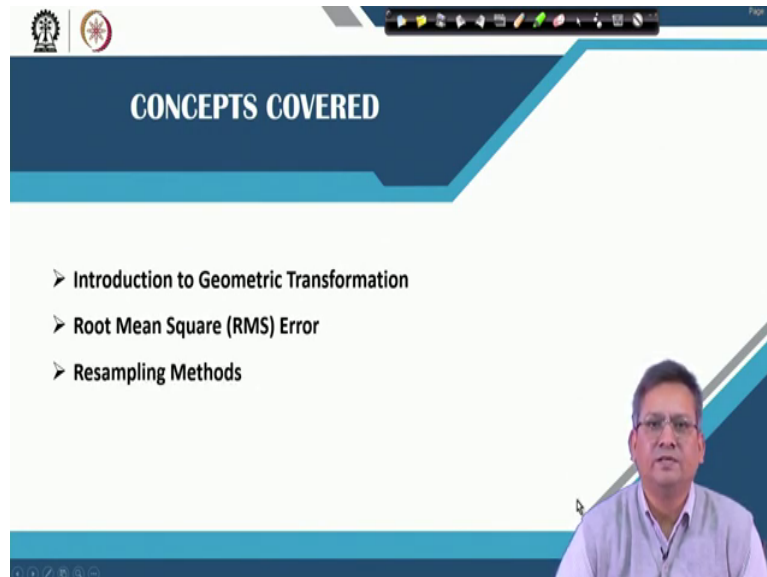


Geo Spatial Analysis in Urban Planning
Prof. Saikat Kumar Paul
Department of Architecture and Regional Planning
Indian Institute of Technology, Kharagpur

Module - 01
Introduction to Geographic Information System and Geographic Distribution
Lecture - 03
Geometric Transformation

So, welcome dear students, we are in the 3rd lecture which is titled as Geometric Transformation in our module 1 and this is the NPTEL course when which is a certification course and this course is titled as Geo Spatial Analysis in Urban Planning. So, we will look into the aspects of GIS operations and analysis. So, today we will look into how basically the data can be transformed, in the last lecture what we had done is we had seen that the earth is basically an oblate sphere.

(Refer Slide Time: 00:47)



The image shows a presentation slide with a dark blue header containing the text "CONCEPTS COVERED". Below the header, there is a list of three topics, each preceded by a right-pointing arrowhead: "Introduction to Geometric Transformation", "Root Mean Square (RMS) Error", and "Resampling Methods". In the bottom right corner of the slide, there is a small video inset showing a man with glasses and a light-colored shirt, presumably the professor, speaking. The slide also features logos of the Indian Institute of Technology Kharagpur and NPTEL in the top left corner, and a navigation toolbar in the top right corner.

So, I mean we had talked about the projections so, in this particular lecture we would be covering the geometric transformation, we would talk about root mean square error and then we would talk resampling methods for the data.

(Refer Slide Time: 01:09)

Geometric Transformation

Map Projection & Transformation

Projectional and coordinate transformation

Image Source : NASA (<https://www.nasa.gov/vision/earth/lookingatearth/earthshape.html>)

IIT Kharagpur

NPTEL

1

Now, as we had said that oblate earth is an oblate sphere. So, we had in the last lecture we had talked about different types of projections I mean we have talked about cylindrical projection, we had talked about conning projection, we had talked about azimuthal projection and each of these projection had their own properties.

Now, we are also I mean tasked with doing a coordinate transformation say when we peel this particular orange that we see in this image and if we have the earth's different continents

drawn on this particular image and we when we open up this particular image it I mean comes as a 2 dimensional map.

All the continents can be put as 2 dimensional maps, but you can see due to this transformation from this 3 dimensional plane to a 2 dimensional plane there are basically I mean lot of errors which crops in I mean error could be regarding scale, it could be regarding directions, it could be regarding measurement of lines. So, we had talked about different properties of the projections in our last class.

