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Lecture-44
Analysis of Water Distribution Networks

Hi friends and welcome back. So, we have started discussing about water distribution systems this week and in the last couple of classes we discussed about the basics of water distribution system and what are the different layout schemes of water distribution networks.

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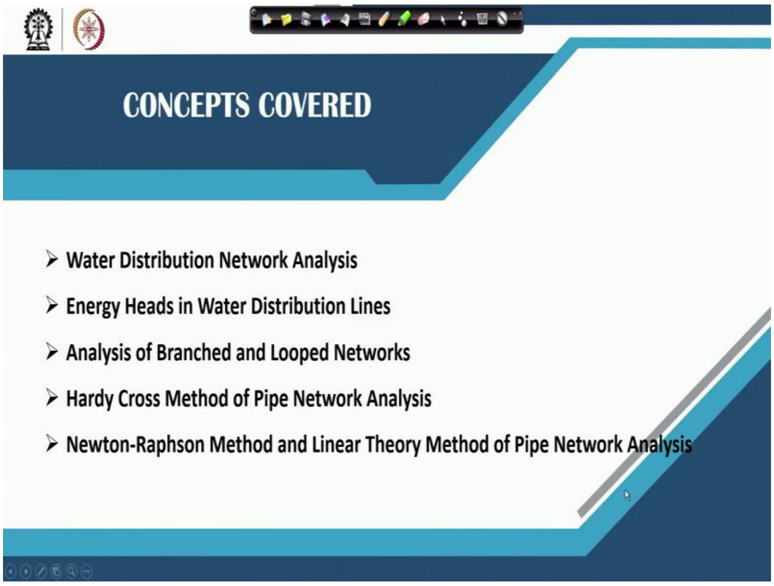

So, this class we are going to talk about the analysis of the water distribution networks mainly. So, what will be discussing about how do we analyze the systems. What are the different energy heads in the water distribution lines? We will talk about the analysis of branched and loop network. Hardy cross method which is the one of the most popular method for pipe network analysis we will discuss that.

And then we will touch upon the couple of other methods like Newton-Raphson method and linear theory methods which are also used mostly in the soft computing tools.

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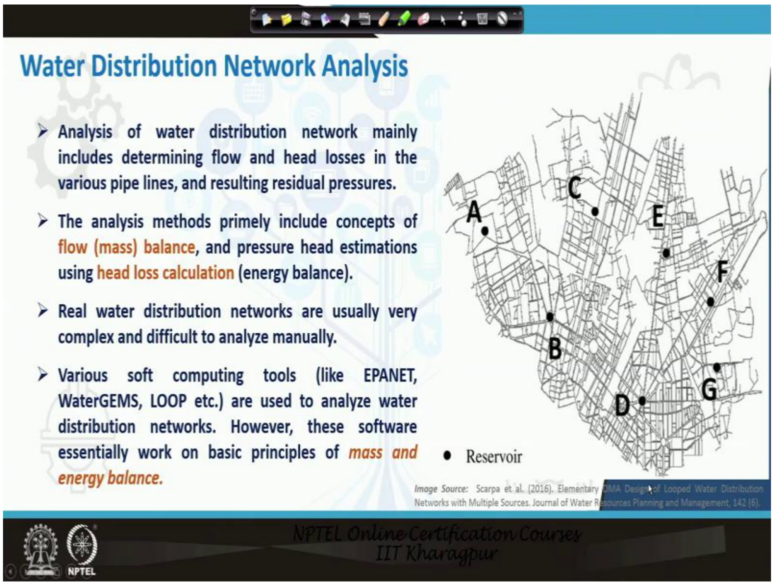

So, if you see a like when we plan to analyze a water distribution network we must understand that what is the objective of this analysis. So, generally the objective of water distribution network analysis is mainly determining the flow and head loss in the various pipelines. So, the idea is to basically what different head losses are occurring at the different in the different pipelines and what are the flow that is being transported through the different pipelines.

So, the head losses also kind of give us the residual pressure at the different nodes. Since we are interested in analysis of the water flow and the pressure, so, primarily the concepts of the mass balance for water which is actually the flow balance is used and pressure head estimate lessons are done using the head loss calculations which is a form of energy balance. So, we try to balance the energy and balance the mass in the system and in that way we tend to analyze what are going to be the flow in the different pipes and what are going to be the head losses across the different pipes.

Now analysis see us are standard but if we are going for a real water distribution networks particularly of larger cities, so these networks are very complex. You can see this can be considered at an example now because of the complexity involved number of pipe number of connections low formed there would be some branches. So, that way there are so many components in a network that it becomes very difficult to analyze such network manually.

And for that purpose we may use soft computing tools various software's like EPANET waterGems, Loops. So, these are some of the software's which can be used for analyzing water distribution network. However these software also essentially work on the basic principles of that mass and energy balance what we were just talking about.

