Geo Spatial Analysis in Urban Planning
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Lecture – 12
Spatial Interpolation

Welcome dear students. We are in lecture 12 now of module 3. So, today we shall discuss about the Spatial Interpolation methods which is used in case of disaggregated data, when you have distributed data and it is not a continuous data. So, in that case, we use different types of methods to interpolate the intermediate values. So, we shall see the different methods by which we can do this spatial interpolation.

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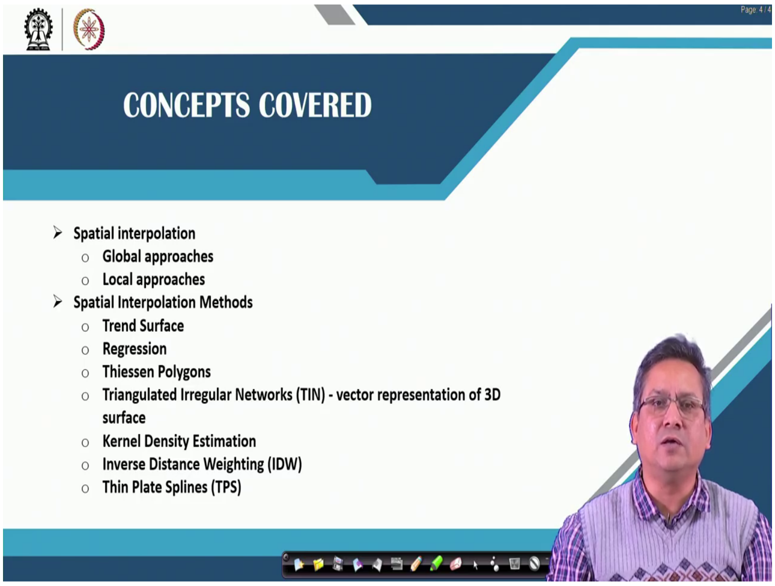

So, the topics or concept that we are going to cover today is we are going to introduce what is spatial interpolation; we are going to see what is spatial interpolation, we shall see the global approaches to spatial interpolation. We shall also see what are the local approaches and how they are different from the global approaches. We shall look into the different spatial interpolation methods.

These methods includes Trend Surface, Regression, Thiessen Polygons, Triangulated Irregular Networks that is the vector representation of the 3D surface. We shall look into Kernel Density Estimation and Inverse Distance Weighing. We shall also finally, look into the Thin Plates Spline in this particular lecture.

So, there is a topic which is Kriging which is I mean for constraint of time we may not be able to cover in this particular lecture. So, we shall continue with this same topic and we shall look into the process of Kriging in the subsequent lecture.

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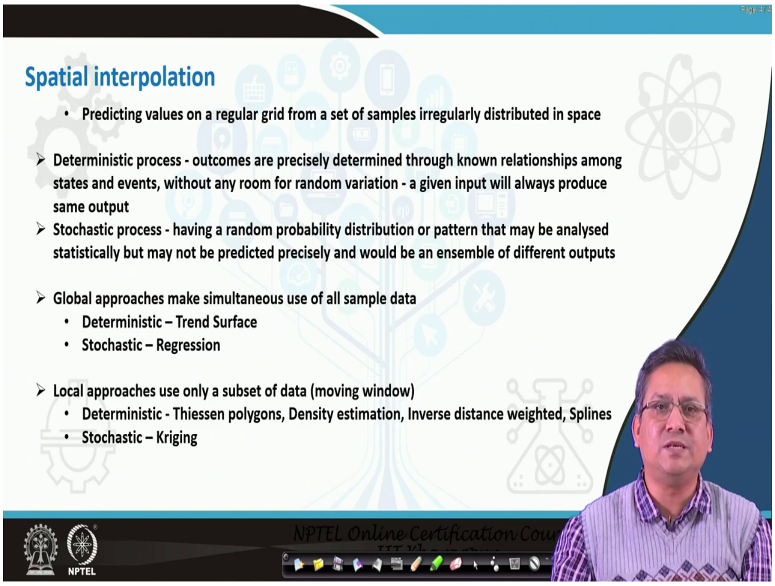

So, let us see what is spatial interpolation, it what does it mean. So, if we have a regular grid and in that particular grid, we know only some few values like we had talked about raster data which is an array of numbers. So, in that particular array, we may know only few data points or we may have a point coverage a GIS point coverage for which we may not we may know the data, we may have the data.

For example, you can talk about say weather observatories which might be having say temperature data values. So, we would want to create a grid, continuous grid of temperature data values based on few sets of observation distributed in the space. So, I mean it would be a regular or a irregular distribution of these data points in space. So, there are different methods by which we can do it.

First, we can use a deterministic process in which the outcomes are determined specifically and we can establish a relationship among the events and the states and there is no random variation in the outcome. So, whether you do the analysis or I do the interpolation, the results will always be the same or if you do this interpolation across different iterations, you given the same inputs your outcome would be the same. So, these processes are the deterministic processes.

Now, the second one is the stochastic process which has a random probability distribution associated with this process or the pattern may be analysed statistically, but it is not predicted precisely. So, it would be an ensemble of different outputs. So, this is what we call it as stochastic process. So, the outcome of these processes may vary depending on the iteration. I mean if you do a second, I mean iteration based on the same sample points, your output may slightly vary ok. So, I mean it is not precisely I mean exactly determined as you would be having, I mean determining these values in the to the deterministic processes.

Now, there are different ways of doing the spatial interpolation. So, first approach is the Global approach. So, it uses the entire data at one go simultaneously. So, that is a global approach. So, the first approach in the global approach is a deterministic surface which is known as a trend surface. So, if we have scatter data points, I mean irregularly distributed data points, we can create a trend circle surface based on deterministic methods using the global approaches.

We can also use the stochastic process to I mean project the values of the points in a using a global approach. So, these type of approaches would use the regression functions in the for the stochastic approach. Now, talking about the local approaches we would be using only a subset of the data or it could be referred to as a moving window that I mean we have a small window and in that particular window, whatever data points are selected that is interpolated, those values are interpolated within that particular window and then, we shift the window and take I mean do it iteratively.