So, now we are going to talk about the geometric transformation say suppose we are having a satellite which is moving around this earth and it acquires an image at a given particular point say at this point it acquires an image over when it moves over India. Now when it acquires an image it does not have any latitude or longitude information. So, that the scaling is also not proper the image may be rotated because the satellite may not be moving exactly in a north south polar orbit. So, it may be a bit inclined.

So, we may see there may be a some kind of rotation when we get that image acquire that image and in some cases there could be flipping also, in a way that as the images acquired either in ascending node or a descending mode flipping might occur. So, basically we are tasked with doing all these corrections and restituting this data in the final transformed form I mean this is the case of an example of an image.

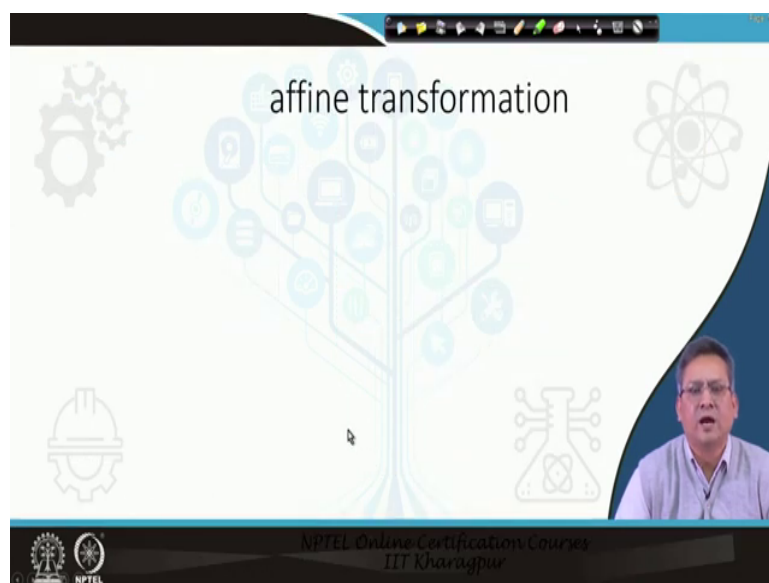
So, we can do a transformation for images as well as we can do transformation for vector data sets. Now the first challenge is to scale up this data. So, these are basically pixels or basically an array of number a matrix of number which is the reflectance of these points that the satellite would have scanned. So, what we can do is, we can scale up this to see that the measure on the screen for the distance that we measure across this map is true as we measure it on the ground.

Now, any shear that this image has I mean because of the scanned geometry there should be some shearing. So, that needs to be corrected, we said that there should be some rotation in the image and there could be some flipping and the translation also could be there that the

image as it should sit on this particular surface with respect to the geographic lat long may have translated I mean may have might have moved and may not sit exactly on this given point.

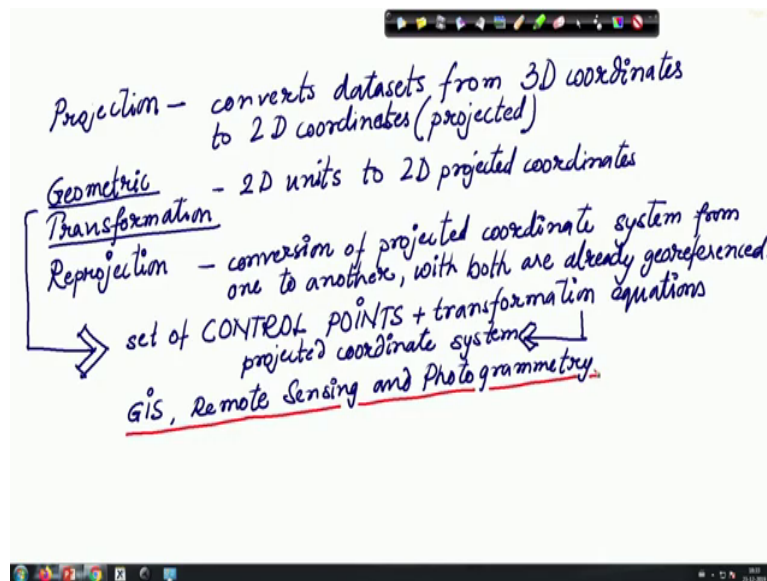
So, we have to I mean do all these operations of scaling, shearing, rotation, flipping and translation to come to the finally, transformed image and in this particular image you can see that this image is not similar to the original one. In a way that you can see there is a slight bend in this particular image and those bend is not uniform across this 2 parallel surfaces that we had in the original image, in this particular image in the original image we had this 2 parallel sides, but in the finally, transformed image you can see there is a slight curvature ok.