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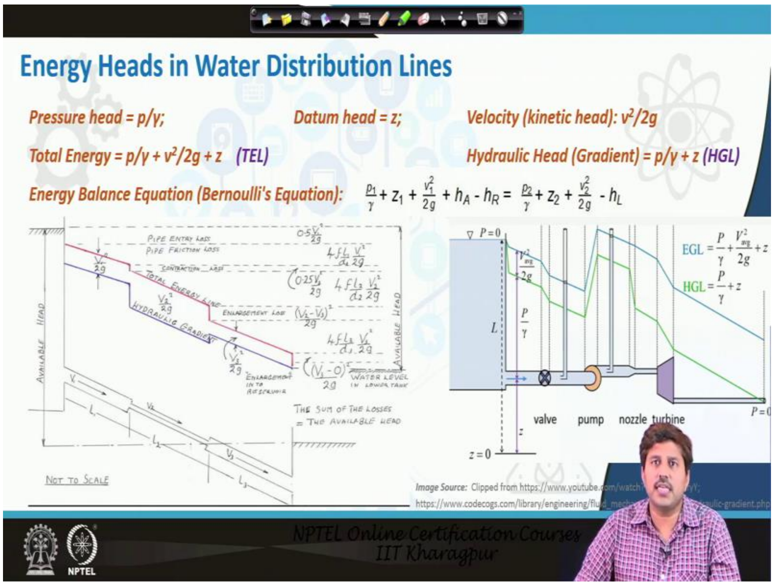

So, if you see the different energy heads different type of energy heads that we get in a water distribution line. So, there is pressure head when water flows under pressure so there is some pressure like there is energy embedded in the water flow due to the pressure under which it is flowing so that is pressure head which is P by gamma simply P by gamma. In open channel flow this pressure head particularly at the top of the water label this pressure head is negligible.

Not negligible means it is a one atmospheric pressure so pressure head is unity in the open channel flow but in pipe flow the water may be flowing under pressure. And in that case whatever is the pressure we can divide that with the gamma and get the pressure head in metres in fact. Then there is a datum head which is basically what is the height from the reference point. So, let us say if this is our reference point so height of a particular point that is basically going to be the datum head.

So, if say this is value of j is equal to 0 and if our pipe is here or pipe is here so if pipe is here so this becomes our datum head. If pipe is if we take pipe here then this becomes our datum head. So, the term head will also be basically depending on from a reference point what is the height at which the pipe is placed that is essentially because of the gravity. So, a pressure head is essentially because of the pressure and datum his is because of the gravity.

Then there is a kinetic head or velocity head which is equal to v square by 2g again in meters so this is all normalized to the meter that is the kinetic head means when water flows so the kinetic energy of the flow that also must be counted when we are basically counting the total energy. So, the total energy will be summation of these three heads so there will be pressure head there will be velocity head or kinetic head and there will be datum head.

So that is known as the total energy and if we like in a pipe flow if we measure the total energy at each point and put that through a line so that is known as the total energy line or TEL. If we exclude the velocity head of the total energy so we basically we take only pressure head and datum head that gives us the hydraulic gradient line our HGL this is known as hydraulic head or hydraulic gradient and if we basically try to monitor that so that will be known as the HGL.

So, HGL essentially is the pressure head and datum head whereas the EGL or energy gradient line or total energy line whatever you call that is basically the sum of the all 3 energy and as if we can apply the like energy balance equation or conservation of energy which is essentially given by the Bernoulli's equation. So, in a system at any point like the total energy has to be balanced so like if say we have one point as a so if at point A or if at point 1 we are having pressure as say P1 datum is g1 and velocity as V1.

So then we will have pressure head as Rho P1 by gamma datum head as g1 and velocity as V 1 square by 2g. Similarly at the second point also we will have all these 3 forms of energy. so, these are the 3 forms of energy which are available at say like in a pipe flow if water is flowing so if say this is our section 1 and this is our section 2. So, this belongs to section 1 and this belongs to section 2 now.

Because the energy is going to remain conserved so if there is no loss or no gain so the summation should be equal if there is velocity is more than pressure will drop or if like there might be difference in height. So, essentially the summation of all three should be same in case of there is no loss or no gain. But in case of there is loss or gain say if there is energy addition if say you are pumping additional power is being pumped at any particular point so you say if you have a pump here so then there is an additional energy.

So, the additional energy input if there is some energy removal so like if you are say having a turbine or wall something which reduces the energy. So, there will be energy decreasing energy will decrease because of that way. So, in accounting all these also like we should have the total energy same across the line and the difference in the energy if there is any difference in the energy is there so that is essentially known as head loss.