So, it also has a deterministic; there a deterministic method for local approaches as well for spatial interpolation. So, these methods deterministic methods are Thiessen polygons, Density estimation, Inversed distance weighted methods and Splining. Now, there are also stochastic methods. Stochastic methods, we have said that it is a probabilistic method which may create a similar values, but not precise values.

So, this in the stochastic method for local approach, we have one method which is known as Kriging. Now, depending on your requirements or the need of your study we would be using either of these methods. We had talked about the global methods, we had talked about the local methods. In this, we have talked about the deterministic methods and we have also talked about the stochastic methods. So, we shall see these processes sequentially.

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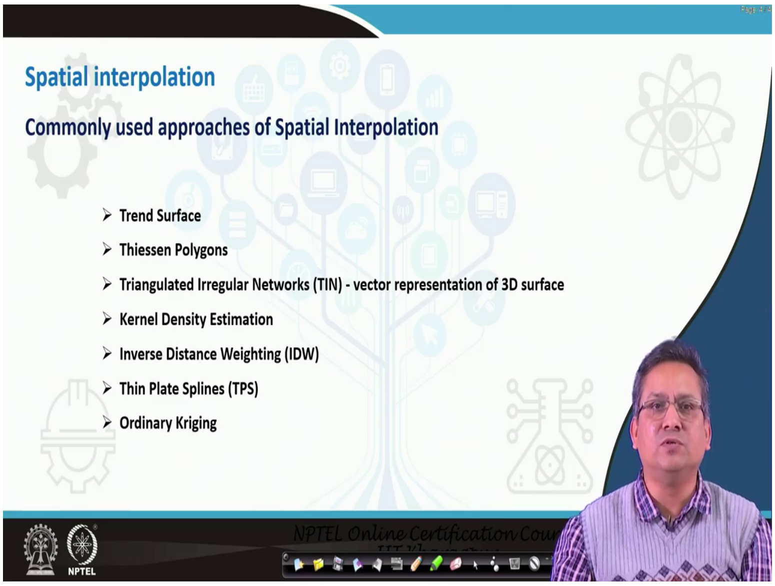

So, the commonly used approaches that are I mean use for spatial interpolation is trend surface. We had already talked about it, we can use Thiessen polygon that is used in the local context. Triangulated Irregular Networks that is vector representation of 3D surface. We can do a Kernel Density Estimation, we can use the process of Inverse Distance Weighing, we can use Thin Plate Splining and we can use Ordinary Kriging method.

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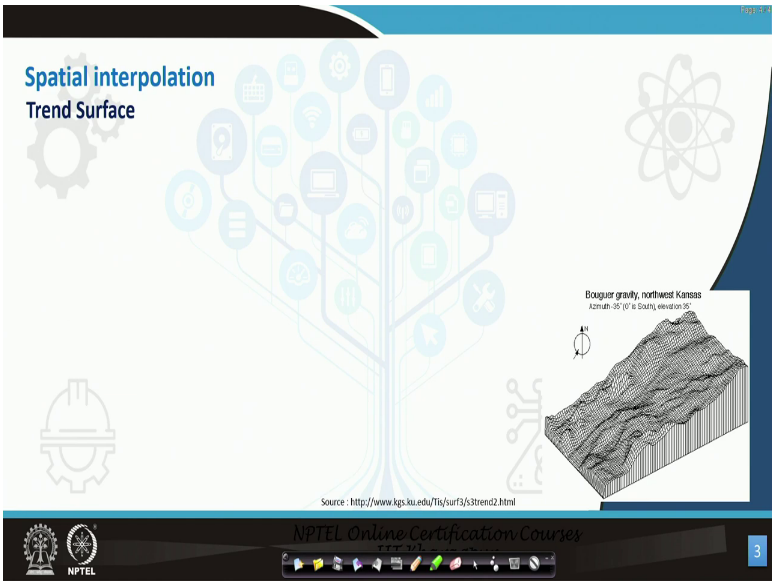

So, let us see what are the different methods of spatial interpolation one by one. So, first let us look at the trend surface. Now, this is a surface which has the gravity data for different I mean different data points which is known. So, it has been interpolated using trend surface.

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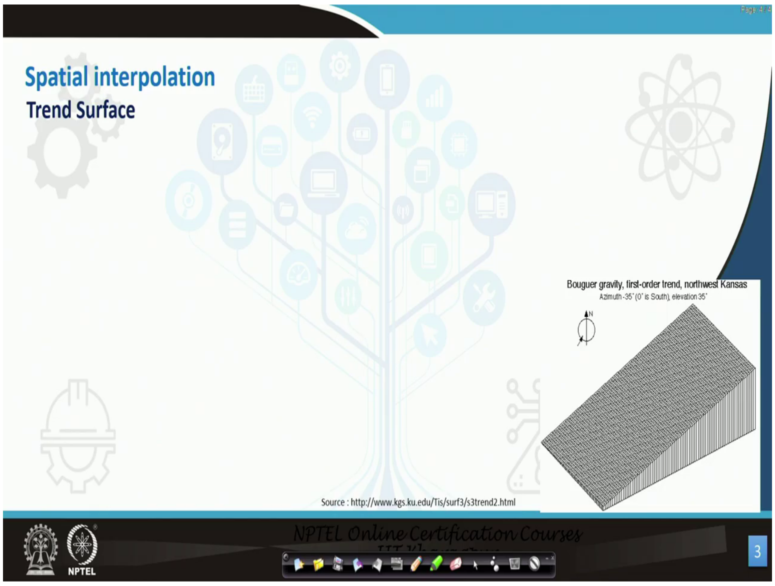

So, when we use a first order trend, you can see you can find out the slope of this particular I mean terrain or I mean the gravity values across this particular terrain. Now, we can also find out the residual, I mean that that is the difference between the initial actual values the and the interpolated values, the first order interpolated values. So, we can find out the difference and this gives us the residual value.