(Refer Slide Time: 05:23)



So, I mean we need to look at the process how we do the transformation.

(Refer Slide Time: 05:34)



So, basically this projection it converts data sets from 3 D coordinates to 2 D coordinates, which are basically the projected coordinates which we which are not the original coordinate. Now whereas, this geometric transformation would convert this data set from 2 D digitizer units or 2 D units 2 dimensional units to 2 D 2 dimensional projected coordinates.

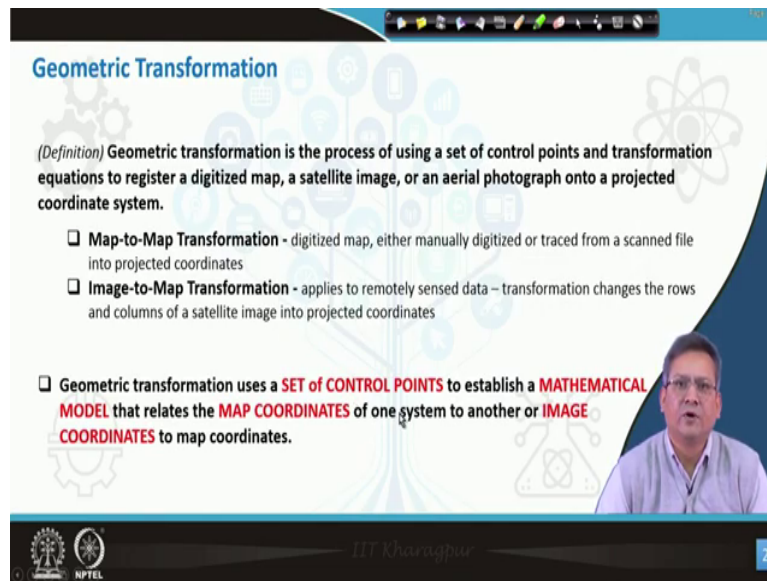
Now, we have a term which is known as reprojection which basically is the conversion of projected coordinate system from one to another when both of them are already georeferenced like in our earlier class we had talked about the different projection. So, say suppose we have projected a given map or an image to say UTM projection that are Universe Transverse Mercator projection we can reproject it to say Lambert conformal chronic conic projection. So, that is what is known as the process of reprojection.

Now, the geometric transformation basically uses this geometric transformation what it does is it basically uses a set of points which is known as control points and the transformations equations are used and this could be satellite image or an aerial photograph or any other digitized map.

If you have a scanner you can digitize it I mean you can scan it so, you get this kind of a digitized map and this set of control points is used in conjunction with transformation equations which will we will have a look at. So, it is basically projected to a basically to a projected co ordinate system. So, we use this transformation equation on the control points and get the projected coordinate system from the row coordinate systems that we have initially.

Next it is basically a this geographic transformation is a very common operation it is used in GIS. So, this process of geometry transformation is used in GIS, it is used in remote sensing and it is also used in photogrammetry. So, you would see that these processes are common to all these different softwares, this process of geometric transformation is common to all the softwares of remote sensing, photogrammetry or GIS.

(Refer Slide Time: 11:38)



Geometric Transformation

(Definition) Geometric transformation is the process of using a set of control points and transformation equations to register a digitized map, a satellite image, or an aerial photograph onto a projected coordinate system.