And as we discussed in the previous lecture that head loss is there in pipe flow because of friction which is known as the major head loss and then there are minor head losses because of various bands expansion contracts and so for those things alright. So, if you try to kind of see the energy line if you say you are starting from here this is your pipe system so you are starting red is the total energy line or TL or EGL.

This is known as EGL also energy gradient line. So, if you can see here this is since here there is again the pipe dia is getting reduced. So, pipe dia is getting reduced means the velocity head will be more because velocity will increase, so V square by 2g will increase. Now so the remaining part like the datum is already there whatever value is there and then the pressure will also decrease because velocity has it is increasing.

So this is our hydraulic gradient line and this is our total energy line. So, you can see that fall in the total energy is less here because that is the lost of energy loss of energy due to entry like in this pipe they due to the entry basically smaller dia pipe. But loss in the hydraulic gradient line is far magnificent because the like of course the total energy loss has been this much but since this portion is having higher velocity gradient.

So pressure will decrease in this pipe there will be less pressure here and as a result we will see that the hydraulic gradient will decrease velocity gradient would be rather higher. So, the difference with between the two which is basically the velocity at V square by 2g was less here but will be more here because velocity is more here. Again say if it expends again when it in interferes again the total energy will fall but this time fall in hydraulic gradient line may be smaller than the total energy.

Because some of the velocity head is will compensate that total fall so velocity will be basically increased and as a result pressure head means velocity will be decreased in larger dia pipe. So, the difference will decrease and pressure will increase. So, that way we can see this is say another example so you start and then as soon as water enters here there this is your total energy line this is your say hydraulic gradient line.

If there is a wall this will dissipate some energy so you will see there is some fall some steeper fall in the energy remaining fall is because of friction. So, because of friction also you will see there is a constant fall happening in here then if it goes through some pump intermediate so pump will push more energy so you will see that energy like total energy as well as because pump is putting pressure, so the pressure head has increased and total energy has increased.

And then again it will start losing because of the friction if say there is no cell where there is a some country happening again you will see that fall in the total energy as well as the hydraulic gradient line. So, that way we can analyze how the energy is changing in the system.

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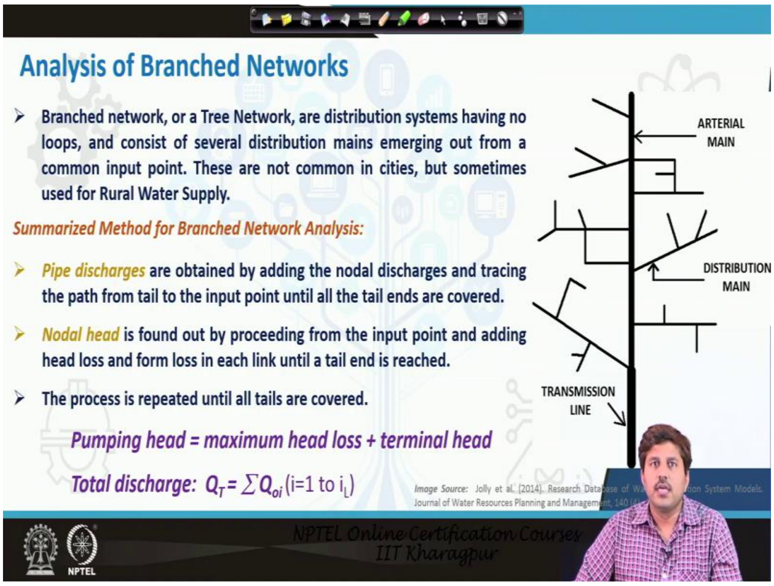

Now let us talk about the analysis of these water distribution networks so as we discussed there are two types of network there are branch network and there are looped Network primarily. Again the loop network could be of different types gridiron or circular or radial that that is another thing but there is possibility of like branch network or loop network. So, for branch network analysis is very simple it is kind of a tree type of network as we discussed earlier.

So, there will be basically several distribution mains which will be emerging out from a common input point these are not very common in the cities but sometime used for rural water supply system. Now there are summarized methods for branch network analysis. So, the pipe discharges are obtained by adding the like we add various nodal discharges tracing the path from the tail to the input point.

So if discharge here is say q1 discharge here is say q2 discharge here is say q3 discharge here is say q4 that way like from each node we will get the discharge. So, for this pipe up till there here we need q1 flow for this pipe we need q1 q2 plus q3 flow for this pipe if say this is q4 this is q5 so this is q6 so flow here should be q4 + q5. Then flow here should be basically q6 also means it will be added q6.

So similarly like we start from the tail and keep on adding keep on adding and by the time like we reach the main transmission line we would be having what discharge that main transmission line should be carrying. Then heads are found out by proceeding from the input point so whatever like it is reverse of discharge like discharge we calculate from the tail ends. How much flow is going to go through those pipelines what is the demand from these dim then that flow must be carried through the different pipes.

