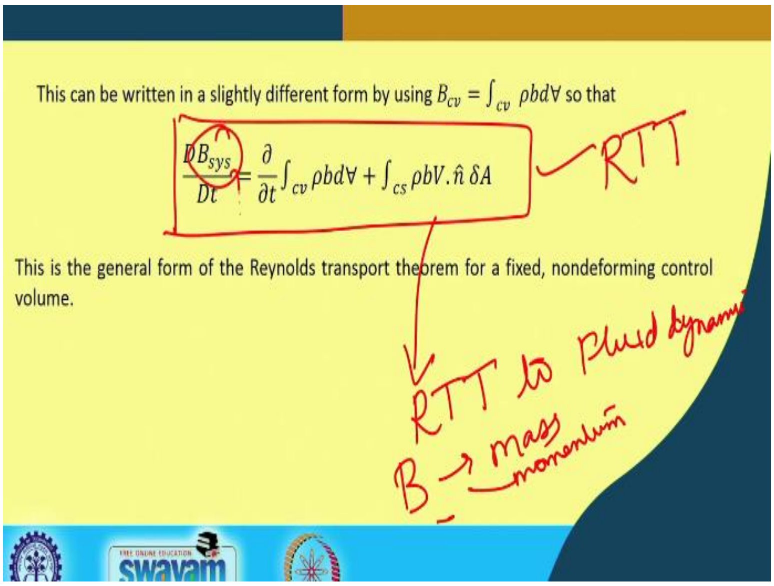

This can be written in a slightly different form by using so, that Bcv is equal to rho b dV. We can also write this in this for del del t of control volume. So, what we did? We just this, one we wrote in this format. So, now, this is the general form of Reynolds transport theorem for a fixed non deforming control volume.

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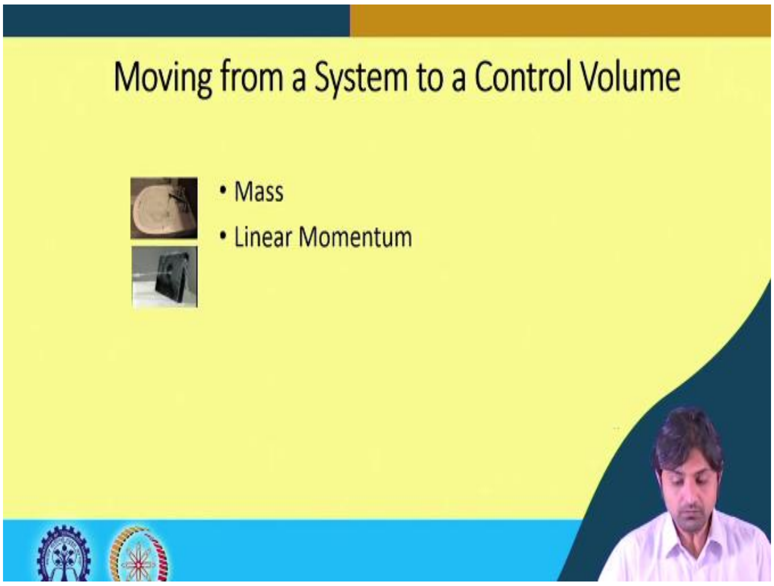

So, now if we move from a system to a control volume, you saw. We are going to cover this sometimes later. I think this is time now and the next step would be moving on from this Reynolds transport theorem this is a generalized form. I hope you would have understood that, just in case you have not you please look at your books. I have tried to derive it to in the most simplest form.

So, in the next class what we are going to see, we are going to apply this Reynolds transport theorem to fluid dynamics where this property B, you know, can either be mass or momentum and then we are going to work, this can be anything because we have derived it for a general system. So, B sys can be anything. So, I think, this is enough for today and the lectures will conclude that I hope to see you in the next class. Thank you so much.