- ❑ **Map-to-Map Transformation** - digitized map, either manually digitized or traced from a scanned file into projected coordinates
- ❑ **Image-to-Map Transformation** - applies to remotely sensed data – transformation changes the rows and columns of a satellite image into projected coordinates

❑ Geometric transformation uses a **SET of CONTROL POINTS** to establish a **MATHEMATICAL MODEL** that relates the **MAP COORDINATES** of one system to another or **IMAGE COORDINATES** to map coordinates.

Dr. Khurshid

NPTEL

2

So, let us see the definition of geometric transformation once again. So, geometric transformation is the process of using a set of control points and as we had said it there is a set of transformation equations which are basically applied to this control points to register a digitized map or satellite image or an aerial photograph on a coordinate system projected coordinate system.

So, we can have 2 types of transformation, one could be a map-to-map transformation and the second could be a image to map transformation. So, a map to map transformation would refer to a digitized map which is either digitally or I mean scanned or manually digitized as a scanned file and it is projected into the projected coordinate system.

Second type is the image - to - map transformation which applies to remotely sense data and we basically transform the changes in the row and the column of the satellite image, I mean

we this data sets from the rows and columns are transferred and basically transcribed into a projected coordinate system.

So, we use a set of control points as we have already said that we use a set of control points and we establish a mathematical model that relates the map coordinates from one system to the image coordinates or map coordinates of another system.

(Refer Slide Time: 13:22)

Geometric Transformation
Transformation Methods

- EQUAL AREA** allows rotation of the rectangle and preserves its shape and size. **Equiarea** [Diagram: A rectangle is transformed into a rotated rectangle of the same area.]
- SIMILARITY TRANSFORMATION** allows rotation of the rectangle and preserves its shape but not size. **Similarity** [Diagram: A rectangle is transformed into a smaller, rotated rectangle.]
- AFFINE TRANSFORM** allows angular distortion of the rectangle but preserves the parallelism of lines (i.e., parallel lines remain as parallel lines). **Affine** [Diagram: A rectangle is transformed into a parallelogram.]
- PROJECTIVE TRANSFORMATION** allows both angular and length distortions, thus allowing the rectangle to be transformed into an irregular quadrilateral. **Projective** [Diagram: A rectangle is transformed into a trapezoid.]

DT Khanna

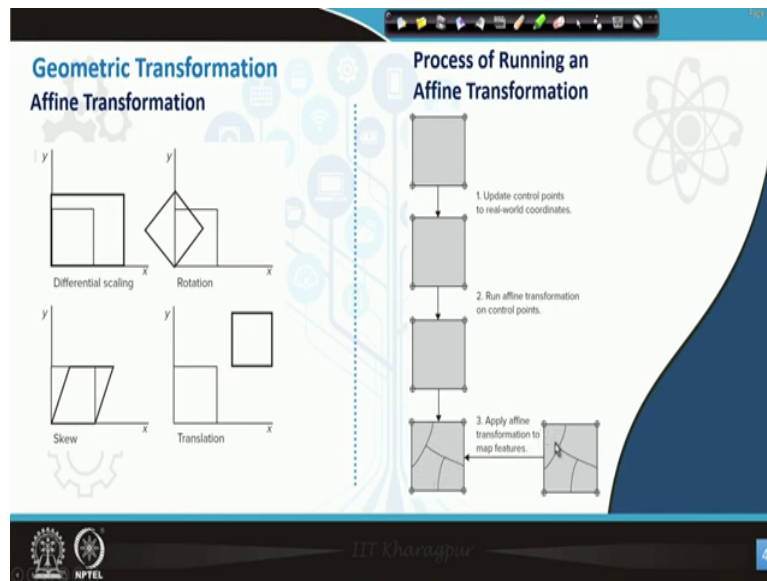
So, what are the different methods of transformation? Now when we are talking about transformation, transformation also can be done in several ways. So, first method is the equal area method which allows for rotation of the rectangle and preserves its shape and size. The second method is the similarity transformation wherein it allows for rotation of the rectangle and preserves its shape, but the size is not preserved you can see the size could be increased or the size could be decreased.