But for pressure purpose we see what is the basic pressure which is being supplied to the mains supply mains and then like at this point what is the head loss happening till this point so what is the pressure available here again from here when water flows. So, what is going to be the pressure available here what is going to be the pressure available at these points through the head losses. So, for pressure because pressure will be reduced so what is the pressure here and then if at some point we see that pressure had reduced too much then we may actually put in some like additional pumping head may need to be provided.

So, maximum head loss that is happening plus terminal head which we need at the end that much must be the pumping head in a branch network. So, if say total maximum head loss at flow you are starting from this point say by the time water reaches here or here the total head loss is say 25 meter or total head loss is say 35, 30 meter. And you want to ensure that the delivery head at this point remains say 10 meter or 12 meter. If you want to say 12 meter or say 15 meter delivery head you want at this point.

So, the net head that you must be pumping at this point should be summation of this total head loss which is happening and terminal head which is needed. So, that means 45 meters head minimum 45 meters head must be pumped at this point. And total discharge as we said it is going to be the sum of all the discharges throughout the various pipe lines.

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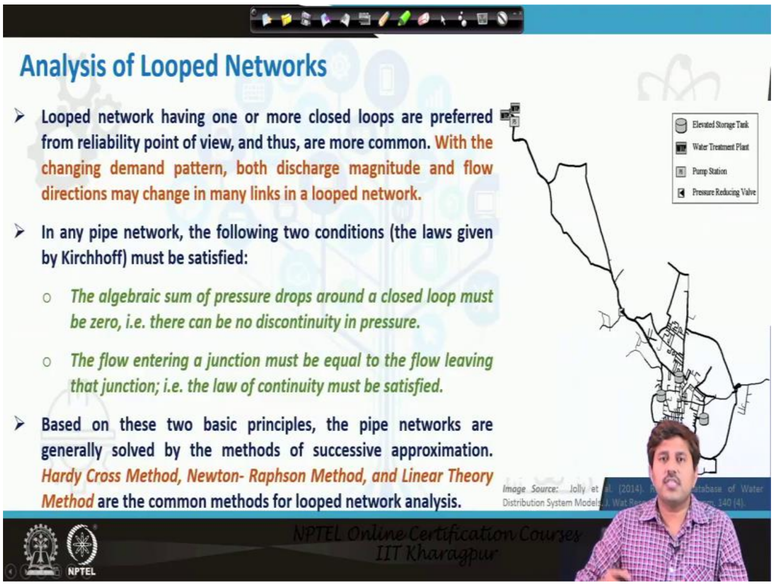

So, this is a simple analysis for branch network. For the loop network it is rather more complicated so loop networks have one or more closed loops and they are basically more reliable in say because if water supply is shut down from one end there might be like other end might cover the water supply in this region but for the loop network because of this particular feature because every point is going to be fed with more than one direction.

So with the changing demand pattern both discharge and magnitude of the flow direction may change in the loop. In a particular loop you can see that some time flow is from this direction some time flow is from that direction depending on how much magnitude we are how much water we are providing and what is the demand nodes what is the demand supply. For say let us say you are having a loop network of this kind with all the demand nodes.

Now you are pumping water from here say if your demand here is very high so part like this part of this will go here and you may see that actually this part is also feeding here whereas if demand is very demand here is reduced and demand at this node is increased then you may see that actually this is being fed from this direction as well as from this direction. So, in this particular pipe you may see that if demand is higher here it might actually be this node might be feeding here.

If demand is lower here and hire here so you can see the reverse flow. So, same pipe may have basically like change in the direction as well as discharge magnitude that that possibility is there. Now in pipe networks there are two conditions which is basically as particular jobs law must be satisfied in these loop network. So, the algebraic sum of the pressure drops around a closed loop must be 0 there cannot be no discontinuity in pressure.

So across a loop whatever is the head loss or pressure drop is happening. So, summation of everything across all the loop because there will be of course we are talking about the algebraic sum so we need to take the sign also so overall net sign with sign this net summation must be 0. And the next one is the flow entering at Junction must be equal to the flow leaving that Junction so that basically law of continuity is satisfied.

That means if let us say we are putting in 10 liters per second water here so that it is not possible that this gets 4 liter per second and this gets 4 liter per second then where are that where is the remaining 2 liter per second water is going. So, in order for law of continuity to be valid if a junction is basically getting feeded with 10 liter water per second having 4 liter per second. So, next one would obviously have 6 liter per second in order to maintain that law of continuity.