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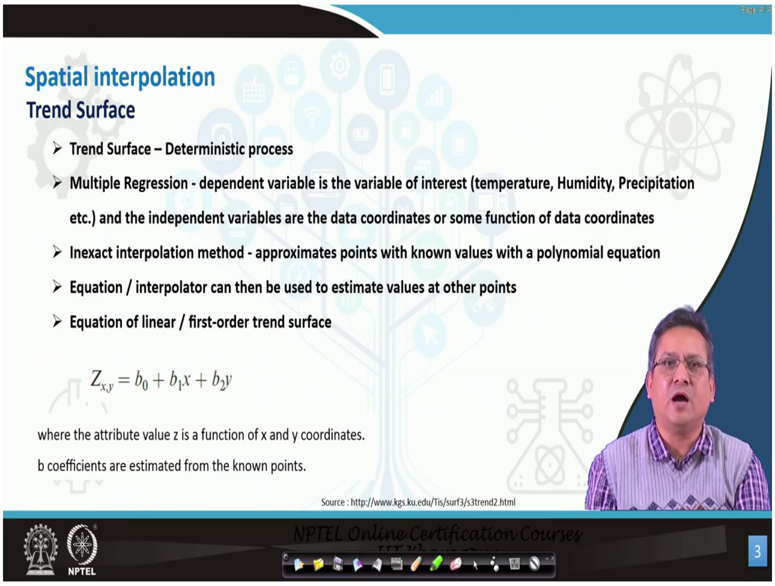

We have seen that this trend surface is a deterministic process. I mean calculating the trend surface is a deterministic process. So, we can use multiple regression; wherein, we can have a dependent variable that is the variable of interest. We said as an example we could be working out or interpolating the values of temperatures which has been recorded by different data points like weather stations in a given terrain as a regular network, irregular network. We can also work out other parameters like humidity precipitation etcetera and we would be having the independent variables which are the data coordinates and or sum function of the data coordinate.

Now, this method is an in exact interpolation method; this is not a exact method of interpolation and it would approximate the points using a polynomial equation. I mean the synthesised points, I mean the data values of unknown points would be the result of implementing a polynomial equation. So, this is used, this equation are the polynomial equation or interpolator, it would be used estimate the values at the other points for which the values are not known and when I mean this equation polynomial equation is of first order then the equation is linear.

So, this is an example of a linear equation; wherein, Z which is the variable of interest, it could be a it could be represented as a first order equation using this equation that is b 0 subscript 0 plus b subscript 0 x plus b subscript 2 and into y. So, in this case your the attribute value Z is the function of x and y coordinates. So, we had earlier seen I mean we had earlier talked about that this the dependent variable is a function of the data coordinates ok. So, and in this case the b coefficients are the estimates from the unknown data points.

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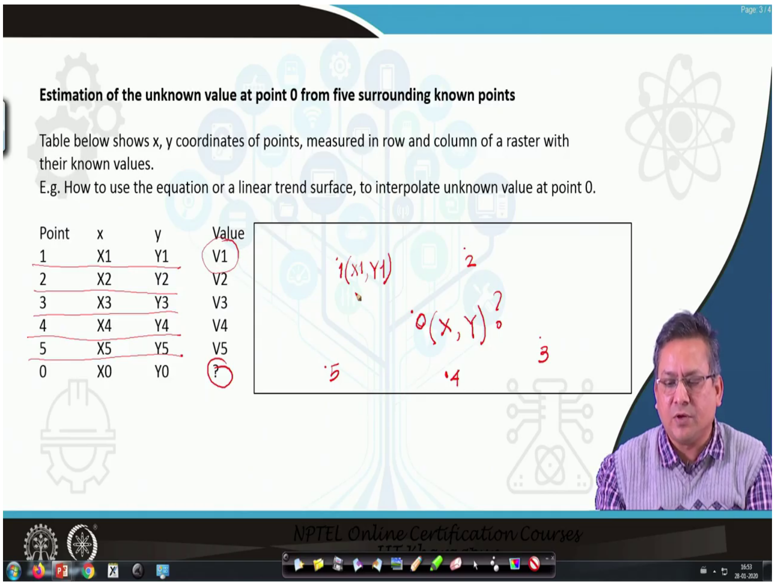

So, what we do is say suppose we have 5 data points in this particular example and we say suppose, we have 5 data points and the 6th one is the 0.0. So, in this case what we do is for the 0.1, we have x and y values that is X 1 and Y 1. So, similarly for 0.2, 0.3, 0.4 and 0.5, we have these data values of I mean X 1 Y 1, X 2 Y 2. So, these are the different data values for your X 1 x and y for the corresponding points for which the values are known; these values are known.

So, you can get the latitude and longitude coordinates of your weather station, you can plot in a GIS. So, you will know the latitude and the longitude or the projected coordinates as X 1 and Y 1 and you would have data points 0 for which you do not know the X and the Y. So, this is a unknown and your X and sorry X and Y is known to you, but the value is unknown. So, in this case the value is unknown. So, we work with the known values of your x and 0.1, 0.2, 0.3, 0.4 and 0.5 and make an equation out of it.