Now, the next one is the affine transform which is the most versatile of all the different transforms that we would be looking at and is the most commonly used transform which is used in most of the GIS operations. So, in this case it allows for angular distortion you can see in this original image to the transformed image you can see there is an angular distortion this square this angle has changed.

So, there is a angular distortion, but it preserves the parallelism of line, you can see that both the lines in the opposite sides are parallel in this case though there is a angular distortion the lines in the opposite sides are parallel. So, this is the most commonly used transformation system which is known as the affine transform and we will be detailing it out we will be looking at what is an affine transform subsequently.

So, next is the projective transform. Now, you can see it basically incorporates all the different aspects that we are talked about in the equal area, the similarity projection or the affine projection that it I mean allows for both angular as well as length distortion, we can see I mean there is a angular distortion as well as length distortion there is a angular distortion as well as length distortion. So, it allows the rectangle to be transformed into an irregular quadrilateral.

(Refer Slide Time: 15:36)



So, we can see the different your different operations that we were talking about in the last slide. That it uses differential scaling and by differential scaling we mean that it the changes in scale by expanding or reducing in x and y direction is permitted in this method of affine transformation. Rotation we can rotate the objects x and y axis with respect to the origin, in this case you can see this square has been rotated you can rotate it clockwise or counterclockwise.

So, basically these are all matrix based operations. So, we can have matrix of numbers; matrix of numbers and we can do a rotation of the matrix. So, it can be either done clockwise or counterclockwise in a given angle. Now we have the next one which is the skew which basically allows for non perpendicularity between the 2 axes.

So, you can see this perpendicular axis in the original image which is shown by this particular square has become I mean the angular nature has changed now. So, it changes its shape to a parallelogram and we said I mean the parallel lines of the opposite sides are preserved then we have the translation. So, you can see the originally the image would have been here or the data set would have been here.

So, these transformations are possible on vector data sets as well as raster data sets. So, you can see that origin has shifted there is a translation of the origin. So, this is the fourth operation which is I mean allowed and it shifts the origin of the to a new location in the output image.

Now there are 3 processes of running an affine transformation there are 3 processes which are involved. So, what we do is, we first acquire the control point. So, in this case in the 4 corners of this given image you can see if there are 4 points which are basically the control points which can be map to the real world. Probably we can have the GIS coordinates your GPS coordinates in terms of it is latitude longitude or we can also have projected coordinates in a reference frameworks such as UTM or say a polyconic or LCC Lambert conformal conic or any other projection system.

Now, in the second process we run this affine transformation on the control points. So, these control points are run through a set of equations and it undergoes this transformation on the control point and finally, we apply the affine transformation to the map features. So, the map features which are inside this particular 4 control points in within this particular area. This affine transformation is applied on all the features that is there on the map and finally, we get to realize the transformed map.

(Refer Slide Time: 19:02)

affine transformation
1st order & 2nd order transformation equation

NPTEL Online Certification Courses
IIT Kharagpur

(Refer Slide Time: 19:04)

AFFINE TRANSFORMATION -

Expressed as pair of first order polynomial equation

1st Order Polynomial Equation

$$\begin{aligned} x' &= a_0 + a_1 x + a_2 y \\ y' &= b_0 + b_1 x + b_2 y \end{aligned}$$

$x, y \sim$ source coordinates
 $x', y' \sim$ rectified / transformed coordinates

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \end{bmatrix}$$

$a_0, a_1, a_2, b_0, b_1, b_2$ are the transformation coefficients

2nd Order Polynomial Equation

$$\begin{aligned} x' &= c_0 + c_1 x + c_2 y + c_3 xy + c_4 x^2 + c_5 y^2 \\ y' &= d_0 + d_1 x + d_2 y + d_3 xy + d_4 x^2 + d_5 y^2 \end{aligned}$$

c_0, c_1, c_2, c_3, c_4 & c_5
 &
 d_0, d_1, d_2, d_3, d_4 & d_5

So, let us talk about affine transformation, let us talk about we were talking about the models mathematical model. So, let us see, what are the different mathematical models which are used in the affine transformation? So, this is expressed as a pair of first order polynomial equation x' equals to a_0 plus $a_1 x$ plus $a_2 y$ and y' equals to b_0 plus $b_1 x$ plus $b_2 y$.