So these two conditions must be satisfied for when we go for analysis of these loop networks. So, based on these two basic principles there are several methods which has been developed for the analysis of the looped network. Majority of these work on the successive approximation some numerical methods like Newton-Raphson or linear theory methods are rely on the numerical method where each like entire network can be solved together whereas the Hardy-Cross kind of method which are more basic try to basically consider loop wise every loop wise how the pressure and flow can be estimated within a loop.

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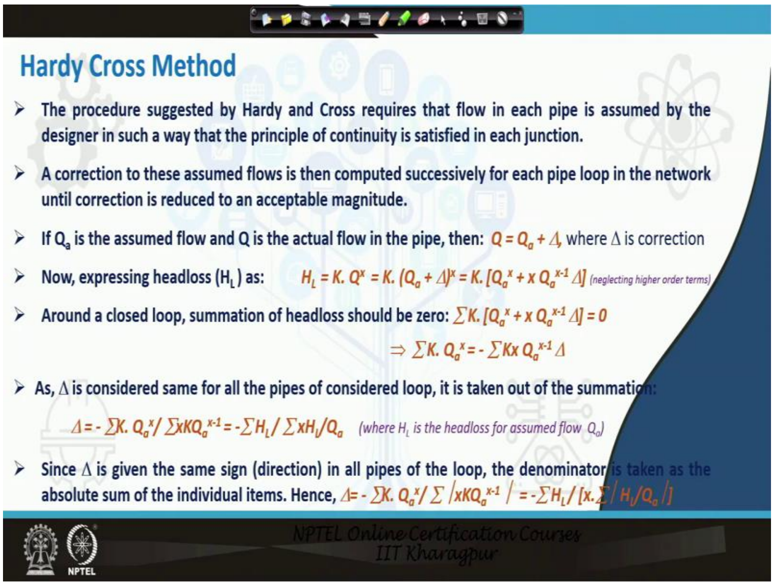

So, the hardy-cross method which is generally the most popular one for the loop network analysis in particularly for manual kind of analysis. So, this is simply the procedure based on the assumption that the principle of continuity and the like net head losses has to be 0 across a closed loop are satisfied. So, they are the basic assumptions here so what is done here practically we first assume a flow in each pipe given the demand or given the flow. We assume a flow in each pipe which satisfied the basic principle of the continuity and then we work out a correction to these flows.

We compute a correction to these flows in the successive steps in several step and then we apply these corrections. So, for each pipe loop in the network until the correction is reduced to an acceptable magnitude or to acceptable number or the correction has become very low. So, that the flows that are there in the pipe appears to be kind of consistent or constant then we stop this iteration and consider that as a final solution right.

So if let us say Qa is the assumed flow if we say assume the flow of Qa in a pipe and Q is the actual flow in up in that particular pipe then Q is going to be equal to Qa + Delta where Delta is our correction which we need to estimate right. Now the head loss through a pipe can be given as we discussed earlier also can be given as the K times Q to the power X where basically X is an empirical factor that depends on which method we are choosing all right.

So, normally like Hazen William or whatever method we are choosing we will assume a value of x for that purpose 1.85 are around 2. So, we have K is equal to Q to the power X that means total head loss is equal to K into Q to the power X. Now Q as we said it is basically

QA + Delta so it is a QA + Delta to the power X. Now if we expand this so we get we can basically get in this form where we neglect the higher order terms of course there will be higher order terms but they will be very small because Delta value is very small.

So, that way if we just consider first and second terms so this is what we are getting. Now around a closed loop the summation of head loss is has to be 0 as one of the principle that we discussed earlier. So, summation if we take summation of these entire number this has to be actually equal to 0 and that means the summation of K into Q a to the power X is going to be equal to minus summation of K X Q a to the power X - 1 by Delta right.

Now Delta is considered same for all pipe considered in a loop. So, if we are actually looking at a loop so the correction that are going to make in each pipe is of the same magnitude. So, since Delta is same so we can take Delta out of the summation and then like our Delta will be basically coming here and then what we get is that Delta is equal to -K cubed to the power X divided by summation of XK Qa to the power X – 1.

Now this part is the summation of the head loss in the pipe summation of head loss in the individual pipe and this part is actually like this is XK Qa to the power X - 1 all right. So, again like if we multiply this with the Q so this becomes Qa to the power X this will become Q a to the power X divided by Q means Q - Q a to the power X - 1 can be written as Q a to the power X divided by Q way that way now we know that whatever had whatever flow we have assumed for that purpose K into this can be written as head loss.

So similarly K into this can be written as a head loss and because this portion will actually become you can like more clearly see this. So, we are talking about XK Q a to the power X – 1. Now we can write Q a to the power X - 1 as q a to the power X divided by Q and this portion can be written as head loss. So, this will become X into head loss divided by Q. So, this is what we get X into a head loss divided by Q.