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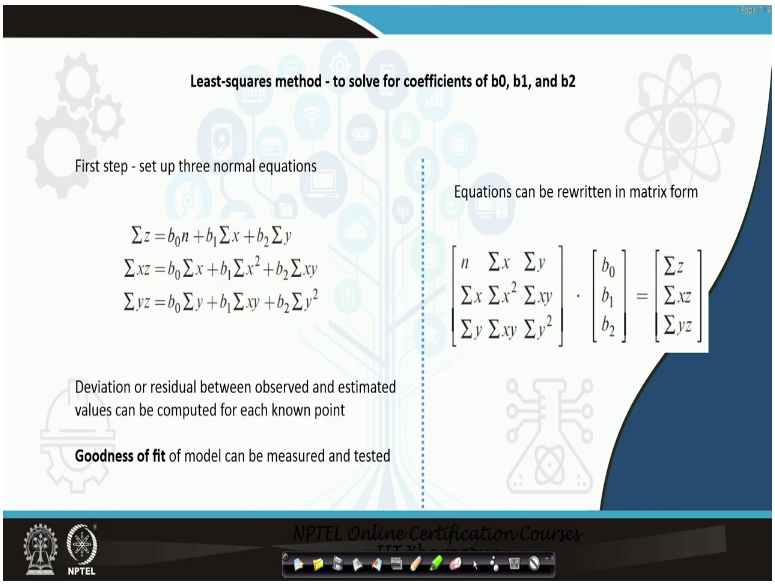

So, if we see this, we use a least square method to solve the coefficients b 0, b 1 and b 2 in the equation that we had talked about. So, that is the first order equation. So, in the first step, what we do is we setup 3 normal equation. So, on the left hand side you have sigma z which is the function of b 0 n plus b1 into sigma y plus b2 into sigma. So, b1 into sigma x plus b 2 into sigma y. Now, the second equation is some is product of x and z and in the third equation, we have the product of y and the z and we have the this two corresponding equation.

So, we set up this first as a first step, we setup this three normal equations, then we can write it in the form of a matrix. These three equations can be written as a matrix that you see out here. So, in this part you can see the values, I mean the I mean known's that is n that is number of points, sigma x, sigma y and then, you have again sigma x, sigma x square comes over here and next element is sigma x, y and similarly these elements come as the third row in this particular matrix.

So, we can write the coefficients b 0, b,1 and b,2 as the next matrix and then, on the right hand side, we have the I mean sigma x, sigma sigma z, sigma x z and sigma y z as three elements. So, we can solve these equation and calculate the value of these coefficients b 0, b 1 and b 2. So, the deviation or the residual would be there between the observed value and the estimated value.

So, I mean if you have more number of points, you can see that the I mean derived surface or the I mean the surface that you simulate would be having a deviation or you can work out the residual. I mean in the last slide, we had seen how we can work out the residual between two different surfaces; one is the known surface and one is the projected surface. So, we can work out the or compute the residual for the two surfaces at each of the known points and we can do a measure of goodness of it and in this model can be tested.

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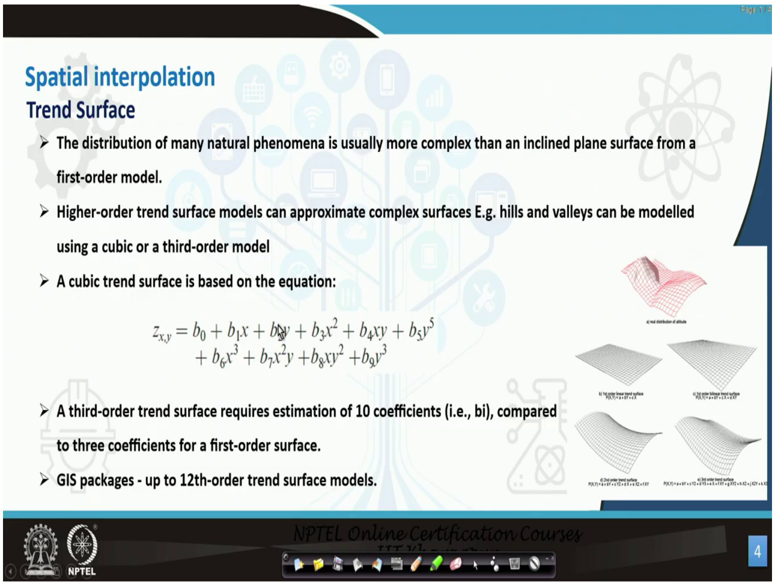

Now, talking about the trend surface, the distribution of many natural phenomena, it is complex, it is more complex than the first order inclined surface that we had seen. So, in this case you can see the first one is the actually the natural surface and when we do first order interpolation, you get a plain surface. Second order you get some semblance of curvature; third second order polynomial you see a saggy kind of thing and from third order polynomial, I mean it creates semblance of this particular surface.

So, I mean this natural phenomena is generally more complex than the first order or second order models. So, what we can do is we can do a higher order surface model fit and it would be able to I mean model the complex surfaces such as if you have hills which has undulating surfaces in a given area and it can be this I mean complex terrains, these can be modelled using third order model or higher models.

So, or cubic trend surface is based on a equation which looks like this. It is the third order polynomial equation. Now, your third order polynomial equation, here you can see that there are ten coefficients which are unknown. So, in our earlier equation, we had seen we had few coefficients; but as you increase the order of polynomial, the number of unknowns would be increasing.

So, what it means that the number of observations also has to be increased so that I mean this solution to this system of equation becomes tractable. In GIS package, generally I mean there are computational limitations, but they would allow up to 12th-order trend fitting for different types of surface model.

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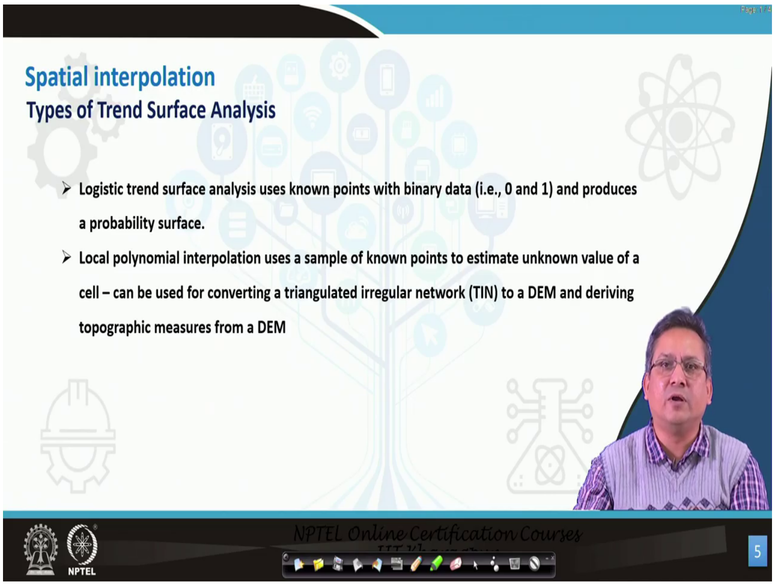

Then, there are different types of trend surface analysis the way we do it. So, first one is the logistic trend; wherein, the known points will be having only having binary values and it produces a probability surface. The next one it is a local polynomial interpolation which uses a sample of known points to estimate your unknown value in the given set and it can these values can be converted into a irregular network, a triangulated irregular network that is known as tin and which is the vector data model for representing a three-dimensional surface.