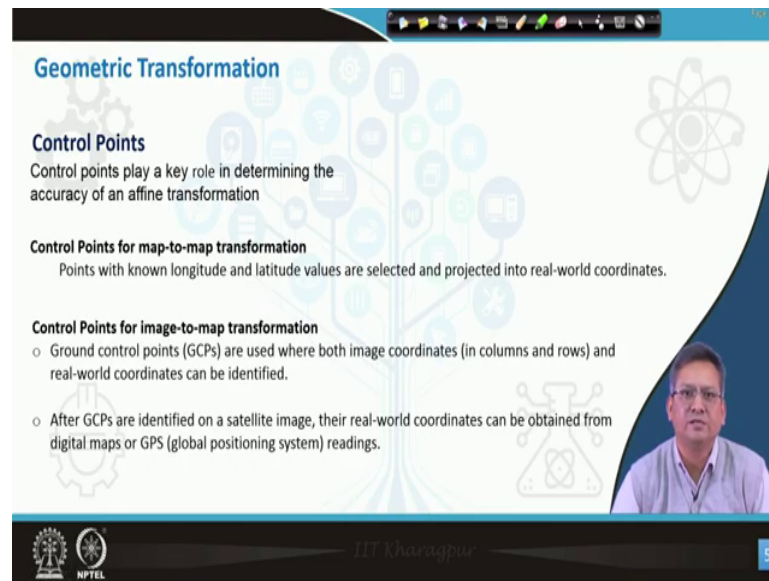
So, these are our two points which are I mean in the image this x and y and you can see that we have this x and y these are the points measured on the image and x' and y' are the transformed coordinates those are the coordinates on which those transformations related to scaling rotations skew or translation has been I mean it under those transformations would have been done on these input points.

Now, this x and y , x and y are the source coordinate and your x' and y' are the rectified coordinates rectified or transformed coordinates. So, if we write it in the form of a matrix we can write it as x' and y' , we can write a_0 , a_1 and a_2 , in this portion we can write b_0 , b_1 and b_2 and it would be a product of the input coordinates. So, in the I mean the first coefficient we can write it as 1 because we have written a_0 out here.

So, we then have the x and the y so, we write the x and the y in the matrix to see that it is in the form of a equation. Now in this case; in this case your a_0 , a_1 , a_2 , b_0 , b_1 and b_2 are the transformation coefficients. So, we can this is the first order polynomial equation, we can write the second order polynomial equation, this is the first order equation. We can write the second order polynomial equation as x' complement x' equals to say c_0 plus $c_1 x$ plus $c_2 y$ plus $c_3 x y$ plus $c_4 x^2$ plus $c_5 y^2$.

Now, and we can write similarly y' as d_0 plus $d_1 x$ plus $d_2 y$ plus $d_3 x y$ plus $d_4 x^2$ plus $d_5 y^2$. So, similarly like we said these are the coefficients we said this values are the coefficients. So, in this particular equation also we see the c_0 , c_1 , c_2 , c_3 , c_4 and c_5 and d_0 , d_1 , d_2 , d_3 , d_4 and d_5 are basically the transformation coefficient.

(Refer Slide Time: 26:07)



Geometric Transformation

Control Points
Control points play a key role in determining the accuracy of an affine transformation

Control Points for map-to-map transformation
Points with known longitude and latitude values are selected and projected into real-world coordinates.

Control Points for image-to-map transformation

- Ground control points (GCPs) are used where both image coordinates (in columns and rows) and real-world coordinates can be identified.
- After GCPs are identified on a satellite image, their real-world coordinates can be obtained from digital maps or GPS (global positioning system) readings.