So summation of this comes in the denominator summation of total head loss comes into the numerator for whatever flow we are assuming. Now here because the Delta is given in the same direction in all pipe loops. So, if like let us say we are considering so Delta that we consider is in the same direction so the denominator that we take is basically absolute sum of the individual items so we take mod in order to get the absolute sum in the denominator.

So essentially we will get basic we will take a mod of this term or mod of this term. And if X is constant we can take X also out of the summation.

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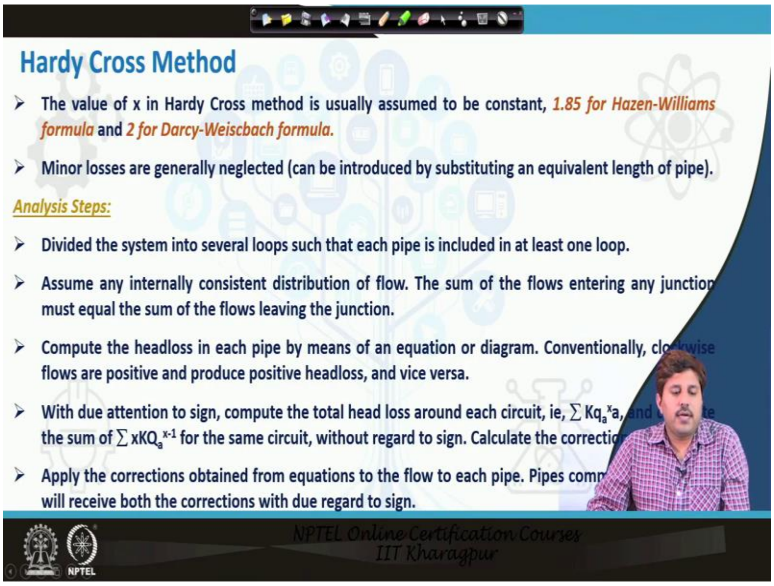

Now so the value of X is taken two in a hardy cross method and one point means two in a hardy cross method for if we are going to use the Darcy Westbeach formula and it is taken as 1.85 if you are going to use the Hazen William formula. And minor losses are generally neglected because they are the magnitude of the minor losses is much smaller as opposed to the major losses or the frictional losses.

But if you want to include the minor losses they can be introduced by the substituting an equivalent length of the pipe. How we do the analysis what are the analysis steps? So, the first step is we divide the system into several loops and each pipe basically we make sure that each pipe is included in at least one loop. Then we assume an internally consistent distribution of flow which follows the law of continuity.

And some of the flow entering at any Junction must be equal to the sum of flow leaving that particular junction. We compute the head loss in each pipe by means of an equation or diagram conventionally the clockwise flows are considered positive and produce positive head loss and the anti clockwise flows are considered negative and they produce negative head loss. Now with considering all this positive and negative and considering the sign whether the flow is clockwise or the flow is anti-clockwise we compute the total head loss around each circuit which is basically KQa to the power X.

And then we compute the XK Q to the power X - 1 for the same circuit without regard to the sign because this as we discussed we take an absolute number of this. And then calculate the correction Delta and then we apply this correction to the flow in the each pipe and we do it in basically successive iterations until unless we reach the point where basically we are satisfied.

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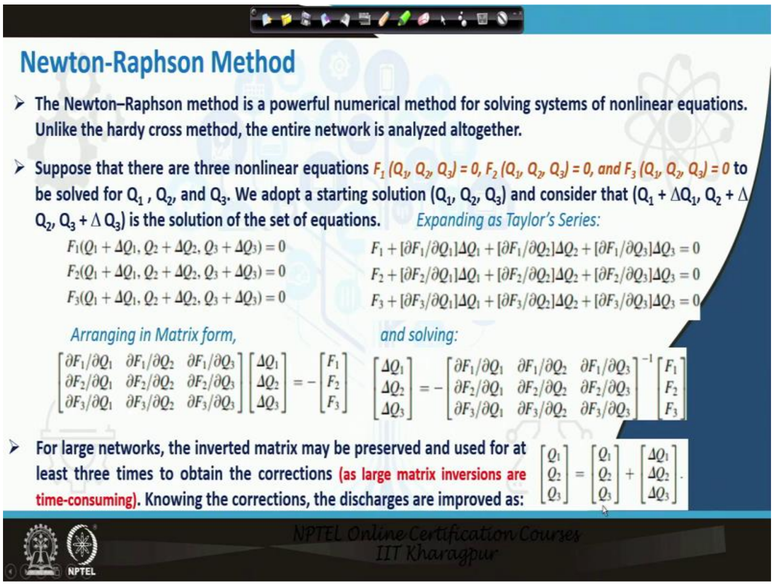

Or the corrections becomes very low or less than the like whatever limit me said if it goes less than that then we stop it there. Another method is Newton-Raphson method which is basically a powerful numerical method for solving a system of non-linear equation. And like in hardy-cross method we have to do a loop wise iteration here we do not need to do that we do the entire network we analyze the entire network altogether.