And this polynomial equation which I mean can be used to take the vertices of the triangulated irregular network as points and it can be extrapolated into a digital elevation model. So, we use the local polynomial interpolation to derive the digital elevation model from a your tins surface, Triangulated Irregular network surface

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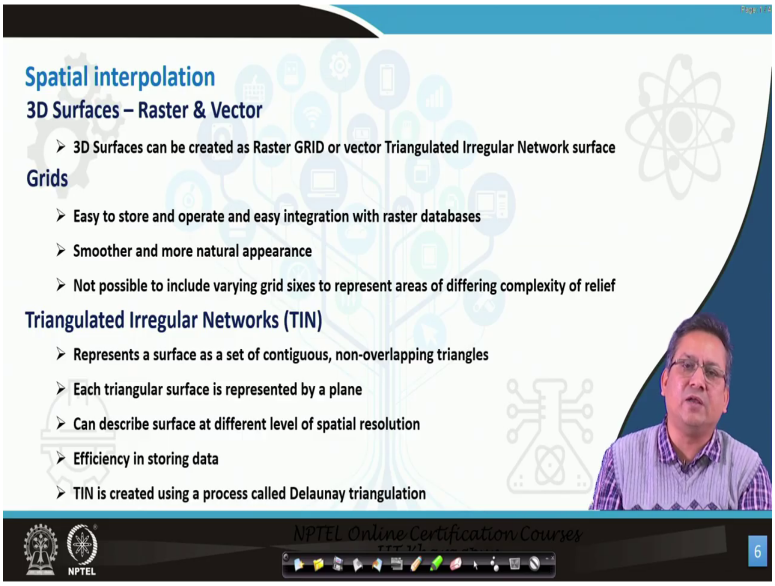

Now, we were talking about the triangulated irregular network which is basically a vector surface and we can also have a dm which is a raster or a GRID. So, when we are talking about the differences between this two surfaces, when it comes to storing your 3D data points, for the grids, it is easy to store and operate with raster database and we can integrated with the raster database model.

So, it is more smoother because you have regular array of data points. So, it would be more smoother and it would have a more natural appearance compared to tin surface. It is not possible to have varying grid fixes to represent areas; wherein, we have complex relief where the relief is very complex where there is a drop in the edge. So, in those cases or there is a projecting surface wherein, I mean it would be like a cantilever surface a hill which projects outwards. So, in those cases it would be difficult to represent those kind of areas using the grid surfaces.

Now, the triangulated irregular networks represents a surface which is non overlapping triangles and which is continuous in nature. So, the each of these surfaces are basically plains. So, this I mean each of the surfaces would be triangular plains and we can define or we can describe the surface at different levels of your spatial resolution and it is also an efficient way of storing data point, I mean 3D data points.

So, how this tins are created grids we can assume that these are matrix an array of numbers? So, which is regular grid; but this triangulated irregular networks are triangles which are not uniform or which are not I mean have which do not have the same density across a space when you create it. So, it is created using a process which is known as Delaunay triangulation.

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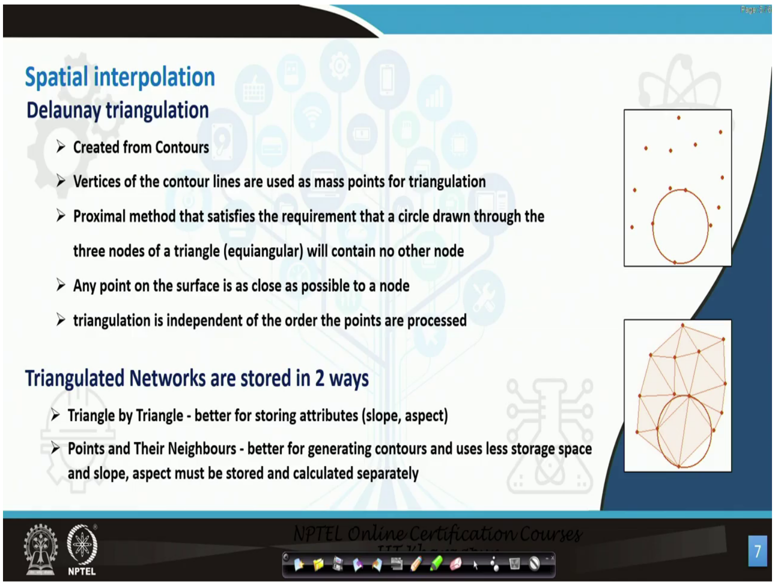

Now, what is Delaunay triangulation? Delaunay triangulation is either created from contours or data points. So, the vertices of the contour lines, they are used to mass produce this points which are then used for triangulation. So, you may have two different levels of consecutive contours. So, say first is the 0 level contours, 0 metre contours; next contours could be a 10 meter contours. So, suppose we have 2 contours. So, first one is a level 0 contours, this is a 0 meter contours and this is at 10 meters.

So, these are two contours data that we have. So, what we do is this contours will have vertices and these two contours will have vertices. So, these points are joined together to create triangulated facets representing the surface. So, this they are used as mass points the vertices of contours for the triangulation. Now, we use proximal method, which this method basically uses 3 points, 3 nodes of a triangle and fits a circle through this 3 points which are almost I mean these triangles are so derived that they are equiangular in nature and will it should not contain other nodes.