The slide features a background with various icons related to technology and mapping. A video inset of a man in a light blue shirt is visible in the bottom right corner. The footer includes the IIT Kharagpur and NPTEL logos, the name 'IIT Khargapur', and a small blue square with the number '5'.

So, when we are talking about control points control points plays a key role in determining the accuracy and these are the points for which we know the latitude and longitude and these are these values are selected the latitude and longitude values and these are projected into the real world coordinates. So, I means there could be 2 types I mean in which we do a map-to-map transformation or image- to-map transformation.

So, in a map to map transformation we know we do the transformation from points wherein we know the latitude and longitude and basically they are projected to the real world coordinates. And for the image to map transformation what we do is we take ground control points which are known as GCPs where the both the image and the real world coordinates can be identified.

So, it could be with respect to 2 images for one image the coordinates are already known it is already georeference, the other image may not be georeference. So, for the first image which is not georeference we can identify points such as road intersections or some a corner of buildings and identify those points in the second image which is already georeferenced and we can relate and build a I mean relationship a mathematical relationship as we had seen in the earlier equation and apply a affine transformation either a first order polynomial or a second order polynomial transformation to give us the geometrically transformed images.

So, after this GCPs are identified in the satellite image and the real world coordinates I mean we can get the GPS coordinates readings or from say topographic sheets you can get the readings of the your latitude and longitude coordinates and it can be transform.

(Refer Slide Time: 28:11)

No. of GCP's are determined based on the order of polynomial transformation (2, 3 or higher order)

$$\text{Total no. of GCP's} = \frac{(t+1)(t+2)}{2}$$

$t \sim$ order of transformation.

GCP Evaluation - Residuals
Error per GCP
RMSE
Error contribution by point.

The image shows a whiteboard with handwritten notes in blue ink. At the top, it states that the number of Ground Control Points (GCPs) is determined by the order of polynomial transformation used, which can be 2, 3, or higher. A formula is provided: Total no. of GCP's = (t+1)(t+2) / 2, where t represents the order of transformation. Below the formula, the text 'GCP Evaluation' is underlined, followed by a list of metrics: Residuals, Error per GCP, RMSE, and Error contribution by point. The whiteboard is displayed in a video call window, with a small inset of a man's face in the bottom right corner.

Now, for selection of the GCPs the number of GCPs we said are the ground control points are determined based on the polynomial transformation that we are going to use order of polynomial transformation. So, it could be a second order transformation it could be a third order or higher order. So, more points would be required if we do I mean higher order transformation.

So, we can find out the minimum number of points using the equation $t + 1$ into $t + 2$ divided by 2 as the total number of GCPs, where t is the order of transformation. So, we can then once we have taken the number of GCPs and we have done this process we can do a GCP evaluation we can do a GCP evaluation by finding the residuals, we can find out the error per GCP per GCP or we can also find out the RMSE and we can also find out the error contribution of the different points, I mean the point that we had taken the GCPs what is the contribution of these error points.

(Refer Slide Time: 31:02)

Geometric Transformation

Root Mean Square (RMS) Error

- common measure of the goodness of the control points;
- measures the deviation between the actual (true) and estimated (digitized) locations of the control points;
- Affine transformation is fit using a statistical method that minimises **root mean square error (RMSE)**

Residual error - input or output error for a control point
- difference between input and transformed coordinate

$$\sqrt{(x_{act} - x_{est})^2 + (y_{act} - y_{est})^2}$$

Average RMS error (avg. RMSE) can be computed by averaging errors from all control points

$$\sqrt{\frac{\sum_{i=1}^n (x_{act,i} - x_{est,i})^2 + \sum_{i=1}^n (y_{act,i} - y_{est,i})^2}{n}}$$

source GCP, X residual, Y residual, RMS error, retransformed GCP

NPTEL

Dr. Kharijpur

6

Now, we can work out the root mean square error as we had talked about in the earlier slide, it is basically a measure of the goodness of the fit of the control point and it would measure the deviation between the actual value I mean that is to be the final value where the point should locate should be located to the estimated locations of the control point.