So it is basically a like we formulate the equations and then Newton-Raphson is just a numerical method to solve the set of those equations. So, suppose like we are having a set of nonlinear equation F 1 F 2 F 3 that way and we want to solve it for Q 1 Q 2 Q 3. So, we take in a starting solution again we take a guess and then the Delta value that we tend to estimate ok so Q 1 + Delta Q 1 Q 2 + Delta Q 2 is going to be the basically the set of the solution.

So, if we replace that these are the let us say 3 functions and we can expend these following this standard Taylor series this is how basically the Newton-Raphson method works and then we can arrange them in the form of matrix after the expansion. And then it comes in a form of matrix like AX is equal to b or that way and then we can solve this matrix. So, the discharges here would be inverse of this multiplied by the F1 F2 F3.

So for larger Network the inverted matrix like may be preserved for at least 3 iterations many times like more than 3 iterations are used but minimum 3 iterations people use it. Why do we do that because the large matrix inversions are very time-consuming so in like in order to save the time the; whatever the inverse matrix has been prepared the same matrix is used in 3 or more iterations. So, with this we estimate these corrections Delta 1 Delta Delta Q 1 Delta Q 2 Delta Q 3 and then we modified our Q as basically by adding these corrections to the like the value that we have assumed and that way we can get.

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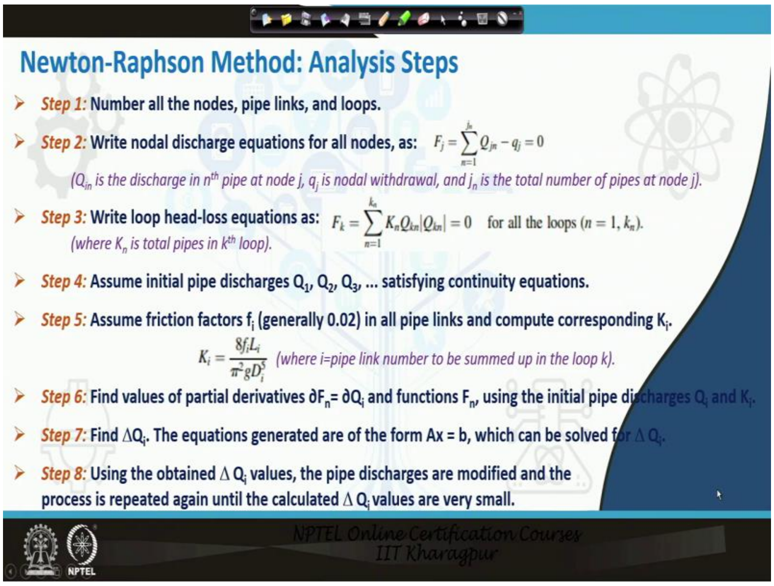

So, if you see the various steps followed here we need to number all the nodes loops everything then we write the nodal discharge equation for all nodes ok whatever is coming in and whatever is going out. Then we need to basically write the loop head equations for all loops then we assume initial pipe discharges which satisfy the continuity condition we assume the friction factors generally 0.02 is taken in all pipe links.

And then compute the corresponding Ki we are basically Ki is the pipe coefficient. So, we can basically compute the Ki this way and then find the value of these partial derivatives the functions that we have taken with respect to Qi and that function so we can use this initial pipe discharge Qi and Ki for this. And then find the Delta Q by solving this and the matrix will be in the form of AX is equal to b which can be solved for this Delta Qi.

And then once we obtain the Delta Qi value we basically modify the discharges with this until this becomes Delta Qi becomes very small or below the limit to which we are ready to accept it.

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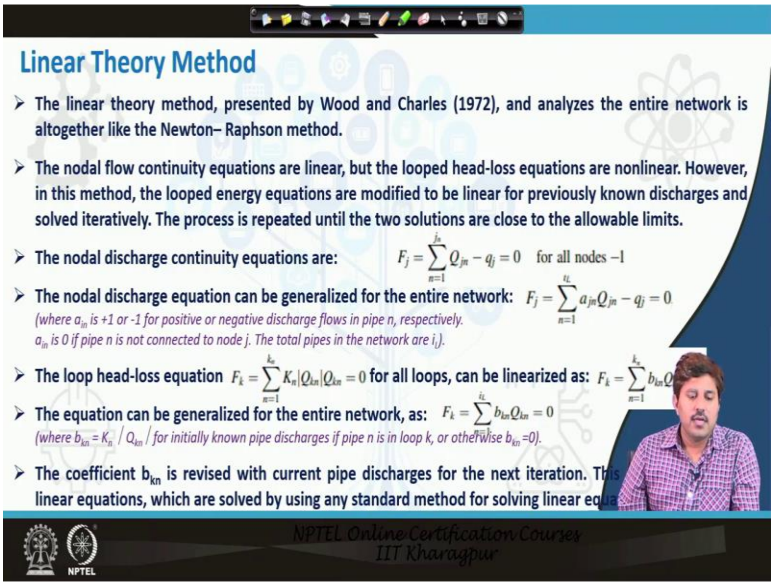