So, any point on the surface is as close as possible to the given node. Now, the triangulation it is a independent process and the points are processed like in this case you can see this 3 data points, they are lying on this particular triangle. So, this 3 data points are lying on this on this particular triangle. So, likewise we generate the triangles, I mean which are almost equiangular based on these data points.

So, these triangles triangulated networks are stored in two ways. It can be stored either by Triangle by Triangle method. It is it I mean provides a better solution for storing the attributes that is you can also include other parameters such as slope or aspect or you can save this triangle triangulated irregular network as Points and Their Networks. So, when you save this triangulated points as points and networks, it is useful when you want to generate contours and it uses less space.

So, but the limitation in this case is it does not it cannot store slope or aspect data along with this point values. So, it has to be stored or calculated separately. So, which is the advantage while we are using a triangle by triangle method.

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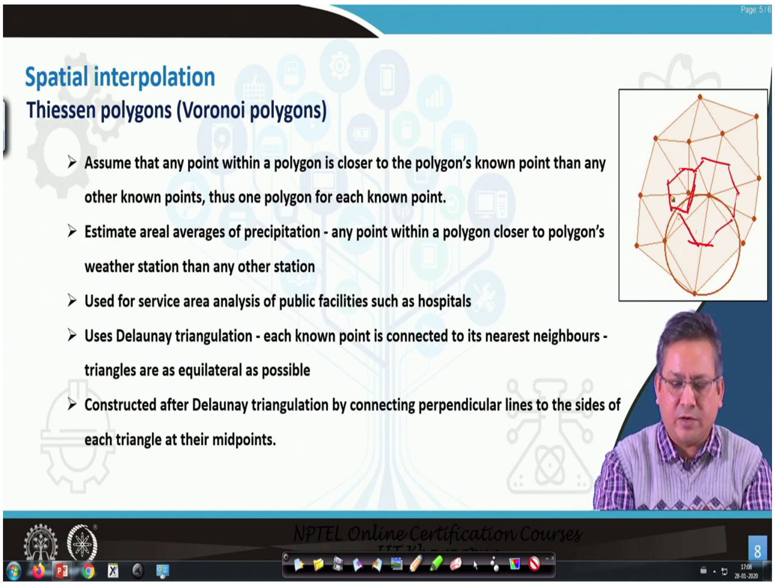

Now, we make Voronoi polygons based upon this Thiessen polygons I mean Thiessen polygons or Voronoi polygons based on this triangles that we had talked about. So, what we do is we create the midpoints of these triangles which we have I mean calculated using your Delaunay triangulation process, wherein we try to create equiangular triangles using 3 points that I mean are lying or located on a circle.

So, in this case what we do is we take the midpoints of these lines and draw perpendicular bisectors. So, you see that these points will intersect each other and will create a polygon. So, your polygons will be created based on these Voronoi's I mean Voronoi polygons based on the Delaunay triangles.

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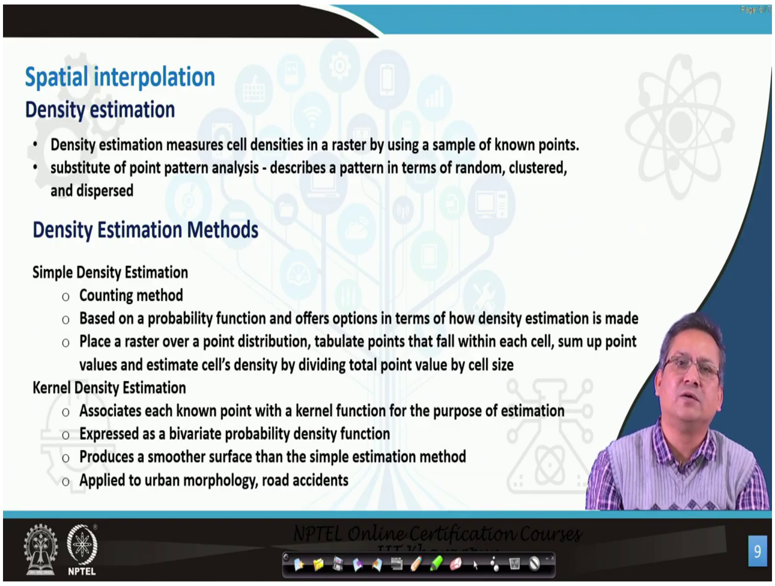

Next, we come down to the density estimation. So, we measure the density using a sample of points and we do the point pattern analysis, it could be the points could be random, they could be clustered or they could be dispersed. So, we have different methods to do the density estimation. The first one is the Simple Density Estimation. So, its basically a counting method.

So, it uses a probability function depending on how dense, I mean what is the density of the estimate and we place a raster over a point distribution and we tabulate the points, I man we calculate the number of points that fall within itself. We then sum up the point values and we calculate the density of the cell by dividing the total point value by cell size. So, this is the first method which wherein, we use a simple density estimation process to find out the density estimation for spatial interpolation.

Now, the next one is the Kernel Density Estimation process, which would associate each point with a kernel function. So, this is expressed as a bivariate probability density function. Now, it generally would produce a smoother surface in comparison to the simple density estimation and we can have several types of application of this kernel density estimation, like we can do a density estimation of areas which are prone to traffic accidents or we can I mean estimate or apply it to the I mean working out on urban morphological parameters.