So, there would be a difference between the actual location value and the estimated value. So, that error would be given as the root mean square error and this affine transformation basically is a fit which is used wherein we use a statistical method that would minimize the RMSE the root mean square error.

So, you can have a look at this particular equation that we have the source GCP and the retransform GCP wherein either the values are already known to us it could be of different images or it could be data acquired by your GPS coordinates. So, we have the source GCP

from a given image and the residual the transformed image. Now we can see that there is a gap there is a difference.

So, we can work out; we can work out the x residual and we can work out the y residual and we can calculate this residual as a function of distance between these 2 points using this particular equation I mean which is similar to a Pythagorean equation. And the average RMSE that is a your root mean square error can be worked out by working out the RMSE of the different points and taking a summation of that taking a sigma from i 1 to n iterating it and dividing it by the number of observations that is n.

(Refer Slide Time: 32:57)

Geometric Transformation Resampling

□ filling each pixel of the new image with a value or a derived value from the original image

Nearest Neighbor - fills each pixel of the new image with the nearest pixel value from the original image

Bilinear Interpolation -

- four close pixels to coordinates(x,y) obtained by geometric transformation are involved in such an interpolation, which is able to define new value
- Output image has less pronounced contrasts with progressive steps

Cubic Convolution -

- uses the average of the 16 nearest pixel values from five cubic polynomial interpolations
- produces a smoother generalized output than bilinear interpolation but requires a longer processing time

The slide includes diagrams for each method showing the original image grid and the corrected image grid. A small video inset of a speaker is visible in the bottom right corner of the slide.

Now, once we have done it the problem next that arises is the resampling that how we basically restitution of the image how it would be done, because it would have undergone

some kind of modification. So, if it were a satellite image or a scanned map it would contain pixels.

So, there could be 3 ways of doing it, the first method is known as the nearest neighbor method which fills each pixel of the new image that is you can see this is the original image that is shown as dotted and the correct image is shown as firm lines and the grids are basically the cells or pixels of the image.

So, basically the nearest neighbor method fills the each pixel of the new image the corrected image the firm line image with nearest pixel values from the original image. So, whichever it would work out the distance to the centroids of all the adjoining images and it would try to work out which is the least distance and it would pick out the pixel value and assign it to that corresponding pixel in the transformed image. Second is the bilinear interpolation it is a very similar method, but it does a averaging of the 4 closest pixels to the transformed pixel of the output image.

So, in this what happens is there are less pronoun contrast and with progressive steps. So, what happens is you can see the contrast will diminish in a image which has been resampled using a bilinear interpolation or a cubic interpolation.

Now for the last part that is the cubic interpolation we see we use an average of the 16 nearest pixel values from the cubic for the cubic polynomial interpolation and it still generates a smoother output in comparison to the bilinear interpolation the cubic convolution image is still smoother, but the problem with this type of convolution is that the processing requires a longer amount of time. So, in a on an average it is almost 7 times of working on a nearest neighbor algorithm.

So, the cubic convolution is I mean it smooths out the image. So, we can see in a just I mean this is what happens in a nearest neighbor I mean method of resampling bilinear interpolation or the cubic convolution.

(Refer Slide Time: 35:39)

Recapitulation

- Introduction to Geometric Transformation
- Root Mean Square (RMS) Error
- Resampling Methods

NPTEL Online Certification Courses
IIT Kharagpur

So, recap of what we have done today, we had talked about what is a geometric transformation and how it is done, we had talked about the GCPs the ground control points, we had talked about an input image and output image and how it can undergo different types of transformation. We had talked about the errors that could come up and how it could be measured using a metric known as root mean square error.

And finally, we had talked about how this data is to be restituted I mean after doing the transformation how the final image is to come up I mean how it is to be transformed and it is done using resampling methods. So, in the resampling methods we had seen there are 3 options in which the nearest neighbor option basically does not do any sort of I mean modification to the original pixel values and the other 2 methods basically it would I mean smoothen out the given image. So, that is for today.

Thank you.