The another method is the Linear Theory method again like this is also a method which is similar to somewhat Newton-Raphson method. We can solve the entire network together we see that the nodal flow continuity equations are linear generally. But the head loss equations are nonlinear. So, we call this linear theory method because in this method we the looped energy equations we modify them to be linear for that previously known discharges and then solve it iteratively solve this set of linear equations instead of going for the non linear equation.

So we repeat this process until two solutions are too close to be kind of we see that 2 solutions are too close to the allowable limit. So, nodal discharge continuity equation again remains the same the nodal discharge equation is this about now this is the like the if you the loop head loss equation. So, this is the loop head loss equation for all the loops which is a nonlinear equation but we linearize it in this form where basically we introduce a factor b kn.

And this equation then can be generated for the entire network in this form where basically b kn is basically kn times if you see this equation so your b kn is this factor. So, for initially known pipe discharges because we know the discharge so we linearize this we convert this to a constant otherwise this 2kn and is also a factor. So, like it is instead of going for K into Q square the head loss equation we write it K Q into Q and this Q we take the previous one and then consider this as a constant factor for that pipe considering it from previous discharge. So it is basically just a function of Q that is how we linearize it.

So that is the concept of linearization over here we consider this as a constant b kn and then b kn we use Q K and for initially discharge pipe if the pipe is in loop K or otherwise we consider this value as a 0 because if it is not in that loop so anyway it is going to be 0. Now we take a sum of like these functional head losses as 0 and then they basically the coefficient b kn is received for the current pipe discharge for the next iteration.

So, this result is kind of set in a linear equation which are solved using any standard method for solving linear equation so that is what is the linear theory method.

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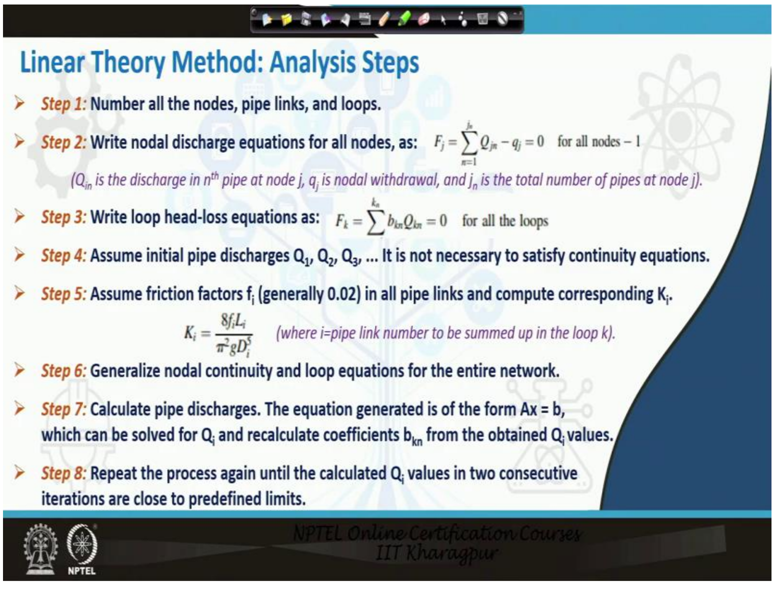

If you see the different step so again similar to the Newton-Raphson method we number all the nodes pipe links will write first the nodal discharge equation will write the loop head loss equation then we use assume the initial pipe discharge. Now here initial pipe discharge is not necessarily to satisfy the continuity equation. Again because we are using previous discharge and we are converting it so that criteria is not like strictly followed here.

And then we assumed a friction factor can compute the ki value we generalize nodal community and loop equation for the entire network and then using that we calculate the pipe discharge following that linear equation linearized equation in the form of again AX is equal to b and we can solve this compute the coefficient like we leak we can recalculate the coefficient that b kn which will be used to obtain the next value of the Qi.

So, we repeat it until the Qi values in 2 consecutive iterations are very close to the desired limit. So, that is how basically we adopt linear theory method for the analysis. So, with this we conclude this class we have talked about the water distribution network analysis in this particular class. So, practically we lend the discussions here and in the next class we will take up some practice problems so like on Hardy-Cross method or energy aspect so that it the concepts can become more clear. So, thank you for joining see you in the next class.