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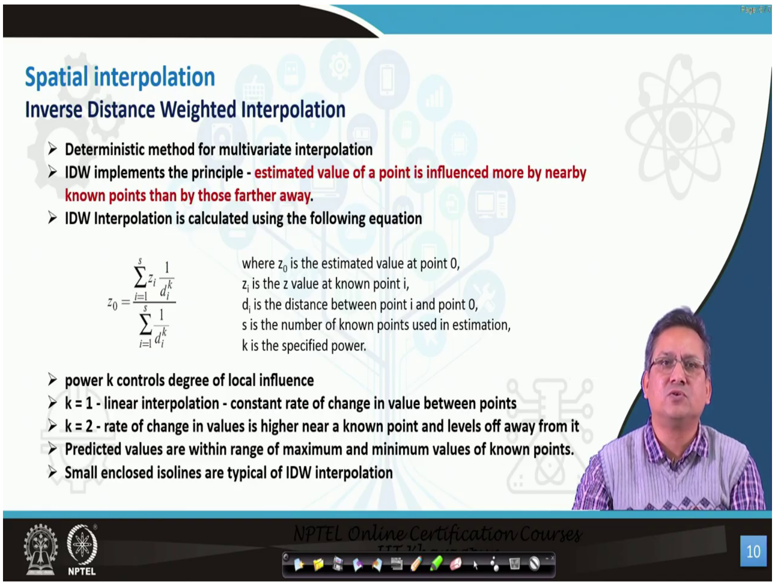

The next method is the Inverse Distance Weighted Interpolation method which is a deterministic method for multivariate interpretation. Now, this are the principle of IDW is to estimate value of point and it is the principal is that this estimated value of a given point is influenced by the nearby points, I mean more influenced by the nearby points than those which are line further away.

So, is it calculates I mean the IDW interpolation is calculated using the this particular equation in which we have a ratio, ratio is the product of the estimated value, the known value at point I and summation of that into 1 upon kth power of distance between the point I and the point O, that is the point O is the unknown where we are estimating the value of z.

So, and in this case your s is the number of points which are used in the estimation and k is the specified power. So, this power k, it would control the degree of influence of the local points. Now, if this value of k is 1, it becomes a linear interpolation that is there is a constant rate of change in the values between the points. So, the interpolation is linear interpolation. Now, if this k assume the value of 2, then the rate of change in values are higher near a known point and it levels of when it is these values are away from it, these points are away from it.

So, the predicted values of IDW interpolation are within the range of maximum and minimum values of the known points. So, the predicted values won't go beyond or below the minimum and the maximum values of the points that you are using for interpolation. So, I mean we it creates a small enclosed isolines, lines of similar values. So, that is typical of Inverse distance weighted Kriging.

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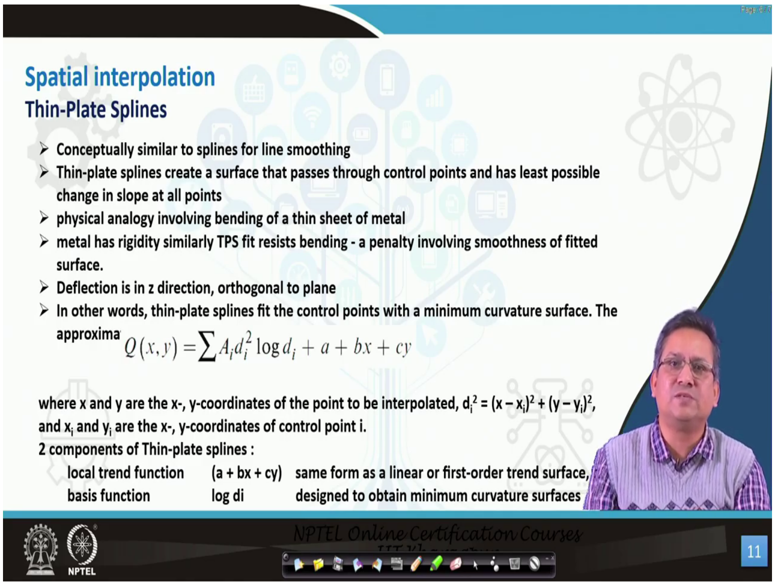

Now, talking about thin plate spline; it is similar to splining. You would have seen your software's, CAD software's wherein, you have data points, you create polylines; wherein, you create polylines using points and then, you fit a spline function on that which smoothens that particular line. So, in this case, it is a similar concept, but we are going to spline a surface. So, I mean these surface would go through or pass through the control point and has the least possible change in slope at all points.

Now, it has a I mean similar analogy to that of a thin sheet metal being bent over different data points say suppose you have nails of different heights; nailed into a plywood and then you put a thin sheet metal plate over this and you try to fit a kind of a surface over this different heights of the nails. So, this metal has rigidity.

So, similarly thin plate spline also I mean fits and it resist bending. So, there is a penalty function which involves smoothness of the fitted surface. Now, the deflection that we see in the plane is in the direction z that is the I mean third dimension which is orthogonal to the plane. So, I mean it controls the curvature of the surface and it is an approximation of thin plane spline. So, and it assumes equation I mean Q equals to Ai di square sigma of log di plus a plus bx plus cy.

In this case x and y are the coordinates of the point which are to be in interpolated and di square is x minus xi square plus y minus yi square; where, x and y are the coordinate of the point, wherein I mean we are going to find out the value that is find out the value of z in the third dimension and x and y are the x coordinates of the control points.

There are two components of thin plates spline; first is the local trend function which is your ax plus bx plus cy. This component which is known as local trend function. It is same form as linear or first-order trend surface that we had seen earlier. Now, there is another term which is known as the basic basis function which is the log of d. So, it is used to I mean design to obtain the minimum curvature surface.

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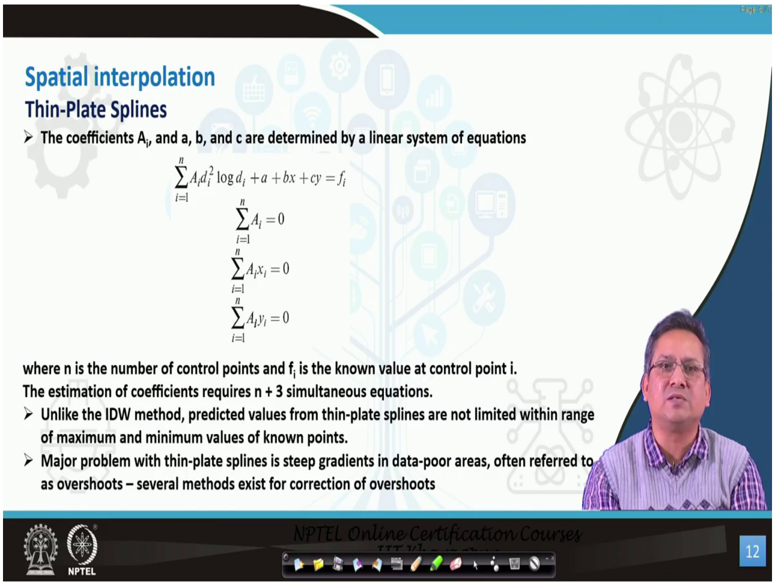

Now, the coefficients Ai, a, b and c how do we determined it; determine it? If we determine it using a system of linear equations, so we again can convert it into a matrix form and then, we can calculate the coefficients of capital Ai, a, b and c; wherein, in this particular case you can see sigma i is from 1 to n; where, n is the number of control points and fi is the known value at the control point i. So, we have this I mean this sums up to value fi. So, this is the known value at control point fi. So, we can have based on the observations, we can have simultaneous equations by which we can convert it into a matrix and we can solve for the coefficients of Ai, a, b and c like we have done earlier.

So, the estimations of the these coefficients would require n plus 3 simultaneous equation and unlike the IDW method, the predicted values of this TSP that is the thin plates splain, it won't be limited within the maximum and minimum range values that we had said for the IDW method. The I mean inverse distance weighted method of interpolation, we had seen those values the outcome values would be ranging between the maximum and minimum values which is not the case for TSP interpolation.

The major problem with TSP is that you may have steep gradience, since the outcome of the values are not within the maximum or the minimum values; sometimes you may encounter steep gradience, especially in areas where in the data are not there, your data sample points are not there. So, it this steep gradients are generally referred to as overshoots and there are different numerical methods and approaches to correct these overshoots.

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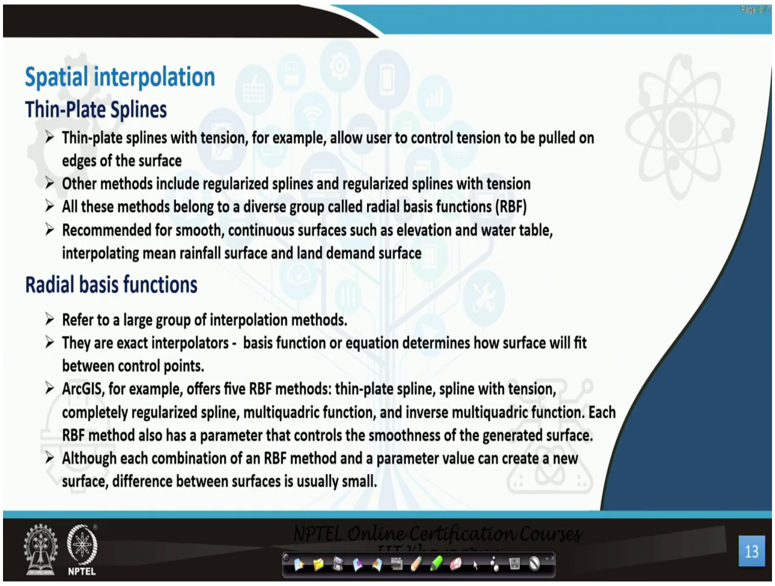

I mean, we can also have thin plates spline with tension, it controls the tension and it pulls the surface I mean this TSP to the edges of the surface. Now, there are other methods in the TSP which are regularised spline and regularised and splines with tension and they have a I mean diverse group. These methods are belong to a diverse group of functions which are known as radial basis function. Now, these this method of interpolation that is the TSP, thin plate spline is recommended when you want to create a very smooth or continuous surface like the surface of a water table or a elevation or if you want to interpolate the rain.

So, if you have such type of problems, then the TSP is the suggested method. So, you have the radial basis function which refers to a group of interpolation methods, they are I mean exact interpolation. So, equation functions or equation determinants that governs how the surface will fit between the control point. So, this is the these are the basis function that we had talked about in the earlier slide.

So, you are ArcGIS software's it offers five different methods of your Radial Basis Function; different methods, you can explore them, explore this methods. So, the difference between these points I mean difference method could be very small.

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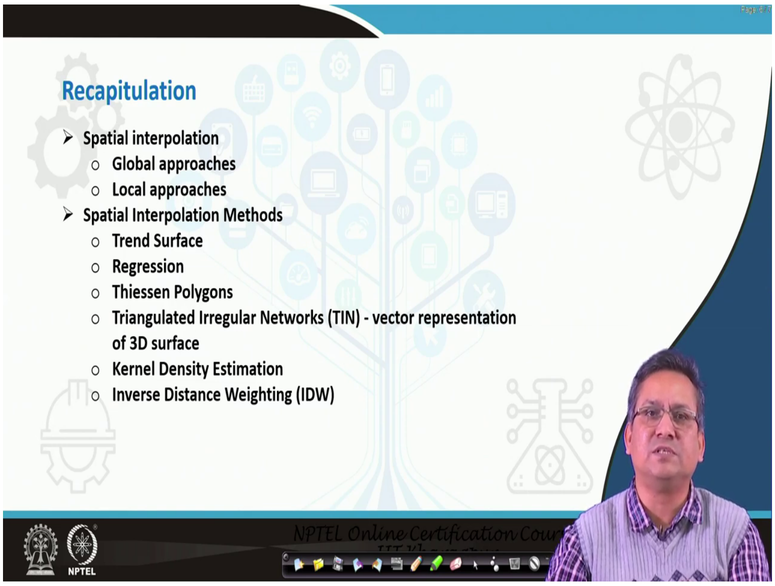

Now, so, we have seen different methods of spatial interpolation, a recap of what we have done in today's lecture. We have seen the different methods of spatial interpolation, we had seen the global approaches, we had gone through the local approaches, we had seen the different spatial interpolation method such as Trend Surface, Regression, Thiessen Polygon, Triangulated Irregular network, Kernel Density Estimation, IDW that is a Inverse Distance Weighing and Thin Split Plain.

So, thanks and we will continue with the Kriging which is the which is another method of interpolation in the next lecture.

Thank you.